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Approved as a Text-Book in **Burmah.**

Matriculation Arithmetic

WITH

NUMEROUS EXAMPLES AND SIMPLE GRAPHS

FOR THE USE OF SCHOOLS AND COLLEGES

BY

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PREFACE.

In this book will be found all that is required in Arithmetic of the students of our Indian Universities. It will be useful to students who may afterwards have to serve in Mercantile offices. Any one, who intends to learn Arithmetic thoroughly, will find in it a safe and complete guide. It differs from the existing treatises in the greater prominence given to the *Unitary Method* and *Arithmetical Equations*. The Unitary Method (called *Subhankar's* method in this country) is practically more useful than the method of Rule of Three. The solution of a problem by the Unitary method gives a greater insight into it than the method of Rule of Three, the use of which in most cases is merely mechanical. The Arithmetical Equations require only certain axioms which are common to all branches of Mathematics.

The Examples in this book are more numerous and of greater variety in the belief that a sound knowledge of the analytical part of Mathematics requires a sound knowledge of Arithmetic, and this can be effected only by the student being drilled with home exercises of at least four sums of Arithmetic every day from the 8th to the 3rd class. The collection of examples in this book is sufficiently large to obviate the necessity of buying another book of Arithmetic. Typical examples of every variety have been worked out, and no pains have been spared to make them really instructive.

One-third of the more important examples in each set should be worked out in the class and the remaining two-thirds may be given as home exercises. The more difficult examples in each set and the Miscellaneous Examples may advantageously be left for a revisional course. The Oral Examples should not be neglected.

Typographical errors are likely to have crept in this the first edition. I shall, therefore, feel highly obliged if any one using this book would be good enough to point them out either to me or to the publishers.

In conclusion, I have to thank many friends who have assisted me in the verification of the Answers of the examples of this book, and especially Babu Chunilal Sil, late principal Mathematical Teacher of the General Assembly's Institution and author of several mathematical works, who has materially helped me in the preparation of this work and without whose help it would perhaps not have been possible for me to complete it.

38/2, Nilmony Mitter's Street, *Calcutta : the 15th December, 1897.* } GAURI SANKAR DE.

PREFACE TO THE SECOND EDITION.

I AM very grateful to the Heads of Institutions, and the reading public for the very cordial reception given to this book, the first edition of which has been sold off in the very brief space of two months.

I also take this opportunity of acknowledging the help given me by several of my friends in pointing out errors, verifying answers of examples, and making valuable suggestions.

In this edition only slight alterations have been made here and there, and errors corrected. About 200 of the less important miscellaneous examples have been omitted from the latter part to reduce the size of the book.

38/2, NILMONY MITTER'S STREET,
20th April, 1898. } GAURI SANKAR DE.

PREFACE TO THE THIRD EDITION.

IN this edition the book has been thoroughly revised and only slight alterations and additions have been made in certain places Almost all the examples have been worked anew in the course of preparing the Key to this book which has been out about a month ago. I hope that few errors are left in this edition.

I have to tender my thanks to my friends and correspondents who have pointed out errors and communicated suggestions for the improvement of the book. Any communication for the improvement of the book will be thankfully received.

38/2, NILMONY MITTER'S STREET,
The 29th December, 1898. } GAURI SANKAR DE.

PREFACE TO THE SEVENTH EDITION.

IN this edition the book has been thoroughly revised and several alterations and additions have been made. Many unimportant articles and examples have been omitted to reduce the bulk of the book and at the same time great pains have been taken to ensure accuracy in the examples and the answers.

38/2, NILMONY MITTER'S STREET,
The 2nd February, 1905. } GAURI SANKAR DE.

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MATRICULATION ARITHMETIC.

CHAPTER I.

Definitions, Names of Numbers, Notation and Numeration.

I. DEFINITIONS AND PRELIMINARY NOTIONS.

1. Anything that is capable of increase or diminution is called a **magnitude**.

2. A magnitude may be *whole and undivided*, as the length of a stick, a period of time; or it may consist of *separate and distinct* parts, as a heap of pebbles, a herd of oxen, a pack of dogs.

3. When a magnitude is whole and undivided, we select some well-marked magnitude of that kind which we call its **unit**, and by counting this unit a sufficient number of times, we make up the given magnitude, but if the magnitude be made up of distinct objects, we select an object of that kind as our **unit**, and see how many of these units are to be taken to make up the given magnitude.

4. Hence, a **unit**, or as it is generally called **unity**, is the representation of a thing considered in its *individual* capacity, without regard to the *parts* of which it may be made up, and it is the *Base* or *Element* of all our computations.

Thus, each of the terms, *a* man, *a* house, *a* pound, &c. denotes one individual of its kind, being the same as *one* man, *one* house, *one* pound, &c respectively; and these are the bases or elements by means of which *several* men, *several* houses, *several* pounds, &c., may be computed.

5. A magnitude represented as made up of one or more of its unit, is called a **quantity**, and the result of the comparison of the given magnitude with its unit respecting how many times it contains its unit is called **number**.

Thus, the length of a stick, a heap of pebbles are *magnitudes*; ten yards, a hundred pebbles are *quantities*; ten and a hundred are *numbers*.

6. Hence, **number** signifies *one* or *more* units, or denotes one or more *distinct* objects of the same kind.

Thus, *one* man, *two* houses, *three* pounds, &c. which are represented by the numbers, *one*, *two*, *three*, &c. denote one or more individuals of the same kind.

7. Numbers thus viewed or considered are termed **whole numbers** or **integers** ; and the *unit* is considered as the *first* or *least* integer.

8. The **measure** or **numerical value** of any quantity is the *number* of times the quantity contains the unit.

Thus, when a foot is used as the unit of length, and we speak of a rod as four feet long, the number *four* represents the measure of the stick.

9. Hence, the measure of a quantity represents its *relative* magnitude, but the measure and the unit together indicate its *absolute* magnitude.

10. Numbers are either **abstract** or **concrete**.

A *concrete* or *applicate* number is a number of objects or units of any kind ; an *abstract* number is a number considered separately and without any relation to objects.

Thus, *five* apples, *ten* pounds, *four* men are *concrete* numbers ; *five*, *ten*, *four* are *abstract* numbers.

11. Hence, an *abstract* number is a number in its literal sense, giving the idea of times or repetitions; but a *concrete* number is simply a quantity.

12. **Arithmetic** is the Science of numbers. It investigates their properties, and points out methods of calculations by means of them.

II. NAMES OF NUMBERS.

13. The *Symbol* or *Representation* of unit or unity is 1 ; but instead of other numbers being expressed by assemblages or multitudes of units *placed* together, which would soon become embarrassing, other characters or symbols have been invented, by means of which every number however great may be expressed ; again, instead of a different symbol being adopted for every different number, which would soon become equally inconvenient, *all* numbers are expressed by means of the following *ten* symbols, or as they are usually termed **figures**, and sometimes **digits**, which have their names respectively annexed :—

1,	2,	3,	4,	5,	6,	7,	8,	9,	0
one,	**two,**	**three,**	**four,**	**five,**	**six,**	**seven,**	**eight,**	**nine,**	**zero,**

the first *nine* of which are all defined by their names : thus, one and one is **two** ; two and one is **three** ; three and one is **four** ; four and one is **five** ; and so on ; and the last which is variously denominated **zero, cipher,** or **nought,** when standing by itself has no signification or at most, denotes the absence of number, and is to be regarded merely as an *auxiliary* digit, for the purposes hereafter to be explained. These nine digits are called *simple numbers*, and *units of the first order.* Their names are perfectly arbitrary,

14. The next number is *nine* and *one*, and we give it the name **ten**. Ten forms a single *unit of the second order*, and by counting by *ten*, as we before counted by *one*, we have

one-ten, two-ten, three-ten, four-ten......nine-ten ;

or more briefly, remembering that "ty" is equivalent to ten, and treating ten as a simple number, we say

ten, twenty, thirty, forty........ ninety.

The names of the nine numbers between ten and twenty, are eleven, twelve, thirteen, fourteen, fifteen,..... ...nineteen.

The names of the nine numbers between twenty and thirty, thirty and forty,......as also the nine numbers that follow ninety are formed by placing in order the names of the first nine numbers after twenty, thirty,......,ninety. Thus we get at last ninety-nine.

15. The number which follows ninety-nine is ninety-nine and one, or *ten tens*, and is called a **hundred**. It is a single *unit of the third order*, and by counting by *hundreds* as we counted by simple units, we have

one-hundred, two-hundred, three-hundred,......nine-hundred.

The names of the ninety-nine numbers between one hundred and two hundred, two hundred and three hundred,........., as also the ninety-nine numbers that follow nine hundred, are formed by placing in order the names of the first ninety-nine numbers after one hundred, two hundred,......nine hundred. Thus we get at last nine hundred and ninety-nine.

16. The number which follows nine hundred and ninety-nine is nine hundred ninety-nine and one or *ten hundred*, and is called **a thousand**. It is a single *unit o the fourth order*. Proceeding as before, we get *ten thousand* as forming a single *unit of the fifth order*, and *ten ten-thousands* or a *hundred thousand* as a single *unit of the sixth order* ; but there being no independent names for these units we call a *thousand* as a *second* principal unit, and count by units, tens and hundreds of thousands.

The names of the nine hundred and ninety-nine numbers between one thousand and two thousand, two thousand and three thousand,......, as also the nine hundred and ninety-nine numbers that follow hundreds of thousand are formed by placing in order the names of the first nine hundred and ninety-nine numbers after one thousand, two thousand,...... hundreds of thousands.

17. The next number is a *thousand thousands*, and forms a single *unit of the seventh order*. It has an independent name and is called a **million**. Considering a million as a *third* principal unit, we count by units, tens, hundreds, thousands, ten-thousands, and hundred-thousands of millions.

18. Lastly, we come to a *million millions*, which is called a **billion**, and forms a single *unit of the thirteenth order*. Proceeding

in this way, we get a *million billions*, which is called a **trillion**, a *million trillions*, which is called a **quadrillion**, and so on

The **periods** which follow the above in succession are **quintillion, sextillion, septillion, octillion, &c.**

In France and some of the United States of America a *thousand* millions is called a billion, a *thousand* billions a trillion, and so on ; hence a billion in England is a trillion in America, &c.

19. From what has been said above, it appears that we practically employ not more than *thirteen* independent words :—*one, two, three, four, five, six, seven, eight, nine, ten, hundred, thousand, million*, and that *ten* units of any order always make one unit of the next higher order.

III. NOTATION.

20. Notation is the method of expressing by certain symbols or characters, any proposed number expressed in words.

21. Beginners have already learnt from Art. 13, how to express the numbers from one to nine by **one** figure ; the following Article will teach them to express numbers from ten to ninety-nine by the use of **two** figures.

22. When a figure is placed on the *right* of the same or any other figure it has by *universal agreement* the effect of increasing the value of the last mentioned figure *tenfold*, at the same time that it retains its own value.

Thus, beginning with the auxiliary digit 0, we have the following numbers and their representations :—

10 ten
11 eleven
12 twelve
13 thirteen
14 fourteen
15 fifteen
16 sixteen
17 seventeen
18 eighteen
19 nineteen
20 twenty
21 twenty-one
22 twenty-two
23 twenty-three
24 twenty-four
25 twenty-five
26 twenty-six
27 twenty-seven
28 twenty-eight
29 twenty-nine
30 thirty
31 thirty-one
32 thirty-two
33 thirty-three
34 thirty-four
35 thirty-five
36 thirty-six
37 thirty-seven
38 thirty-eight
39 thirty-nine
40 forty
41 forty-one
42 forty-two
43 forty-three
44 forty-four
45 forty-five
46 forty-six
47 forty-seven
48 forty-eight
49 forty-nine
50 fifty
51 fifty-one
52 fifty-two
53 fifty-three
54 fifty-four
55 fifty-five
56 fifty-six
57 fifty-seven
58 fifty-eight
59 fifty-nine
60 sixty
61 sixty-one
62 sixty-two
63 sixty-three
64 sixty-four
65 sixty-five
66 sixty-six

67 sixty-seven	78 seventy-eight	89 eighty-nine
68 sixty-eight	79 seventy-nine	90 ninety
69 sixty-nine	80 eighty	91 ninety-one
70 seventy	81 eighty-one	92 ninety-two
71 seventy-one	82 eighty-two	93 ninety-three
72 seventy-two	83 eighty-three	94 ninety-four
73 seventy-three	84 eighty-four	95 ninety-five
74 seventy-four	85 eighty five	96 ninety-six
75 seventy-five	86 eighty-six	97 ninety-seven
76 seventy-six	87 eighty-seven	98 ninety-eight
77 seventy-seven	88 eighty-eight	99 ninety-nine

which is the largest number that can be expressed by **two** digits.

23 The use of **two**, either the *same* or *different* figures, will not enable us to go beyond this number, but a repetition of the contrivance in the last Article will by means of *more* figures supply the defect.

Thus, supposing the effect of any figure's being placed on the right of symbols formed as above, to be to increase all their values *tenfold*, we shall have

100 one hundred	200 two hundred
101 one hundred and one	201 two hundred and one
102 one hundred and two	202 two hundred and two
&c. &c.	&c. &c.

so likewise of succeeding numbers ; thus, we have

345 three hundred and forty-five	750 seven hundred and fifty
586 five hundred and eighty-six	946 nine hundred and forty-six

and again 999 will be *nine hundred* and *ninety-nine*, which is the largest number capable of being expressed by **three** figures.

Here, the *first* figure on the right hand is said to occupy the *units' place*, the *second*, the place of *tens*, and the *third*, that of *hundreds*.

Of the auxiliary digit 0, the sole use is in the effect specified in the last two Articles ; and all figures to the *right* of it will therefore be unaffected by it.

24. In estimating numerical magnitudes, we proceed in order from *hundreds* to *thousands*, *tens of thousands*, and *hundreds of thousands* ; *millions*, *tens of millions*, and *hundreds of millions* ; in precisely the same manner as we have done above from *units* to *tens*, and from *tens* to *hundreds*.

25. Agreeably to the principle of Art. 22, it is *assumed* that "*any* figure placed on the right of one or more figures, has the effect of increasing every one of them tenfold without altering its own value" ; and this enables us to express with facility any number whatever.

Thus, 1000 will represent one thousand.
5493 will represent five thousand, four hundred and ninety-three.
23456 will represent twenty-three thousand, four hundred and fifty-six.
729054 will represent seven hundred twenty-nine thousand and fifty-four.
1803205 will represent one million, eight hundred three thousand, two hundred and five.
32754081 will represent thirty-two million, seven hundred fifty-four thousand and eighty-one.
473025004 will represent four hundred seventy-three million, twenty-five thousand and four.

26. If the first three figures beginning from the right hand be denominated so many *units*, tens of *units* and hundreds of *units*, it follows that the next three figures taken in the same way will be *thousands*, tens of *thousands*, and hundreds of *thousands*, the next three in order will be *millions*, tens of *millions*, and hundreds of *millions* ; and so on.

Whence, to express in figures any number proposed, we have only to consider in which of these divisions each part of it ought to be found, observing that *three* figures from the right must be taken to make each division *complete*, before we proceed to the next. Thus

Ex. 1. Express by means of figures : *Thirty-five thousand eight hundred and nineteen.*

Here eight hundred and nineteen belongs to the *first* division on the right, and is written 819 : also, thirty-five thousand must be found in the *second* division from the right, and is 35 : whence the proposed number will be expressed by 35,819.

Ex. 2. Write down in figures the number : *Five million twenty-five thousand, six hundred and seven.*

In this case, the *first* division on the right will be 607 ; the *second* will be 025, the digit 0 being affixed to the left of the others without altering their values, to make up the required number of *three* ; and the third is 5 ; so that the expression required will be 5,025,607.

Ex. 3. Express by figures the following number : *Five hundred and seventy million, two hundred six thousand and fifty-four.*

Here, the *first* division is 054, the 0 altering only the values of the figures in the *subsequent* divisions ; the *second* division is 206 ; and the *third* is 570 ; whence the number proposed is correctly expressed by 570,206,054.

27. This method of notation can never present any difficulty, provided it be carefully remembered that every division of figures as we proceed from the right hand towards the left must be *completed* as far as it is possible ; and by a little practice, we shall be enabled to write down any number by beginning at the *left hand.*

Ex. 1. To write down *Six hundred and thirteen million five hundred and twenty-seven*, we observe that the division of *millions* will be 613; that of *thousands* will be 000, and that of *units* 527; so that the number is expressed by 613,000,527.

Ex. 2. To represent *Ten thousand million* by figures; for the *fourth* division we have 10, and for each of the *third*, *second* and *first* 000, so that the representation required is 10,000,000,000.

Examples I.

Represent the following numbers in figures:—

1. Forty-three; seventy-nine; sixty-five; eighty-four; fifty-eight; ninety-seven; sixty; eighty-seven.

2. Four hundred and forty-nine; five hundred and ninety-eight; seven hundred and four; four hundred and five; two hundred and thirty-five; nine hundred and fifty-eight, seven hundred and twenty-five; eight hundred and thirty five.

3. Four thousand; seven thousand, eight hundred and four: eighty-nine thousand and sixty-three; fifty-three thousand, two hundred and twenty-three; eight thousand and forty-six; six hundred three thousand, two hundred and forty; five hundred thousand, five hundred and five; nine hundred nine thousand and nine.

4. Three hundred forty one thousand three hundred and twenty-three; two hundred thousand and seventy-five; seven hundred seven thousand and seventy; five hundred thousand; eighty thousand and eight; four hundred two thousand and seven hundred.

5. Nine million, forty-three thousand, six hundred and two; seven million, eight hundred fifty-nine thousand, six hundred and thirty-two; three million, forty thousand and twenty; one million four hundred and three thousand; five million, five hundred thousand six hundred and seventy-six; eleven million and five; one million, three hundred, seventy-eight thousand, two hundred and sixty-seven; one million, ten thousand and one.

6. Forty-five million, three hundred eighty-seven thousand and twenty-five; ninety-two million, five hundred sixty-eight thousand, nine hundred and eighty-five; eleven million, five hundred sixty-five thousand, four hundred and thirty seven; forty million, forty thousand and five; ninety-six million, ninety-six thousand and ninety-six.

7. Three hundred forty nine million, four thousand and sixty-five; one hundred million, thirteen thousand and one; nine hundred nine million, nine thousand and ninety-nine: eight hundred forty two million, two hundred forty-six thousand, four hundred and eighty-four; three million, four hundred fifty-two thousand, one hundred and sixty-one; four hundred ninety-four million.

8 Ninety-nine million, ninety-nine thousand and ninety-nine; one hundred eleven million, six hundred fifty thousand and fifty;

six hundred forty million, sixty-four thousand and six hundred; five hundred million, seven hundred three thousand and two; six hundred nine million, one thousand, two hundred and eight.

9. Two thousand, eight hundred four million, two hundred fifty-two thousand and ninety-seven; twelve thousand, thirty-six million, fifty-four thousand and seventy-nine; four thousand million, nine hundred thousand and five; six thousand three hundred four million, five hundred six thousand, five hundred and six; forty thousand, two hundred eighty million, five hundred thirty thousand, two hundred and fifty-nine.

10. Four hundred thousand million and ten thousand; eight hundred thirty-six thousand, five hundred and seventy-three million, two hundred forty-four thousand and six; nine hundred thousand, nine hundred million, nine hundred thousand and nine; six hundred thousand, sixty million, six thousand and six.

11. Nine thousand four hundred five million four thousand, five hundred and fifty, four hundred thirteen thousand, seven hundred twenty-three million, nine thousand and four; five thousand, and eight hundred eight million, sixty-eight thousand and eighty.

12. Eight billion, two hundred seven thousand and five; three billion, four thousand, seven hundred two million, one hundred sixty-four thousand, seven hundred and twenty-two; one billion, three hundred thousand and five.

13. Ninety-nine billion, ninety million, ninety-nine thousand, nine hundred and nine; one hundred billion, one hundred ninety-six thousand, four hundred million, ten thousand and nine.

14. Six hundred fifty-four thousand, three hundred twenty-three billion, four thousand, twenty-one million, fifty thousand, three hundred and one; forty-seven thousand, five hundred twenty-six billion, eight hundred seventy thousand, seven hundred forty-four million, one hundred three thousand, two hundred and eighty-four.

15. Nine trillion, four billion, six hundred forty million, three hundred and sixty-five.

16. Write in figures the least number of six digits and the greatest number of eight digits. How many numbers are represented by three digits?

17. Write down in figures all the numbers between eighty-seven and ninety-three, between six hundred and eleven to six hundred and twenty, and between nine hundred and forty-seven to nine hundred and seventy.

18. When told to write five million, five hundred five thousand five hundred and five in figures, one boy wrote 550555, and another wrote 50550505; what mistakes did they commit?

28. It will be observed, from what has been said, that each of the nine figures or digits, 1, 2, 3, 4, 5, 6, 7, 8, 9, has a **simple**

absolute or **intrinsic** value of its own, whereas the auxiliary digit 0 has no such value and on this account the former are termed **significant** figures, in contradistinction to the last. It will moreover have occurred to the reader, that every one of these significant digits in addition to its **simple** value, which is fixed and certain, possesses also a **local** or **accidental** value dependent upon the situation in which it is placed.

Thus, in the expression of the number, *Four thousand three hundred and twenty-one*, which will be 4321, the 1 in the first place on the right hand, retains its *simple* value; the second figure 2, in its situation denotes two *tens* or *twenty*; the third is three *hundreds*, and the fourth is four *thousands*; so that the *local* values of 2, 3 and 4 here, are respectively, *ten* times, a *hundred* times and a *thousand* times, as great as their *simple* values; and it is the circumstance of assigning to each of the significant figures a **local** as well as a **simple** value, which confers upon the system the immense powers it possesses.

29. The characters 1, 2, 3, 4, 5, 6, 7, 8, 9, 0 and the mode of representing numbers by their combinations were first invented by the Hindus. The word *Digit* (denoting a *Finger*) usually applied in these characters, seems to point out the means originally employed in estimating numerical magnitudes; the number 10, which is called the *Base* or *Radix* of the system, and by which the *local values* of the *digits* are regulated, being that of the *Fingers* of *both* hands. Thus came the name **Decimal System** of Notation. The system was borrowed from the Hindus by the Arabs, who introduced it into Europe about the 11th century Hence the Europeans call it the *Arabic Notation*. The Notation appears to be as *complete* and *convenient* as can well be imagined, and in its present state may certainly be regarded as one of the greatest and most successful efforts of human ingenuity ever exhibited to the world.

IV. NUMERATION.

30. Numeration is the art of reading or estimating the value of a number expressed by figures, and is therefore the *reverse* of Notation.

31. From the circumstance of every figure possessing a *local* as well as a *simple* value, it follows that the value of each figure must be estimated by the place which it occupies; hence, a figure standing by itself expresses so many *units*; a figure in the second place from the right, denotes so many *tens*; a figure in the third place, so many *hundreds*, and so on; consequently, if we suppose any numerical expression to be divided into **periods**, or portions each consisting of three figures as far as they go, the figures of the **period** on the right will be **units**, and tens and hundreds of **units**; those of the next will be units, tens and hundreds of **thousands**; these of the third will be units, tens and hundreds of **millions**; and so on.

Thus, 25 is read twenty-five.
304 is read three hundred and four.
5,287 is read five thousand, two hundred and eighty-seven.
60,539 is read sixty thousand, five hurdred and thirty-nine.
207,385 is read two hundred seven thousand, three hundred and eighty-five.
1,739,204 is read one million, seven hundred thirty-nine thousand, two hundred and four.
35,024,376 is read thirty five million, twenty-fcur thousand, three hundred and seventy-six.
275,008,005 is read two hundred seventy-five million eight thousand and five.

32. In each of the above instances, we conceive the expression to be separated into *periods* of three figures each as far as they go, beginning at the right hand. But if the number contains more than nine figures, then instead of supposing that each division consists of *three* figures, if we include *six* figures as far as we can in each division from the right hand, the first may be regarded as so many hundreds of thousands of **units** ; the next as so many hundreds of thousands of **millions** ; the next as so many hundreds of thousands of what are called **billions** and the succeeding divisions, of so many hundreds of thousands of what are termed **trillions, quadrillions,** &c.

Thus, 34,567008,093402 is read thirty-four billion, five hundred sixty-seven thousand and eight million, ninety-three thousand four hundred and two.

33. The last two Articles will be rendered more clear by the following scheme, called the **Numeration Table**.

&c.	hundreds of thousands of billions	tens of thousands of billions	*thousands* of billions	hundreds of billions	tens of billions	**billions**	hundreds of thousands of millions	tens of thousands of millions	*thousands* of millions	hundreds of millions	tens of millions	**millions**	hundreds of thousands	tens of thousands	**thousands**	hundreds	tens	**units**
&c. &c.	6	5	4	3	2	1	6	5	4	3	2	1	6	5	4	3	2	1

34. In reference to what was said in Art. 32, it may be proper to observe that the method of proceeding differs from that adopted by the *French* and some other *European* Arithmeticians, who adhere throughout to divisions of *three* figures, according to the principle of Art. 31, and after the division of *millions*, proceed directly to that of *billions*, tens of *billions*, and hundreds of *billions*: then to *trillions*, tens of *trillions*, and hundreds of *trillions*, and so on: and this method certainly possesses some advantages in point of simplicity; but as numbers of these magnitudes are not of very frequent occurrence, it has not been thought necessary to depart from the *Notation* and *Nomenclature* established in England.

Examples II.

Write down in words the following numbers :—

1. 17 ; 24 ; 35 ; 46 ; 27 ; 48 ; 59 ; 76 ; 84 ; 95 ; 66 ; 75 ; 89.

2. 217 ; 319 ; 583 ; 695 ; 725 ; 308 ; 406 ; 846 ; 932 : 725.

3. 3406 ; 5260 ; 4236 ; 3298 ; 5678 ; 2405 ; 9286.

4. 43201 ; 87054 ; 34002 ; 49803 ; 58030 : 76503.

5. 903756 ; 903284 : 827109 : 319420 ; 243065 ; 123456.

6. 2714325 ; 8047328 ; 4010010 ; 8004640 ; 1234007.

7. 12870045 : 20084216 ; 79030284 ; 43002005.

8. 321408653 ; 408076032 ; 314159265 ; 123456789.

9. 571268405 ; 3179040601 ; 31968020907 8.

10. 1234567654321 ; 5020040003060 : 4302500764009.

11. 200900600002 ; 4328700006321 ; 6400000264600 2.

12. 319080259417 ; 2360459782134 78.

13. 1527875430029 ; 543217698900 7.

14. Write the largest and the smallest numbers possible with the symbols 5, 4, 9, 2, 7.

15. Give the local value of each of the significant digits in the following numbers :—

95 ; 64 ; 575 ; 8297 ; 40276 ; 3205 ; 478296 ; 40302605 ; 50003029 ; 70300006 ; 9786002030.

16. Express in words the greatest number of five figures and the least number of seven figures.

17. Write down *all* the numbers that can be formed by the digits 2, 3, 4, taken all together.

V. THE HINDU METHOD OF NUMERATION.

35. The following is the **Indian Numeration Table** in common use :—

&c.	&c.	hundreds of crores वृन्द	tens of crores अर्बुद	crores कोटि	tens of lacs नियुत	lacs लक्ष	tens of thousands अयुत	thousands सहस्र	hundreds शत	tens दश	units एक
		4	5	6	7	5	2	8	3	6	4

The above number is read thus :—Four hundred and fifty-six crores, seventy-five lacs, twenty-eight thousand, three hundred and sixty-four.

The Hindu names of places of figures are as follow :—**eka**, *dasha*, *shata*, **sahasra**, *ajuta*, **laksha (lac)**, *nijuta*, **koti** (**crore**), *arbuda*, *brinda*, *kharva*, *nikharva*, *sankha*, *sagara*, *padma*, *padmanava*, *mohapadma*, *kshuni*, *akshuhini*, *dhuli*, *mohadhuli*, *antya*, *parardha*.

Examples III.

Write down in words the following numbers according to the Indian Numeration :—

1. 19237 ; 60081 ; 49027 ; 167208 ; 200753 ; 830005.

2. 7090709 ; 8001025 ; 3905086 ; 24050008 ; 4001745.

3. 402178l5 ; 4030024340 ; 4780230016 ; 23456000.

4. 123456789 ; 6450300000 ; 760242900.

5. 4500002430 ; 8000785000 ; 4020504008.

Express in figures :—

6. Four lacs, fifteen thousand, two hundred and eight ; fifty-six lacs, four thousand and twenty-nine ; eight hundred forty-three lacs, seventy-four thousand, two hundred and nine ; eight lacs and five ; seventy-five lacs ; thirty lacs, seven hundred and eight.

7. Two crores, fifteen lacs and four ; thirty-seven crores, seven lacs, four thousand and twelve ; one hundred and forty-five crores, nineteen lacs, and seven ; five thousand and ninety-nine crores, four lacs, five thousand, six hundred and seven.

8. Eighty crores, thirty lacs, one thousand and eleven ; four thousand two hundred and ninety-five crores, fourteen lacs, and eighty-five ; seventy-five thousand four hundred and ten crores, fourteen lacs, nine thousand and nine.

9. How many lacs are there in twenty millions ? How many thousands are in ten lacs ? How many millions in forty crores ?

10. Read according to the Indian numeration the number—four hundred five million, seventy-five thousand, nine hundred and four.

11. Express a *billion* in Indian, and a *akshuhini* in English Notation.

12. A boy was told to write nine crores, five lacs, four thousand, seven hundred and fifty-six, and he wrote 905407056. Find out his mistakes.

VI. THE ROMAN SYSTEM OF NOTATION.

36. A different system of Notation was in use among the Romans. long before the introduction of the Arabic Notation into Europe by the Moors in Spain.

In this system the characters chiefly used are I, V, X, L, C, D and M which denote respectively the numbers 1, 5, 10, 50, 100, 500 and 1000 in the Arabic system. Again when a *bar* or *line* is placed over a character, it increases its value a *thousandfold.*

Thus $\overline{V}$ stands for 5000, $\overline{C}$ represents 100000.

The following table gives a full view of the method of expressing numbers in the **Roman System** :—

I	1	XV	15	CC	200
II	2	XVI	16	CCC	300
III	3	XVII	17	CD	400
IV	4	XVIII	18	D	500
V	5	XIX	19	DC	600
VI	6	XX	20	DCC	700
VII	7	XXX	30	DCCC	800
VIII	8	XL	40	CM	900
IX	9	L	50	M	1000
X	10	LX	60	MCD	1400
XI	11	LXX	70	MCM	1900
XII	12	LXXX	80	MM	2000
XIII	13	XC	90	MDCCCLXXXVI	1886
XIV	14	C	100	$\overline{DLX}$DCCCXLIV	560844

Examples IV.

Express in Arabic Notation each of the following numbers :—

1. VII, XVII, XXI, LIV, XXVIX, XXXIX.
2. LXLV, XLVIII, XCV, CCXIV, DXIV, CDXIX.
3. MIX, MDCCCIV, MDCL, MDCCLXVI, $\overline{MC}$, DCV.
4. $\bar{V}$DLV, $\dot{V}$IDL, $\overline{CCX}$CDXL, $\overline{CCXC}$DXL, $\bar{M}$X, $\overline{MMDM}$C.

Express in Roman Notation each of the following numbers :—

5. 9, 16, 35, 46, 68, 75, 89, 99, 105, 148.
6. 32, 28, 49, 69, 78, 95, 215, 327, 433, 549.
7. 745, 923, 567, 1234, 1567, 1853, 1918.
8. 1231, 1262, 1862, 1877, 1999, 2001, 1769.
9. 15497, 20015, 200150, 651002, 1000001, 2003450.

CHAPTER II.

The Four Fundamental Operations.

37. We now proceed to the consideration of the **Four Fundamental Operations** that can be performed upon numbers which are those of **Addition**, **Subtraction**, **Multiplication** and **Division** each of which will be defined, explained and exemplified in its order.

I. ADDITION.

38. **Addition** consists in finding a number equal to *two* or *more* numbers taken together.

The several numbers given to be added are called **summands,** and the single number obtained by adding them is called their **sum** or **amount**.

In addition the several numbers to be added must be either all *abstract* numbers or all *concrete* numbers of the *same kind*.

39. *Addition* is of two kinds, **simple** and **compound**.

(i) *Simple Addition* is one in which the numbers to be added together are either all *abstract* numbers, or all *concrete* numbers *of the same denomination* (*e.g.*, all *rupees*, or all *pounds*, or all *miles*, &c.).

(ii) *Compound Addition* is the method of collecting into one sum several *concrete* numbers of the same kind, but not expressed in *one* denomination of that kind.

40. It is usual, in the application of Arithmetic, to express the operation of *Addition* by the *sign* + invented for the purpose. It is read **plus**.

Thus, the sum of 4 and 5 is expressed in the form 4+5, wherein the sign + between 4 and 5 denotes the addition of the latter number to the former, and is read four *plus* five.

The sign = is called the **the sign of equality**. It is read **equals** or **is equal to.**.

Thus 4+5=9 expresses the result of the addition of 4 and 5 to be 9, or the *equality* between the *sum* of the numbers 4 and 5 and the *number* 9. It is read four *plus* five *equals* nine.

41. To effect the operation of *Addition*, it is merely necessary to know from *memory* or by *practice*, the sums of every two single figures. The following Table, called the **Addition Table** should be carefully committed to memory by beginners :—

1 and	2 and	3 and	4 and	5 and	6 and	7 and	8 and	9 and
1 are 2	1 are 3	1 are 4	1 are 5	1 are 6	1 are 7	1 are 8	1 are 9	1 are 10
2......3	2......4	2......5	2......6	2......7	2......8	2......9	2.....10	2......11
3......4	3......5	3......6	3......7	3......8	3......9	3.....10	3.....11	3......12
4......5	4......6	4......7	4......8	4......9	4.....10	4.....11	4.....12	4......13
5......6	5......7	5......8	5......9	5.....10	5.....11	5.....12	5.....13	5......14
6......7	6......8	6......9	6.....10	6.....11	6.....12	6.....13	6.....14	6......15
7......8	7......9	7.....10	7.....11	7.....12	7.....13	7.....14	7.....15	7......16
8......9	8.....10	8.....11	8.....12	8.....13	8.....14	8.....15	8.....16	8......17
9.....10	9.....11	9.....12	9.....13	9.....14	9.....15	9.....16	9.....17	9......18

This Table can easily be carried on for numbers larger than 10; for instance, since 4 and 1 make 5, 4 and 11 make 10 more than 4 and 1, *i.e.*, make 15. Again, since 8 and 7 make 15, 8 and 17 will make 25, and so on, the result in each case, being 10 more than in the corresponding case in the Table. Also 3 and 46 make 49, 9 and 56 make 65, 8 and 87 make 95, and so on, the results in the several cases being respectively 40, 50, 80, more than the corresponding results in the Table.

Ex. 1. Add together 4, 8, 5, 0, 9.
We add thus, 4 and 8 make 12, 12 and 5 make 17,
17 and 0 make 17, 17 and 9 make 26,
$\therefore 4+8+5+0+9=26.$

or thus,

$$\begin{array}{r} 4 \\ 8 \\ 5 \\ 0 \\ 9 \\ \hline 26 \end{array}$$

Ex. 2. Find the sum of 24, 13, 15, 42.
24 and 13 make 37, 37 and 15 make 52,
52 and 42 make 94,
$\therefore 24+13+15+42=94.$

or thus,

$$\begin{array}{r} 24 \\ 15 \\ 13 \\ 42 \\ \hline 94 \end{array}$$

Examples V. (MENTAL ADDITION.)

1. Write down the sums of :—

(1) 2 and 4 ; 2 and 10 ; 3 and 5 ; 4 and 7 ; 5 and 9; 8 and 7.
(2) 9 and 10 ; 8 and 8 ; 7 and 3 ; 7 and 6 ; 9 and 1 ; 5 and 9.
(3) 2 and 9 ; 0 and 7 ; 4 and 9 ; 9 and 7 : 4 and 11 ; 9 and 14.
(4) 7 and 7 ; 7 and 9 ; 8 and 10 ; 9 and 6 ; 4 and 12 ; 7 and 13.
(5) 8 and 2 ; 8 and 5 ; 9 and 14 ; 8 and 13 ; 7 and 15 ; 6 and 14.
(6) 10 and 6 ; 10 and 9 ; 11 and 5 ; 13 and 6 ; 14 and 3.
(7) 4 and 17 ; 3 and 19 ; 12 and 12 ; 13 and 13 ; 16 and 12.
(8) 8 and 0 ; 12 and 13 ; 12 and 15 ; 11 and 16 ; 10 and 19.
(9) 15 and 8 ; 11 and 15 ; 18 and 12 ; 16 and 15 ; 13 and 16.
(10) 18 and 16 ; 15 and 15 ; 14 and 14 ; 16 and 16 ; 11 and 17.
(11) 10 and 11 ; 10 and 12 ; 11 and 13 ; 11 and 18 ; 12 and 19.
(12) 17 and 17 ; 18 and 19 ; 16 and 18 ; 19 and 19 ; 16 and 19.

2. (1) Add 6 to 28, to 38, to 48, to 58, to 68, to 78, to 88, &c.
(2) Add 8 to 25, to 35, to 45, to 55, to 65, to 75, to 85, &c.
(3) Add 15 to 39, to 49, to 59, to 69, to 79, to 89, to 99.

3. Add together :—

(1) 12 and 37 ; 13 and 25 ; 14 and 84 ; 14 and 26 ; 14 and 76.
(2) 19 and 75 ; 17 and 87 ; 16 and 56 ; 18 and 75 ; 18 and 52.
(3) 26 and 64 ; 36 and 85 ; 49 and 24 ; 39 and 75 ; 27 and 31.
(4) 39 and 42 ; 49 and 99 ; 26 and 37 ; 75 and 94 ; 53 and 84.
(5) 16 and 85 ; 17 and 54 ; 45 and 33 ; 64 and 89.

4. Count aloud by increments of 7 up to 100, starting at 6, at 9, at 13, at 15, at 17, at 19, at 21, at 23, at 25, and at 29.

5. Find the sums of :—

(1) 1, 3 and 5 ; 2, 5 and 3 ; 3, 9 and 7 ; 8, 4 and 6 ; 7, 7 and 7.
(2) 9, 9 and 2 : 7, 3 and 6 ; 8, 5 and 9 ; 5, 5 and 9 ; 7, 5 and 9.
(3) 3, 3, 3 and 3 ; 4, 6, 1 and 9 ; 8, 0, 9 and 6 ; 8, 8, 8 and 8.
(4) 5, 5, 8 and 4 ; 9, 8, 7 and 6 ; 4, 7, 2 and 6 ; 6, 7, 8 and 9.
(5) 4, 0, 3, 5 and 9 ; 6, 0, 5, 0 and 9 ; 7, 2, 8, 8 and 5.

6. Find the values of :—

(1) $3+4+9+3+3+5$; $3+6+8+5+6+4$; $6+0+4+7+0+5$.
(2) $9+5+7+8+3+4$; $6+9+9+7+7+5$; $5+8+9+7+5+6+3$.

7. Ram has 6 books, and his brother 5 ; how many books have they together ?

8. A boy has 8 marbles in one pocket, and 5 in another ; how many marbles has he ?

9. Bepin has 4 marbles, Gopal 7 and Bejoy 5 ; how many have they together ?

10. In a garden there are 4 mango trees, 6 cocoanut trees, 5 jack trees and 8 plum trees ; how many trees are there in all ?

11. Shyam paid 3 pice for a loaf, 4 pice for sugar, and 2 pice for butter. How much did he pay altogether ?

12. One boy gained 3 prizes, another 2, and another 5. How many prizes did the three boys gain ?

13. Hari has 8 marbles, and Bhuban 7 more than Hari. How many have they both together ?

14. One dovecot has 8 pigeons, another has 10, and a third has 12. How many pigeons have the three dovecots ?

15. A boy paid 4 pice for a pencil, 2 pice for a pen-holder, 14 pice for a slate and 7 pice for quills ; how many did he pay for the whole ?

16 Ram's age is 4 years, Gopal is 2 years older than Ram ; Shyam's age is the sum of the ages of the other two. Find the sum of all their ages.

17. In a school there are four classes. In the first class there are 6 boys ; in the second class 7 boys ; in the third 2 more than in the first class ; in the fourth 5 more than in the second class. How many boys are there in the school ?

18. Ram, Hari, and Gopal went to fish. Ram caught 9 lobsters, Hari caught none, and Gopal caught 12. How many lobsters did the three boys catch ?

19. Ram has a line 6 feet long, Shyam one 10 feet long, and Bhuban one 9 feet long. If the three lines were joined, how long a line would they make ?

20. Jogin got a prize of 5 rupees, Upendra got 6 rupees more than Jogin ; how many rupees did they get altogether ?

21. A farmer has 8 cows, 6 calves, and 5 sheep. How many animals has he altogether ?

22. Hari got from his father 9 pice, his two brothers 7 and 8 pice respectively, and his sister 5 pice ; how much did the father give in all ?

23. A man's age is 38 years ; how old will he be after 12 years ?

24. From a rope are cut off 15 yards and there are 6 yards left ; what was the length of the rope ?

25. After giving away 15 rupees, I have 8 rupees still left ; how many rupees had I in all ?

26. What number is that from which if I take first 8, and then 5, there will remain 24 ?

27. A man has a son whose age is 10 years ; he is older than his son by 26 years ; what is his age ?

28. I have 25 nuts in my pocket, and my father gives me 15 more ; how many have I in all ?

29. A rupee contains 64 pice ; how many pice are there in two rupees ?

30. A woman sold 4 mangoes to *A*, to *B* 5 more than to *A*, to *C* as many as to *A* and *B*, to *D* 9 more than to *B* ; had *C* bought as many more mangoes as he did buy, the woman would have sold all her mangoes ; how many mangoes had she to sell ?

SIMPLE ADDITION.

42. The principle usually termed **carrying** in the Rule given below is "*that the tens of any order in a partial sum may te* **carried** *as units to the next higher order*," for ten units of any order are equivalent to one unit of the next higher order.

43. The following is the Rule for the addition of large numbers :—

RULE. Place the numbers under one another in such a manner that units may stand under units, tens under tens, hundreds under hundreds, and so on, and draw a line below all the horizontal rows of figures. Then add up the figures in the first vertical row on the right-hand, find the numbers of *tens* and *units* in their sum, and put down the number of *units*, whether it be zero or any of the nine other digits. **Carry** as many *units* as there are *tens* thus found to the next vertical row and add them up as before, observing the numbers of *tens* and *unit* contained in the sum. Place the number of *units* under the row added, and carry the number of *tens* to the next ; proceed in the same manner till the last row is added, when put down the numbers both of *tens* and *units*, as there are no more figures of higher denominations. The entire sum thus put down will be the **sum** of the separate numbers

Ex. Add together 6254, 893, 48 and 5487.

Arrange the numbers according to the Rule given above, and proceed to add the columns beginning from the column of units.

```
 6254
  893
   48
 5487
-----
12682
```

The sum of 4, 3, 8 and 7 is 22. Place the **2** units under the row of units, and carry on the 2 tens units to the row of tens.

The sum of 2, 5, 9, 4 and 8 is 28. Place the **8** tens under the row of tens, and carry on the 2 hundreds units to the row of hundreds.

The sum of 2, 2, 8 and 4 is 16. Put down the **6** hundreds under the row of hundreds, and carry on the 1 thousand units to the row of thousands.

The sum of 1, 6 and 5 is 12. Put down the **12** under the row of thousands. Thus the entire sum is <u>12682.</u>

44. A **Proof** is a second operation which serves as a test of the correctness of the first.

The *Proofs of Addition* depend on this principle—The **sum** of several numbers is not affected by the **order** in which they are added together ; thus $4+8=8+4$.

45. To ascertain whether the operation is correctly performed, various expedients might be resorted to :—first, that of adding the numbers *downwards* instead of *upwards*, which, because the *same* set of numbers cannot have two *different* sums, must give the same result : second, that of omitting one of the horizontal rows of figures in a *second* operation, and afterwards adding it to the result of the rest obtained by the Rule : third, that of *casting out the nines* from the sum of the digits in the *summands* and the sum of the digits in the amount ; if the two results coincide the operation may be *presumed* to be correct. (*Casting out the nines* is explained in Art. 79.)

Examples VI.

1. Add together :—

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
37	90	57	24	98	68	79	12	87	97
42	45	68	56	55	48	27	56	68	59
23	73	75	35	60	99	94	48	59	68

(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)	(19)
85	78	310	704	345	2969	787	347	3214
92	69	46	450	903	4868	678	238	6786
99	75	147	979	450	6787	425	410	2345

(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)
889	654	8888	6748	415	293	814	325	4028
803	546	5173	5555	278	75	326	748	354
519	465	7421	7864	614	409	628	493	95
745	824	7643	5408	932	3	459	869	2076

(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)	(37)
736	9806	785	6045	736	8076	459	9542	17384
402	1932	8756	4500	405	432	3687	876	12345
4159	6580	9540	8068	8159	5431	7468	4093	5432
47	9889	8559	9647	49	458	243	7777	946
2468	7885	386	9407	7204	9327	5907	4685	24607

(38)	(39)	(40)	(41)	(42)	(43)	(44)
71407	15161	96748	33456	67895	796210	894142
90781	8098	25003	84771	56789	34728	378523
68943	958	84067	66854	98765	514344	66666
32605	41978	95674	72984	87956	858521	8524
72777	78368	98765	99999	78965	936266	981234

(45)	(46)	(47)	(48)	(49)	(50)
659873	5678912	2345678	1234567	4893054	9876543
487	4567891	3192463	2345671	9876543	9999999
6935	3456789	7283642	8742015	2483109	4602
70415	2144124	9234925	8888888	7834510	341025
8796	7878787	8364774	4310943	3469146	46

(51)	(52)	(53)	(54)	(55)
466779	897654	9466495	768402	27591046
878987	987763	7545478	95320089	5768004
365363	123456	29099	6949	39039587
432698	789099	2988607	84982759	596459
756545	789789	9292929	700897	78534842
487988	437977	7833210	78563412	19827634

2. Find the values of :—

(1) 567+90+48+39+4728+1000+6489+327+4578

(2) 37045+6879+3724+4562+82971+37256+409

(3) 5971096+7266440+5846666+5325863+5755621+5656219+2754013+4036957.

(4) 48678+53232+48214+87292+93246+37527+40752+53033+35002+15382+1128+5404.

(5) 1541061+1891484+1817881+2265380+2323979+379153+2010958+1476985+1774013+1764304+1076539+847590.

(6) 795824+1049700+1279605+593411+949908+8204+208513+1250687+974983+1267694+2038505+801986+608592+1007740+7292.

3. Find the sum of :—

(1) 774145, 999455, 1016062, 1797223, 1854905, 1681274, 74952, 3467035 and 1226612.

(2) 5971096, 1756856, 2124682, 1964909, 2582060, 2633447, 51027, 2280382 and 1721608.

(3) 36530, 4179, 1899, 52773, 130079, 17801, 15235, 118940, 101665, 35584, 5057 and 12162.

(4) 925682143, 832563297, 4327568, 98526342, 753291424, 643263, 71952875 and 2147397.

(5) 441698853, 37519162, 599678437, 4840, 5128697, 20304009, 679821345, 172564 and 4263721.

4. Add together seven hundred and six ; twenty-five thousand and eighty-four ; nineteen thousand and ninety-nine ; seven thousand, four hundred and three ; ten thousand ; ninety-nine thousand and ninety-nine ; and eight hundred and eighteen.

5. Add together five hundred sixty thousand, two hundred and eighteen ; ninety thousand and eighty-five ; three hundred six thousand, five hundred and sixty-seven ; seventeen thousand, eight hundred and nine ; seventy-eight thousand and eight ; twelve thousand and fifty ; six hundred twenty thousand, six hundred and twenty-six ; and nine thousand and twelve.

6. Add together seven hundred seven thousand, four hundred and fifty-nine ; ninety-eight thousand and seventy-four ; six thousand, eight hundred and seven ; five hundred thousand, three hundred and nine ; seven thousand, nine hundred and seventy-eight ; and nine hundred nine thousand, nine hundred and ninety-nine.

7. Add together fifty-five millions, seven hundred thousand and five ; seven hundred millions, nine hundred eight thousand, two hundred and five ; seventy-six millions, fourteen thousand and fifty-nine ; eight hundred seventy-seven millions, nine hundred two thousand and forty-seven ; seven millions, eight hundred four thousand, five hundred and twelve ; and five hundred seventy-five millions, eight hundred one thousand and ninety-nine.

8. Add together three hundred nine millions, four hundred seventeen thousand and eighty seven ; six hundred seventy-five thousand and forty-nine ; seven thousand ninety-seven millions, eight hundred fourteen thousand, three hundred and five ; seventy-nine millions, five hundred four thousand and forty-nine ; six thousand seventy-eight millions, four hundred thirty-nine thousand, six hundred and forty-seven ; and seven thousand millions, eight hundred seventy-six thousand, four hundred and twenty-nine.

9. Find the amount of five thousand, six hundred and ninety-two ; four lacs, thirty-five thousand and eleven ; eighty five lacs, four hundred and ninety-nine ; forty-three lacs and forty-three ; and five hundred and four.

10. Find the total of six lacs, six thousand and six ; four crores, twenty-five lacs, six hundred and thirty-five ; nine hundred and ninety-three crores, seventy five lacs, and seventy-five ; eighty-five crores, eighty-five lacs, eighty-five thousand and eighty-five ; twenty-three crores, five lacs, fifty-eight thousand and eighty-nine ; and four hundred sixty-three crores, nineteen lacs, four thousand and ninety-five.

11. One apple-tree had 816 apples on it, and another had 638 ; how many apples were on both trees ?

12. There are 129 boys, 308 girls, and 60 infants in a school ; how many children are there altogether in the school ?

13. A train contains 63 first-class passengers, 120 second-class and 154 third-class ; how many passengers are in the train ?

14. A man has been working five days. On Monday he earns 25 annas, on Tuesday 34, on Wednesday 16, on Thursday 38, and on Friday 27 ; how much does he earn in the five days ?

15. In one book there are 525 pages, in another 144, and in another as many as in the other two ; how many pages are there the three books ?

16. Figures were used by the Arabs in the year 890 and decima fractions were invented 574 years later ; in what year were they invented ?

17. Five mango-trees produced as follows : the first 657 ; the second 231 more than the first ; the third 892 ; the fourth 11 more than all the first three ; the fifth as many as all the others. How many mangoes were there on all the trees ?

18. A gentleman left his property by will, thus : to his wife nine thousand and eighty rupees ; to each of his two younger sons, five thousand, eight hundred and ninety-four rupees ; the rest of his property in two equal shares between his three daughters, and eldest son : the eldest son's share was fifteen hundred and twenty rupees more than the mother's share ; what did the gentleman die worth ?

19. Europe contains 3807195 square miles, Asia 17805146, Africa 11647428, America 13542400, and Oceania 3347840, what does this make the extent of the land on the surface of the globe ?

20. The number of Mahomedans in the Burdwan division is 957630, in the Presidency division 4063137, in the Rajshahye division 4885165, in the Dacca division 5531869, and in the Chittagong division 2425610 ; find the total Mahomedan population for Bengal Proper.

21. Bought a lot of ground for 675 rupees ; erected a house upon the same, at a cost for carpenter's works 2540 rupees ; mason's work 637 rupees ; painter's works 242 rupees and for grading the lot 293 rupees ; what was the cost of the whole ?

22. A man bought four chests of oranges. In the first chest there were 589 oranges ; in the second 215 more than in the first ; in the third 197 more than in the first ; in the fourth as many as there were in the first and third. How many oranges did he buy ?

23. A man has two thousand, and eighty-one sovereigns, three thousand and sixty-eight rupees, one thousand, one hundred and eleven dollars, and two hundred and sixty-nine half-rupees. How many *coins* has he altogether ?

24. Find the sum of six numbers each equal to 7903856.

25. A man was born in 1764 ; in what year was he 83 years old ?

26. In a dictionary there are 869 words beginning with the latter A, 742 with the letter B, 1061 wiih the letter C, and 1154 with the letter D How many words begin with the letters A, B, C and D ?

27. Add together the sum of five numbers each equal to 4597 and the sum of four numbers each equal to 89796.

28. January has 31 days, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30 and December 31. How many days are there in the whole year ?

29. From a sum of money I first took away 71407 rupees, and then 90781 rupees and had still 69843 rupees left ; what was the sum ?

30. The number of soldiers in an army of six regiments are 895, 976, 884, 937, 949 and 982 respectively ; the first, third and fifth regiments are respectively joined by 246, 145, and 102 soldiers. Find the whole number of soldiers in the six regiments.

II. SUBTRACTION.

46. Subtraction is the method by which we find what number is left when a smaller number is taken from a greater.

The greater number is called the **minuend,** the smaller one the **subtrahend,** and the number left the **remainder.**

47. The number left is the **difference** between the two given numbers ; it is also the **excess** of the greater number over the less ; it is also the number which must be **added** to the less number to make it equal to the greater. Hence **Subtraction** is sometimes called **complementary addition.**

48 Like *Addition, Subtraction* is of two kinds, **simple** and **compound.**

(i) *Simple Subtraction* is one in which the numbers are both *abstract* numbers or both *concrete* numbers of the *same* kind.

(ii) *Compound Subtraction* is the method of finding the difference between two *concrete* numbers of the *same* kind, but of *different* denominations of that kind.

49. The operation of *Subtraction*, is indicated or expressed by the sign —, which is read **minus,** with the use of the sign =.

Thus, the *excess* of 7 above 3 will be expressed in the form $7-3=4$, which is read seven *minus* three *equals* four : where the sign — between 7 and 3 denotes the subtraction of the latter from the former and the sign = between 3 and 4 shows the *equality* of the excess to 4.

50. To effect the operation of *Subtraction*, it is necessary to *recollect* the difference of every two numbers less than 20. The following Table, called the **Subtraction Table,** should be committed to memory by beginners.

1 from	2 from	3 from	4 from	5 from	6 from	7 from	8 from	9 from
1 leave 0	2 leave 0	3 leave 0	4 leave 0	5 leave 0	6 leave 0	7 leave 0	8 leave 0	9 leave 0
2....... 1	3....... 1	4....... 1	5....... 1	6....... 1	7....... 1	8....... 1	9....... 1	10.... 1
3....... 2	4....... 2	5....... 2	6....... 2	7....... 2	8....... 2	9....... 2	10.... 2	11.... 2
4....... 3	5....... 3	6....... 3	7....... 3	8....... 3	9....... 3	10.... 3	11.... 3	12.... 3
5....... 4	6....... 4	7....... 4	8....... 4	9....... 4	10.... 4	11.... 4	12.... 4	13.... 4
6....... 5	7....... 5	8....... 5	9....... 5	10.... 5	11.... 5	12.... 5	13.... 5	14.... 5
7....... 6	8....... 6	9....... 6	10.... 6	11.... 6	12.... 6	13.... 6	14.... 6	15.... 6
8....... 7	9....... 7	10.... 7	11.... 7	12.... 7	13.... 7	14.... 7	15.... 7	16.... 7
9....... 8	10.... 8	11.... 8	12.... 8	13.... 8	14.... 8	15.... 8	16.... 8	17.... 8
10.... 9	11.... 9	12.... 9	13.... 9	14.... 9	15.... 9	16.... 9	17.... 9	18.... 9
11.... 10	12.... 10	13.... 10	14.... 10	15.... 10	16.... 10	17.... 10	18.... 10	19.... 10

This Table can easily be extended further ; for instance, since 2 from 3 leave 1, 2 from 13, *i. e*, from $3+10$, leave $1+10$, or 11, the result being 10 more than in the corresponding case in the Table. Also since 7 from 15 leave 8, 7 from 45, *i e.*, from $15+30$, leave $8+30$, or 38, the result leaving 30 more than in the corresponding case in the Table. Also since 9 from 14 leave 5, 9 from 54, *i e*, from $14+40$

leave $5+40$, or 45, and 9 from 99, *i.e.*, from $19+80$, leave $10+80$ or 90; and so on.

Examples VII. (MENTAL SUBTRACTION).

1. (1) Take 2 from 4, from 7, from 11, from 6, from 12, &c.
(2) Take 3 from 4, from 3, from 6, from 8, from 13, &c.
(3) Take 4 from 6, from 9, from 13, from 15, from 19, &c.
(4) Take 8 from 12, from 15, from 19, from 21, from 25, &c.
(5) Take 9 from 15, from 18, from 20, from 24, from 36, &c.

2. (1) Subtract 6 from 20, 47, 32, 70, 63, 55, 81, 71 and 99.
(2) Subtract 7 from 18, 22, 49, 33, 84, 51, 94, 88 and 38.
(3) Subtract 5 from 18, 25, 53, 61, 70, 82, 67, 93 and 90.

3. How many does

(1) 9 leave from 15; 5 from 14; 7 from 12; 9 from 71; 8 from 21?
(2) 7 leave from 44; 8 from 38; 9 from 88; 6 from 94; 5 from 47?

4. Find the difference between :—

(1) 13 and 18; 3 and 14; 20 and 25; 30 and 45; 15 and 11.
(2) 89 and 47; 46 and 12; 34 and 68; 14 and 31; 14 and 95.

5. What must be added to 11 to make 15, 7 to make 18, 6 to make 15, 4 to make 11, 9 to make 17, 21 to make 49, 31 to make 44, and 30 to make 82?

6 By how much does 13 exceed 7, 17 exceed 8, 19 exceed 8, 26 exceed 14, 29 exceed 13, 69 exceed 26, 95 exceed 32, 98 exceed 36, 82 exceed 64, and 89 exceed 72?

7. Count by decrements of 3, 5 and 7, commencing at 100.

8. How much is 33 less 7; 84 less 5; 49 less 6; 67 less $5+2$; 96 less $4+0+4$; and $67+16$ less $15-4$?

9. Take $5+3$ from 11; $7+2$ from 17; 12 from $14+11$; 25 from $48+11$; $9+6$ from $12+5$; $3+8$ from $2+9$; and $1+4$ from $2+7$.

10. How many times can 5 be taken from 15; 6 from 18; 9 from 27; and 12 from 48?

11. A girl has 8 oranges. She gives 3 to her sister. How many has she left?

12. Shyam has 6 pice. He pays 1 pice for a top, 2 pice for a whistle, and 2 pice for a kite. How many has he over?

13. A boy has 18 pice in his pocket. He loses 7 and spends 4. How many pice has he left?

14. If you buy 18 yards of ribbon, and find that you have 3 yards too much, how many yards should you have bought?

15. A man planted 25 trees; 8 of them died. How many lived?

16. Jadu has 19 apples, and Bhuban has 8. How many has Jadu more than Bhuban?

17. I bought 6 pice worth of apples, and 4 pice worth of pears. What money had I over out of 15 pice?

18. A baker's boy sets out with 21 rolls. He leaves 5 in one house, 4 in another, 6 in a third and 5 in a fourth. How many rolls has he left?

19. Ram is 19 years old; Gopal is 8 years old. How many years is Gopal younger than Ram?

20. A man had 26 sheep; he sold 10, and 6 were stolen. How many were left?

SIMPLE SUBTRACTION.

15. The following are the Rules for the subtraction of large numbers.

(i) When none of the figures of the *Subtrahend* exceeds the corresponding figures of the *Minuend.*

RULE. Place the less number under the greater, so that units may stand under units, tens under tens, hundreds under hundreds, and so on; then draw a line below the lower number. Begin at the units' place and subtract each figure in the lower line from the corresponding figure in the upper, taken by itself, and put down the remainder below the line just drawn, units under units, tens under tens, hundreds under hundreds, and so on. The entire difference or remainder, so put down, will be the **difference** or **remainder** of the proposed numbers.

Ex. 1. Subtract 425 from 1679.

```
1679
 425
----
1254
```

Place the smaller number 425 under the greater 1679, and draw a line below it. First take 5 from 9, and place the difference **4** under the units' figure below the line drawn; next take 2 from 7 and set down the remainder **5** in the tens' place below the line; next take 4 from 6 and put down the difference **2** in the hundreds' place under the line. Lastly bring down **1** since there is nothing below it. Thus the remainder is 1254.

Ex. 2. Subtract 5634 from 9657.

```
9657
5634
----
4023
```

As before, put 5634 below 9657, and draw a line. Take 4 from 7, the remainder is **3**; 3 from 5 leaves **2** as remainder; 6 from 6 leaves *nothing* or **0** as remainder; lastly 5 from 9 leaves **4** as remainder. Thus the entire remainder is 4023.

(ii) If the units of any order in the *Subtrahend* exceed those of the *Minuend.*

In this case we avail ourselves of the following principle, usually termed **borrowing** :—"*The Minuend and Subtrahend may be increased by the same number without altering their difference.*"

Hence we may increase the number of units in any order of the *Minuend* by 10, if we increase that of the next higher order in the *Subtrahend* by 1.

RULE. Place the numbers as in (i) and draw a line below. Begin at the units' figure, but if the said figure in the lower line exceed that in the upper, increase the upper figure by **ten** and then subtract the lower figure from the upper *figure thus increased.* Put down the remainder as in (i), and **carry 1** to the *next higher* figure in the lower line. Proceed with the remaining figures as in (i), observing that whenever **ten** units have been **borrowed**, or added to the upper line, **one** unit must be **carried**, or added to the next higher denomination in the lower line.

Ex. Subtract 5634 from 7483.

7483
5634
1849

Since 4 is greater than 3, 3 is made 13 by adding 10 to it; from 13 take 4 and put down the remainder **9**. Now add 1 to the next lower figure 3; the sum is 4, which subtracted from 8 leaves 4. Put down **4**. Next 6 is greater than 4; so 10 is added to 4, and from the sum 14, subtract 6. The remainder is **8**. Lastly, add 1 to the next lower figure 5; the sum is 6, which subtracted from 7 leaves the remainder **1**. Thus the difference is 1849.

52. In the preceding Example, the same result would be obtained, if we have **borrowed** ten units of the *next* denomination from the *Minuend*, as is usual in France. For whether we suppose 1 to be *added* to the *lower* line, or *subtracted* from the *upper*, the remainder will evidently be the same on both suppositions. In *practice*, however, the former method is convenient.

53. Subtraction being the *reverse* of Addition, it follows, that if we add together the remainder and the less of the numbers proposed the sum ought to be equal to the greater; and the operation of subtraction may be *presumed* to be correct when this is the case. Another method of testing the correctness of the result is this: *Cast out the nines* from the sum of the digits in the *minuend*, and also from the sum of the digits in the *subtrahend* and the *remainder*; if the two results coincide, the operation may be *presumed* to be correct.

Examples VIII.

1. Perform the following subtractions :—

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
59	79	85	70	98	428	526	702	650	912
42	45	69	54	89	274	317	504	56	707

(11)	(12)	(13)	(14)	(15)	(16)	(17)
7046	7825	4286	9821	8943	6789	5959
807	4976	3097	6935	4573	697	999

(18)	(19)	(20)	(21)	(22)	(23)	(24)
23456	56785	76325	62831	7080001	542657	204087
4987	39876	59876	48072	39508	214958	76498

(25)	(26)	(27)	(28)	(29)	(30)
6829019	1531335	1287657	78602045	493827156	8539410
6599341	1456516	1000958	59763567	246913578	3438148

(31)	(32)	(33)	(34)	(35)	(36)
74147863	370489000	68539582	650030042	13456789	352100435
9701297	269579235	45947895	94090096	8765432	79213679

(37)	(38)	(39)	(40)	(41)
777722233	909009099	453870250	1000100010	765007005
38945635	842248484	39004065	999999999	400827054

2. Find the difference between :—

(1) 75011 and 6012 ; 3095 and 80131 ; 8019 and 18018.

(2) 110111 and 11012 ; 916553 and 1683452 ; 251483 and 77777.

(3) 20470932 and 80476325 ; 613020303 and 420536075.

(4) 12785462 and 1842567 ; 92603745298 and 25402987609.

3 Find the values of :—

(1) 5124060 − 5083959 ; 1056789 − 967899 ; 4060124 − 3951035.

(2) 6284503 − 4995629 ; 7014062 − 6985172 ; 6001004 − 5480018.

(3) 1010102 − 956784 ; 3601020 − 3598642 ; 5490206 − 4301218.

(4) 500120456 − 499296845 ; 40602136097 − 2846545789

4. What is the excess of 12795 above 8096? How much greater is 2600509050 than 433418175?

5. By how much is 87719808 greater than 68440260?

6. What is the excess of 9497605 above 8688516?

7. By how much is a lac greater than ninety-five thousand, nine hundred and nine, and less than a million?

8. What number must be added to each of the following numbers to make the sum equal to ten millions?—8423458, 457685, 9032401, 7612345, 5040289, 904507 and 9003465.

9. What number must be taken from each of the numbers 999999, 425078, 8725900, 6420587 and 428905 to leave 245678?

10 Required the excess of three hundred five million, two hundred and four, above seventy-five thousand, three hundred and eighty-six.

11. From seven hundred eighteen million, fourteen thousand and fifty-six take ninety-eight million, seven hundred three thousand, six hundred and seventeen.

12. Subtract thirteen lacs, four thousand and fifty-six from seventy-five crores, two hundred and three.

13. Take eleven thousand eleven hundred and eleven from twelve thousand one hundred and twelve.

14. A box contains 4074 oranges ; 2386 of them were sold. How many remained ?

15. In 1882 a man was 86 years old. In what year was he born ?

16. William the Conqueror began to reign in the year 1066 ; how many years elapsed between that period and the battle of Waterloo, which was fought in 1815 ?

17. A tea merchant has 4680 maunds of tea. He sells 1000 maunds to one customer, 999 to a second, and 354 to a third. How many maunds of tea has he left ?

18. Jadu has 829 marbles ; he gives away 618 and then buys 206. How many has he now ?

19. A man was born in 1845 ; what was his age in 1896 ?

20. A man was 25 years old at the birth of his son ; what is the son's age when the father is 74 years old ?

21. A merchant bought a certain quantity of goods for 6246 rupees and sold them for 7137 rupees. How much did he gain ?

22. One mountain is 15732 feet high, another is 3571 feet high. How much is the one higher than the other ?

23. A railway receives in a year 2684040 rupees. Of this sum 1786064 rupees are for goods and the rest for passengers. How much was received for passengers ?

24. Of 17254120 Hindu population for Bengal Proper, 8624022 are males and the rest females ; find their number.

25. Queen Victoria was born in 1819 How old was she in 1895 ?

26. Three boys A, B and C at marbles won together 105 ; if the numbers that B and C won be added together they will make 82, and of this number B won 47. What did each boy win at play ?

27. A gentleman gave 12462 rupees for a house and some land ; the house alone was worth 9375 rupees ; what was the value of the land ?

28. The answer to a subtraction sum is 1026 and the top line 4387. What is the second line ?

29. A man has 826 sovereigns in one box and 682 in another ; he takes 176 from the former and puts them in the latter. How many are in each box now ?

30. When will the Prince of Wales, who was born in the year 1841, be as old as the Queen was in the year 1878, who was born in the year 1819 ? How old will the Queen then be ?

54. A number preceded by the sign + (*plus*), is called a **positive** number, and a number preceded by the sign − (*minus*), is called a **negative** number. When *no* sign is affixed to a number, it is considered as *positive*.

55. An **expression** is one in which two or more numbers are connected by the sign + or − ; and the numbers thus connected are called its **terms**.

Thus, $4-3+2+1$ is an *expression* ; 4, 3, 2 and 1 are *terms* ; 4, 2 and 1 are *positive* ; and 3 is *negative*.

56. If an addition and a subtraction, or *vice versâ*, have to be performed in succession, we may invert their **order**, provided the resulting expression be possible.

Thus, since $9+5-3=11$ and $9-3+5=11$; $\therefore 9+5-3=9-3+5$.

57. Hence it is easily shown that additions and subtractions may be performed in any *order* ; and that the value of an expression made up of additions and subtractions may be obtained by taking the *difference* of the sums of *all* the positive and the negative numbers separately.

Ex. Find the value of $365-101+2+18-267$.

Here, $365+2+18=385$; $101+267=368$; also $385-368=17$.

Therefore the value required $=\underline{17}$.

58. The **complement** of a number is its defect from 10 units of the number's highest order.

Thus, the *complement* of 6 is 4, and of 659 is 341, for $10-6=4$, and $1000-659=341$.

Examples IX.

Find the value of each of the following expressions :—

1. $16-4+12-25+7-2$.
2. $751-9+1786-235-12-672$.
3. $18+6-31+537-628-19+209$.
4. $467-84+49-36$.
5. $1246-362-371+495+156-386+256$.
6. $3210-67+59+401-342+491-382+459-87$.
7. What number must be added to the sum of 750 and 3287 to make the result equal to the sum of 505, 650, 19 and 9003 ?
8. What is the difference between $23047+175-368+495-132$ and $10000-8406-704+7305$?
9. From the difference between 3285 and 456 subtract the difference between 19011 and 17455.

10. A basket contained oranges, nuts and mangoes, in all 1769; there were 1696 oranges and nuts, and 1262 nuts and mangoes. How many more nuts were there than oranges?

11. Gopal goes up 16 steps of a ladder, which has 45 steps, then down 7 steps, then up 10, then down 2, then down 4, then up 11, then down 9, then up 7, then up 5, then down 8; what step from the top and bottom will he then be standing upon?

12. Write down the complements of 4; 7; 43; 86; 574; 998.

III. MULTIPLICATION.

59. Multiplication is the method by which we find the sum of a given number repeated as many times as there are units in another given number.

60. The number to be *repeated* is called the **multiplicand**, the other the **multiplier**, and the sum found the **product**. The *multiplicand* and the *multiplier* are both called **factors** or *makers* of the *product*.

61. From the mode in which results are obtained in multiplication, it is manifest that Multiplication is merely a *compendious* method of performing the addition of two or more *equal* numbers.

Thus, to multiply 7 by 4 being the sum arising from the number 7 repeated *four* times, we may determine the product as $7+7+7+7$ or 28. Here 7 is the *multiplicand*, 4 the *multiplier*, and 28 the *product*; also 7 and 4 are *factors* of 28.

62. *Multiplication* is either **simple** or **compound**.

(i) When the multiplicand is either an *abstract* number, or a *concrete* number of *one* denomination, it is called *Simple Multiplication.*

(ii) When the multiplicand is a *concrete* number of *more than one* denomination, but all of the *same* kind, it is called *Compound Multiplication.*

63. The operation of *Multiplication* is expressed by the sign $\times$, which is read **into** or **times** or **multiplied by**. Sometimes a dot is used instead of a $\times$.

Thus, 5×7 denotes the product of 5 and 7, and is read 5 *into* 7, or 5 *times* 7, or 5 *multiplied by* 7. Also $5.7=5\times 7$. This must not be confounded with a dot placed near the top, as 5·7. (Art. 332).

64. The operation intended by the word *Multiplication*, is defined in Art. 59; and in the first place we will shew that the conclusions which it leads to, may be safely depended upon, as far as the *order* of the *factors* may influence the *product.*

Thus, to multiply 7 by 5, write down 1 in a horizontal line 7 times, and repeat this line 5 times. The sum of each horizontal line is 7, and there are 5 such lines, therefore the sum of all the ones is 7 × 5. Again, the sum of each vertical line is 5 and there are 7 such lines, therefore the sum of all the ones is 5 × 7; that is, 7 × 5 is the same as 5 × 7.

1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1
1	1	1	1	1	1	1

By reasoning of this kind it is made to appear that the product has a *similar* or *symmetrical* relation to both its factors, because it remains the same if we interchange the *Multiplicand* and the *Multiplier*.

65. A number multiplied by 0 is 0, as also 0 multiplied by a number is 0; for a number taken *no* number of times is *nothing*, also *nothing* taken any number of times is *nothing*.

Thus, 5 × 0 = 0, as also 0 × 5 = 0.

66. The following Tables, which are termed the **Multiplication Tables**, present at one view the product arising from the multiplication of any two numbers not exceeding 20; and though the products of the *nine digits* form the *basis* of those of all numbers whatever, it is here extended for the sake of *practical* convenience, and should be carefully committed to memory.

Table 1.

	1	2	3	4	5	6	7	8	9	10
Once ...	1	2	3	4	5	6	7	8	9	10
Twice ...	2	4	6	8	10	12	14	16	18	20
Thrice ...	3	6	9	12	15	18	21	24	27	30
4 times ...	4	8	12	16	20	24	28	32	36	40
5 times ...	5	10	15	20	25	30	35	40	45	50
6 times ...	6	12	18	24	30	36	42	48	54	60
7 times ...	7	14	21	28	35	42	49	56	63	70
8 times ...	8	16	24	32	40	48	56	64	72	80
9 times ...	9	18	27	36	45	54	63	72	81	90
10 times ...	10	20	30	40	50	60	70	80	90	100

Table 2.

	1	2	3	4	5	6	7	8	9	10
11 times ...	11	22	33	44	55	66	77	88	99	110
12 times .	12	24	36	48	60	72	84	96	108	120
13 times ...	13	26	39	52	65	78	91	104	117	130
14 times ...	14	28	42	56	70	84	98	112	126	140
15 times ...	15	30	45	60	75	90	105	120	135	150
16 times ..	16	32	48	64	80	96	112	128	144	160
17 times ...	17	34	51	68	85	102	119	136	153	170
18 times ..	18	36	54	72	90	108	126	144	162	180
19 times ...	19	38	57	76	95	114	133	152	171	190
20 times ...	20	40	60	80	100	120	140	160	180	200

Table 3.

	11	12	13	14	15	16	17	18	19	20
11 times ...	121	132	143	154	165	176	187	198	209	220
12 times ...		144	156	168	180	192	204	216	228	240
13 times ...			169	182	195	208	221	234	247	260
14 times ...				196	210	224	238	252	266	280
15 times ...					225	240	255	270	285	300
16 times ...						256	272	288	304	320
17 times ...							289	306	323	340
18 times ...								324	342	360
19 times ...									361	380
20 times ...										400

67. In Multiplication, one of the factors, namely, the *multiplier* must necessarily be an *abstract* number.

Thus, if the factors are 7 rupees and 8 rupees, we could easily multiply together the abstract numbers 7 and 8, whose product is 56 ; but the *denomination* of this result as the product of 7 rupees and 8 rupees cannot be ascertained, and the *operation* is altogether *absurd*. Hence, the multiplication of *concrete* numbers *as such*, is altogether impossible. We can, however, multiply 7 rupees by the *abstract* number 8, and interpret the product 56 rupees as how many rupees there are in 8 times 7 rupees.

It is also absurd to speak of 7 multiplied by 8 rupees, but not of 7 times 8 rupees. Of the two factors that make 56 rupees, one must be *abstract*, the other *concrete*, but it does not matter which, for 7 times 8 rupees = 8 times 7 rupees. In no case do we multiply by rupees.

In certain cases, however, as will be seen hereafter, the meaning of multiplication may be so extended as to include some concrete multipliers. (Art. 378.)

Examples X. (MENTAL MULTIPLICATION.)

1. How much is
(1) 7 times 6 ; 11 times 8; 9 times 7; 11 times 11; 8 times 9 ; 7 times 15 ?
(2) 10 times 3; 9 times 12; 7 times 7; 12 times 14; 4 times 18; 6 times 8 ?
(3) 8 times 11 ; 5 times 12 ; 11 times 12 ; 5 times 17 ; 6 times 19 ?

2. What is the product of—
(1) 13 by 12 ; 8 by 9 ; 15 by 14 ; 18 by 17 ; 0 by 4 ; 12 by 4 ; 11 by 15 ?
(2) 15 by 19 ; 17 by 12 ; 6 by 0 ; 0 by 11 ; 20 by 15 ; 16 by 18 ; 14 by 18 ?

3. How many are 16×19 ; 13×15 ; 19×19 ; 12×12 ; 17×19 ; 20×13 ; 13×14 ; 14×18 ; 17×15 ; 15×20 ?

4. One book has 12 pages. How many pages will 8 such books have?

5. There are 11 boys in a class ; each works 8 sums in an hour. How many sums do they all work together ?

6. If one knife costs 14 pice, how many pice will 9 knives cost ?

7. If there are 9 desks in a room, and 6 boys at each desk, how many boys will there be in the room ?

8. What will 9 stools cost at 9 rupees each ?

9. How many trees are in 18 rows, each row having 9 trees ?

10. If I give 5 boys 8 marbles each, how many will be left out of 81, and out of 100 ?

11. A boy wrote 12 lines of dictation and there were 9 words in a line ; how many words did he write altogether ?

12. How many more are 9 tens than 4 twenties ? 10 tens than 6 tens ?. 9 nines than 5 nines ?

13. In one foot there are 12 inches ; how many inches are there in 6, 8, 9, 11 feet ?

14. There are 7 days in a week; how many days are there in 8, 11, 12 weeks?

15. A boy walks 3 miles in an hour. How many miles will he walk in 6 hours?

16. How many legs have 14 horses? How many feet have 9 ducks?

17. Ram is 8 years of age; his father is 4 times as old. How old is his father?

18. A man walked 4 miles in one hour. How many miles would he walk at the same rate in 16 hours?

19. Multiply 8 by 4 and take away 10; how much remains?

20. A window has 9 rows of panes, and 12 panes in each row. How many panes are there in the window?

SIMPLE MULTIPLICATION.

68. When the Multiplier does not exceed 20, the multiplication is called **Short Multiplication.**

69. When the Multiplicand is a large number and the Multiplier a number of **one** figure, we have the following Rule :—

RULE. Write down the multiplier under the units' figure of the multiplicand, and draw a line underneath. Begin at the units' figure of the multiplicand, and multiply each figure in succession by the multiplier, setting down and *carrying* precisely as in Addition.

Ex. Multiply 3468 by 7.

$$\begin{array}{r} 3468 \\ 7 \\ \hline 24276 \end{array}$$

Here 7 times 8 is 56. Set down **6** in the units' place and carry 5; 7 times 6 is 42, and $42+5=47$; set down **7** in the tens' place and carry 4; 7 times 4 is 28, and 4 carried is 32; put down **2** in the hundreds' place and carry 3; lastly 7 times 3 is 21, and $21+3=24$; set down **24**. The product is therefore 24276.

70. Writing down the *local* values of the figures, the process will stand thus :—

3468 = 3 thousands + 4 hundreds + 6 tens + 8 units;

∴ 3468 × 7 = 7 × 3 thousands + 7 × 4 hundreds + 7 × 6 tens + 7 × 8 units,
= 21 thousands + 28 hundreds + 42 tens + 56 units,
= 21 thousands + 28 hundreds + 47 tens + 6 units,
= 21 thousands + 32 hundreds + 7 tens + 6 units,
= 24 thousands + 2 hundreds + 7 tens + 6 units,
= **24,276**.

71. When the multiplier is greater than 9 but does not exceed 20, the multiplication can be effected easily in one line, by the help of the Rule in Art. 69.

Ex. 1. Multiply 59867 by 15.

```
  59867
     15
 898005
```

Here, 15 times 7 is 105; put down **5** and carry 10; then 15 times 6 is 90, and 90+10=100; put down **0** and carry 10; then 15 times 8 is 120, and 120+10=130; put down **0** and carry 13; then 15×9=135 and 135+13=148; set down **8** and carry 14; lastly, 15×5=75 and 75+14=89; set down **89**. Thus the product is 898005.

Ex. 2. Multiply 350974 by 18.

```
 350974
     18
6317532
```

Here, 18×4=72; put down **2** and carry 7; then 18×7=126, and 126+7=133; put down **3** and carry 13; then 18×9=162, and 162+13=175; set down **5** and carry 17; then 18×0=0 and 0+17=17; place **7** and carry 1; then 18×5=90, and 90+1=91; put down **1** and carry 9; lastly 18×3=54, and 54+9=63; put down **63**. Thus the product is 6317532.

72. When the multiplier is a simple number followed by **one** or **more** ciphers, we have the following Rule :—

RULE. Multiply the multiplicand by the simple number, and to the right of the product place as many ciphers as there are ciphers to the right of the multiplier.

Ex. Multiply 5867 by 70; and by 18000.

```
     5867      (1) Here 5867×70=5867×7 tens,
       70                       =41069 tens,
   410690                       =410690.

     5867      (2) Here 5867×18000=5867×18 thousands,
    18000                          =105606 thousands,
105606000                          =105606000.
```

Examples XI.

1. Multiply 284 by 2; 1475 by 3; 2867 by 4; 9048 by 2; 6057 by 4; 80965 by 5; 49508 by 8; 33069 by 7; 91537 by 12.

2. Multiply.

(1) 5849 separately by 2, 3, 4, 5, 6, 7, 8, 9 and 11.
(2) 38476 separately by 3, 5, 7, 9, 11, 13, 14, 15 and 19.
(3) 3870492 separately by 2, 5, 3, 7, 4, 9, 6, 8, 11, 12 and 15.
(4) 6508794 separately by 8, 7, 9, 11, 13, 15, 17 and 19.
(5) 987654321 separately by 2, 3, 4, 5, 6, 7, 8, 9, 11 and 12.

3. Find the values of

(1) 48508×8; 69360×6; 49216×11; 69432×12.
(2) 38476×9; 876549×12; 378125×16; 456932×18.
(3) 43275×17; 46059×16; 30748×19; 600954×20.
(4) 4609758×14; 4609758×19; 56380477×18.

4. Multiply

(1) 980989 separately by 10, 100, 1000 and 10000.
(2) 72051 separately by 30, 40, 70, 90 and 100.

(3) 91357 separately by 20, 200, 300, 5000 and 9000.
(4) 790785 separately by 120, 1500, 17000, 1300 and 190000.
(5) 900968 separately by 800, 1600, 14000 and 180000.

5. By how much does 18 times 1118 exceed 17 times 1050?

6. Find the sum of 19 times 2304 and 15 times 2045.

7. Multiply 123456789 separately by 1, 2, 3, 4, 5, 6, 7, 8 and 9, and add the several products together.

8. An estate contains 45068 bighas. Each bigha is worth 18 rupees. What is the value of the whole estate?

9. A railway train consists of 17 trucks. Each truck carries 12644 maunds weight. How many maunds does the whole train carry?

10. A man bought 305 cows at 12 rupees a head, and having spent on them for food 95 rupees, sold them at 16 rupees a head; what did he gain by his bargain?

11. Ram bought of Jadu 15 books at 13 annas each, and Jadu bought of Ram 19 books at 19 annas each; how many annas had Jadu to give to Ram?

12. Two persons start from the same place, and travel (i) in the same direction, (ii) in opposite directions. One travels at the rate of 93 miles a day and the other at 79 miles a day. How far will they be apart at the end of 7 days?

---

73. When the multiplier is greater than 20, the multiplication is called **Long Multiplication.**

74. When the multiplicand and multiplier are both large numbers, we have then the following general Rule :—

RULE. Place the multiplier under the multiplicand, so that units of the same order may be under one another and draw a line under the whole. Begin at the units' figure of the multiplier, and multiply by each of its figures in order, writing down each *partial* product so that its first figure shall be under the figure of the multiplier that produces it. *Add* together these *partial* products and the *sum* is the product required.

Ex. Multiply 7823 by 645.

```
   7823
    645
-------
  39115
 31292
46938
-------
5045835
```

Here, first multiply 7823 by 5 and set down the product 39115. Then multiply 7823 by 4, and put down the product 31292, so that 2 may come under the tens' place of the first partial product, 9 in the hundreds' place and so on. Lastly, multiply 7823 by 6 and set down the product 46938 so that 8 may be in the hundreds' place of the first partial product, and so on. Add up the three lines of figures already obtained and their sum 5045835 is the required product.

75. The reasoning above employed can be rendered more clear by the following consideration.

Since the above product is the sum of 7823 repeated 645 times

and $645 = 600 + 40 + 5$; therefore by the use of Arts. 69 and 72 we have the following process :—

7823	$645 = 600 + 40 + 5$.
645	
39115	$7823 \times 5 =$ 39115
312920	$7823 \times 40 =$ 312920
4693800	$7823 \times 600 =$ 4693800
5045835	5045835

76. If one or more of the figures of the multiplier be o, it is evident that the corresponding *partial product* will be o (Art. 65) and the lines may be entirely omitted after placing down each o *once*, to give the proper value to the product arising from the next figure.

Ex. Multiply 4968 by 709.

```
  4968
   709
 44712
347760
3522312
```

Here, in multiplying by 709, we first multiply by 9 and put down the result ; then when we multiply by 7, we really multiply by 700, but not by 70, and so place the first figure of the second partial product under the hundreds' figure of the first, affixing one cipher in the ten's place.

77. If the multiplicand, or multiplier, or both, end in ciphers, the ordinary process of Multiplication may be *shortened* or *facilitated* by the following Rule :—

RULE. Suppose the ciphers at the right of multiplicand, or multiplier or both omitted, find the product of the resulting numbers, and to the right of this product place as many ciphers as were supposed to be omitted in multiplicand, or multiplier or both together.

Ex. Multiply 47600 by 47 ; 257 by 64000, and 7900 by 83000.

Here, omitting the ciphers on the right, or *supposing* them to be omitted, we have

```
47600          257         7900
   47        64000        83000
 3332         1028        237
1904         1542        632
2237200    16448000    655700000
```

Where the ciphers are *annexed* at last to the right of the products obtained in the ordinary way, to give the other figures their proper local values.

Thus, in the first case, when we multiply 6 by 7, we really multiply 600 by 7, and 600 multiplied by 7 gives 4200 ; therefore two ciphers are annexed after 2 in the product.

In the second case, when we multiply 7 by 4, we really multiply 7 by 4000 and 7 multiplied by 4000 gives 28000 ; therefore three ciphers are annexed after 8 in the product.

In the third case, when we multiply 9 by 3, we really multiply 900 by 3000, and 900 multiplied by 3000 gives 2700000, therefore five ciphers are annexed after 7 in the product.

78. If the multiplicand and multiplier change places, the product must be the *same* as before, otherwise the *same* numbers would have *more* products than *one* (Art. 64). Hence, it is convenient to make the larger number the multiplicand and the smaller number the multiplier.

79. The following **Proofs** are generally adopted in Multiplication.

(1) Interchange multiplicand and multiplier; the product ought to be the *same.*

(2) **By Casting out the nines.** We cast the nines out of a number thus: add together all its figures omitting every 9, and if the sum be greater than 9, replace it by the sum of its figures, and if the new sum be greater, replace it by the sum of its figures, and so proceed till we have a sum less than 9.

Cast the nines out of multiplicand and *multiplier.* Multiply the results, and *cast the nines out* of their product, noting the new result; now *cast the nines out* of the *product*, and if the result *coincide* with the one previously noted, we presume that the work is *correct.*

80. We may mention here that the above test fails in the *three* following cases:—

(1) If the order of figures in the product be misplaced as 86 for 68.

(2) If the errors made compensate each other, so far as the sum of the digits is concerned as 65 for 83.

(3) If 9 be written for 0, or 0 for 9, or either be inserted or omitted too often; as, 59 for 50, or 597 for 57, or 708 for 78, and so on.

Ex. Multiply 5867 by 3478, annexing the proof.

```
 5867...8                  Multd.                     3478
 3478...4          \        and        /              5867
 ————...5           \      Multr.     /              —————
46936                \       5       /               24346
41069       Multd. 8.  \   /   Multr. 4              20868
23468                  /   \                        27824
17601                 /  5   \                      17390
————————             /       \                     ————————
20405426...5        /  Prodt. \                    20405426
```

Beginning at the left hand, we cast the *nines* out of the

(1) Multiplicand thus:—13, 19, 26; replace 26 by the sum of 2 and 6 or **8**.

(2) Multiplier thus:—7, 14, 22; replace 22 by the sum of 2 and 2 or **4**.

Now multiply 8 by 4, giving 32, which replace by the sum of 3 and 2 or **5** and note this result.

(3) Product thus:—6, 11, 15, 17, 23; replace 23 by the sum of 2 and 3 or **5**.

As this result coincides with the previous one, we presume the work is correct.

Examples XII.

1. Multiply 946 by 61 ; 869 by 89 ; 917 by 46 ; 909 by 88 ; 463 by 608 ; 417 by 739 ; 3259 by 497, and 692 by 73.

2. Multiply

(1) 47691 by 27 ; 28573 by 35 ; 716281 by 48 ; 39265 by 39.
(2) 129385 by 66 ; 138476 by 81 ; 480765 by 97 ; 829741 by 59.
(3) 8241763 by 123 ; 921846 by 158 ; 827941 by 376.
(4) 5086927 by 495 ; 254037 by 2980 ; 4785328 by 7802.
(5) 56380477 separately by 35, 48, 72 and 132.
(6) 67836479 separately by 356, 4378 and 78539.
(7) 70870096 separately by 404, 3009 and 900807.
(8) 279420 by 7350 ; 678000 by 87600 ; 80108 by 7770.
(9) 56348 by 50601 ; 876000 by 678000 ; 896385 by 6687400.
(10) 57483000 by 40, 900, 430, 24500 and 4670000.

3. Find the product of :—

(1)	(2)	(3)	(4)	(5)
45678	3124791	436712	1100785	4532815
9128	89023	45678	71053	751283
(6)	(7)	(8)	(9)	(10)
447002	8913243	110375009	110200570	275642
578648	234567	198075	200570	125255
(11)	(12)	(13)	(14)	(15)
447529123	4465348	79094451	84964270	123456789
8901234	7000608	7640950	8743590	123456789

4. Find the values of :—

(1) 704745 × 615 ; 469830 × 369 ; 391525 × 861.
(2) 1174575 × 2214 ; 3523725 × 2583 ; 926196 × 7896.
(3) 920685 × 7098 ; 4465348 × 7000608 ; 7650329 × 600509.
(4) 400905703206 × 7008130502 ; 8070906050493 × 64032000905.
(5) 6709802607508 × 2005032057 ; 1310275031496 × 20456300170.

5. What is the difference between 23456 multiplied by 996, and the remainder in subtracting 4 times 23456 from 23456000 ?

6. A bigha of land costs 784 rupees, what will 203 bighas cost ?

7. If there are 432 pages in a book, how many will there be in 80704 such books ?

8. If I give 125 boys 79 marbles each, how many shall I have left out of 10000 ?

9. 79432 copies of a newspaper are printed daily. How many are printed in a year of 314 days ?

10. The cost of constructing a Railway is 61303 rupees per mile ; what will 701 miles cost ?

11. An army consists of 295 battalions of 34618 men each ; what is the whole number of men in the army ?

12. In a town there are 734 houses ; 345 of them contain, on an average, 11 persons each and the rest 15 each. How many persons reside in the town ?

13. If a master employs 73 workmen, each of whom receives 34 rupees per month, how many rupees does he pay away per month ?

14 If of 20000 shells used in war, 3648 are 36 pounders, 11275 are 24 pounders, and the rest 18 pounders ; what is the total weight (in pounds) of the whole ?

15. A clock strikes 114 times in a day. How often will it strike in 365 days ?

16. A town has 436 streets. Each street contains on an average 6422 inhabitants. What is the population of the town ?

17. A directory contains 798 pages. There are 72 names in each page. How many names are in the directory ?

18. 343 paving-stones are required for every yard in a street. There are 18742 yards in the street. How many paving-stones will the whole street require ?

19. The distance of the Earth from the Sun is found to be 11608 times the Earth's equatorial diameter, and that diameter is 7926 miles. Required the distance between the Earth and the Sun.

20. India contains about 1466576 square miles and the population is reckoned to be about 189 persons to every square mile ; what is the whole population of the country ?

---

81. To find the product of *more* than two numbers, multiply the product of two of the numbers by the third, the result by the fourth, and so on. The final result is called the **continued product** of so many **factors**.

Thus, the *continued product* of 3, 5, 8 and 47 $= 3 \times 5 \times 8 \times 47 = 15 \times 8 \times 47 = 120 \times 47 = 5640$, and 3, 5, 8 and 47 are *factors* of 5640.

82. The continued product of any numbers will remain the *same*, however we may change the *order* of its factors.

Thus, since $4 \times 2 \times 5 \times 7 \times 3 = 8 \times 5 \times 7 \times 3 = 40 \times 7 \times 3 = 280 \times 3 = 840$ and $5 \times 4 \times 2 \times 3 \times 7 = 20 \times 2 \times 3 \times 7 = 40 \times 3 \times 7 = 120 \times 7 = 840$;

$$\therefore 4 \times 2 \times 5 \times 7 \times 3 = 5 \times 4 \times 2 \times 3 \times 7.$$

Ex. Find the continued product of 3471, 7 and 52.

```
  3471
     7
 -----
 24297
    52
 -----
 48594
121485
-------
1263444
```

Here, we first multiply 3471 by 7, and the product is 24297 ; again multiply 24297 by 52, and the product is 1263444 ; thus the continued product of the several factors is 1263444.

83. If *one* or *more* of the factors in any continued product be 0, the whole product is 0. (See Art. 65.)

Examples XIII.

1. Find the continued products of :—

(1) 4, 7, 25. (2) 13, 15, 17. (3) 18, 19, 20. (4) 407, 18, 5.
(5) 729, 8, 61. (6) 7184, 6, 12. (7) 35, 32, 14, 29. (8) 35, 29, 43, 87.
(9) 33, 13, 15, 4, 56. (10) 27, 57, 35, 1277. (11) 156, 13, 365, 78.
(12) 18, 19, 35, 24, 12, 17. (13) 340, 255, 783. (14) 675, 225, 180, 125.

2. A library contains 3275 volumes, and each volume on the average 493 pages, and each page 39 lines. How many lines are there?

3. If the earth moves round the Sun at the rate of 68000 miles an hour, how far will it move in 365 days of 24 hours each?

4. If every page of a book contains 36 lines, and each line on an average 11 words, how many words would there be in 157 pages?

5. If each of 36 trucks in a luggage train contains 18 barrels of cement, and each barrel 36 maunds, how many maunds is the train carrying?

6. How many yards of silk are there in 9 packages, each containing 8 parcels, each parcel 26 pieces, and each piece 53 yards?

7. In a school there are 10 classes: each class has 4 desks, each desk holds 18 boys; how many boys are there in the school?

8. If 37 labourers earn 39 rupees each per day; how many rupees do they all earn in 36 working days?

9. If every man lived to marry and have 8 male children, how many great-great-grand children of the male sex could every one expect to have?

10. A Railway passenger train consists of 32 carriages; each carriage is divided into 12 compartments; in each compartment there are 5 benches and on each bench there is space for 8 persons; how many persons can the train carry?

84. When a number is multiplied by itself *once*, *twice*, *thrice*, *four*, *&c.*, times, the product is called the **second, third, fourth, fifth**, &c., **power** of that number respectively. The *second* and *third powers* of a number are commonly termed its **square** and **cube** respectively. The number itself is called its **first power**.

85. These *powers* are often indicated by small numerals 2,3,4,5, &c., placed above the number to its right, which express how often the number is repeated in the product. The small numerals so used are therefore called the **indices** or **exponents** of the several *powers*.

Thus, $5^2=5\times5=25$; $\therefore$ 25 is the *second power* or *square* of 5.
$5^3=5\times5\times5=125$; $\therefore$ 125 is the *third power* or *cube* of 5.
$5^4=5\times5\times5\times5=625$; $\therefore$ 625 is the *fourth power* of 5, and so on.

86. If the three signs +, −, ×, occur in an expression, the

operation of Multiplication is to be performed first and then that Addition or Subtraction.

Thus, $4\times4\times3+3\times3\times2-4\times2\times1+2\times1\times0=48+18-8+0$
$=66-8=58.$

Examples XIV.

1. Find the squares of :—

(1) 1, 2, 3, 4, 5,...25 ; 39, 46, 54, 86, 99. (2) 172, 237, 906, 987.
(3) 729, 873, 1043, 5496. (4) 7342, 9384, 8796, 1234.

2. Find the cubes of :—

(1) 1, 2, 3, 4, 5,...25 ; 37, 48, 68, 77. (2) 88, 97, 123, 456.
(3) 308, 876, 765, 999. (4) 987, 5386, 9876, 1234.

3. Find the fourth powers of :—

(1) 678, 305, 987, 988. (2) 908, 3271, 8004, 9999.

4. Find the values of :—

(1) $1^2+2^2+3^2+4^2+5^2+6^2+7^2+8^2+9^2$. (2) $23^2+15^2-3^3$.
(3) $5^3-4^2-8^2$. (4) $1^3+2^3+3^3+4^3+5^3+6^3+7^3+8^3+9^3$.
(5) $2^4+3^4-1^4$. (6) $25^2+28^2-20^2-18^2+15^2$.

5. Simplify the following expressions :—

(1) $8\times4-3\times6+4\times3-2\times1+5\times2+3\times7$.
(2) $5\times6\times3+4\times3\times0-2\times1\times4+3\times6\times4-2\times2$.
(3) $8\times6\times3\times1-3\times6\times2\times4+4\times6\times7\times4-7\times8\times2\times0$.
(4) $-9\times6\times2\times3+7\times4+4\times6\times3\times5-3\times6\times7\times0\times5+2\times3\times4$.
(5) $7^2+2\times3^2+3\times5^2+4\times9^2$. (6) $3^2\times2+2\times3-2^2\times3+6\times1^3$.
(7) $23^2-11^2+115\times11^2-110^2+112^2$.
(8) $3^3+3\times4\times5+5^3-4^3-2\times4-2^3+6^2-3^3$.

IV. DIVISION.

87. Division is the method of finding *how many times* one given number is contained in another given number. The former of these numbers is called the **divisor**, the latter the **dividend**, and the number telling *how many times* the **quotient**. The number left after the operation is finished, is termed the **remainder**.

88. In dividing one number by another, we obviously take the latter number from the former, as often as we are able, according to the principle of Subtraction before explained. Hence *Division* bears the same relation to *Subtraction*, as *Multiplication* bears to *Addition*.

Thus, to divide 26 by 8, means that we are to find how many times 26 contains 8, and the operation at the side shews that 26 contains 8, 3 times with a remainder 2. Here 26 is called the *dividend*, 8 the *divisor*, 3 the *quotient* and 2 the *remainder*.

```
26(1, 1, 1
 8
18
 8
10
 8
 2
```

89. Hence, by division we break up a given number into as many equal parts as there are units in another given number, and thus find one of these parts.

90. *Division* is of two kinds, *simple* and *compound.*

(i) When the dividend and divisor are both *abstract* numbers, or both *concrete* numbers of one and the same denomination; or when the divisor as an *abstract* number, and the dividend a *concrete* number of one denomination, it is called *simple Division.*

(ii) When the dividend is a *concrete* number of the same kind, but of different denominations of that kind, and the divisor an *abstract* number; or when both the dividend and divisor are *concrete* numbers of the same kind but of different denominations of that kind, it is called *Compound Division.*

91. When there is no remainder, the division is said to be **exact** and since the Quotient tells how many times the Dividend contains the Divisor, it follows that *Dividend = Divisor × Quotient.* But when there is a Remainder, the division is called **inexact**, and the *Dividend = Divisor × Quotient + Remainder.*

92. The operation of *Division* is expressed by means of the sign ÷ and sometimes /, which is read **divided by** or simply **by**. It is also denoted by writing the dividend above the divisor with a line between them.

Thus, 42 ÷ 7 denotes that 42 is to be divided by 7, and is read 42 *divided by* 7 or simply 42 *by* 7. Also 42/7 and $\frac{42}{7}$ mean 42 ÷ 7.

93. In division, the quotient is an *abstract* number, if the dividend and divisor are both abstract or both concrete numbers, but the quotient is a *concrete* number, if the dividend is a concrete number and the divisor an abstract number. The divisor, if concrete, must be of the same kind as the dividend.

Thus, 45 divided by 5, or 45 rupees divided by 5 rupees, gives the *abstract* number 9 as quotient, for 5 or 5 rupees taken 9 times gives 45 or 45 rupees, and 45 rupees divided by 5 gives the *concrete* number 9 rupees as quotient, for if 45 rupees be divided into 5 equal parts, each of these parts will contain 9 rupees. Also 45 rupees divided by 5 yards has no meaning, according to the definition of Division in Art. 93.

94. As Division is the *reverse* of Multiplication, it follows that by a reversed process, the Multiplication Tables must furnish the means of obtaining the quotient, when the divisor does not exceed 20 and the dividend 400.

Ex. 1. Divide 96 by 8.

Since 8 × 12 = 96; therefore 96 ÷ 8 gives 12 as quotient.

Ex. 2. Divide 259 by 17.

Since 17 × 15 = 255, and 259 − 255 = 4; therefore 259 ÷ 17 gives 15 as quotient and 4 as remainder

Examples XV. (Mental Division.)

1. How many times does 8 contain 2? 36 contain 3? 20 contain 4? 35 contain 5? 24 contain 6? 56 contain 7? 81 contain 9?

2. Divide 14 by 2 12 by 3; 48 by 4; 20 by 5; 42 by 6; 49 by 7; 32 by 8; 108 by 9; 90 by 10; 77 by 11; 96 by 12.

3 Divide

(1) 56 separately by 2, 3, 4, 5, 6, 7, 8, 9, 12 and 14.
(2) 98 separately by 2, 5, 7, 9, 13, 15, 17, 18 and 19.
(3) 168 separately by 2, 7, 8, 9, 6, 12, 11, 15 and 18.
(4) 288 separately by 4, 7, 9, 10, 6, 8, 12, 15 and 17.
(5) 342 separately by 3, 6, 8, 9, 4, 11, 13, 15, 16 and 18.
(6) 172 by 9; 141 by 11; 128 by 14; 257 by 16; 195 by 19.

4. In 54, how many times is 8, and how many over? How many times is 15 contained in 195? In 240, how many times is 18, and how many over?

5. If 16 be taken 14 times from 228, what is left?

6. what is the 9th part of 36, 54, 108 and 144?

7. To how many boys can I give 9 marbles if I have 153?

8. At a cricket match 11 players make 132 runs. If each made the same number of runs, how many did each make?

9. A Patsala consists of 128 boys and they are made to stand in 8 rows; how many are there in each row?

10. If 320 rupees are shared equally among 16 men; how many does each man receive?

11. Divide 132 oranges equally among 7 girls and 5 boys.

12. Divide 96 pencils equally among 8 boys.

13. Bhuban spent 180 pice in oranges, buying them at the rate of 6 for 3 pice; how many oranges did he buy?

14. A boy, having a basket containing 214 oranges, distributed them equally between his 8 school-fellows and himself; the number which remained he gave to his school-master; how many did the school-master receive?

15. A man bought 11 cows at 18 rupees each, and sold them so as to gain 99 rupees; what did he sell each cow for?

16. How many seers of sugar at 5 annas each can be bought for 330 annas?

17. A woman bought 180 eggs at 3 for 2 pice and 275 more at 5 for 3 pice, and sold the whole lot at 13 for 19 pice; what does she gain or lose?

18. If 5 men can do a piece of work in 18 days, how long will it take 9 men to do the same work?

19. How many pen knives, worth 8 annas each, ought to be exchanged for 144 pen-holders at one anna each?

20 A man walked 306 miles in 18 days; how many miles did he walk per day?

SIMPLE DIVISION.

95. When the dividend is a large number, but the divisor less than 20, the division is called **Short Division** and can be done by the following Rule.

RULE. Place the divisor and dividend thus :
divisor) dividend.

From the left of the dividend cut off a number not less than the divisor but less than 10 times the divisor, giving the first partial dividend. Find by the aid of the Multiplication Tables how often the divisor is contained in this dividend ; put down the quotient under the units' figure of this dividend, and take notice of the remainder (whether it be any number or 0). On the right of this remainder, conceive in your mind to be placed the least number of the figures next following in the dividend which, affixed to the remainder, will make a number not less than the divisor. Proceed, as above, with this new partial dividend to find the next figure of the quotient : taking care to place after the first figure in the quotient a cipher for every figure brought down from the dividend which, affixed to the remainder, makes a number less than the divisor.

Continue this process till all the figures of the dividend have been thus brought down ; and if there be any remainder at the end of the operation, write it as a remainder distinct from the quotient.

Ex. 1. Divide 612459 by 7.

7)612459
87494 rem. 1.

From the left of the dividend cut off a number not less than 7 but less than 70 : that is, cut off 61, our first partial dividend. Now 7 is contained in 61, 8 times and 5 over ; put the 8 under the 1 in 61, and to the right of the remainder 5 affix the next figure of the dividend 2, making 52, the second partial dividend. But 7 is contained in 52, 7 times and 3 over ; put 7 in the quotient, and to the right of the remainder 3 affix the next figure 4 making 34, the third new dividend ; and so proceed.

The above operation is usually performed in saying :—
7 in 61, **8** and 5 over ; in 52, **7** and 3 over ; in 34, **4** and 6 over ; in 65, **9** and 2 over ; in 29, **4** and **1** over (as remainder).

Thus the quotient is 87494, and the remainder 1.

Ex. 2. Divide 61245 by 15.

15)61245
4083

Here 15 in 6 goes no times, but 15 in 61 goes 4 times and 1 over ; write **4** under the 1. Then 15 in 12 goes no times, but 15 in 124 goes 8 times and 4 over ; write **0** under the 2 and **8** under the 4 ; lastly, 15 in 45 goes 3 times, write **3** under the 5.

Thus the quotient is 4083.

96. The truth of the above method may be shewn thus :—
Since 61245 = 61 thousands + 2 hundreds + 4 tens + 5 units,
= 60 thousands + 12 hundreds + 4 tens + 5 units,
= 60 thousands + 124 tens + 5 units,
= 60 thousands + 120 tens + 45 units.

∴ 61245 divided by 15 gives as quotient **4** thousands + **8** tens + **3** units or 4083.

Examples XVI.

1. Divide

(1) 462 separately by 3, 6, 8, 9, 10, 11 and 12.
(2) 682 separately by 3, 4, 6, 8, 9, 11, 14 and 15.
(3) 8425 separately by 5, 7, 8, 10, 13, 16, 17 and 19.
(4) 6876 separately by 2, 3, 7, 9, 11, 12 and 14.
(5) 35298 separately by 3, 5, 9, 7, 10, 12 and 18.
(6) 348 by 2 ; 4596 by 3 ; 276284 by 4 ; 84375 by 5.
(7) 53844 by 5 ; 536074 by 7 ; 95832417 by 8 ; 3158367 by 10.
(8) 7163253651 by 9 ; 1234567890 by 11 ; 9876543 by 12.
(9) 27643532 by 14 ; 35762445 by 15 ; 47623554 by 18.
(10) 34672352 by 16 ; 987654321 by 17, by 18, by 19, by 20.

2. If 1674 men are drawn up in 18 columns, how many men are there in each column ?

3. I distributed 2160 marbles among a number of boys, and gave each boy 12 marbles ; how many boys were there ?

4. What is the 15th part of 135090 ? the 11th part of 101112 ?

5. A farmer has 1786 sheep divided into 19 equal flocks. How many sheep are there in each flock ?

6. A farmer spent 1872 rupees in the purchase of oxen. Each ox cost 12 rupees. How many oxen did he buy ?

7. If the sum of 18 and 30 be divided by their difference, and the quotient be multiplied by the product of 16 and 27, what is the result ?

8. A man gives 14 cows and 35 sheep for 55 bags of potatoes worth 7 rupees per bag ; if each sheep was worth 3 rupees, what did he get for each cow ?

---

97. When the dividend and divisor are both large numbers, the division is called **Long Division** and can be performed by the following general Rule.

RULE. On either side of the dividend draw curved lines ; place the divisor on the left and the figures of the quotient as they arise on the right ; thus

divisor)dividend(quotient

Then try to find how often the first one or two figures on the left hand of the divisor are contained in the first one or more of those of the dividend, and place the result on the right as the first figure of the quotient ; and the product arising from the multiplication of the divisor by this figure being subtracted from the dividend, *bring down* or *annex* to the right of the remainder the next figure of the dividend. Proceed as before, and continue the process till all the figures of the dividend have been brought down ; then the quotient, and the remainder if any, will be obtained.

If at any stage of the process, the divisor is greater than the *partial* dividend, affix a *cipher* to the quotient and bring down the next figure of the dividend. Continue this process till the partial dividend is greater than the divisor and then proceed as before.

Ex. 1. Divide 75035 by 349.

```
349)75035(215
    698
    ---
     523
     349
     ---
     1745
     1745
     ----
```

Here, the first figure **2** in the quotient is obtained by inquiring how often 3 is contained in 7, or 34 in 75; then, after multiplying 349 by 2, which, from the places of the figures, represents 2 *hundreds*, and subtracting the product which is 698, from 750, we have a remainder 52; to this the next figure 3 of the dividend is *annexed* to form the partial dividend 523. Now seek how often 3 is contained in 5, or 34 in 52, and the quotient being **1**, 1 *ten* is annexed to the 2 *hundreds* already obtained; multiplying 349 by 1, which means 1 *ten*, and subtracting the product 349 from 523, we get the remainder 174. Bring down the last figure 5 of the dividend to form the partial dividend 1745, and we find the corresponding quotient to be **5** units exactly, for 349 multiplied by 5 produces 1745, and the operation is then completed, leaving no remainder. Therefore the whole quotient is 215.

98. Supplying the auxiliary digits omitted in the above operation the process would stand thus :—

```
349)75035(200+10+5
    69800
    -----
     5235
     3490
     ----
     1745
     1745
     ----
```

Ex. 2. Divide 39875365 by 8654.

```
8654)39875365(4607
     34616
     -----
      52593
      51924
      -----
        66965
        60578
        -----
         6387
```

Here 3987 is less than 8654, but 39875 is greater; therefore take 39875 for the first partial dividend. It contains the divisor 4 times; put **4** in the quotient, multiply 8654 by 4, placing the product 34616 under 39875, and subtract, leaving 5259. To the remainder 5259 *annex* the next figure of the dividend 3, giving 52593, the second partial dividend. It contains the divisor 6 times; put **6** in the quotient, multiply 8654 by 6, placing the product 51924 under 52593, and subtract, leaving 669. Again, to 669 bring down the next figure 6, giving 6696 the third partial dividend. It contains the divisor 0 times; put **0** in the quotient, and the remainder is now 6696. Lastly to 6696 bring down the last figure 5, giving 66965, the fourth partial dividend. It contains the divisor 7 times; put **7** in the quotient, multiply 8654 by 7, placing the product 60578 under 66965, and subtract, leaving a remainder 6387. Thus the quotient is 4607 and the remainder is 6387.

99. **When the divisor is terminated by one or more ciphers, we use the following Rule.**

RULE. Cut off all the ciphers on the right of the divisor and as many figures from the right of the dividend :—for the quotient, divide the remaining figures of the dividend by the remaining figures of the divisor (Arts. 95, 97), and for the final remainder bring down to the particular remainder the figures cut off from the dividend.

Ex. Divide 20573296 by 80 and by 345000.

```
(1) 8,0)2057329,6              (2) 345,000)20573,296(59
        257166 - 16                         1725
                                             3323
                                             3105
                                              218296
```

In the first example, in dividing by 8 the remainder is **1**, to which we bring down the figure cut off **6**, giving 16 for the final remainder, and 257166 for quotient.

In the second example, the remainder in dividing by 345 is **218**, to which we bring down the figures cut off **296**, giving 218296 for the final remainder and 59 for quotient.

* **100.** The **Proofs** usually adopted in division are the following :—

(1) To the product of the divisor and quotient add the remainder (if any) ; if the result coincides with the dividend, we presume that the work is correctly performed.

(2) By **casting out the nines.**

(*a*) From the sums of the digits in the *divisor* and the *quotient* subtract 9 as many times as possible, and set down the remainders to the left and right of a cross sign.

(*b*) Multiply the two remainders and from the product subtract 9 as often as possible. Put down the remainder below the cross sign.

(*c*) Lastly subtract the *remainder* from the *dividend* and from the sum of the digits of this difference subtract 9 as many times as possible and set down the remainder above the cross sign. If the upper and lower figures agree, it is presumed that the operation is correct.

Ex. Find the quotient and remainder when 275487 is divided by 736.

Division.

```
736)275487(374
    2208
     5468
     5152
      3167
      2944
       223
```

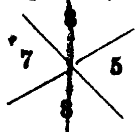
Thus the quotient is 374 and the remainder 223.

Proofs.

(1)

```
   374
   736
  2244
 1122
2618
275264
   223
275487
```

(2) $7+3+6=16$, *rem.* **7** and $3+7+4=14$, *rem.* **5**.



$7\times5=35$, *rem.* **8**.
Also $275487-223=275264$, and $2+7+5+2+6+4=26$, *rem.* **8**.

101. If all the four signs +, −, ×, ÷ are used together in an expression, the operations of *Division* and *Multiplication* are to be performed first and next those of *Addition* and *Subtraction*.

Ex. Find the value of 14+12÷6×4−3×2+6×72÷12.

The expression = 14+2×4−3×2+6×6 = 14+8−6+36
= 58−6 = 52.

Examples XVII.

1. Divide :—

(1) 92483 by 23. (2) 79958 by 39. (3) 79796 by 79. (4) 588168 by 84.
(5) 79512587 by 43. (6) 69637856 by 32. (7) 67001228 by 49.
(8) 144157246 by 83. (9) 47073256 by 37. (10) 7417784 by 88.
(11) 579826952 by 76. (12) 9009196416 by 96. (13) 58762347 by 99.
(14) 17587694293 by 54. (15) 14528340631 by 84.
(16) 3708501975 by 81. (17) 96790123458 by 98.

2. Find the values of :—

(1) 419352633÷123 (2) 1721034655÷144. (3) 47123419361÷132.
(4) 3577926÷506. (5) 27291888÷478. (6) 87624792÷843.
(7) 48310567÷549. (8) 6430776444÷876. (9) 137090807÷996.
(10) 630762540981÷652. (11) 632798014÷7243.
(12) 519387042÷2731. (13) 140167329÷7038.
(14) 395494875÷6007. (15) 2106144185÷2375.
(16) 25413286÷7960. (17) 8327976÷5730.
(18) 64157660÷1480. (19) 935384767÷4836.
(20) 900370575÷54321. (21) 183920748÷37246.
(22) 2828882701578÷38706. (23) 2919333978682÷76913.
(24) 61190852817674÷873156. (25) 163034794788÷321567.
(26) 4872643258 76÷5678909. (27) 876824985621÷90956845.
(28) 56400003227÷76589451. (29) 32899438654÷100104325.
(30) 191776658604÷68589649. (31) 4676705026675÷1543212353.
(32) 121932631112635269÷123456789.
(33) 1630188053103649203285÷2837154309.
(34) 5602119750149670530000÷7000020305 06.
(35) 1630188053103649203285÷574585614865.

3. Divide :—

(1) 237876093 by 5605, by 9089, by 40857, and by 57085.
(2) 81229 separately by 10, 20, 30, 40, 50, 80, 90.
(3) 342604 separately by 100, 400, 600, 800, 900.
(4) 78534826 separately by 800, 12000, 3200, 475000.
(5) 3854269734 separately by 310, 5900, 587000, 90900.
(6) 25413286 by 7900 ; 19054832 by 83000 ; 26799534687 by 7890000.

4. Find the values of :—

(1) 192÷16+720÷18+795÷15−1786÷19.
(2) 3871÷49+6935÷95−5432÷56−1375÷25+4590÷45.
(3) 56+81÷3+8×7×9÷12×136÷17−72÷18+6×3.
(4) 12×16÷8+17×6−18×32÷8−27÷9×7+8×30÷15+56÷14.
(5) 15×37153+73474−67152÷4+40734×2−5485×7.

5. If a bag contains 103 potatoes, how many will be required to hold 7432274 potatoes ?

6. If each carriage contains 57 passengers, how many carriages are there in a train carrying 969 passengers ?

7. Each of 156 boys uses 12 pen-nibs, and a box contains 144 nibs. How many boxes are required ?

8. A confectioner sells 23475 maunds of sweetmeats in a year of 313 days ; how many maunds does he sell in a day ?

9. Supposing a Railway train to travel from Calcutta to Delhi, a distance of 924 miles, in 44 hours, what is the average speed per hour ?

10. The population of a country is 3083220 and its area is 7341 square miles. How many people are there on an average to each square mile ?

11. Find the number of pages in a book which has on an average 207 words on a page, and contains 201411 words altogether ?

12. How many minutes will a wheel be in turning round 895702 times, if it turn 158 times in a minute ?

13. What number multiplied by 79 will give the same produc— as 257 multiplied by 553 ? er

14. A shopkeeper sold 267 shawls for 4005 rupees, gaining therat by 4 rupees on each shawl ; what had each shawl cost him ?

15. The population of a certain village is 21510, and one out of 45 dies annually. How many die in a year ?

16. Find how many times the numbers 11, 15, 19, and 23 must be equally repeated to make 13668.

17. Find the 532nd part of 1004416. What is the 365th part of 36865365 ?

18. How many pages contain 30888 words, every page having 52 lines of 9 words each ?

19. If 168465 maunds of rice be distributed equally among 11231 famine-stricken men, how many maunds will each receive ? And if the family of each consist of 5 persons, what will be the share of each person ?

20. The rays of light comes from the Sun to the Earth in 498 seconds ; at what rate does light move per second, the distance of the Sun from the Earth being 93000000 miles ?

V. THE USE OF BRACKETS.

102. Brackets, which are of several kinds, as ( ), { }, [ ], are used to denote that all numbers included within any pair of them are to be considered as forming but one number and are therefore to be equally affected by any number not included within the same pair of brackets.

Thus (2+3+7) denotes that 2, 3 and 7 are to be taken as making one number, *i. e.*, whatsoever, outside the brackets affects 2 in any way, must also affect 3 and 7 in the same way.

A **vinculum** is a sign sometimes used instead of brackets. It consists of a *line* drawn over the numbers to be considered as forming one number.

Thus $\overline{2+3}$ express the same thing as $(2+3)$.

103. When two or more numbers, connected by the signs of operation are enclosed in a pair of brackets the operations of arithmetic indicated inside the brackets are to be performed before the brackets are removed. Thus,

Ex. 1. $7-\overline{5-3}=7-2=\underline{5}$.

Ex. 2. $22-(4\times 3+5-6\div 2)=22-(12+5-3)$
$=22-(17-3)=22-14=\underline{8}$.

104. When a number immediately precedes an expression included in a pair of brackets, this number is to be multiplied by the number obtained after removing the brackets.

Thus, $7+4(5-2)-6\times 3=7+4\times 3-18=7+12-18=19-18=1$.

105. When an expression is included in more than one pair of brackets, it is convenient to remove the innermost bracket first, then the innermost of those that remain, and so on, till all the brackets are removed.

Thus, $25-[\{15\times 10-2\times 12-8(2\times 12-10)\}\div 2]\times(15-\overline{10+2})$
$=25-[\{150-24-8(24-10)\}\div 2]\times(15-12)$
$=25-[\{150-24-8\times 14\}\div 2]\times 3$
$=25-[\{126-112\}\div 2]\times 3$
$=25-[14\div 2]\times 3=25-7\times 3=25-21=4$.

106. If the sign + *plus* precedes a bracket, the bracket may be removed without affecting the result.

Thus, since $7+(5-3)=7+2=9$ and $7+5-3=12-3=9$,
therefore, we have $7+(5-3)=7+5-3$.

107. If the sign − (*minus*) precedes a bracket, the bracket may be removed, provided the signs of all the numbers, inside the bracket be changed from + to −, and from − to +.

Thus, since $29-(7-5+3)=29-(2+3)=29-5=24$,
and $29-7+5-3=34-10=24$,
therefore, we have $29-(7-5+3)=29-7+5-3$.

108. The sign ∴ signifies *therefore*, and is often used in stating a method by which an answer has been obtained. The sign ∵ stands for *because* or *since*, and is used in stating a reason.

Examples XVIII.

1. Find the values of :—

(1) $10+(5-3)-(17-8)+(16-11)+25-(6-3+4)$.

(2) $20-\overline{10-3-6}+(15-3)-(16-9)-(5+6)+(4+9)$.

(3) $8+4(12-7)-3(9-5)+7(16-19+5)-(18-\overline{6+7})$.

(4) $3\{8+25-3(20-12)\}$. (5) $3\{8+(25-3)\overline{20-12}\}$.

(6) $287-[\{15\times 10-2(12-8)(2\times 12-10)\}\div 2]\times 15-\overline{10-2}$.

(7) $1520-\{610+\overline{703-608}\}$. (8) $605-\{(95-11-19)+237\}$.

(9) $86-\{(59-48)+16-(59-49)\}$. (10) $168-\{(70-39)+(90-83)\}$.

(11) $1246-(362-156)-\{371-(495-386)\}$.

2. Find the values of :—

(1) $(1536-487)-1392\div 29+7\times 5$. (2) $5880\div(167-132)\times 6$.

(3) $(194+65)\times 7+(352-220)\div 11-952\div(91-35)$.

(4) $(67893-8637)\div 823+7546\times(2356-945)-(9870\times 170)$.

(5) $\{(312570\times 598+76125\times 47+318\div 3)-155146\}\div(6139\times 15)$.

3. If the sum of 274 and 108 be multiplied by their difference and the product be divided by 166, what will be the quotient ?

4. If the sum of 103, 29, and 267 be divided by 19, and the quotient be multiplied by 57, and the product be diminished by 197, what will be the remainder ?

5. Multiply $(325-293)$ by $(306\div 17)$ and to the product add $(1000+99)$.

6. From 34856 subtract (763×41) and to the remainder add $\{1998\div(663-441)\}$.

7. Find the difference between 876 and $459-368+149$.

8. What number subtracted from $(2471+56)$ will leave $(3863-1498)$ as remainder ?

9. Find the difference between

$$3210+401-(67-59) \text{ and } 342-(491-382).$$

10. From the sum of the greatest numbers of 9 and 10 digits subtract the difference of the least numbers of 10 and 11 digits.

11. From the sum of the greatest numbers of 4, 5 and 6 digits subtract the sum of the least numbers of 3, 4 and 5 digits.

12. Find the values of :—

(1) $6+8[3\times 6+\{3+7-(8+3-6)-(2\times 6\div 3+3-2)\}]$.

(2) $66\times 37-8[(9-7)\times 6-(27+12)\div 13+(17+15+39-50)\times 5-9\times 7]$.

(3) $\{(7+75)\times 43+(4698+171)\div 9\}+\{(73+14-2)-(16\div 2+4\times 7)\}$.

(4) $8[4\times\{(360\times 120)-(47+13)\div 3\}]\div\{(360\times 120)+(65-25)\div 5\}+5401$.

(5) $108\div 9\times[76-9\{63-7(9\times 3-4\times 8+5^2-10\times 2)-(2^3\times 9-2^4)\}]$
-11×12.

(6) $23\times 11\times 3+7[206\times(8+6-13)-\{(14-8)\times 7-(15+5-11)\times 2^3$
$+(6^3-13\times 2^3)+(6\times 8\times 15\div 5)\}]+(2\times 5+3^2-3\times 4\times 7)$.

(7) $84-7[-11-4\{-17+3(8-\overline{9-5})\}]$.

(8) $5\times\{4-2[4-2(4+3)]\}-4\times\{4-2[4-2(4+3)]\}$.

(9) $19+12\times 15-120-4+\{29-13\times 2+(14-9)\times 3\}$.

(10) $9\times[125\div 5(7-2)\times 8(9-7)+4\{7+2(3+8)\}]$.

VI. MISCELLANEOUS PROPOSITIONS.

(IN THE FUNDAMENTAL OPERATIONS.)

109. Sum, difference, &c.

(1) Given the difference between two numbers and the greater, to find the smaller number.

RULE. *Subtract the given difference from the greater number, and the result is the required smaller number.*

Ex. If 34060 be the difference between two numbers, and the greater number is 48752, what is the less number?

The less number $= 48752 - 34060 = \underline{14692}$.

(2) Given the difference between two numbers and the smaller, to find the larger number.

RULE. *Add together the given difference and the smaller number, and the sum is the required larger number.*

Ex. The difference between two numbers is 14610 and the less is 4007; what is the larger number?

The larger number $= 14610 + 4007 = \underline{18617}$.

(3) Being given the sum and difference of two numbers, to find the numbers.

RULE. *To find the larger number, add together the given sum and difference, and divide the result by 2. To find the smaller number, subtract the given difference from the given sum and divide the result by 2.*

Ex. 1. The sum of two numbers is 25264, and their difference is 736; what are the numbers?

The larger number $= (25264 + 736) \div 2 = 26000 \div 2 = \underline{13000}$.

The smaller number $= (25264 - 13000) = \underline{12264}$,

or, the smaller number $= (25264 - 736) \div 2 = 24528 \div 2 = \underline{12264}$.

Ex. 2. The price of a carriage with horse is 1590 rupees, and the price of the carriage is 324 rupees more than that of the horse. Find the price of each.

Here, the sum of the two prices is 1590 rupees and the difference 324 rupees.

$\therefore$ the price of the carriage $= (1590 + 324) \div 2 = \underline{957}$ rupees.

And the price of the horse $= (1590 - 957)$ or $\underline{633}$ rupees.

(4) Being given the sums of every two of three given numbers, to find the numbers.

RULE. *Add together the three given sums, divide the result by 2, and from the quotient subtract separately the three given sums. The several differences are the required numbers.*

Ex 1. The sum of the first and second of three numbers is 59; that of the first and third is 53; and that of the second and third is 42. Find the numbers.

$(59+53+42)\div 2=77,$
$\therefore$ the first number $=77-42=35,$
the second number $=77-53=24,$
and the third number $=77-59=18.$ } *Ans.*

Ex. 2. At a game of cricket *A* and *B* together score 75 runs; *B* and *C* together score 90 runs; and *A* and *C* together score 51 runs; find the number of runs scored by each of them.

Here, *A*, *B* and *C* together score $(75+90+51)\div 2$ or 108 runs.

$\therefore$ *A* scored $(108-90)$ runs $=18$ runs,
B scored $(108-51)$ runs $=57$ runs,
and *C* scored $(108-75)$ runs $=33$ runs. } *Ans.*

(5) Having given the sum of three numbers, the excess of the first over the second, and the excess of the second over the third, it is required to find the numbers.

RULE. *Subtract the sum of the excess of the second over the third and of the first over the third (which may be obtained by adding the two given excesses from the given sum, and divide the result by 3. The quotient is the least of the three required numbers.*

Ex. Divide 53 rupees among *A*, *B* and *C*, so that *A* may receive 7 rupees more than *B*, and *B* 8 rupees more than *C*.

Here, the sum of the three shares is 53 rupees, and the excess of *A*'s share over *C*'s is 8+7 or 15 rupees,

and $53-(8+15)=53-23=30.$
$\therefore$ *C*'s share $=(30\div 3)$ rupees $=10$ rupees,
B's share $=(10+8)$ rupees $=18$ rupees,
and *A*'s share $=(18+7)$ rupees $=25$ rupees. } *Ans.*

110. Product, Quotient, Remainder, &c.

(1) Given the product of two numbers and one of them, to find the other.

RULE. *Divide the product by the given number, and the quotient thus obtained is the other required number.*

Ex. The product of two numbers is 890368, and the smaller number is 256; what is the larger number?

The larger number $=890368\div 256=\underline{3478}$.

(2) Given the divisor, the quotient and the remainder, to find the dividend.

RULE. *Multiply together the divisor and the quotient, and to the product add the remainder. The result is the dividend.*

Ex. If the divisor be 3857, the quotient 489, and the remainder 1305, what is the dividend?

The dividend $= 3857 \times 489 + 1305 = \underline{1887378}$.

(3) Given the dividend and the quotient, to find the divisor.

RULE. *Divide the dividend by the quotient, and the result is the divisor.*

Ex. The dividend is 342604 and the quotient 883, find the divisor.

The divisor $= 342604 \div 883 = \underline{388}$.

(4) Given the dividend, the quotient, and the remainder, to find the divisor.

RULE. *From the dividend subtract the remainder, and divide the difference by the quotient. The result is the divisor.*

Ex. 1. The dividend is 119376, the quotient 25 and the remainder 2076; what is the divisor?

The divisor $= (119376 - 2076) \div 25 = 117300 \div 25 = \underline{4692}$.

Ex. 2. A farmer having 2316 sheep, on putting an equal number of them into each of 25 fields, had 16 remaining. How many did he put into each of the fields?

The required number $= (2316 - 16) \div 25 = 2300 \div 25 = \underline{92}$.

(5) To find the *least number* which must be added to a given number to make it exactly divisible by a second given number.

RULE. *Divide the first given number by the second, and subtract the remainder from the second given number. The difference is the required number.*

Ex. What least number must be added to 4856752 to make it exactly divisible by 2163?

$4856752 \div 2163$ gives 2245 as quotient and 817 as remainder.

$\therefore$ the number to be added $= 2163 - 817 = \underline{1346}$.

(6) To find the *least number* which must be subtracted from a given number to make it exactly divisible by a second given number.

RULE. *Divide the first given number by the second, and the remainder is the required number.*

Ex. What least number must be subtracted from 90625 that it may be divisible by 727?

$90625 \div 727$ gives 124 as quotient and 477 as remainder.

$\therefore$ the number to be subtracted $= \underline{477}$.

(7) To find the *greatest* number of a given number of digits which is divisible by a given number.

Proceed as in the following example:—

Ex. Find the greatest number of five digits which is divisible by 529.

The greatest number of 5 digits is evidently 99999.

99999 divided by 529 gives 189 as the quotient and 18 as the remainder.

∴ the reqd. greatest number = 99999 − 18 = 99981.

(8) To find the *least* number of a given number of digits which is divisible by a given number.

Proceed as in the following example :—

Ex. Find the least number of six digits which is divisible by 4325.

The least number of 6 digits is evidently 100000.

100000 divided by 4325 gives 23 as the quotient and 525 as the remainder, and 4325 − 525 = 3800.

∴ the reqd. least number = 100000 + 3800 = 103800.

111. Equidifferent Series.

The numbers 1, 2, 3, 4, 5, etc., are called *natural* numbers, of which 1, 3, 5, etc., are *odd*, and 2, 4, 6, etc., are *even* numbers.

(1) To find the sum of any number of the *natural* numbers beginning with 1.

RULE. *Multiply the last number by the next higher number; and divide the result by 2. The quotient is the required sum.*

Ex. Add together 1 + 2 + 3 + 4 + 5 + ... + 40.

Here, the last number is 40, and the next higher number is 41.

∴ the required sum = 40 × 41 ÷ 2 = 820.

(2) To find the sum of any number of *odd* numbers beginning with 1.

RULE. *The square of the number of times the numbers ar repeated, is the required sum.*

Ex. Add together 1 + 3 + 5 + 7 + 9 + ... + 25.

Here, the number of times the numbers are repeated is 13.

∴ the sum required = 13^2 = 169.

(3) To find the sum of any number of *even* numbers beginning with 2.

RULE. *Multiply the number of times the numbers are repeated by the same increased by 1. The product is the required sum.*

Ex. Add together 2 + 4 + 6 + 8 + ... + 30.

Here, the number of times the numbers are repeated is 15.

∴ the sum required = 15 × 16 = 240.

(4) To find the sum of any given numbers increasing or decreasing by a fixed number.

RULE. *Multiply the sum of the two extreme numbers by the number of terms (or times repeated), and divide the result by 2. The quotient is the required sum.*

Ex. Add together $2+5+8+11+\ldots\ldots\ldots\ldots+47$.
Here, the number of terms will be found to be 16.
$\therefore$ the sum $=16\times(2+47)\div2=16\times49\div2=392$.

Examples XIX.

1. What number subtracted from 850967 will leave 3876?

2. The difference between two numbers is 84489 and the larger is 123456, what is the smaller?

3. The smaller of two numbers is $3087+56299$ and their difference is 22371; what is the larger number?

4. The greater of two numbers is the sum of 505, 650, 19 and 9003 and the difference between them is $3287-750$. What is the less number?

5. The sum of two numbers is 12640 and their difference 1608; what are the numbers?

6. The sum of the ages of two men is 173 years and the difference between them is 15 years; what are their ages?

7. The sum and difference of two numbers are 1426 and 384 respectively; find the numbers.

8. A man bought a pair of horses and a carriage for 857 rupees; the carriage was worth 165 rupees more than the horses; what was the price of each?

9 Two men having met on a journey, found that they had travelled 1200 miles, and that one had travelled 360 miles more than the other; what distance had each travelled?

10. Divide 168 marbles between two boys, giving to one 42 more than the other.

11. Ram, Gopal and Hari began to play at marbles. Ram and Gopal have 77 marbles between them, Gopal and Hari 63, and Ram and Hari 70. How many marbles has each?

12. A basket containing oranges, apples and plums, has 15 more oranges than apples, and 8 more apples than plums. The whole number of fruits in the basket is 112. Find the number of each kind in the basket.

13. Three persons *A*, *B* and *C*, are possessed of certain sums of money, such that *A* and *B* together have 120 rupees; *A* and *C* together have 140 rupees; and *B* and *C* together have 150 rupees. What is the sum possessed by each?

14. Divide 4680 rupees, after giving away 180 rupees to the poor, between *A*, *B* and *C*, giving *B* 216 rupees more than *A*, and *C* 336 rupees more than *B*.

15. The product of two numbers is 17037006 and one of them is 4858, what is the other?

16. If the divisor be 3857, the quotient 489 and the remainder 1305, what is the dividend?

17. A dividend is 16322853, the quotient is 1754 and the remainder is 129; what is the divisor?

18. The quotient arising from the division of 183926157 by a certain number is 4938 and the remainder is 5409. Find the divisor.

19. What least number must be added to 34568135 that the sum may be exactly divisible by 357?

20. What least number must be subtracted from 56854327 that the difference may be exactly divisible by 7323?

21. By what number must 109109109 be divided so that the quotient may be 51784, and 221 over?

22. What number multiplied by 1617 will give 50696184?

23. What least number must we subtract from 57385 so that it can be exactly divided by 387? And what least number must we add?

24. The sum of the product of two numbers and 355 is 87403; one of the numbers is 216, find the other number.

25. What least number must be added to 30984051, that the sum may be exactly divisible by 388?

26. Add together :—

(1) $1+2+3+4+\ldots\ldots\ldots+60$.
(2) $1+2+3+4+\ldots\ldots+100$.
(3) $2+5+8+11+\ldots\ldots\ldots+29$.
(4) $1+3+5+7+\ldots\ldots+31$.
(5) $2+4+6+8+\ldots\ \ldots+30$.
(6) $2+6+10+14\ldots\ldots+78$.
(7) $5+8+11+14+\ldots\ldots+53$.
(8) $100+97+94+\ldots\ldots+43$.

27. A debt can be discharged in 52 weeks by paying one rupee the first week, 3 rupees the second week, 5 rupees the third week and so on. Required the amount of the debt.

28. A person goes 3 miles on the first day, 5 miles on the second, 7 miles on the third, and so on. How far has he travelled in a month of 30 days?

29. How many times will a clock strike in a day of 24 hours?

30. Write down 576987, and under it write the eighth succeeding number, and under this latter the next eighth succeeding number and so proceed till nine numbers have been written down; find their sum.

31. Find the greatest and least numbers of 5 digits which are divisible by 327.

32. Find the least number of 6 digits which is divisible by 273.

33. Find the product of the two greatest numbers of 5 digits.

34. Divide the greatest number of 7 digits by the least number of 4 digits.

35. Find the sum of the greatest and the least number that can be formed by the digits 3, 2, 0, 1, 5, 8 and 9 taken all together.

112. Addition, Subtraction, &c.

(1) To subtract a number from another consisting of 1 followed by ciphers only.

RULE. *Put down as many nines as there are ciphers in excess of the number of figures in the subtrahend ; then (beginning from the left) write down in order the differences of each of the figures from 9 except the units' figure, which subtract from 10.*

Ex. Subtract 5736428 from 10000000000.

Here are 10 ciphers in the minuend, and 7 figures in the subtrahend ; hence put down **999**. Again 5 from 9 is **4**, 7 from 9 is **2**, 3 from 9 is **6**, 6 from 9 is **3**, 4 from 9 is **5**, 2 from 9 is **7**, and 8 from 10 is **2**. Therefore the required difference is 9994263572.

(2) To subtract *mentally* the sum of several numbers from a given number.

Proceed as in the following example :—

Ex. Subtract the sum of 1286, 495, 4758, 984 from 15812.

15812	
1286	*Mentally* thus : 4, 12, 17, 23 and **9**=32 ;
495	*carry* 3, 11, 16, 25, 33 and **8**=41 ;
4758	*carry* 4, 13, 20, 24, 26 and **2**=28 ;
984	*carry* 2, 6, 7 and **8**=15.
8289 *Ans.*	

(3) To subtract *mentally* from a number the product of two other numbers one of which is less than 20.

Proceed as in the following example :—

Ex. Subtract 8 × 549 from 6567.

6567	*Mentally* thus : 8 × 9 = 72, and **5** = 77 ;
549	*carry* 7, add 8 × 4, 39, and **7** = 46 ;
8	*carry* 4, add 8 × 5, 44, and **1** = 45 ;
2175 *Ans.*	*carry* 4, 4, and **2** = 6.

113. Multiplication by factors.

To multiply one number by another which can be resolved into factors each less than 20.

RULE. *Multiply the given number by each of the factors in succession, and the final product is the required one.*

Ex. 1. Multiply 31729 by 648.

648 = 9 × 9 × 8,

31729	285561	2570049
9	9	8
285561	2570049	20560392 *Ans.*

Ex. 2. Multiply 43896 by 357, and by 735 ; making in each case only two partial multiplications.

(1)		(2)	
	43896		43896
	357		735
	307272		307272
35=7×5	1536360	35=7×5	1536360
	15670872 *Ans.*		32263560 *Ans.*

Ex. 3. Multiply 567224 by 48872; and 48872 by 567224; making in each case only three partial multiplications.

(1)		(2)	
	567224		48872
	48872		567224
	4537792		342104
48=8×6	27226752	56= 7×8	2736832
72=8×9	40840128	224=56×4	10947328
	27721371328 *Ans.*		27721371328 *Ans.*

Examples XX.

1. Subtract 57364 from 1000000; 542056 from 1000000000 7859064 from 1000000000; and 79854 from 10000000.

2. Subtract

(1) 3671+45+467+2073 from 10608.
(2) 469+10876+2468+13972 from 38709.
(3) 1234567+1234+123+12345 from 4567208.
(4) 3843+396+428+1543+2897 from 12964.

3. Subtract *mentally* :—

(1) 4×2016 from 8124; 6×1632 from 9798; 8×4506 from 46325.
(2) 9×18764 from 198765; 7×53197 from 3690756.
(3) 15×14567 from 3567824; 18×51987 from 37373784.

4. Add 4×123 to 878; 9×2345 to 4675; 8×1071 to 8795

5. Multiply by factors :—

(1) 98989 by 44; 98909 by 72; 89088 by 96; 79797 by 63.
(2) 9785643 by 128; 6301246 by 256; 8725364 by 432.
(3) 9457283 by 792; 8465729 by 512; 5374896 by 588.
(4) 13245 by 1188; 246785 by 1872; 989045 by 15015.

6. Multiply in *two* lines :—

(1) 4016 by 637; 3543 by 648; 47862 by 1629; 31127 by 14412.
(2) 324567 by 486, by 936, and by 13212; 617635 by 1089.

7. Multiply in *three* lines :—

(1) 765389 by 64164, by 189279, and by 83256.
(2) 92135 by 10813212; 459896 by 864729; 1234567 by 4321089.
(3) 7893261 by 5678109; 5710987 by 105613212.

8. Multiply 876043 by 1449117 and by 28917136 in *three* lines.

114. Abbreviated methods of Multiplication.

(1) To multiply a number by 5.

RULE. *Annex one cipher to the right of the multiplicand, and divide the result by 2. The quotient is the required product.*

Ex. 1. Multiply 879324 by 5.

2)8793240

4396620=the required product.

Ex. 2. Multiply 6508 by 15.

2)65080=product by 10........(1)

32540=product by 5....... (2)

97620=the required product, adding (1) and (2).

(2) To multiply a number by 25.

RULE. *Annex two ciphers to the right of the multiplicand, and divide the result by 4. The quotient is the required product.*

Ex. 1. Multiply 57943 by 25.

4)5794300

1448575=the required product.

Ex. 2. Multiply 7575 by 35.

4)757500

189375=product by 25...... (1)

75750=product by 10........(2)

265125=the required product, adding (1) and (2).

Ex. 3. Multiply 6213 by 75.

4)621300=product by 100......(1)

155325=product by 25 .....(2)

465975=the reqd. prod., subtracting (2) from (1).

(3) To multiply a number by 125

RULE. *Annex three ciphers to the right of the multiplicand, and divide the result by 8. The quotient is the required product.*

Ex. Multiply 860978 by 125.

8)860978000

107622250=the required product.

(4) To multiply a number by a number all the figures of which are *nines.*

RULE. *Annex as many ciphers to the right of the multiplicand as there are nines in the multiplier, and from the result subtract the number itself. The difference is the required product.*

Ex. Multiply 6875 by 999.

6875000=product by 1000......(1)

6875=product by 1 .....(2)

6868125=the reqd. prod., subtracting (2) from (1).

(5) To multiply a number by a number which differs by a small number from 100, 1000, 10000, &c., or from 50, 500, 5000, &c.

Proceed as in the following examples :—

Ex. 1. Multiply 423571 by 98 and by 9997.

(1) 98 = 100 − 2.
423571 × 100 = 42357100
423571 × 2 = 847142
∴ the product = 41509958.

(2) 9997 = 10000 − 3.
423571 × 10000 = 4235710000
423571 × 3 = 1270713
∴ the product = 4234439287.

Ex. 2. Multiply 6854 by 496.

Here, 496 = 500 − 4.
6854 × 5,00 = 68540,00 ÷ 2 = 3427000
6854 × 4 = 27416
∴ the required product = 3399584.

(6) To multiply a number by 11.

RULE. *Add each figure to the figure on its left, beginning with 0 on the right, carrying 1 when necessary. The number thus formed is the required product.*

Ex. Multiply 75384 by 11.

75384
11
829224

Here, 0+4=4 ; 4+8=12, carry 1 ; 1+8+3=12, carry 1 ; 1+3+5=9 ; 5+7=12, carry 1 ; 1+7=8 ; but all the necessary wordings are 4, 12, 12, 9, 12, 8.

(7) To multiply a number by 625.

RULE. *Annex four ciphers to the right of the multiplicand and divide the result by 16. The quotient is the required product.*

Ex. Multiply 4837 by 625.

16)48370000
3023125 = the required product.

115. Squares, Cubes, &c.

(1) To find the square of a number of two figures.

RULE. *Increase and diminish the number by the complement of its units' figure and to the product of the two results thus obtained add the square of the complement. The number thus formed is the required square.*

Ex. 1. Find the square of 84 and 95.

Here, the complement of 4 is 6 and of 5 is 5.

(1) 84 + 6 = 90 and 84 − 6 = 78.
∴ the reqd. square = 90 × 78 + 6^2
= 7020 + 36
= 7056.

95 + 5 = 100 and 95 − 5 = 90.
∴ the reqd. square = 100 × 90 + 5^2
= 9000 + 25
= 9025.

Ex. 2. Find the square of 467.

$467+67=534$; $467-67=400$.
$\therefore 467^2=534\times 400+67^2$
$=213600+67^2$.

Again, $67+7=74$; $67-7=60$
$\therefore 67^2=74\times 60+7^2$
$=4440+49=4489$.

Hence, $467^2=213600+4489=218089$.

(2) To find the difference of the squares of two numbers.

RULE. *Multiply the sum of the numbers by their difference, and the product is the required difference.*

Ex. Find the value of $(339)^2-(319)^2$.

Here, $339+319=658$ and $339-319=20$.
$\therefore$ the required difference $=658\times 20=\underline{13160}$.

(3) To express the product of two numbers as the difference of two squares.

RULE. *Find the sum and difference of the numbers and divide each result by 2. The difference of the squares of the two quotients is the required difference of two squares.*

Ex. Express 81×53 as the difference of two squares.

Here $(81+53)\div 2=134\div 2=67$ and $(81-53)\div 2=28\div 2=14$,
$\therefore$ the required difference $=(67)^2-(14)^2$.

Examples XXI.

1. Multiply :—

(1) 879326 separately by 5, 25, 75, 125 and 625.
(2) 63945 separately by 15, 35, 75 and 125.
(3) 87911365 separately by 5, 25, 75, 125 and 625.
(4) 4439854 separately by 99, 999, 9999 and 99999.
(5) 5792 separately by 96, 996, 9994 and 9998.
(6) 8734652 separately by 11, 121, 1331 and 99994.

2. Find the squares of :—

(1) 37, 45, 48, 55, 65, 75, 64, 71, 83, 96 and 125.
(2) 108, 149, 156, 183, 215, 391, 478, 456 and 524.

3. Express the following products as the difference of two squares :—65×53 ; 96×74 ; 126×84 ; 245×197 ; 478×316.

4. Find the values of :—

(1) $(575)^2-(425)^2$; $(101)^2-(99)^2$; $(1639)^2-(739)^2$; $(1811)^2-(689)^2$.
(2) $(753)^2-(625)^2$; $(1723)^2-(277)^2$; $(2731)^2-(269)^2$; $(678)^2-(638)^2$

5. Divide :—

(1) $(8133)^2-(8131)^2$ by 16264 ; $(5874)^2-(3795)^2$ by 2079.
(2) $(2259)^2-(1759)^2$ by 4018 ; $(3156)^2-(968)^2$ by 2188.

6. Find the greatest number of 8 digits which is divisible by 5293.

7. Find the least number of 7 digits which is divisible by 7293.

8. Find the least number of 9 digits and the greatest number of 8 digits which are divisible by 37213.

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116. Division by factors.

When the divisor is the product of two or more factors, we use the following Rule :—

RULE. *The quotient is obtained by dividing in succession by each of the factors of the divisor, and the final remainder at each step is obtained by multiplying its particular remainder by all the divisors preceding its own, and adding the preceding final remainder.*

Ex. 1. Divide 25872 by 56.

$56 = 7 \times 8$.

```
7)25872
8)3696
   462
```

Dividing in succession by 7 and 8, the quotient is 462.

Ex. 2. Divide 96500093 by 105.

$105 = 3 \times 5 \times 7$.

```
3)96500093
5)32166697...2
7) 6433339...2...8
    919048...3...53.
```

Dividing in succession by 3, 5 and 7, the particular remainders are 2, 2 and 3. The final remainder at the first step is 2 ; the final remainder at the second step is found thus : $2 \times 3 + 2 = 6 + 2 = 8$. The final remainder at the third step is found thus : $3 \times 5 \times 3 + 8 = 45 + 8 = 53$.

Thus the quotient is 919048 and the remainder 53.

117. Abbreviated methods of Division.

(1) To divide a number by 5, 15, 35, 45, 55 or 65.

RULE. *Multiply the number by 2 and divide the product respectively by 10, 30, 70, 90, 110, or 130, as in Art. 99. The result in each case gives the quotient and for the true remainder divide the remainder so obtained by 2.*

Ex. Divide 86246 by 5 and 15623 by 45.

(1)

```
   86246 × 2
1,0)17249,2
    17249...2
```

Thus the quotient is 17249 and the remainder $2 \div 2 = 1$.

(2)

```
   15623 × 2
9,0)3124,6
    347...16
```

Thus the quotient is 347, and the remainder $16 \div 2 = 8$.

(2) To divide a number by 25, 75, 175, 225, 275 or 325.

RULE. *Multiply the number by 4, and divide the result by 100, 300, 700, 900, 1100, or 1300 as in Art. 99. The result in each*

case gives the quotient and for the true remainder divide the remainder so obtained by 4.

Ex. Divide 37057 by 25, and 905785 by 175.

(1) 37057 × 4
1,00)1482,28
1482...28

Thus the quotient is 1482, and the true remainder 28 ÷ 4 = 7.

(2) 905785 × 4
7,00)36231, 40
5175...640

Thus the quotient is 5175, and the remainder 640 ÷ 4 = 160.

(3) To divide a number by 125, 375 or 875.

RULE. *Multiply the number by 8, and divide the product respectively by 1000, 3000, or 7000, as in Art. 99. The result in each case gives the quotient and for the true remainder divide the remainder so obtained by 8.*

Ex. Divide 905785 by 125, and 1607708 by 375.

(1) 905785 × 8
1,000)7246,280
7246...280

Thus the quotient is 7246 and the remainder 280 ÷ 8 = 35.

(2) 1607708 × 8
3,000)12861,664
4287...664

Thus the quotient is 4287, and the remainder 664 ÷ 8 = 83.

(4) To divide a number by 625.

RULE. *Multiply the number by 16, and divide the result by 10000, as in Art. 99. The quotient is the required quotient and for the true remainder divide the remainder so obtained by 16.*

Ex. Divide 3023173 by 625.

3023173 × 16
1,0000)4837,0768
4837...768

Thus the quotient is 4837, and the remainder 768 ÷ 16 = 48.

(5) The method of Long Division may be much shortened by the use of Art. 112 (3).

Ex. Divide 15218125 by 3854.

3854)15218125(3948 *Ans.*
36561
18752
33365
2533 rem.

Mentally thus : 3 × 4 = 12 and 6 = 18 ;
carry 1, add 3 × 5, 16 and 5 = 21 ;
carry 2, add 3 × 8, 26 and 6 = 32 ;
carry 3, add 3 × 3, 12 and 3 = 15.

Then bringing down the next digit in the dividend, repeat the process for the next digit in the quotient.

Examples XXII.

1. Divide by factors :—

(1) 1461408 by 32 ; 347808 by 56 ; 1556334 by 162.
(2) 7825687 by 64 ; 6598769 by 84 ; 8791605 by 88.
(3) 7654325 by 96 ; 12345678 by 68 ; 36925814 by 82.
(4) 76538959 separately by 28, 64, 72, 96 ; 39541234 by 256.
(5) 87625432 by 726 ; 179276I8 by 476 ; 5213742 by 1142.
(6) 3790603808 separately by 132, 196, 378 ; 3246541 by 792.

2. Divide :—

(1) 37964 separately by 5, 50, 500, 5000 and 25.
(2) 8754316 separately by 5, 15, 25, 35, 45, 55, 65 and 75
(3) 90273189 separately by 125, 175, 225 and 275.
(4) 154725876 separately by 125, 375, 625 and 875.
(5) 68015637 by 8654 ; 57300652 by 5129 and 36942536 by 4204.

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118. Average, Shares, Barter, &c.

(1) To find the average of two or more numbers.

RULE. *Divide the sum of the numbers by their number and the quotient is the required average.*

Ex. 1. The attendance at a school was 254 on Monday, 326 on Tuesday, 204 on Wednesday and 192 on Thursday. Find the average daily attendance of the 4 days.

On Monday the attendance was	254.	976÷4=244.
Tuesday	326.	
Wednesday	204.	∴ the average was 244.
Thursday	192.	
∴ in 4 days	976.	

Ex. 2. The yearly expenses of a person during the first 4 years was *Rs.*675, during the next 5 years *Rs.*825, and during the following 7 years *Rs.*977. What was his average expenses ?

In 4 years the exp. amt. to	(*Rs.*675 × 4) or *Rs.*2700,	16)*Rs.*13664.
5	(*Rs.*825 × 5) or *Rs.*4125,	*Rs.*854.
7	(*Rs.*977 × 7) or *Rs.*6839,	∴ the average
∴ in 16 years	*Rs* 13664	was *Rs.*854.

(2) To divide a given number into parts, having certain given relations among them.

Proceed as in the three following Examples.

Ex. 1. Divide 184 oranges between Ram and Gopal; giving Ram 7 times as many as Gopal.

If Gopal gets 1 orange, Ram gets 7 oranges ; and $1+7=8$.

$\therefore$ Gopal's share $=(184 \div 8)$ or 23 oranges,
and Ram's share $=(23 \times 7)$ or 161 oranges.

Ex. 2. Divide 384 rupees among *A*, *B*, *C* and *D*, so that for every 5 rupees given to *A*, *B* gets 7 rupees, *C* 8 rupees, and *D* 12 rupees.

$5+7+8+12=32$. *Rs.*$384 \div 32=$ *Rs.*12.

$\therefore$ *A* gets *Rs.*$12 \times 5=$ R60,
B ... *Rs.*$12 \times 7=$ R84,
C gets *Rs.*$12 \times 8=$ R 96,
and *D* ... *Rs.*$12 \times 12=$ R144.

Ex. 3. Divide 1351 nuts among 13 men, 17 women, and 30 children, giving each woman 5 times the share of each child, and each man the share of a woman and a child.

If each child gets 1, a woman gets 5 and a man $5+1$ or 6. Therefore 30 children get 30, 17 women get 5×17 or 85, and 13 men 6×13 or 78.

Now, $30+85+78=193$; and $1351 \div 193=7$.

$\therefore$ the children will have 7×30 or 210 nuts,
the women ... 7×85 or 595 nuts,
and the men ... 7×78 or 546 nuts.

Ex. 4 How many horses worth 132 rupees each, must be given for 1476 sheep worth 11 rupees each ?

The cost of 1476 sheep at *Rs.*11 each $=$ *Rs.*$1476 \times 11=$ *Rs.*16236.

And $16236 \div 132=123$. $\therefore$ the required no. of horses $=$ 123.

Ex. 5. If a man can travel 2440 miles in 4 weeks, how many miles can he travel in 9 weeks ?

In 4 weeks the man travels 2440 miles.
$\therefore$ in 1 week ... ... $2440 \div 4$ or 610 miles.
$\therefore$ in 9 weeks ... ... 610×9 or 5490 miles.

119. Backward Process.

In a backward process, beginning from the last number, we change Addition into Subtraction, Subtraction into Addition, Multiplication into Division and Division into Multiplication.

Ex. What number is that which if I divide by 6, to the quotient add 25, from the sum take 36 and multiply the remainder by 4, the product is 40 ?

The required number $=(40 \div 4+36-25) \times 6=(46-25) \times 6$
$=21 \times 6=126$.

Examples XXIII.

1. What is the average age of 4 men whose ages are 47, 55, 29 and 77 respectively ?

2. In a school register of daily attendance the numbers for a certain week were—Monday 83, Tuesday 80, Wednesday 75, Thursday 80, Friday 78, Saturday 72. What was the average daily attendance ?

3. At a competitive examination there were 4 candidates at the age of 19, 3 at 20, 2 at 22, and 3 at 24. Find the average age.

4. A man's income for 3 years is *Rs.*250 a year, for the next 5 years it is *Rs.*294, and for the next 4 years *Rs.*309. What is his average income for the 12 years ?

5. In the month of April, a man slept 7 hours on each of 16 nights, 6 hours on each of 8 nights, 8 hours on each of 5 nights and 10 hours on the last night. How long did he sleep each night on an average during the month ?

6. Divide 1008 rupees among *A*, *B* and *C*, so that for every 2 rupees *A* gets, *B* shall get 3 rupees and *C* 4.

7. Divide 2624 apples among *A*, *B* and *C*, so that for every 5 apples given to *A*, *B* may get 11, and *C* 16.

8. The price of a carriage with horse is 1590 rupees, and the price of the carriage is 5 times that of the horse. Find the price of the horse.

9. If 23 men earn 1380 rupees in a month, how many men will earn 1980 rupees in the same time ?

10. A gentleman left 225,000 rupees to be divided amongst his 4 sons and 3 daughters in such a way that each son would receive three times as much as each daughter. How much did each son and each daughter receive ?

11. Divide 33775 rupees among 13 men, 17 women and 30 children, giving each woman 5 times the share of each child, and each man the share of a woman and a child.

12. 24 cows are worth 864 rupees, and 45 horses are worth 28350 rupees ; how many of such horses ought to be exchanged for 2520 such cows ?

13. Divide 2954 rupees among *A*, *B*, *C* and *D*, so that for every 2 rupees given to *A*, *B* shall get 3 rupees, *C* 4 and *D* 5.

14. A farmer had a horse worth 375 rupees and exchanged it for a yoke of oxen and three cows ; the oxen he sold for 125 rupees, two of the cows at 85 rupees each and the other for 76 rupees. How much did he lose by the bargain ?

15. Find a number such that if I divide it by 3, and then add 4, then divide the result by 2 and add 3, then multiply the result by 4 and subtract 5, the result of the whole will be 19.

Examples worked out.

Ex. 1. I have to divide 750 rupees among a number of boys and girls, giving 3 rupees to each boy and 2 rupees to each girl; there are as many boys as girls; how many boys are there?

Here, 1 boy + 1 girl receive (3+2) or 5 rupees.

∴ 150 × (1 boy + 1 girl) receive 5 × 150 or 750 rupees, for 750 ÷ 5 = 150. ∴ the number of boys = 150. *Ans.*

Ex. 2. A man living at the rate of 750 rupees a year for 6 years finds that he is exceeding his income, and reduces his expenditure to 540 rupees a year; at the end of 4 years he finds that he is just out of debt; what is his income?

In 6 years his expenses amount to *Rs* 750 × 6 = *Rs*.4500.
In 4 years ... ... *Rs*.540 × 4 = *Rs*.2160.
∴ in 10 years his income amounts to *Rs*.6660,

∵ his debts of the first 6 years are paid off by the savings of the last 4 years,

∴ his yearly income = *Rs*.6660 ÷ 10 = *Rs*.666. *Ans.*

Ex. 3. Two persons started at the same time from *A* and *B*. One left *A* for *B* travelling 5 miles an hour, and the other from *B* for *A* travelling 7 miles an hour. The distance between *A* and *B* is 108 miles. When and where did they meet?

While the first walks 5 miles, the second walks 7 miles, and the distance to be travelled by both before they meet is 108 miles.

Now, 5 + 7 = 12; and 108 ÷ 12 = 9. ∴ they meet after 9 hours.

Also, the distance from *A* where they meet = 5 × 9 or 45 miles.

Ex. 4. A man bought 75 cows at 50 rupees each, 94 cows at 43 rupees each and 106 cows at 48 rupees each; at what price per head must he sell the cows, so as to gain 595 rupees by his bargain?

The cost of 75 cows at *Rs*.50 each = *Rs*. 75 × 50 = *Rs*. 3750.
... ... 94 cows at *Rs*.43 ... = *Rs*. 94 × 43 = *Rs*. 4042.
... ... 106 cows at *Rs*.48 ... = *Rs*.106 × 48 = *Rs*. 5088.
∴ the cost of 275 cows = *Rs*.12880.
gain = *Rs*. 595.
∴ the selling price of 275 cows = *Rs*.13475.
∴ the selling price of a cow = *Rs*.13475 ÷ 275 = *Rs*.49.

Ex. 5. If 30 men can build a wall in 12 days, how many men can build it in 18 days?

In 12 days the work can be done by 30 men.

∴ in 2 days the work ... (30 × 6) or 180 men.
∴ in 18 days " ... (180 ÷ 9) or 20 men. *Ans.*

Ex. 6. Reduce 7 men, 12 women and 5 children to an equivalent number of children, supposing 2 women equivalent to a man, and 3 children equivalent to a woman.

1 woman = 3 children ; ∴ 12 women = 3 × 12 or 36 children.
Again, 1 man = 2 women = 2 × 3 or 6 children.
∴ 7 men = 7 × 6 or 42 children.
Hence, 7 men + 12 women + 5 children = (42 + 36 + 5) or 83 children.

Ex. 7. A tank has three pipes attached to it. By two of these 482 and 516 maunds of water respectively enter into it every hour, while by the third 322 maunds go out in the same time. When all the pipes are opened together the tank becomes full in 320 hours; how many maunds of water can the tank hold?

The quantity of water remaining in the tank per hour when all the pipes are opened together = (482 + 516 − 322) or 676 maunds.

∴ in 320 hours, the water remg. = 676 × 320 or 216320 maunds.

Hence the tank can hold 216320 maunds of water.

Ex. 8. A man at his death directed in his will that his property should be divided among his four sons as follows:—The eldest to receive *Rs.*1032 more than the second; the second *Rs.*1023 less than what the third and fourth together receive; the third and the fourth together to receive *Rs.*3251; but the third to receive *Rs.*31 less than the fourth. Find the value of his whole property and the share of each son.

Since the third and the fourth together receive *Rs.*3251, and the third *Rs.*31 less than the fourth, therefore the third's share = (3251 − 31) ÷ 2 or *Rs.*(3220 ÷ 2), *i. e.*, *Rs.*1610.

Therefore the fourth's share = *Rs.*(1610 + 31) or *Rs.*1641.
... second's ... = *Rs.*(3251 − 1023) or *Rs.*2228.
... eldest's ... = *Rs.*(2228 + 1032) or *Rs.*3260.

Hence, the whole estate = *Rs.*(1610 + 1641 + 2228 + 3260)
= *Rs.*8739. *Ans.*

Ex. 9. Prove that the sum of the six numbers that can be formed by different arrangements of the three digits 2, 5 and 7 taken all together can be represented by the expression $2 \times (2+5+7)(10^2+10+1)$.

The six numbers that can be formed by different arrangements of the digits 2, 5, 7 are the following:—

257, 275, 527, 572, 725, 752.

In the above numbers, we see that

(1) 2 occurs twice in the hundreds' place, twice in the tens' and twice in the units', and their sum $=2\times(200+20+2)=2\times2\times(100+10+1)$ $=2\times2\times(10^2+10+1)$.

(2) 5 occurs twice in the hundreds' place, twice in the tens' and twice in the units', and their sum $=2\times(500+50+5)=2\times5\times(10^2+10+1)$.

(3) 7 occurs twice in the hundreds' place, twice in the tens' and twice in the units', and their sum $=2\times(700+70+7)=2\times7\times(10^2+10+1)$.

Hence the sum of all the six numbers
$=2\times2\times(10^2+10+1)+2\times5\times(10^2+10+1)+2\times7\times(10^2+10+1)$
$=2\times(2+5+7)\times(10^2+10+1)$.

Miscellaneous Examples I.

1. What number must be added to 7965499 to give 541850036?

2. How much is the difference between 628716 and 79019 greater than the sum of 56095, 2800, 10009, 7097, 159, 3000 and 90829?

3. The sum of two numbers is 125678, and their difference is 1422; find the numbers.

4. The sum of two numbers is 15678, and the larger number exceeds the smaller by 1234; find the numbers.

5. What number multiplied by 1256 will give the same product as (i) 314 by 476; (ii) 7536 by 378?

6. A man having bought an estate, sold it again for *Rs.*21128 losing thereby *Rs.* 1878; what did the estate cost him?

7. The population of a certain village is 1254 and one out of 33 dies annually. How many die in a year?

8. The product of two numbers is 2225808 and one of them is 936; what is the other number?

9. The difference of two numbers is 6782, and the greater is 178962; what is the smaller number?

10. What number is that which being divided by 4, the quotient increased by 6, the sum multiplied by 4, the product increased by 16, and the sum divided by 44, the quotient will be 10?

11. What number must be multiplied by 327 to produce 1203033?

12. How much greater is the product of 17 and 15 than the product of their sum and difference?

13. Of a town containing 434611 inhabitants 57569 more are females than males. Find the numbers of males and females.

14. Express 6789×1231 as the difference of two square numbers.

15. The divisor is 712, the quotient 31 and the remainder 699. What is the dividend?

16. What least number must be added to 58667, that the sum may be divisible by 2564?

17. Subtract the value of the second and fourth digits from that of the third and fifth digits in the number 123456.

18. The quotient arising from the division of 256329 by a certain number is 354, and the remainder is 387. Find the divisor.

19. A person who was born in 1779, died at the age of 46 years; his son died 27 years afterwards, and his daughter died 13 years after his son; in what year did the daughter die?

20. Of what number is 7036 both divisor and quotient?

21. What number is that, which being divided by 24, the quotient increased by 26, the sum diminished by the difference between 40 and 27, the remainder multiplied by 4, and the product divided by 11 will give 12 for a quotient?

22. The quotient being = 5 times divisor = 7 times remainder = 105; find the dividend.

23. The quotient being 958 and the divisor 607, find the dividend. What would the dividend be, had there been a remainder 44?

24. What least number must be subtracted from 2346, that the remainder may be divisible by 135? By what least number must the same be multiplied that the product may be divisible by 36?

25. A house and its furniture cost *Rs.*570600; the house is 8 times the furniture. What is the cost of the house?

26. Find the number, which if I multiply by 7, then subtract 31, then divide the result by 3, then add 5, and then multiply by 4, the result is the square of 10.

27. A merchant has three sorts of sugar; the first and second together weigh 12356 maunds; the third 7152 maunds less than the sum of the first and second; also the second weighs 1647 maunds less than the third. Find the quantity of each sort.

28. The product of two numbers is 1270374, and half of one of them is 3129; what is the other number?

29. There were 2244 pears on a tree. The owner gathered 46 daily for 14 days: he divided the remainder between his son and daughter, giving the former 5 for every 3 that he gave the latter; how many pears did the son receive more than the daughter?

30. The Duke of Wellington died in the year 1852, aged 83; Napoleon was born in the same year as the Duke, and died in 1821; what was Napoleon's age at the time of his death?

31. A speculator gained *Rs.*3560, and afterwards lost *Rs.*3479; he then gained *Rs.*6283, and then lost first *Rs.*1089, and then *Rs.*2361; by how much did his gains exceed his losses?

32. What least number must be subtracted from $72347 + 11 \times 7$ that the remainder may be divisible by $17 \times 9 + 3 \times 6$?

33. A merchant bought 122 maunds of oats at *Rs.*2 per maund,

and 256 maunds of an inferior sort at *Re.*1 per maund and mixing the two sorts sold the whole for *Rs.*525. How much did he gain or lose?

34. A man dies worth *Rs.*2427498 to be divided among his three sons. He directed in his will that the eldest and second together shall get *Rs.*1937734, and the second and third together *Rs.*1196570. How many does each receive?

35. How many words are there in a book of 347 pages, if there are 13 words in each line, and 40 lines in each page?

36. A water-tub has two pipes attached to it. The first discharges 14 seers and the second 15 seers of water per minute. When the tub is full, both the pipes are opened at once, and the tub becomes empty in 15 minutes. Find the content of the tub.

37. *A* is 27 years older than *B*, and 15 years younger than *C* who is 54 years of age; *D* is as old as the sum of *A*'s and *B*'s ages. Is *C* older or younger than *D*? How much?

38. *A* has 74 marbles, *B* has 34 more than *A*, and *C* has 16 more than *B*; *A* gives *B* and *C* each 19, *B* gives *A* and *C* each 34, and *C* gives *A* and *B* each 10. How many marbles have *A*, *B* and *C* respectively after these exchanges?

39. A person bought 68 bales of cloth containing 67048 yards; each bale contained 34 pieces, and each piece contained the same number of yards; find the number of yards in each piece.

40. The nuts in a bag were divided among 59 boys and 27 girls; each boy had 3 times as many as each girl; there were just nuts enough and one over to give the girls 7 nuts apiece. How many nuts did the bag contain?

41. A man's annual income is *Rs.*7836. His expenditure in January is *Rs.*632, in February and March *Rs.*1146, in April, May and June *Rs.*1698, and in each of the remaining 6 months *Rs.*595 on an average. How much does he save in the year?

42. A man divided his property worth *Rs.*12547 among his 4 sons in such a manner that the eldest received *Rs.*126 more than the second, the second *Rs.*131 more than the third, and the third *Rs.*121 more than the fourth. How much did each receive?

43. Three pipes are attached to a water-tub. By two of these 36 and 24 maunds of water respectively enter into it every hour, while by the third 33 maunds go out in the same time. If the tub can hold 2673 maunds of water, when will it be full, if all the pipes are opened together?

44. Express 19191×1225 as the difference of two square numbers.

45. If 256512 be divided by 105, using its factors 3. 5 and 7, find the true quotient and the true remainder.

46. A gentleman left *Rs.*123600 to be divided among his two

sons, four daughters and one sister, in such a way that each daughter would receive twice as much as the sister, and each son one-half of what the three daughters would receive. What did the sister receive ?

47. A man worth 30 lacs of rupees, having no heirs, divides his whole property among his four faithful servants *A*, *B*, *C* and *D*. He gives to *B* twice as much as he gives to *A* and *Rs.*1234 more ; to *C* twice as much as *A* less *Rs.*2284, and to *D* *Rs.*32000. Find his bequest to *A*.

48. *A* and *B* walk at the rates of 10 and 13 miles per hour respectively. If they are walking towards each other, and if the distance between them be 207 miles, find when they will meet.

49. *A* says to *B* and *C*, I have *Rs.*1650 ; *B* replies, if I had *Rs.*753 more than I have, I should have as much as you have ; *C* adds, if I had *Rs.*105 more than I have, I should have as much as both of you. How many more rupees has *C* than *B* ?

50. To what number must 28 be added that the sum being multiplied by 25, the product will be 125625 ?

51. From what number must 302 be subtracted that the remainder being multiplied by 125, the product will be 321000625 ?

52. Divide *Rs.*40 between *A* and *B* in such a way that if *A* gets *Rs.*5, *B* shall get *Rs.*3.

53. Divide *Rs.*30 among *A*, *B* and *C* in such a way that if *A* gets *Re.*1, *B* shall get *Rs.*2 and *C* *Rs.*3.

54. If the sum of 250 and 173 be multiplied by their difference, and the product be divided by 33, find the result.

55. Add together the six numbers you can form with the three figures 3, 4 and 5, taken all together, and multiply the sum by 597.

56. Add together all the numbers that you can form with the four digits 1, 2, 3 and 0 taken all together

57. Arrange the nine digits 1, 2, 3, 4, 5, 6, 7, 8, 9, in three lines with three digits in each line, so that the sum of these digits may, taken in every possible direction, be 15.

58. Find the sum of all the numbers that you can form with the digits 1, 5, 7, only two digits being taken at a time.

59. By what number must 123456 be divided that if 15328 be added to the quotient and the sum divided again by 8, the quotient will be 7060 ?

60. *A*, *B* and *C* have between them 1467 marbles. *B* has three times as many as *A*, and *C* 131 marbles more than the sum of *A* and *B*. How many has each ?

61. Divide *Rs.*5000 among *A*, *B*, *C* and *D* in such a manner, that if *A* gets *Rs.*2, *B* shall get *Rs.*3, *C* *Rs.*4 and *D* *Rs.*11.

62. Simplify—

(1) $920 \div 23 \times 720 + (42 \div 7) \times (78 + 13) \div (5 \times 4)$.

(2) $1250 \times (72 \div 4) \div (20 \times 5) \times (64 + 16) \div (111 \div 37)$.

63. What least number must be added to $3243 \div 3 \times 9$ that the sum may be divisible by $15 + 5 \times 8 \times 7$?

64. What number less than 365 added to 730320 will make the number exactly divisible by 365 ?

65. A man spends *Rs.*1485 annually for 6 years and runs into debt. He then reduces his expenses to *Rs.*1109 a year, and in 10 years just manages to clear off his debts. What is his yearly income?

66. Multiply 765389 by 64164, and by 189279, and by 83256, making in each case only three partial multiplications.

67. A volume of a work contains 6 parts of 128 pages each, and there are 46 lines in each page and 58 letters in each line. How many letters are there in 9 volumes ?

68. A man spends *Rs.*600 a year for 5 years and saves some money; he then raises his expenditure during the next 7 years to *Rs.*720 a year, and finds all his savings spent. What does he earn each year?

69. The sum of the product of two numbers and 1420 is 349612 ; one of the numbers is 864. Find the other number.

70. Find the number which being divided by 24 gives a quotient which if increased by 36 and the sum multiplied by 24 gives a product that will be greater than 876 by 300.

71. If in dividing a number by 336, the operation be performed by short division by employing the factors 6, 7 and 8 in succession and the several remainders be 1, 2 and 3 ; find the complete remainder.

72. If two men start from the same place and travel in opposite directions, the one at the rate of 42 miles and the other 45 miles a day, how far apart will they be at the end of 12 days ?

73. If two men start from the same place and travel in the same direction, the one at the rate of 512 miles and the other 540 miles a week, how far apart will they be at the end of 8 weeks ?

74. A dividend is 4637064283, the quotient is 80496 and the remainder is 11707, what is the divisor ?

75. If 20 men can do a piece of work in 11 days, how many days will it take 22 men to do it ?

76. *A*, *B*, *C* and *D* have among them *Rs.*69 ; *A*, *B* and *C* have among them *Rs.*48 ; *B* and *C Rs* 31, *B* having *Rs.*15 more than *C* ; how many more rupees have *A* and *B* than *C* and *D*?

77. The product of three numbers is 535500 ; one of the numbers is 75, another is 68. What is the third ?

78. The product of three numbers is 8937992 ; the third number is double the second, and the sum of the second and third is 906. Find the first number.

79. Divide *Rs.*3975 among A, B, C and D so that B may have *Rs.*23 more than A, C *Rs.*45 more than A and B together, and D *Rs.*29 less than B and C together.

80. A grazier bought a certain number of bullocks for *Rs.*4900, and sold a part of them for *Rs.*3840 at *Rs.*32 a head, and gained on those he sold *Rs.*480. How much did he gain a head, and how many did he buy at first?

CHAPTER III.

Compound Quantities.

120. If one quantity contains another of the same kind an exact number of times, the first is said to be a **multiple** of the second, and the second a **submultiple** or **aliquot part** of the first.

121. We have already seen that in considering quantities of the same kind, we take an arbitrary but well-defined quantity of that kind as our **unit**, and finding *how many* times it is contained in each of them, we express them as whole numbers. But in this way very large quantities will be expressed by very high numbers, which give by inspection little idea of their relative values; to obviate this inconvenience we take such multiples of the *unit* as will enable us to avoid very high numbers. Thus, of length, we take a **yard** as our unit, but to measure long lengths we use the **mile**, a high multiple of the yard. Hence has arisen the custom of using large units for large quantities and small units for small quantities. Thus, we say that the price of a chair is 8 **rupees**; that of a book is 14 **annas**, and that of a pencil is 2 **pice**.

122. Since it is the custom to use more than one unit for things of the same kind, it would be convenient to select one quantity as the principal or **standard** unit and thence derive the various minor or **auxiliary** units, either by dividing this unit into a number of equal parts or by multiplying it a number of times. The *standard* unit of any quantity and its *auxiliary* units are called its **denominations**.

123. In the preceding Chapter we have considered only such *abstract* numbers, or such *concrete* numbers of one denomination as are formed by figures whose local values are always regulated by the same fixed number *ten*; but the rules given can easily be extended to *concrete* numbers of different denominations, wherein the local values of the figures are connected by more numbers than one; as, for instance, to **rupees**, **annas** and **pies**, where twelve pies are equivalent to *one* anna, which is the next higher denomination; *sixteen* annas to one rupee, which is the next denomination in order; the *different* numbers 12 and 16 connecting the denominations, in the same manner, as the *fixed* number 10 was supposed to connect the denominations of Integers.

Here, the standard unit **rupee** is divided into 16 equal parts to obtain the auxiliary unit *anna*, and into 16 × 12 or 192 equal parts to obtain the auxiliary unit *pie*. Thus, the rupee, anna and pie are the various denominations of money.

124. The processes employed in cases of this nature are **Reduction**, and the fundamental operations are then called **Compound Addition, Compound Subtraction, Compound Multiplication** and **Compound Division**, each of which will be exemplified in order ; and the various **Tables**, which furnish us with a list of the relative magnitudes of the different *auxiliary* units, and by means of which the above operations are conducted, are given below in order.

TABLE I. MONEY.

British Indian Money.

125.

3 Pies (*p.*) or 2 half-pice	make	1 Pice (*ps.*)
2 Pice	"	1 Half-anna.
4 Pice or 12 pies	"	1 Anna (1*a.*)
16 Annas	"	1 **Rupee** (*Re.* 1 or 1/.)
15 Rupees	"	1 Sovereign.

126. Accounts in Bengali are kept by the following Table.

4 Cowries	make	1 Ganda	4 Pans	make	1 Chouk
5 Gandas	"	1 Buri (Paisa)	4 Chouks	"	1 Kahan or Rupee.
4 Buris	"	1 Pan (Anna)			

Also 1 *Cowry* or *Bat*=3 *Krantis*=4 *Kags*=5 *Tals*=7 *Dwips*=9 *Dantis*=72 *Jabs*=80 *Tils*=320 *Ranus*=1280 *Bahars* or *Ghuns*=25600 *Bindus*.

Therefore 1 *Cowry*=4 *Kags* ; 1 *Kag*=20 *Tils* ; 1 *Til*=16 *Ghuns* ; 1 *Ghun*=20 *Bindus*.

127. The following Tables are in use in different parts of India.

IN BEHAR, N.-W. P. AND PUNJAB

5 Cowries	make	1 Adhi
2 Adhis	"	1 Damri
2 Damris	"	1 Chhadam
2 Chhadams	"	1 Adhela
2 Adhelas	"	1 Paisa
2 Paisas	"	1 Taka.
2 Takas	"	1 Anna.

IN BOMBAY.

100 Raes	make	1 Quarter
4 Quarters	"	1 Rupee.

IN MADRAS.

1 Pagoda=*Rs.*3. 8*a.*

IN CEYLON.

100 Cents make 1 Rupee.

In British India the common medium of exchange is *silver*. The principal coin made of it is called a **Rupee**. The Rupee weighs 1 tola or 180 grains, and consists of 11 parts of silver and 1 of alloy. The weight of a gold *Mohur* is the same as that of a Rupee and is 180 grs. It consists of 11 parts of gold and 1 of alloy. The values of gold coins are variable, and therefore they are not used in mercantile transactions except the Sovereign, whose value is 15 rupees. The *Cowry*

is a shell brought from the Laccadive and Maldive Islands, and is used for very small payments. They vary in value according to supply in market but they are generally rockoned at 80 to a pice.

N. B.—The cowries as shells are now going out of use, but cowries (called karas) are in use in keeping accounts.

Of the copper coins, a half-anna weighs 200 grains ; a pice weighs 100 grains, and a half-pice 50 grains

15 *Sicca* Rupees = 16 Rupees The *Doctor's* Gold Mohur = 16 Rupees ; the *Lawyer's* Gold Mohur = 17 Rupees.

Gold coins (*obsolete*) ; Five-rupee piece ; Ten-rupee piece ; Gold Mohur ; Double Gold Mohur.

Silver coins (*current*) : Two anna piece ; Four-anna piece or Quarter-rupee ; Eight-anna piece or Half rupee ; Rupee.

Copper coins (*current*) : Pie, Half-pice ; Pice or Paisa ; Double paisa or Half-anna.

Note. *Re.* 1 = 2 half-rupees = 4 quarter-rupees or four-anna pieces = 8 two-anna pieces, and 1 anna = 2 double paisas. Also *Re.* 1 = 64 pice = 192 pies.

English Money.

128.

2 Farthings (*q.*)	make	1	Half-penny ($\frac{1}{2}d.$)
2 Half-pence	...	1	Penny (*d.*)
12 Pence	...	1	Shilling (1*s.* or 1/.)
20 Shillings	...	1	**Pound** (£1.)

[1, 2, 3 farthings are usually denoted by $\frac{1}{4}d.$, $\frac{1}{2}d.$, $\frac{3}{4}d.$, respectively.]

Money as expressed by means of these denominations is called **Sterling** money, in order to distinguish it from **stocks, shares,** &c. The **Standard** gold coin of England is made of a metal consisting of 22 parts of *pure gold*, and 2 parts of *copper*. Each of these 24 parts is called a *Carat*. Pure gold is said to be 24 carats *fine* and standard gold 22 carats *fine*. The *Pound sterling* is represented by a gold coin called a **sovereign,** and from 40 pounds Troy of standard gold are coined 1869 sovereigns ; and the value of gold of the *Mint-Fineness*, called 22 carat gold, is £3. 17*s.* 10$\frac{1}{2}$*d.* per ounce.

The **Standard** silver coin consists of 37 parts of *pure silver*, and 3 parts of *copper*. A pound Troy of this metal furnishes 66 *shillings*, and the *Mint-Price* of standard silver is 5*s.* 6*d.* per ounce. The silver coinage is not a *legal tender* for more than 40*s.*, the gold coinage being the *general* standard of value.

In the **copper coinage,** 24 pennies are made from an Avoirdupois pound of copper. This coin is not a legal tender for more than 12*d.* The coins now *current* in England are the following :—

Copper coins : Farthing ; Half-penny ; Penny.

Silver coins : Three-penny piece ; Four-penny piece ; Six-penny piece ; Shilling ; Florin (2*s.*) ; Half-crown (2*s.* 6*d.*) ; Crown (5*s.*)

Gold coins : Half-sovereign ; Sovereign.

The following coins were formerly in use, but now they are *obsolete*.

Silver coins : Groat (4*d.*) ; Tester (6*d.*)

Gold coins : Noble (6*s.* 8*d.*) ; Angel (10*s.*) ; Half-Guinea (10*s.* 6*d.*) Mark or Merk (13*s.* 4*d.*) ; Guinea (21*s.*) ; Carolus (23*s.*) ; Jacobus (25*s.*) ; Moidore (27*s.*)

Note. 1 shilling = 2 six-pences = 3 four-penny pieces = 4 three-penny pieces. Also 1 half-crown = 5 six-pences ; 1 half-guinea = 21 six-pences ; 1 guinea = 42 six-pences.

Also £1 = 4 crowns = 8 half-crowns = 10 florins = 40 six-pences = 80 three-pences = 240*d.* = 960*q.*

I. REDUCTION.

129. When a quantity is expressed in one denomination only it is called a **simple** quantity ; as, 7 rupees ; 5 yards.

When a quantity is expressed in several denominations, it is called a **compound** quantity ; as, *Rs.*8. 2*a* 3*p.* ; 5 yards 2 feet 3 inches.

130. **Reduction** is the process by which we convert or change (1) a simple or a compound quantity into terms of its lower denominations, or (2) a simple quantity into terms of its higher denominations, so that the *real* or *absolute* values remain unaltered.

131. *To express a quantity in terms of its lower denominations.* (**Descending Reduction**).

RULE. Multiply the number in the highest denomination by the number of units of the next inferior denomination contained in one unit of the highest, and to the product add the number (if any) of the inferior denomination in the quantity proposed ; and repeat this for each succeeding denomination till the required one is obtained.

Ex. 1. Reduce *Rs.*315 to *pice.*

*Rs.*315
16
5040*a.*
4
20160*ps.*

Re 1 = 16*a.*
∴ *Rs.*315 = (315 × 16)*a.* = 5040*a.*
Again, 1*a.* = 4*ps.*
∴ 5040*a.* = (5040 × 4)*ps.* = 20160*ps.*
∴ *Rs* 315 = 20160*ps.*

Ex. 2. Reduce *Rs.*5 14*a.* 6*p.* to *pies.*

Rs. 5 14*a.* 6*p.*
16
94*a.* (5 × 16 + 14)
12
1134*p.* (94 × 12 + 6)

*Rs.*5 = 5 × 16*a.* = 80*a.*
∴ *Rs.*5. 14*a.* = 94*a.*
Again, 94*a* = 94 × 12*p.* = 1128*p.*
∴ *Rs.*5. 14*a.* 6*p* = 94*a.* 6*p* = 1128*p.* + 6*p.*
= 1134*p.*

Ex. 3. Reduce £25. 13*s.* 3$\frac{3}{4}$*d.* to *farthings.*

£ 25. 13*s.* 6$\frac{3}{4}$*d.*
20
513*s.*
12
6162*d.*
4
24651*q.*

£1 = 20*s.* ∴ £25 = 25 × 20*s.* = 500*s.*
∴ £25. 13*s* = 500*s.* + 13*s.* = 513*s.*
Again, 1*s.* = 12*d.* ; ∴ 513*s.* = 513 × 12*d.* = 6156*d.*
∴ 513*s.* 6*d.* = 6156*d.* + 6*d.* = 6162*d.*
Again, 1*d.* = 4*q.* ; ∴ 6162*d.* = 6162 × 4*q.*
= 24648*q.*
∴ £25. 13*s.* 6$\frac{3}{4}$*d.* = 24648*q.* + 3*q.* = 24651*q.*

Here, the *denominations* are separated by a point as (.) ; and this is necessary to distinguish them from *ordinary* numbers, which do not require it, because their local values are all fixed and certain.

Examples XXIV.

1. Reduce to *annas* :—

(1) *Rs* 17 ; *Rs*.19 ; *Rs*.42 ; *Rs*.45 ; *Rs*.69 ; *Rs*.84 ; *Rs*.95.
(2) *Rs*.87 ; *Rs*.120 ; *Rs*.245 ; *Rs*.460 ; *Rs*.9. 12*a*. ; *Rs*.20. 14*a*.
(3) *Rs*.36 6*a*. : *Rs*.53. 13*a*. ; *Rs*.87. 11*a*. ; *Rs* 79. 15*a*. ; *Rs*.234. 11*a*.

2. Reduce to *pies* :—

(1) *Rs*.34 ; *Rs* 56 ; *Rs*.97 ; *Rs*.146 ; *Rs*.342 ; *Rs*.496.
(2) *Rs*.84. 5*a*. ; *Rs*.76 12*a*. ; *Rs* 265. 9*a*. ; *Rs*.804. 13*a*. ; *Rs*.945. 6*a*.
(3) *Rs*.15. 8*a*. 3*p*. ; *Rs*.7. 13*a*. 11*p*. ; *Rs*.8. 0*a*. 5*p*. ; *Rs*.9. 10*a*. 9*p*.
(4) *Rs* 425. 7*a*. 9*p*. ; *Rs*.550. 3*a*. 11*p*. ; *Rs*.1250. 5*a*. 7*p*. ; *Rs*.5050. 14*a*. 1*p*. ; *Rs*.456. 14*a*. 11*p*. *Rs*.31. 10*a*. 1*p*. ; *Rs*.343. 8*a*. 7*p*.

3. Reduce (i) to *pice* and (ii) to *pies* :—

(1) *Rs*.52 ; *Rs*.19 ; *Rs* 112. 6*a*. ; *Rs*.36. 11*a*. 2*ps*. ; *Rs*.20. 8*a*. 3*ps*.
(2) *Rs*.87. 10*a*. 1*ps*. ; *Rs*.172. 5*a*. 3*ps*. ; *Rs*.225 9*a*. 2*ps*. ; *Rs*.476. 12*a*. 1*ps* ; *Rs*.782. 0*a*. 3*ps*. ; *Rs*.13. 10*a*. 3*ps*. ; *Rs* 215. 7*a*. 3*ps*.

4. Reduce (i) to *gandas* and (ii) to *cowries* (karas) :—

(1) *Rs*.19 ; *Rs*.34 ; *Rs*.56 ; *Rs*.78 ; *Rs*.105 ; *Rs*.84. 7*a*.
(2) *Rs*.102. 15*a*. 1*ps*. ; *Rs*.24. 14*a*. 3*ps*. ; *Rs*.405. 13*a*. ; *Rs*.75. 7*a*. 5 *gan*.
(3) *Rs*.48. 9*a*. 10 *gan*. ; *Rs*.53. 13*a*. 17 *gan*. ; *Rs*.9570. 14*a*. 16 *gan*.

5. Reduce to *cowries* (karas) :—

Rs.53. 13*a*. 17 *gan*. 2 *cow*. ; *Rs* 68. 9*a*. 11 *gan*. 1 *cow*. ; *Rs*.18. 6*a*. 12 *gan*. 2 *cow*. ; *Rs*.5942. 0*a*. 17 *gan*. 3 *cow*.

6. Reduce (i) to *pice* and (ii) to *pies* :—

(1) 175 half-rupees ; 370 quarter-rupees ; 845 two-anna pieces.
(2) 425 double-paisas ; 3116 two-anna pieces ; 2415 half-rupees.
(3) 34212 quarter-rupees ; 20157 double-paisas ; 67950 four-anna pieces ; 827 eight-anna pieces.

7. Reduce (i) to *half-rupees* ; (ii) to *quarter-rupees* and (iii) to *two anna pieces*.

(1) *Rs*.729 ; *Rs*.925 ; *Rs* 1228 ; *Rs*.1427 ; *Rs*.4243 ; *Rs*.9740.
(2) *Rs*.858. 8*a*. ; *Rs*.9726. 8*a*. ; *Rs*.73246 ; *Rs* 57509.

8. Reduce (i) to *half-annas* and (ii) to *half-pice* :—

Rs.75. 6*a*. ; *Rs*.132. 9*a*. ; *Rs*.150. 0*a*. 2*ps*. ; *Rs*.3005. 10*a*. 2*ps*.

9. Reduce :—

(1) A lac of rupees to *paisás* ; *Rs*.7125. 4*a*. to *four-anna pieces* ; *Rs*.6075. 8*a*. to *two-anna pieces* ; *Rs*.1250. 7*a*. 2*ps*. to *double-paisás* ; *Rs* 9864. 8*a*. to *eight-anna pieces*.
(2) *Rs*.1325. 9*a*. 1*ps*. to *half-paisás* ; *Rs*.3116. 14*a*. 6*p*. to *double-paisás* ; *Rs*.2415. 10*a*. 9*p*. to *half-pice*.

10. Reduce to *shillings* :—

(1) £345 ; £498 ; £795 ; £1402 ; £9086 ; £8092.
(2) £71. 1*s.* ; £490. 18*s.* ; £790. 13*s.* ; £3456. 17*s.* ; £6403. 7*s.*

11. Reduce to *pence* :—

(1) £65 ; £98 ; £156 ; £405 ; £1849 ; £5043 ; £9236.
(2) £134. 15*s.* ; £198. 13*s.* ; £416. 11*s.* ; £526. 5*s.* ; £926. 7*s.*
(3) £2. 6*s.* 8*d.* ; £40. 10*s.* 6*d.* ; £11. 7*s.* 9*d.* ; £374. 11*s.* 8*d.*
(4) £655. 13*s.* 6*d.* ; £71. 13*s.* 5*d.* ; £343. 13*s.* 5*d.* ; £1274. 19*s.* 9*d.*

12 Reduce to *farthings* :—

(1) £4. 8*s.* 4¾*d.* ; £7. 13*s.* 11¾*d.* ; £13. 19*s.* 0¼*d.* ; £29. 10*s.* 11*d.*
(2) £101. 9*s.* 2¼*d.* ; £[illegible]53. 3*s.* 4½*d.* ; £600. 6*s.* 3*d.* ; £83920. 16*s.* 2¾*d.*

13. Reduce (i) to *half-pence* and (ii) to *farthings* :—

(1) 15*s.* 6*d.* ; 18*s.* 9*d.* ; 13*s.* 11*d.* ; 19*s.* 6*d.* ; 8*s.* 10*d.* ; 17*s.* 5*d.*
(2) £4080 ; £8608 ; £8734 ; £726. 18*s.* ; £517. 13*s.* ; £2125. 6*s.*
(3) £79. 14*s.* 8*d.* ; £47. 19*s.* 9½*d.* ; £389. 12*s.* 8½*d.* ; £879. 18*s.* 0¾*d.*
(4) £1560. 10*s.* 4½*d.* ; £2145. 18*s.* 7¾*d.* ; £9136. 15*s.* 9½*d.*
(5) 3899 half-sovereigns ; 4807 crowns ; 8608 half-crowns ; 6530 florins ; 5869 six-pences ; 6958 groats ; 8009 three-penny pieces ; 9076 guineas ; 3089 half-guineas ; 7632 four-penny pieces ; 1445 moidores ; 2047 nobles ; 3286 florins ; 1983 six-pences.

14. Reduce (i) to *three-penny pieces*, (ii) to *four-penny pieces*, and (iii) to *six-pences* :—

(1) £95 ; £128 ; £8076 ; £1857 ; £9083 ; £9072.
(2) £11. 14*s.* ; £144. 17*s.* ; £2145. 11*s.* ; £4265. 15*s.* ; £3264. 17*s.*

15. Reduce :—

(1) 95 guineas 17*s.* 9¾*d.* to *farthings* ; £450. 16*s.* 6*d.* to *six-pences.*
(2) £570. 12*s* to *florins* ; £382. 7*s.* 6*d.* to *half-crowns* ; £589. 15*s.* to *crowns* ; £3500. 17*s.* 6*d.* to *half-crowns.*
(3) £99. 9*s* 9*d.* to *three-pences* ; 5573 half-crowns to *pence.*
(4) 9571 half-crowns to *six-pences* ; 9100 half-crowns to *three-pences.*

16. Reduce to *farthings* :—

(1) 7[illegible] *gui.* 16*s.* 2¾*d.* ; 937 *flor.* 1*s.* 2¼*d.* ; 2902 *cr.* 1*s.* 3¾*d.*
(2) 150 *half sov.* 7*s.* 2½*d.* ; 79924 *gui.* 12*s.* 2¾*d.* ; 7255 *flor.* 1*s.* 3¾*d.*

17. For how many children can a treat be provided with *Rs.*32. 8*a.* at 2 annas a head ?

18. How many two pice stamps can I buy for *Rs.*5. 6*a.* 2*ps.* ?

19. If the cost of a telegram is 3*d.* a word, how many words can be sent for £1. 3*s.* 3*d.* ?

20. A poor woman had only *Rs.*2. 1*a.* 8*p.* to live upon. She spent daily 4 pies for her food. How many days did she live upon it ?

132. *To express a simple quantity in terms of its higher denomination.* **(Ascending Reduction).**

RULE. Divide the number by the number of units which make one unit of the next higher denomination, setting down the remainder (if any) as of the same denomination as its dividend; and continue this process till we come to the required denomination.

Ex. 1. Reduce 1560*ps.* to *rupees, &c.*

4 | 1560*ps.* ∵ 4 pice = 1 anna.
16 | 390*a* ∵ 16 annas = 1 rupee.
Rs. 24. 6*a.* ∴ the result is *Rs.* 24. 6*a.*

Ex. 2. Reduce 30857*p.* to *rupees, annas* and *pies*

12 | 30857*p.* ∵ 12 pies = 1 anna
16 | 2571 − 5*p* ∵ 16 annas = 1 rupee.
Rs. 160 − 11*a.* ∴ the result is *Rs.* 160. 11*a.* 5*p*

Ex. 3. Reduce 97403*q.* to *pounds.*

4 | 97403*q* ∵ 4*q.* = 1*d*
12 | 24350 − 3*q.* ∵ 12*d.* = 1*s.*
20 | 2029 − 2*d.* ∵ 20*s.* = £1.
£101 − 9*s.* ∴ the result is £101. 9*s.* 2¾*d*

Ex. 4 Reduce 36173 half-pence to *guineas.*

2 | 36173 half-pence. ∵ 2 half-penny = 1*d.*
12 | 18086 − 1 *half-penny.* ∵ 12*d.* = 1*s.*
21 { 3 | 1507 − 2*d.* ∵ 21*s.* = 1 *gui.*
21 { 7 | 502 − 1 } 16*s.* ∴ the result is 71 *gui.* 16*s.* 2½*d.*
gui. 71 − 5 }

Examples XXV

1. Reduce to *rupees, annas* and *pies* :—
(1) 25325*p.*; 57509*p.*; 51039*p.*; 679298*p.*; 37921*p.*; 456786*p.*
(2) 643294*p.*; 732394*p.*; 1982345*p.*; 967573*p.*, 1043324*p.*

2. Reduce to *rupees, annas* and *pice* :—
987945*ps.*; 1234567*ps.*; 547321*ps.*; 894956*ps.*; 5537792*ps.*

3. Reduce to *rupees, annas, &c.* :—
(1) 8320 gandas ; 7680 cowries (karas) ; 379498 gandas ; 40768 buris.
(2) 1045673 double-paisás ; 2067544 half-paisás ; 1077760 cowries (karas).
(3) 342876 buris ; 596824 paisás ; 23679 double-paisás ; 103678 half-paisás ; 1155440 cowries (karas).

4. Reduce to *rupees* :—
(1) 1648 half-rupees ; 1892 quarter-rupees ; 2530 two-anna pieces.
(2) [illegible]896 annas ; 5952 paisás ; 920320 gandas ; 24320 cowries (karas).

5. Reduce to *pounds*, *shillings* and *pence* :—

(1) 69132*d.*; 60948*d.*; 90231*d.*; 2733*d.*; 89900*d.*; 157362*d.*
(2) 147040*q.* ; 2840619*q.* ; 123290*q.* ; 350000*q.* ; 805639799*q*
(3) 345679*q.* ; 130013*q.* ; 10000199*q.*; 284079*q.* ; 415739*q.* ; 36503244*q.*

6. Reduce to £. *s. d.* :—

(1) 899045 half-sovereigns ; 971112 crowns ; 48073 florins ; 886101 half-crowns ; 85730 half-pence ; 13029 three-pences.
(2) 15137 four-penny pieces ; 82556 florins ; 28892 half-crowns ; 2857 four-penny pieces ; 987653 half-pence ; 47285 guineas.
(3) 23645 moidores ; 49726 half-guineas ; 183491 six-pences ; 281062 three-pences ; 40340 farthings.

7. How much money will be required to buy 37528 penny stamps ?

8. A dealer bought 438 cocoanuts at 9 pies each ; how many rupees, &c. had he to pay for them ?

9. I distributed among 1682 beggars a sum of money, giving them two pice a head ; what sum did I spend ?

10. If during a festival 2250 people on an average cross the Hugli Bridge daily, each paying 2 pice, what is the collection of the ferry farmer, if the festival lasts for 16 days ?

133. There are some cases in Reduction where we cannot pass directly step by step from the given denominations to the one proposed. We must in such cases pass through an intermediate denomination common to both, and it will be advisable to keep such common denomination as high as possible. Then, find by division what quantity of the proposed denomination is equivalent to the given quantity.

Ex. 1. Reduce £253. 9*s.* 10*d.* to *half-crowns.*

```
    £253. 9s. 10d.
      20
    5069s.
      12
3,0|608 3,8d.
   |2027 - 28d.
```

1 half-crown = 2*s.* 6*d.* = 30*d.*
∴ the result is 2027 half-crowns and 28*d.* or 2*s.* 4*d.* over.

Ex. 2. Reduce *Rs.*31. 10*a.* 2*p.* to £. *s. d.*, when 1*q.* = 2 *pies.*

```
Rs 31. 10a. 2p.       2|60748p.        4|30374q.
   16                  |30374q.       12|759 - 1q.
  506a.                              2,0|6,3 - 3d.
   12                                   |£3 - 3s.
  6074p.
```

∴ *Rs.*31. 10*a.* 2*p.* = £3. 3*s.* 3½*d.*

134. Proof. Descending and ascending Reductions are inverse processes ; if therefore we perform one process on a given

quantity, and on the result the other process, we ought to get the original quantity.

Thus, if by the descending process we find that £25. 13*s*. $6\frac{3}{4}$*d*. = 24651*q*., we ought by the ascending process to find that 24651*q* = £25. 13*s*. $6\frac{3}{4}$*d*.

Examples XXVI.

1. Reduce (i) to *guineas* and (ii) to *half-guineas* :—
£63 ; £105 ; £96. 16*s*. ; £876. 15*s*. ; £538 ; £10728.

2. Reduce (i) to *crowns* and (ii) to *half-crowns* :—
£265. 10*s*. ; £589. 15*s*. ; £437. 10*s*. ; £620. 5*s*. ; £5189. 15*s*.

3. Reduce to *crowns* :—
10987 guineas ; £89000 ; £36. 17*s*. 6*d*. ; 18756 four-penny pieces

4. Reduce to *half-crowns* :—
£48. 17*s*. 6*d*. ; £382. 7*s*. 6*d*. ; £583. 2*s*. 6*d*. ; 670 half-guineas.

5. Reduce to *guineas* : —
28906 florins ; 107284 half-crowns ; 23810 crowns ; 760 half-crowns ; £647. 0*s*. 11*d*. ; £375. 16*s*. $0\frac{3}{4}$*d*.

6. Reduce to *half-guineas* :—
325 crowns ; 10867 half sovereigns ; 3150 four-penny pieces ; £3240. 10*s*. 6*d*. ; 147 half-crowns.

7. Reduce to £. *s*. *d*., (1*d*. = 11*p*.) :—
Rs.35. 9*a*. 3*p*. ; *Rs*.707. 11*a*. 7*p*. ; *Rs*.2510. 8*a*. 4*p*.

8. Reduce to *Rs*. *a*. *p*., (1*q*. = 2 *pies*) :—
£32. 14*s*. 7*d*. ; £96. 17*s*. 6*d*. ; £903. 17*s*. $6\frac{1}{4}$*d*. ; 54 half-guineas ; 107 florins ; 17 half-crowns.

9 If a guinea be equal to *Rs*.10. 8*a*. ; find the number of two-anna pieces contained in 1760 guineas.

10. Reduce 7500 Sicca rupees to *current rupees* and 6432 rupees to *Sicca rupees*.

II. COMPOUND ADDITION.

135. Keeping in mind what was said in Art. 123, we need no additional inquiry to inform us that the fundamental operations on *Compound Quantities* must be performed as in *Integers*, with this difference, that instead of carrying and borrowing *tens*, we must do the same with the *different numbers* which connect their parts together ; and we shall therefore merely enunciate the rule for each at the beginning of the portion of the work appropriated to it.

136. **Compound Addition** is the method of finding a single quantity which is equal to two or more quantities of the same kind. This single quantity is called the **sum** of the given quantities

RULE. Arrange the quantities under one another according to their denominations, so that units of the same denomination may be in the same vertical column, and draw a line below them. Add together the numbers of the lowest denomination; reduce the sum to the next higher denomination; set down the remainder, if any, under the column, and carry the quotient to the first figure of the next column. Repeat the process with all the columns.

Ex. 1. Add together *Rs.*14. 15*a.* 10*p.*, *Rs.*54. 14*a.* 9*p.*, *Rs.* 156. 11*a.* 2*p.*, and *Rs.*34. 14*a.* 10*p.*

Rs.	*a.*	*p.*
14	15	10
54	14	9
156	11	2
34	14	10
*Rs.*261	8	7 *Ans.*

10*p.*+9*p.*+2*p.*+10*p.*=31*p.*=2*a.* 7*p.*

Carry 2*a.*; 2*a.*+15*a.*+14*a.*+11*a.*+14*a.* =56*a*=*Rs* 3. 8*a.*

*Carry Rs.*3; *Rs* 3+*Rs.*14+*Rs.*54+*Rs.*156+*Rs* 34. =*Rs.*261.

Ex. 2. Add together £156. 8*s.* 9¾*d.*, £33. 15*s.* 11¾*d*, £204. 0*s.* 1½*d.*, £5275. 17*s.* 8*d.* and £105. 18*s.* 6½*d.*

£.	*s.*	*d.*
156	8	9¾
33	15	11¾
204	0	1½
5275	17	8
105	18	6½
£5776	1	1½ *Ans.*

3*q.*+3*q.*+2*q.*+2*q.*=10*q.*=2½*d.* *Carry* 2*d*;
2*d.*+9*d.*+11*d.*+1*d.*+8*d.*+6*d*=37*d.*=3*s.* 1*d.*
Carry 3*s.*; 3*s.*+8*s.*+15*s.*+17*s.*+18*s.*=61*s.*=£3. 1*s*
Carry £3; £3+£156+£33+£204+£5275+£105 =£5776.

Examples XXVII.

1. Add together:—

(1)		(2)		(3)		(4)		(5)		(6)	
As.	*p.*	*As.*	*p.*	*As.*	*p.*	*As.*	*p.*	*As.*	*p.*	*As.*	*p.*
9	7	12	3	9	8	13	4	12	3	15	4
12	3	13	7	11	2	7	8	14	4	11	10
9	4	4	9	13	4	9	10	3	7	4	5
11	10	7	10	3	7	13	8	2	6	8	11

(7)			(8)			(9)			(10)		
Rs.	*a.*	*p.*	*Rs.*	*a*	*p.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*
2	2	2	8	11	9	42	10	9	67	10	6
3	4	3	10	5	9	54	12	6	71	12	9
5	7	9	9	12	5	67	9	3	62	14	9
8	10	6	12	11	6	75	11	6	73	13	8
9	12	7	15	6	8	72	6	7	85	7	5

(11)			(12)			(13)			(14)		
Rs.	*a.*	*p.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*
5	11	3	12	12	3	6	14	9	47	5	2
9	10	10	19	4	10	14	0	3	1	15	9
2	14	9	4	15	8	15	15	5	65	6	0
3	5	11	7	9	5	27	12	11	88	15	3
1	6	7	23	7	6	7	14	4	14	15	10
11	13	6	25	0	2	29	0	5	34	14	10
15	7	7	8	14	3	104	13	1	54	14	9

(15)			(16)			(17)			(18)		
Rs.	*a.*	*ps.*	*Rs.*	*a.*	*ps.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*
7	11	2	27	11	2	378	9	10	98	0	9
8	14	3	9	14	3	4	7	4	448	6	5
13	12	1	4	10	1	56	8	8	3839	4	0
315	10	2	156	8	2	464	0	3	97	3	2
23	7	2	215	13	2	368	6	8	136	3	7
625	15	3	18	7	1	535	7	1	4837	4	9
24	0	1	106	14	0	97	3	2	28	10	9
129	13	3	315	0	2	893	15	9	234	11	6
56	8	1	57	14	3	14	10	7	536	12	11

(19)			(20)			(21)			(22)		
Rs.	*a*	*p.*	*Rs.*	*a.*	*p*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*
1135	4	3	1325	10	9	3004	7	6	74037	9	4
1243	6	9	7602	11	3	907	5	2	80668	12	0
1575	8	8	3006	7	7	1235	10	7	50087	13	4
2007	7	7	4040	8	6	2727	11	5	136	7	4
3445	9	10	3050	12	5	3647	12	9	3270	2	8
4002	10	11	2225	13	8	7532	9	8	5971	14	8
997	11	10	110	6	6	2121	13	10	58065	9	4
1005	9	9	965	14	11	3333	15	8	360	2	8
2220	13	7	1097	13	4	2025	7	6	943	5	4
997	15	3	2110	6	9	1605	0	10	72459	4	0

2. Add together :—

(1)		(2)		(3)	
s.	*d.*	*s.*	*d.*	*s.*	*d.*
3	$7\frac{3}{4}$	19	$8\frac{1}{2}$	19	$10\frac{1}{4}$
14	$6\frac{1}{2}$	1	$9\frac{3}{4}$	18	$4\frac{1}{2}$
2	11	15	9	9	5
15	$8\frac{3}{4}$	13	$3\frac{1}{2}$	15	$8\frac{3}{4}$
13	4	10	$9\frac{1}{2}$	14	$9\frac{3}{4}$

(4)			(5)			(6)		
£.	*s.*	*d.*	*£.*	*s.*	*d.*	*£.*	*s.*	*d.*
37	13	6	3	7	$6\frac{1}{2}$	18	15	$7\frac{1}{4}$
29	12	4	69	11	10	76	14	2
6	3	9	13	0	$4\frac{1}{4}$	25	10	$2\frac{3}{4}$
55	17	2	37	13	$2\frac{3}{4}$	13	13	$3\frac{1}{2}$
7	10	10	26	15	7	66	4	$7\frac{1}{4}$

(7)

£.	s.	d.
456	14	$8\frac{3}{4}$
9	16	$4\frac{1}{2}$
83	18	$10\frac{1}{2}$
17	19	7
686	7	$9\frac{1}{4}$
8	15	$6\frac{1}{2}$
3548	19	$9\frac{1}{2}$
95	8	$8\frac{3}{4}$

(8)

£.	s.	d.
8	19	$10\frac{3}{4}$
1379	17	$6\frac{3}{4}$
897	16	$9\frac{1}{2}$
89	18	11
4357	8	$11\frac{3}{4}$
52765	15	$8\frac{3}{4}$
99	19	$11\frac{1}{4}$
67	5	10

(9)

£.	s.	d.
7	19	$3\frac{1}{2}$
46	12	4
276	4	$7\frac{1}{2}$
77	7	$9\frac{3}{4}$
8760	10	6
795	15	$3\frac{1}{4}$
20	4	4
813	11	$7\frac{3}{4}$

(10)

£.	s.	d.
2769	10	$8\frac{1}{2}$
36	11	$2\frac{3}{4}$
472	13	10
4792	18	$4\frac{1}{2}$
3279	15	$8\frac{1}{4}$
24	8	11
429	17	$5\frac{3}{4}$
4198	15	$4\frac{1}{4}$

3. Add together :—

(1)

Rs.	*a.*	*p.*
3672	6	9
4278	13	6
236	4	1
5982	14	6
3716	8	4
410	7	10
6759	0	5
4917	0	0
427	12	6
218	8	5
29	15	8
6374	8	11
7109	15	7
492	7	5

(2)

Rs.	*a.*	*p.*
8274	5	7
329	8	6
415	2	9
42	5	10
2736	7	4
9	15	7
8138	14	4
725	4	6
87	9	11
234	15	4
9027	5	9
4378	9	3
274	2	5
42	9	7

(3)

Rs.	*a.*	*p.*
527	9	8
8436	10	2
4167	9	8
429	8	3
927	7	7
8	1	2
72	7	9
429	0	5
7283	8	6
5432	12	3
710	10	6
636	8	2
42	3	7
9245	8	6

(4)

£.	s.	d.
7214	18	$7\frac{1}{4}$
829	2	$1\frac{1}{2}$
3484	19	11
151	3	$9\frac{1}{2}$
40	14	$3\frac{3}{4}$
2607	17	$10\frac{1}{2}$
263	6	6
90	18	$8\frac{1}{4}$
485	13	7
7324	7	$4\frac{3}{4}$
934	16	$1\frac{1}{4}$
78	15	$10\frac{1}{2}$

(5)

£.	s.	d.
4614	13	$3\frac{1}{2}$
12	4	$5\frac{1}{4}$
6078	11	3
85	7	$3\frac{3}{4}$
843	19	10
7913	5	$8\frac{1}{2}$
24	6	8
1012	14	$6\frac{1}{2}$
820	12	4
537	9	$11\frac{3}{4}$
125	16	8
8416	15	$4\frac{1}{4}$

(6)

£.	s.	d.
9241	12	$5\frac{1}{2}$
159	3	$9\frac{1}{2}$
63	17	$10\frac{1}{4}$
4375	19	4
88	6	$7\frac{3}{4}$
797	15	9
972	13	3
2356	11	$6\frac{1}{2}$
38	5	$8\frac{1}{4}$
125	18	5
6316	4	$2\frac{3}{4}$
244	3	7

4. A cash-box contained 89 sovereigns, 35 half-sovereigns, 19 half-crowns, 25 florins, 31 shillings and 15 six-penny bits; find the value of the coins in £. *s.* *d.*

5. A tradesman bought goods to the value of £1368. 12*s*. 6*d* he paid for carriage, £25. 16*s*. 9*d*., and other charges, £2. 15*s*. 8½*d*.; he gained by the sale of the goods £269. 15*s*. 3½*d*.; how much did he sell the goods for ?

6. A stationer bought some books for *Rs*.79 12*a*. 6*p*., some paper for *Rs*.161. 4*a*. 3*p*., some pens for *Rs*.14. 10*a*. and some envelopes for *Rs*.12. 8*a*. 6*p*. How much must he charge for all these articles, so as to gain exactly *Rs*.100 by his bargain ?

7. A collection was once made in a district for a charitable purpose. The following coins were obtained : 99 gold mohurs, 1875 rupees, 990 eight-anna pieces, 5891 four-anna pieces, 1276 two-anna pieces, 90617 half-anna pieces and 81516 pice. What did the collection amount to in *Rs. a. pies* ?

8. Add together 53 guineas, 107 sovereigns, 161 half-guineas, 55 half-sovereigns, 223 half-crowns, 505 four-penny pieces, and 603 arthings.

III, COMPOUND SUBTRACTION.

137. Compound Subtraction is the method of finding what quantity is left when a smaller quantity is taken from a greater of the same kind. The quantity thus left is called the **difference** of the given quantities.

RULE. Write the less number below the greater, so that units of the same denomination may be under one another, and draw a line below. Begin at the right hand and subtract (if possible) each number in the lower line from the corresponding one in the upper and place the remainder underneath. But if, in any case, the number in the lower line be greater than the one above it, add to the upper one as many units of the same denomination as make one unit of the next higher denomination, and then subtract, taking care to add 1 to the next number in the lower line. Proceed thus through all the columns.

Ex. 1. Subtract *Rs*.47. 12*a*. 9*p*. from *Rs*.72. 15*a*. 3*p*.

Rs.	*a*.	*p*.
72	15	3
47	12	9
Rs.25	2	6

3*p*. is less than 9*p*., so add 12*p*. to 3*p*., and 1*a*. to 12*a*.; 15*p*. − 9*p*. = **6***p*.
15*a*. − 13*a*. = **2***a*., and *Rs*.72 − *Rs*.47 = ***Rs*.25**.

Ex. 2. Subtract £207. 13*s*. 8¾*d*. from £304. 2*s*. 10½*d*.

£.	*s*.	*d*.
304	2	10½
207	13	8¾
£96	9	1¾

2*q*. is less than 3*q*. so add 4*q*. to 2*q*. and 1*d*. to 8*d*.:
6*q*. − 3*q*. = **3***q*., or ¾*d*.
10*d*. − 9*d*. = **1***d*.
2*s*. is less than 13*s*., so add 20*s*. to 2*s*. and £1 to £207 ; 22*s*. − 13*s*. = **9***s*. ; £304 − £208 = **£96**.

Examples XXVIII.

1. Perform the operation of subtraction in the following :—

(1)			(2)			(3)			(4)			(5)		
Rs.	*a.*	*p.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*ps.*	*Rs*	*a.*	*ps.*
55	15	10	106	12	9	57	6	3	75	11	2	126	3	1
47	8	11	77	15	10	46	9	10	49	12	3	82	8	3

(6)			(7)			(8)			(9)		
Rs.	*a.*	*p.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*
150	4	10	1000	8	4	269	5	11	4172	8	5
24	5	9	488	15	6	189	13	10	2008	14	9

(10)			(11)			(12)			(13)		
Rs.	*a.*	*p.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*
772	13	9	5400	14	7	3406	4	7	4658	7	6
347	15	11	3216	15	10	2958	13	9	4139	9	8

(14)			(15)			(16)			(17)		
Rs.	*a.*	*p.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p*
50			575	10	6	1000			7071	15	10
48	14	11	89	11	9	101	10	8	5707	10	11

2. Perform the operation of subtraction in the following :—

(1)		(2)		(3)		(4)		(5)	
s.	*d.*	*s.*	*d.*	*s.*	*d*	*s.*	*d.*	*s.*	*d.*
17	9	17	$5\frac{3}{4}$	19	$0\frac{1}{8}$	18	$3\frac{1}{4}$	10	$3\frac{1}{4}$
11	$8\frac{1}{4}$	5	$9\frac{1}{8}$	14	$11\frac{3}{4}$	11	$7\frac{1}{2}$	4	$7\frac{3}{4}$

(6)			(7)			(8)			(9)		
£.	*s.*	*d.*	*£.*	*s.*	*d.*	*£.*	*s.*	*d.*	*£.*	*s.*	*d.*
58	15	$3\frac{1}{2}$	95	14	2	586	17	$1\frac{1}{2}$	100	14	7
19	4	$7\frac{3}{4}$	37	6	$3\frac{3}{4}$	298	13	$1\frac{3}{4}$	50	14	$7\frac{1}{2}$

(10)			(11)			(12)			(13)		
£.	*s.*	*d.*	*£.*	*s.*	*d.*	*£.*	*s.*	*d.*	*£.*	*s.*	*d.*
98	6	$2\frac{1}{4}$	100	3	3	611	17	$2\frac{1}{2}$	743	0	$4\frac{3}{4}$
67	11	$4\frac{3}{4}$	95	15	$6\frac{1}{4}$	492	18	$8\frac{3}{4}$	275	15	$5\frac{1}{2}$

(14)			(15)			(16)			(17)		
£.	*s.*	*d.*	*£.*	*s.*	*d.*	*£.*	*s.*	*d.*	*£.*	*s.*	*d.*
525	14	$7\frac{3}{4}$	536	8	$7\frac{1}{4}$	837	14	$2\frac{1}{2}$	86	15	$9\frac{1}{4}$
345	17	$8\frac{3}{4}$	89	13	$9\frac{3}{4}$	358	18	$6\frac{3}{4}$	9	18	$11\frac{1}{2}$

3. Subtract :—

(1) *Rs.*979. 15*a.* 9*p.* from *Rs.* 5707. 15*a.* 7*p.*
(2) *Rs.* 2102. 13*a.* 11*p.* from *Rs.* 4365. 10*a.* 9*p.*
(3) *Rs.* 6779. 14*a.* 8*p.* from *Rs.* 7865. 12*a.* 6*p.*
(4) £554. 12*s.* $7\frac{3}{8}$*d.* from £1739. 7*s.* $6\frac{1}{4}$*d.*
(5) £1975. 13*s.* $9\frac{1}{4}$*d.* from £3003. 10*s.* 4*d.*
(6) *Rs.* 55734. 12*a.* 4*p.* from *Rs.* 88659. 8*a.* 3*p.*
(7) The sum of *Rs.* 14. 3*a.* 5*p.* and *Rs.* 9. 8*a.* 7*p.* from *Rs.* 53. 11*a.* 6*p.*
(8) The sum of £5. 6*s.* $4\frac{1}{2}$*d.*, £31. 15*s.* $10\frac{1}{4}$*d.*, £43. 18*s.* $5\frac{3}{8}$*d.*, and £25. 16*s.* $4\frac{3}{4}$*d.* from £371. 14*s.* $6\frac{3}{4}$*d.*

4. What must be added to £157. 16*s.* $9\frac{1}{2}$*d.* to make £355. 13*s.* 4*d.* ?

5. After spending *Rs.*237. 14*a.* 3*ps.*, how much has a man left out of *Rs* 532. 10*a.* ?

6. A man has 50 guineas in his purse ; what would he have lef after paying bills amounting to £49. 8*s.* $11\frac{1}{4}$*d.* ?

7. A tradesman, in making out a bill, copied 16*s.* 3*d.* for £16. 3*s.* and £10. 8*s.* for 10*s.* 8*d.* By what amount was the bill wrong ?

8. By how much is *Rs.* 803. 11*a.* 3*p.* greater than *Rs.*213. 8*a.* 4*p.*?

9. *A* borrowed from *B* *Rs.*387. 5*a.* 8*p.* and then *Rs.* 39. 9*a.* 1*p.* ; repaid him *Rs.*28. 7*a.* and again borrowed *Rs.* 625. 13*a.* 11*p.* ; find what will be the amount of his debt still due if he makes payment of *Rs.*967. 3*a.* 7*p.*

10. Find the value of *Rs.*20. 15*a.* 11*p.*+*Rs.*28. 11*a.* 3*p.*−*Rs.*17. 12*a.* 5*p.*+*Rs.*59. 13*a.* 6*p.*−*Rs.*13. 10*a.* 4*p.*+*Rs.*18. 3*a.* 7*p.* −*Rs.*28 12*a.* 9*p.*−*Rs.*10. 14*a.* 3*p.*

11. A man has *Rs.*5000 in the bank ; he draws *Rs.*2500 on Monday, *Rs.*1175. 4*a.* on Wednesday, and *Rs.*959. 6*a.* on Saturday. What has he left in the bank ?

12. A boy took the sum of 19*s.* $11\frac{3}{4}$*d.* three times out of a bag containing £5. What was left?

13. A house and furniture are worth *Rs.* 1001. 11*a.* 10*p.* The house costs *Rs.*750. 14*a.* 11*p.* What is the value of the furniture ?

14. *A*, *B* and *C* together owe £107. 11*s.* 8*d.* ; the sum of the debts of *A* and *B* is £70. 5*s.* 5*d.*, and of *B* and *C* £80. 16*s.* 1*d.* How much does each owe ?

15. *A*, who has *Rs.*5. 4*a.*, gives *B* *Rs.*3. 7*a.* 6*p.* and *C* *Rs.*2. 9*a.* ; but he receives from *D* *Rs.*10. 10*a.* 8*p.*, and from *E* *Rs.*3. 11*a.* 6*p.* less than he received from *D* ; how much has he after these payments ?

16. A tradesman's cash in hand on Monday morning was £5. 13*s.* 6*d.* His cash receipts on Monday amounted to £2. 15*s.* $6\frac{3}{4}$*d.*

and on the following days of the week were, respectively, £4. 18*s.* 4*d.*, £3. 13*s.* $6\frac{1}{2}$*d.*, £5. 10*s.* $10\frac{1}{2}$*d.*, £4. 12*s.* 11*d.*, and £16. 9*s.* $8\frac{1}{2}$*d.* His cash outlay during the week amounted to £24. 17*s.* $5\frac{3}{4}$*d.* What cash had he remaining at the end of the week?

IV. COMPOUND MULTIPLICATION.

138. Compound Multiplication is the method by which we find the sum of a compound quantity repeated as many times as there are units in a given number. The sum found is called the **product**.

139. When the Multiplier is not greater than 20.

RULE. Place the Multiplier under the lowest denomination of the multiplicand and draw a line below. Beginning with the lowest denomination multiply by the given multiplier, and find the number of the next higher denomination contained in the product; put down the remainder (if any), and carry the quotient to the next product, and repeat the process till all the denominations are multiplied.

Ex. 1 Multiply *Rs.*72. 11*a.* 9*p.* by 7.

Rs.	*a.*	*p.*
72	11	9
		7
*Rs.*509	2	3

9*p.* × 7 = 63*p.* = 5*a.* 3*p.*; *carry* 5*a.*
11*a.* × 7 = 77*a.*, with 5*a.* = 82*a* = *Rs.*5. 2*a*; *carry Rs.*5.
*Rs.*72 × 7 = *Rs.*504, with *Rs.*5 = *Rs.*509.

Ex. 2. Multiply £9. 19*s.* $7\frac{3}{4}$*d.* by 17.

£.	*s.*	*d.*
9	19	$7\frac{3}{4}$
		17
£169	13	$11\frac{3}{4}$

3*q.* × 17 = 51*q.* = 12*d.* 3*q.* = $12\frac{3}{4}$*d.*; *carry* 12*d.*
7*d.* × 17 = 119*d.*; 119*d.* + 12*d.* = 131*d.* = 10*s.* 11*d.*; *carry* 10*s.*
19*s.* × 17 = 323*s.*; 323*s.* + 10*s.* = 333*s.* = £16. 13*s.*; *carry* £16
£9 × 17 = £153; £153 + £16 = £169.

140. When the Multiplier is a number greater than 20, and can be resolved into two or more factors none of which is greater than 20, multiply by each of these factors in succession, and the last result will be the product required. (Art. 81.)

Ex. 3. Multiply *Rs.*99. 13*a.* 9*p.* by 28 and £9. 19*s.* [illegible] by 42.

28 = 4 × 7.

Rs.	*a.*	*p.*
99	13	9
		4
*Rs.*399	7	0
		7
*Rs.*2796	1	0

42 = 6 × 7.

£.	*s.*	*d.*
9	19	$7\frac{3}{4}$
		6
£59	17	$10\frac{1}{2}$
		7
£419	5	$1\frac{1}{2}$

141. When the Multiplier exceeds or falls short of a product by a small number, multiply by such product and then by this number and add or subtract for the required product.

Ex. Multiply *Rs.*240. 7*a.* 10*p.* by 29, and £17. 8*s.* 5¼*d.* by 139.

$29 = 28 + 1 = 4 \times 7 + 1$

Rs.	*a.*	*p.*	
240	7	10	
		4	
*Rs.*961	15	4	
		7	
*Rs.*6733	11	4	product by 28
240	7	10	 1
*Rs.*6974	3	2	 29

$139 = 144 - 5 = 12 \times 12 - 5$.

£.	*s.*	*d.*	
17	8	5¼	
		12	
£209	1	3	
		12	
£2508	15	0	product by 144
87	2	2¼	 5
£2421	12	9¾	 139

142. When the Multiplier is a very large number.

RULE. Multiply by 10 as many times in succession as there are figures in the multiplier less 1; then multiply the given quantity by the units' figure of the multiplier, the first product by the tens' figure, the second product by the hundreds' figure and so on. The sum of these partial products will give the required product.

Ex. Multiply £16. 12*s.* 9¼*d.* by 7249.

£.	*s.*	*d.*		£.	*s.*	*d.*		
16	12	9¼	×9=	149	14	11¼	product by	9
		10						
£166	7	8½	×4=	665	10	10	....	40
		10						
£1663	17	1	×2=	3327	14	2	... .	200
		10						
£16638	10	10	×7=	116469	15	10	....	7000
				£120612	15	9¼	....	7249

143. When the multiplier is a large number, as in the above example, and we are told to proceed by Compound Multiplication, the following is the simplest method.

£.	*s.*	*d.*
16	12	9¼
		7249
£120612	15	9¼

4)	7249*q.*	= 1*q.* × 7249
	1812 ...1*q.*	
	65241	= 9*d.* × 7249
12)	67053	
	5587...9*d.*	
	86988	= 12*s.* × 7249.
2,0)	9257,5	
	4628...15*s.*	
	115984	= £16 × 7249.
	£120612	

144. In compound multiplication we may reduce the multiplicand to the lowest denomination contained in it, then multiply this result by the multiplier, and then reduce the product back again. This method is generally tedious.

Ex. Multiply £5045. 6*s*. 2¼*d*. by 4342.

£5045. 6*s*. 2¼*d*. = 4843497*q*.

and 4843497*q*. × 4342 = 21030463974*q*.

and 21030463974*q*. = £21906733 6*s*. 1½*d*. *Ans*.

Examples XXIX.

1. Multiply :—

(1) *Rs*. 18. 8*a*. 4*p*. by 2 ; *Rs*. 42. 10*a*. 6*p*. by 3 ; *Rs*. 67. 11*a*. 6*p*. by 8.
(2) *Rs*. 51 11*a*. 7*p*. by 4 ; *Rs*. 67. 13*a*. 9*p*. by 7 ; *Rs*. 58. 2*a*. 7*p*. by 6.
(3) *Rs*. 65. 12*a*. 8*p*. by 5 ; *Rs*. 84. 11*a*. 5*p*. by 11.
(4) *Rs*. 48. 14*a* 10*p*. by 9 ; *Rs*. 66. 3*a*. 4*p*. by 18.
(5) £19. 18*s*. 7½*d*. by 8 ; £3 9*s*. 7¾*d*. by 12 ; £87. 8*s*. 11¾*d*. by 10.
(6) £37. 19*s*. 9½*d* by 9 ; £374. 12*s*. 10¼*d*. by 7.
(7) £549. 13*s*. 7¾*d*. by 11 ; £49. 13*s*. 0¾*d*. by 19.
(8) £497. 19*s*. 7¼*d*. separately by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.
(9) *Rs*. 666 10*a*. 9*p*. ... ... 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.
(10) £6. 12*s*. 5*d*. ... ... 13, 14, 16, 18 and 20
(11) *Rs*. 104. 12*a*. 5*p*. ... ... 13, 14, 15, 17, 19 and 20.
(12) £86. 10*s*. 7¾*d*. ... ... 15, 16, 17, 18 and 19.

2. Multiply (by factors) :—

(1) *Rs*. 194. 8*a*. 7*p*. by 24 ; *Rs*. 320. 14*a*. 10*p*. by 21.
(2) *Rs*. 586. 13*a*. 10*p*. by 64 ; *Rs*. 142. 0*a*. 9*p*. by 132.
(3) *Rs*. 1005. 12*a*. 3*p*. by 72 ; *Rs*. 133. 6*a*. 6*p*. by 75.
(4) *Rs*. 205. 4*a*. 3*p*. by 108 ; *Rs*. 140. 2*a*. 6*p*. by 144.
(5) *Rs*. 249. 15*a*. 5*p*. by 198 ; *Rs*. 8967. 8*a*. 6*p*. by 351.
(6) £98. 18*s*. 3*d*. by 96 ; £68. 7*s*. 4¾*d*. by 35 ; £13. 7*s*. 4¾*d*, by 275.
(7) £99. 0*s*. 7¾*d*. by 77 ; £4. 8*s*. 9¼*d*. by 121 ; £13. 15*s*. 6½*d*. by 132
(8) *Rs*. 277. 5*a*. 2*p*. by 216 ; *Rs*. 1230. 10*a*. 1*p*. by 224.

3. Multiply (by factors and parts) :—

(1) *Rs*. 77. 2*a*. 4*p*. by 23 ; *Rs*. 13. 15*a*. 4*p*. by 62 ; £9. 19*s*. 7¾*d*. by 31.
(2) *Rs*. 3. 15*a*. 10*p*. separately by 67, 71 and 79.
(3) *Rs*. 398. 15*a*. 2*p*. ... ... 69, 59 and 41.
(4) £130. 18*s*. 9¼*d*. ... ... 89, 93 and 113.
(5) £808. 12*s*. 7½*d*. ... ... 79, 83 and 131.
(6) *Rs*. 19. 11*a*. 6*p*. ... ... 379 and 845.
(7) *Rs*. 8. 14*a*. 2*p*. ... ... 1234 and 5678.
(8) *Rs*. 37. 15*a*. 6*p*. ... ... 9803 and 5840.
(9) £5045. 6*s*. 2¼*d*. ... ... 923, 956 and 2765.

4. Multiply :—

(1) £324. 12*s*. 6½*d*. by 394 ; £2. 16*s*. 9½*d*. by 702.

(2) *Rs*. 19. 4*a*. 6*p*. by 3210; *Rs*.23. 6*a*. 2*p*. by 3684.
(3) £9. 15*s*. 10$\frac{3}{4}$*d*. by 4508; £3. 18*s*. 11$\frac{1}{4}$*d*. by 57089.
(4) *Rs*. 42. 4*a*. 4*p*. separately by 3005 and 7082.
(5) £567. 13*s*. 8$\frac{3}{4}$*d*. ... ... 8736 and 98736.

5. Find the values of :—

(1)	19 things at	3*a*. 2*p*.	each.	(2)	156 things at	11*a*. 6*p*.	each.
(3)	96	9*a*. 5*p*.	...	(4)	315	15*a*. 8*p*	...
(5)	428	16*s*. 10$\frac{1}{2}$*d*.	...	(6)	728	7*s*. 7$\frac{3}{4}$*d*.	...
(7)	943	*Rs*.4 2*a*.	...	(8)	625	£1. 13*s* 6*d*	...
(9)	729	*Rs*.7. 5*a* 3*p*.	...	(10)	829	*Rs*. 8 11*a*. 5*p*.	...
(11)	1502	19*s*. 5$\frac{3}{4}$*d*.	...	(12)	2014	17*s* 6*d*	...

6. Make out the following bills :—

(1) 17 yards of calico at 6*a*. 6*p*. per yard : 143 yards of long cloth at 12*a*. 10*p*. per yard ; 14 yards of merino at *Rs* 2 3*a* 6*p* per yard ; 204 yards of flannel at 14*a*. 9*p*. per yard ; 456 yards of linen at *Re*. 1. 12*a*. per yard ; and 755 yards of silk at *Rs* 3 5*a*. 4*p*. per yard.

(2) 40 seers of Assam Tea at *Rs*.3. 3*a*. 4*p*. per seer : 65 lbs. of China Tea at *Rs*.2. 5*a*. 4*p*. per lb. ; 35 seers of coffee at *Re* 1 12*a*. 4*p*. per seer ; 145 seers of sugar at 7*a*. 4*p*. per seer ; and 122 seers of best sugar at 10*a*. 4*p*. per seer

(3) 23 yards of silk at 5*s*. 4$\frac{1}{2}$*d*. per yard ; 5 yards of velvet at 13*s* 6*d*. per yard ; 8 yards of velveteen at 3*s*. 11$\frac{1}{2}$*d*. per yard ; 13 yards of linen at 3*s*. 2*d*. per yard ; 19 yards of flannel at 1*s* 9*d*. per yard ; and 26 yards of calico at 11$\frac{3}{4}$*d*. per yard.

7. A man distributed a certain sum of money to 79 poor persons and gave £17. 12*s*. 9$\frac{1}{4}$*d*. to each ; find the sum of money distributed

8. A bankrupt's estate can pay 14*a*. 10$\frac{1}{4}$*p*. in the rupee, what will a creditor receive who has lent 3125 rupees, and how much will he lose ?

9. How much money must be added to £1000 that each of 33 people may receive £35. 3*s*. 4*d*. ?

10. A gowala exchanges 59 calves each worth *Rs*.15. 10*a*. for 37 cows each worth *Rs*.26. 4*a*. ; ought he to receive, or to pay any money ? How much ?

11. If I spend £2. 7*s*. 1$\frac{1}{4}$*d*. a day, how much is that in a year of 365 days ?

12. There are 53 chests of drawers ; in each chest there are 4 drawers ; in each drawer there are 10 compartments and in each compartment there are deposited £32. 5*s*. 6*d*. How much money is deposited in the chests ?

V. COMPOUND DIVISION.

145. **Compound Division** is the method by which (1) we break up a compound quantity into as many *equal parts* as there are

units in a given number, and thus find the value of one of these parts, (2) we find how many times one compound quantity is contained in another of the same kind. The first method is called **Partition** and the second **Quotition**.

146. In the first case the divisor is an abstract number, and the quotient telling *the value of each part* is a compound quantity of the same kind as the dividend. In the second case the divisor is a compound quantity of the same kind as the dividend, and the quotient telling *how many times* is an *abstract* number.

147. *When the divisor is an abstract number.*

RULE. Place the dividend and divisor as in Simple Division. Find how often the divisor is contained in the highest denomination of the dividend, put down the quotient, and reduce the remainder (if any), to the next inferior denomination. Add to it the number of that denomination in the dividend, and repeat the division. Continue the process step by step through all the denominations.

(1) When the divisor does not exceed 20, the division can be performed *mentally* thus :—

Ex. Divide *Rs.*436. 5*a.* 4*p.* by 11.

```
   Rs.  a.  p.     Rs.436 ÷ 11 is Rs.39 with Rs.7 over.
11)436   5   4     Rs.7 = 112a., with 5a. = 117a. ;
Rs. 39  10   8          117a. ÷ 11 is 10a. and 7a. over.
                   7a. = 84p., with 4p. = 88p., which ÷ 11 is 8p.
```

(2) When the divisor is a number larger than 20.

Proceed as in the following Examples.

Ex. Divide £52. 10*s.* 7½*d.* by 41, and *Rs.*3441. 5*a.* 9*p.* by 129.

```
(1)     £.   s.   d.              (2)      Rs.    a.   p.
   41) 52   10   7½ (1£           129) 3441    5    9 (26Rs.
       41                              258
       11                              861
       20                              774
      230 (5s.                          87
      205                               16
       25                             1397 (10a.
       12                              129
      307 (7d.                         107
      287                               12
       20                             1293 (10p.
        4                              129
       82 (2q.                           3p.
       82
```

∴ the required quotient is *Rs.*26. 10*a.* 10*p.* and 3*p.* over.

∴ the required quotient is £1. 5*s.* 7½*d.*

148. When the divisor is the product of two or more factors, divide by each of them successively, and find the remainder as in Simple Division.

Ex. Divide £1478. 13*s*. 8¾*d*. by 77.

	£.	*s*.	*d*.	
77 { 7)	1478	13	8¾	
11)	211	4	9¾	...... 2*q*. or 44*q*.
	£19	4	0¾	......44*q*.

The final remainder is $6 \times 7 + 2$ 2*q*. or 44*q*. ∴ the quotient is £19. 4*s*. 0¾*d*. and 11*d*. over.

149. When there is a **remainder** after division, we can always find a quotient which is correct **to the nearest pie** or **farthing** by the following Rule.

RULE. Neglect the remainder, if it is *less* than the divisor divided by 2 ; but *otherwise* add 1*p*. or 1*q*. to the quotient.

Ex. Find to the *nearest pie* the result of dividing *Rs*.727. 15*a*. 10*p*. by 67, and to the *nearest farthing* £333. 19*s*. 4¾*d*. by 29.

(1) *Rs*. *a*. *p*.

67) 727 15 10 (10*Rs*.
67
57
16
927 (13*a*.
67
257
201
56
12
682 (10*p*.
67
12

Here $2 \times 12 = 24$, which is less than 67.
∴ Quotient is *Rs*.10. 13*a*. 10*p*

(2) £. *s*. *d*.

29) 333 19 4¾ (11£.
29
43
29
14
20
299 (10*s*.
29
9
12
112 (3*d*.
87
25
4
103 (3*q*.
87
16

Here $2 \times 16 = 32$, which is greater than 29.
∴ Quotient £11. 10*s*. 4*d*.

150. *When the divisor is 10, 100, 1000, &c.*

RULE. Cut off from the right of each succeeding dividend as many figures as there are ciphers in the divisor ; the figures to the left will at each step give the quotient and the figures to the right the remainder.

Ex. Divide *Rs.*1179. 2*a.* 8*p.* by 100, and £9797. 5*s.* 6*d.* by 900.

(1)

```
     Rs.     a.  p.
100) 11,79   2   8
        16
     a.12,66
        12
     p.8,00
```

∴ Quotient = *Rs.*11. 12*a.* 8*p.*

(2)

```
        £.        s.   d.
     9)9797       5    6
100)£10,88       11    8½...6q.
        20
     s.17,71
        12     The final remainder
      d 8,60   is 42×9+6 or 384q.
         4     or 8s.
      q.2,42
```

∴ Quotient = £10. 17*s.* 8½*d.* and 8*s.* over.

Examples XXX.

1. Divide :–

(1) *Rs.*11. 13*a.* 8*p.* by 2 ; *Rs.*393. 14*a.* 4*p.* by 7 ; *Rs.*328. 15*a.* 4*p.* by 5.
(2) *Rs.*5161. 9*a.* 4*p.* by 3 ; *Rs.*440. 5*a.* 6*p.* by 9 ; *Rs.*436. 5*a.* 4*p.* by 11.
(3) *Rs.*5392. 1*a.* 4*p.* by 8 ; *Rs.*576. 8*a.* by 12 ; *Rs.*1721. 7*a.* 10*p.* by 14.
(4) £26. 15*s.* 3½*d.* by 2 ; £87 16*s.* 8½*d.* by 9 ; £614. 2*s.* 6¾*d.* by 7.
(5) £79. 13*s.* 9*d.* by 12 ; £147. 11*s.* 6¾*d.* by 15 ; £95. 2*s.* 3¾*d.* by 11.
(6) £241. 8*s.* 8¾*d.* by 63 ; £1990. 10*s.* 9*d.* by 42 ; £75. 1*s.* 10½*d.* by 45.
(7) *Rs.*8370. 15*a.* separately by 17, 51 and 126.
(8) *Rs.*12342. 12*a.* 2*p.* ... ... 19, 59 and 325.
(9) *Rs.*3253. 15*a.* ... ... 23, 87 and 712.
(10) £1302. 18*s.* by 144 ; £890. 12*s.* 6*d.* by 125.
(11) £75. 6*s.* 4½*d.* by 103 ; £4718. 14*s.* 8*d.* by 132.
(12) £7549. 17*s.* 6*d.* by 859 ; £77573. 18*s.* 9¾*d.* by 4578.

2. Divide by the *short* method :—

(1) £239. 14*s.* 4½*d.* separately by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.
(2) *Rs.*1088. 12*a.* separately by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12.
(3) *Rs.*1877. 7*a.* 4*p.* by 14 ; *Rs.*2757. 9*a.* by 18 ; *Rs.*7023. 2*a.* by 20.
(4) £623. 5*s.* 4½*d.* by 13 ; £318. 10*s.* 7*d.* by 14 ; £139. 13*s.* 8*d.* by 16.

3. Divide by using factors :—

(1) *Rs.*517. 11*a.* 4*p.* by 35 ; *Rs.*34. 11*a.* by 45 ; *Rs.*3868. 3*a.* 6*p.* by 99.
(2) *Rs.*3639. 1*a.* 6*p.* by 81 ; *Rs.*3191. 6*a.* by 132 ; *Rs.*5761. 8*a.* by 14[illegible]
(3) £579. 18*s.* by 45 ; £1328. 13*s.* 6*d.* by 56 ; £453. 11*s.* 6¾*d.* by 77.
(4) £374. 10*s.* 3*d.* by 108 ; £576. 3*s.* by 144 ; £386. 16*s.* 5¾*d.* by 99.

4. Divide :—

(1) *Rs.*2625. 1*a.* 8*p.* by 10.
(2) £176. 16*s.* 8*d.* by 10.
(3) *Rs.*3395. 13*a.* 4*p.* by 100.
(4) £73. 12*s.* 11*d.* by 100.

(5) £876. 2s. 11d. by 100. (6) *Rs.*1151. 9*a*. 2*p*. by 1000.
(7) £9658. 17*s*. $3\frac{1}{2}$*d*. by 1000. (8) *Rs.*4579. 2*a*. 8*p*. by 400.
(9) *Rs.*6925 by 800 ; *Rs.*3625 by 6000 ; *Rs.*11375 by 2400.
(10) £1556. 5*s*. by 3600 ; £513. 8*s*. 9*d*. by 3100 : £2559. 7*s*. 6*d*. by 18900.

5. Divide :—

(1) *Rs.*73298. 3*a*. 8*p*. separately by 842, 912 and 8317.
(2) *Rs.*84566. 2*a*. 8*p*. ... ... 392, 573 and 7856.
(3) *Rs.*56789. 15*a*. 8*p*. by 9357. (4) *Rs.*98767. 5*a*. 2*p*. by 10048.
(5) £6011656. 5*s* $8\frac{1}{4}$*d*. by 2331. (6) £467325. 10*s*. $1\frac{1}{2}$*d*. by 2803.
(7) £530866. 17*s*. 6*d*. by 2772. (8) £4420895. 0*s*. $3\frac{3}{4}$*d*. by 3001.

6. Find, to the *nearest pie* or *farthing*, the result of dividing :—

(1) *Rs.*33. 9*a*. 4*p*. by 9. (6) *Rs.*2684. 2*a*. 9*p*. by 241.
(2) *Rs.*511. 8*a*. 5*p* by 97. (7) *Rs.*523. 6*a*. 8*p*. by 100.
(3) *Rs.*29. 10*a*. 3*p*. by 31. (8) £1867. 16*s*. $8\frac{3}{4}$*d*. by 407.
(4) £150. 4*s*. 9*d*. by 12. (9) £15104. 19*s*. 2*d*. by 100.
(5) £74. 6*s*. $10\frac{3}{4}$*d*. by 23. (10) £2160. 18*s*. 11*d*. by 1000.

7. If *Rs.*2757. 9*a*. be equally divided among 18 people ; how much will each receive ?

8. A man spends *Rs.*5611. 14*a*. in a year of 365 days ; how much does he spend in a week of 7 days ?

9. After buying 15 books I have £2. 15*s*. $7\frac{1}{4}$*d*. left out of £7. What was the price of each book ?

10. The cost of 720 goats is *Rs.*712. 8*a*. ; what is the cost of each goat ?

11. 205 sovereigns, all equally light, are worth £201. 15*s*. $11\frac{1}{4}$*d*. ; find the worth of each.

12. A cattle-dealer bought 11 cows at *Rs.*8. 4*a*. each ; after spending *Rs.*26. 4*a*. in feeding them, he sells 3 of them for *Rs.*11. 4*a*. each ; at what price must he sell each of the others to gain *Rs.*23 by the bargain ?

---

151. *When the divisor is a compound quantity of the same kind as the dividend.*

RULE. Reduce the dividend and the divisor to the same denomination, and then proceed as in Simple Division.

Ex. 1. Divide *Rs.*113. 14*a*. 6*p*. by *Rs.*12. 10*a*. 6*p*.
*Rs.*113 14*a*. 6*p*. = 21870*p*. ; *Rs.*12. 10*a*. 6*p*. = 2430*p*.
∴ the quotient required = 21870 ÷ 2430 = 9. *Ans.*

Ex. 2. How many cricket balls each worth 5*s*. $7\frac{1}{2}$*d*. can I buy with £134. 14*s*. $4\frac{1}{2}$*d*. ?

£134 14*s*. $4\frac{1}{2}$*d*. = 129330*q*. ; 5*s*. $7\frac{1}{2}$*d*. = 270*q*.
∴ the number of balls = 129330 ÷ 270 = 479 *Ans.*

Examples XXXI.

1. Divide :—

(1) *Rs.*175. 9*a.* 4*p.* by *Re.*1. 12*a* 8*p.* ; *Rs.*854. 2*a.* 8*p.* by *Rs.*20. 13*a.* 4*p*

(2) *Rs.*438. 7*a.* by *Rs.*6. 5*a.* 8*p.* ; *Rs.*4012. 2*a.* by *Rs.*25. 11*a.* 6*p.*

(3) £28. 2*s.* 6*d.* by 12*s.* 6*d.* ; £150. 7*s.* 5*d.* by 6*s.* 3½*d.*

(4) £286. 3*s.* 2*d.* by £1. 11*s.* 1¼*d.* ; £144. 13*s.* 11½*d.* by 9*s.* 11¾*d.*

(5) *Rs.*22831. 1*a.* 6*p.* by *Rs.*66. 2*a.* 10*p.* ; £4808. 14*s.* by £7. 8*s.* 5*d.*

(6) *Rs.*200157. 8*a.* 10*p.* by *Rs.*576. 13*a.* 2*p.* ; £131. 4*s.* 4½*d.* by 10*s.* 7½*a*

2. How often is

(1) *Rs.*760. 6*a* 8*p.* contained in *Rs.*6843. 12*a.* ?

(2) *Rs.*3 12*a* 10*p.* ... ... *Rs* 2771. 11*a.* 6*p.* ?

(3) *Rs.*2. 15*a.* 4*p.* ... ... *Rs* 2366. 10*a*, 8*p.* ?

(4) £35 16*s.* 7¾*d* ... ... £9961. 7*s.* 6½*d* ?

(5) £2579. 0*s.* 0¾*d.* ... ... £399745. 9*s.* 8¼*d* ?

3. Find the quotient and the remainder in the division of :—

(1) *Rs* 9607. 15*a.* 10*p.* by *Rs.*26. 5*a.* 2*p.*

(2) *Rs* 1225. 11*a.* 9*p.* by *Rs* 55. 10*a.* 8*p.*

(3) £568. 13*s.* 8*d.* by £1 8*s.* 6*d.*

(4) £339. 14*s.* 7½*d.* by £4. 11*s.* 9¼*d.*

4. How many dollars worth 4*s.* 1½*d* each must be given in exchange for £235 10*s.* 9*d* ?

5. To how many persons may *Rs.*607. 12*a.* be distributed giving *Rs.*46 12*a.* to each ?

6. How many hats each costing £1. 2*s.* 3½*d.* can be bought for £134. 17*s.* 3½*d.* ?

7. How many cows at *Rs.*108. 12*a.* each can I buy with the proceeds of selling 87 horses at *Rs.*1151. 4*a.* each ?

8. How many days must a labourer work at 2*s.* 1½*d.* a day to earn £51 ?

9. I buy a number of books at 2*s.* 9½*d.* each and sell them at 3*s.* 3*d.* each. If I thereby make a profit of £2. 4*s.*, how many books must I buy.

10. I buy 60 gallons of wine at £1. 3*s.* 6*d.* a gallon and £1. 10*s.* is gained by selling it at £1 2*s.* 6*d.* a gallon. How much water is added ?

II. MEASURES OF WEIGHT.

151. Indian Bazar Weight.

4 Sikis	make	1 **Tola**
5 Sikis	,,	1 **Kancha**

4 Kanchas or 5 Tolas	make	1 Chhatak (*ch.*)
4 Chhataks	,,	1 Poa
4 Poas or 16 Chhataks	,,	1 **Seer** (*sr.*)
5 Seers	,,	1 Pasari
8 Pasaris or 40 Seers	,,	1 **Maund** (*md.*)

The weight of a rupee is called a *tola*. A seer = 80 tolas.
49 Bazar maunds = 54 Factory maunds.

BOMBAY LOCAL WEIGHT.

4 Dhans	make	1 Ratika
8 Ratikas	,,	1 Masha
4 Mashas	,,	1 Tank
72 Tanks	,,	1 Seer
40 Seers	,,	1 Maund
20 Maunds	,,	1 Kandi.

MADRAS LOCAL WEIGHT.

180 Grains	make	1 Tola
3 Tolas	,,	1 Palam
8 Palams	,,	1 Seer
5 Seers	,,	1 Vis
8 Vis	,,	1 Maund
20 Maunds	,,	1 Kandi.

49 Bazar maunds = 144 Bombay maunds. 175 Bazar maunds = 576 Madras maunds. 25 Bombay maunds = 28 Madras maunds.

153. English Standard Weight. (*Avoirdupois.*)

16 Drams (dr.)	make	1 Ounce (oz.)
16 Ounces	,,	1 **Pound** (lb.)
28 Pounds	,,	1 Quarter (qr.)
4 Quarters or 112 lbs.	,,	1 Hundred-weight (cwt.)
20 Hundred-weights	,,	1 Ton.

1 Stone = 14 lbs. ; 1 Cental = 100 lbs.

A stone of butcher's meat	= 8lbs.
A sack of Coal	= 2 cwt.
A barrel of Gunpowder	= 100lbs.
A Pack of wool.	= 240lbs.
A Firkin of Butter	= 56lbs.
A Great Pound of Silk	= 24 oz.
A sack of flour	= 280 lbs.
A barrel of ,,	= 196 lbs.
A peck of ,,	= 14 lbs.
A quartern loaf	= 4 lbs.
A pocket of Hops	= 168 lbs.
Two Fodders of Lead	= 39 cwt.

A pound (Avoirdupois) = 7000 grains (Troy) ; 7 Bazar maunds = 576lbs. (Avoir.) ; 1 Bombay maund = 28lbs. (Avoir.) ; 1 Madras maund = 25 lbs. (Avoir.) ; 3 Factory maunds = 2 cwt. ; 35 seers = 72lbs. (Avoir.)

The Jeweller's Tables.

INDIAN JEWELLER'S WEIGHT.

4 Dhans	make	1 Rati (*ra.*)
6 Ratis	,,	1 Anna (*a.*)
8 Ratis	,,	1 Masha (*ma.*)
12 Mashas or 16 annas	,,	1 Tola or Bhari

ENGLISH TROY WEIGHT.

24 Grains (gr.)	make	1 Penny-weight (dwt.)
20 Penny-weights	,,	1 Ounce (oz.Tr.)
12 Ounces or 5760 grains	,,	1 Pound (lb.Tr.)

1 Tola = 180 grs. Troy ; 1 Bazar maund = 100lbs. Troy ; 1 pound = 32 tolas ; a *Carat* = $3\frac{1}{8}$ grs. (for weighing diamonds).

[Gold, silver, jewels and precious stones are weighed by the Troy weight].

Measures of Weight for Medicines.

BENGAL PHYSICIANS' WEIGHT.			ENGLISH APOTHECARIES' WEIGHT.		
4 Dhans	make	1 Rati	20 Grains	make	1 Scruple (℈)
10 Ratis	"	1 Masha	3 Scruples	"	1 Dram (ʒ)
8 Mashas	"	1 **Tola**	8 Drams	"	1 Ounce (℥)
			12 Ounces	"	1 Pound (℔)

[The Apothecaries' weight is now out of use, except in selling drugs by retail].

144lbs. (Avoir.) = 175lbs. (Troy or Apoth.); 1 lb. (Troy or Apoth.) = 5760 grains; 1 lb (Avoir.) = 7000 grs. Troy; 1 lb. (Avoir.) + the weight of a double-pice (200 grs.) = half-a-seer.

Note. The term 'carat' applied to gold has a relative meaning only; any quantity of pure gold, or of gold alloyed with some other metal, being supposed to be divided into 24 equal parts, called *carats*; if the gold be pure, it is said to be 24 carats fine; if 22 parts be pure gold and 2 parts alloy, it is said to be 22 carats fine.

Standard gold is 22 carats fine; *Jeweller's* gold is 18 carats fine.

Ex. 1. Reduce 14cwt. 3qrs. 24lbs. to *ounces*, and 32250 kanchas to *maunds*.

(1)
```
cwt.  qrs.  lbs.
 14    3    24
  4
 59 qrs.
 28
1676 lbs.
 16
26816 oz.  Ans.
```

(2)
```
     4)32250 kan.
    16) 8062 ch......2 kan.
   4,0)  50,3 sr......14 ch.
         12  md.......23 sr.
```

∴ the result = 12 mds. 23 sr. 14 ch. 2 kan.

Ex. 2. Reduce 425095 grs. of gold to lbs. &c., and 11ka. 13 mds. 3 vis 5 palams 2 tolas to *tolas* (Mad.).

(1)
```
24 { 4)425095 grs.
   { 6)106273...3   }
 2,0) 17712 dwts. 1 } 7 grs.
  12)   885 oz......12 dwts.
         73 lbs......9 oz.
```

∴ the result = 73 lbs. 9 oz. 12 dwts. 7grs.

(2)
```
ka.  md.  vis  pa.  tolas
11   13   3    5    2
20
233 mds.
  8
1867 vis
 40
74685 palams
    3
224057 tolas.  Ans.
```

Examples XXXII.

(Indian Bazar and Avoirdupois Weights.)

1. Reduce (i) to *kanchas* and (ii) to *tolas* :—

(1) 20 mds. 13 sr. 7 ch. ; 12 mds. 15 sr. 10 ch. ; 75 mds. 32 sr. 15 ch.
(2) 46 mds. 25 sr. 12 ch. ; 25 mds. ; 45 mds. 12 sr. 8 ch.

2. Reduce to *kanchas* :—

(1) 30 mds. 27 sr. 12 ch. 2 kan. ; 45 mds. 30 sr. 8 ch. 1 kan.
(2) 210 mds. 15 sr. 2 ch. 3 kan. ; 220 mds. 17 sr. 3 kan.

3. Reduce to *maunds &c.* :—

4123000 kan. ; 30205676 kan. ; 15025276 tolas ; 4876235 poas ; 4320578 ch. ; 4362508 tolas ; 782504 poas.

4. Reduce to *tolas* :—

2mds. 5vis 4sr. ; 5kan. 15mds. 4vis ; 8kan. 14mds. 7vis 5palams.

5. Reduce to *dhans* :—

2 mds. 14 sr. 57 ta. ; 8 kan. 16 mds. 25 sr. 55 ta. 3 m. ; 10 kan. 10 mds. ; 39 sr. 16 ta. 3 m. 2 rat.

6. Reduce 156728306 tolas to *kandis* ; 460879025 dhans to *kandis* ; 786250 tanks to *kandis* ; 4586 seers to *kandis.*

7. Reduce :—

(1) 11 cwt. 2 qrs. 17 lbs. 15 oz. to *ounces* ; 3 cwt. 13 lbs. to *ounces.*
(2) 6 tons 5 st. to *ounces* ; 4 tons 15 cwt. 2 qrs. 12 lbs. to *pounds.*

8. Reduce to *drams* :—

(1) 2 tons 10 cwt. 1 qr. 2 lbs. 3 oz. 3 drs. ; 3 tons 14 cwt. 3 qrs. 25 lbs. 11 oz. 9 drs. ; 3 tons 3 qrs. 3 oz. ; 27 lbs.

9. Reduce to *tons, cwt., &c.* :—

(1) 87654 lbs. ; 378539 oz. ; 1693539 drs. ; 65437 drs.

(2) 2345820 drs. ; 1008001 oz. ; 237023 oz. ; 59653007 st.

(3) 21633 lbs. ; 17739853 oz. ; 5390054 drs. ; 713969416 drs.

10. Add together :—

(1)

mds.	sr.	ch.
25	10	5
110	12	3
115	20	12
97	27	15
102	15	7

(2)

mds.	sr.	ch.	kan.
65	10	10	2
72	15	8	2
102	14	10	3
125	30	13	2
207	32	15	1

(3)

mds.	sr.	ch.	kan.
115	30	7	1
202	27	10	2
323	15	12	3
222	9	5	2
313	32	14	3

(4)

tons	cwt.	qrs.	lbs.	oz.
16	0	3	5	15
8	16	0	0	14
28	8	1	27	6
210	6	3	14	11
17	17	0	15	12
[illegible]	15	3	18	13

(5)

cwt.	qrs.	lbs.	oz.
32	2	15	12
47	0	25	7
5	3	17	10
23	1	19	15
1	2	10	8
9	3	0	14

(6)

tons	cwt.	qrs.	lbs.
32	12	2	25
18	15	0	20
23	10	1	16
14	18	1	27
25	4	0	3
35	12	2	19

11. Perform the operation of subtraction in the following :—

(1)

mds.	sr.	ch.
530	10	12
396	27	15

(2)

mds.	sr.	ch.	kan.
672	12	10	0
127	24	14	3

(3)

mds.	sr.	ch.	kan.
427	10	10	1
212	25	14	3

(4)

cwt.	qrs.	lbs.
47	0	12
32	3	22

(5)

tons	cwt.	qrs.	lbs.
75	7	1	16
41	14	2	19

(6)

cwt.	qrs.	lbs.	oz.
112	2	23	8
59	0	27	10

12. Multiply :—

(1) 110 mds. 20 sr. 12 ch. separately by 24, 36 and 72.
(2) 225 mds. 22 sr. 13 ch. 2 kan. ... ... 144, 126 and 360.
(3) 20 tons 3 qrs. 12 lbs. ... ... 132 and 143.
(4) 25 tons 18 cwt. 2 qrs. 15 lbs. ... ... 144 and 1728.
(5) 8 tons 87 lbs. 13 drs. ... ... 18, 29, 47 and 133.

13. Divide :—

(1) 252 mds. 10 sr. 12 ch. separately by 63 and 84.
(2) 1230 mds. 22 sr. 15 ch. ... ... 112 and 336.
(3) 3125 mds. 10 sr. 10 ch. 2 kan. ... ... 167 and 4008.
(4) 48 tons 17 cwt. 3 qrs. 27 lbs. 1 oz. ... ... 9, 17 and 500.
(5) 30 tons 15 cwt. 2 qrs. 15 lbs. ... ... 144 and 864.
(6) 1061 cwt. 2 qrs. by 37 cwt. 3 qrs. 18 lbs.; 89 cwt. 22 lbs. by 3 cwt. 1 qr. 6 lbs.; 404 mds. 35 sr. 2 ch. 2 kan. by 23 mds. 32 sr. 10 ch. 2 kan.

14. If 41 cwt. cost £52. 10*s*. 7½*d*., what is the price of a cwt. ?

15. A chest of tea weighing 1 cwt. 1 qr. 15 lbs. cost £22. 8*s*. 10½*d*. what is the cost of 1 lb. ?

16. At a school feast the children on the average ate 9 oz. of cake a piece, and 84 lbs. 6 oz. of cake were eaten : how many children were there in the school ?

(Indian Jeweller's and Troy Weights.)

1. Reduce to *dhans* :—

25 tolas 10 m. 4 r. 3 dh. ; 150 tolas 14 a. 5 r.; 162 tolas 13 a. 3 r. 2 dh.

2. Reduce to *tolas* :

56430 dhans ; 53426 ratis ; 37484 dhans ; 32458 ratis.

3. Reduce to *grains* :—

(1) 12 lbs. 10 oz. 15 dwts. 14 grs.; 15 lbs. 11 oz. 17 grs.; 9 lbs. 18 dwts.
(2) 16 lbs. Troy ; 9 oz. 17 dwts. 22 grs.; 165 oz. 280 grs.

4. Reduce to *lbs. etc.* (Troy) :—

13600 grs.; 146320 grs.; 400903 dwts.; 6739 oz.; 873521 grs.

5. Add together :—

(1)

tolas	m.	r.	dh.
45	10	7	3
63	8	6	2
58	9	5	1
62	11	7	3
39	8	4	2
112	6	3	1

(2)

tolas	a.	r.	dh.
47	10	3	2
52	9	2	1
65	8	5	3
77	13	4	0
82	14	5	3
75	12	4	2

(3)

lbs.	oz.	dwts.	grs.
64	11	16	14
21	10	12	13
2	0	1	16
12	10	0	18
24	11	12	0
14	1	0	1

6. Perform the operation of subtraction in the following :—

(1)

tolas	m.	r.	dh.
530	8	4	2
327	12	6	3

(2)

tolas	a.	r.	dh.
579	11	3	2
380	14	5	3

(3)

lbs.	oz.	dwts.	grs.
81	10	9	18
14	11	12	19

(4)

lbs.	oz	dwts.	grs.
225	8	14	15
167	11	18	19

(5)

lbs.	oz.	dwts.	grs.
13	0	7	18
11	11	15	23

(6)

tolas	a.	r.	dh.
467	10	3	0
279	13	5	2

7. Multiply :—

(1) 115 tolas 7 m. 5 r. 1 dh. separately by 72 and 80.
(2) 210 tolas 10 a. 2 r. 2 dh. ... ... 132 and 143.
(3) 22 lbs. 7 oz. 12 dwts. 20 grs. ... 64 and 96.
(4) 83 lbs. 17 dwts. 5 grs. ... ... 26, 131 and 257.

8. Divide :—

(1) 1125 tolas 8 m. 6 r. 3 dh. separately by 132 and 144.
(2) 1020 tolas 12 a. 4 r. 2 dh. ... ... 172 and 516.
(3) 606 lbs. 4 oz. 15 dwts. 20 grs. ... ... 131 and 500.
(4) 110 lbs. 10 oz 14 dwts 16 grs. ... ... 136 and 272.
(5) 2025 lbs. 2 oz. 18 dwts. 8 grs. by 5 lbs. 6 oz. 280 grs.

9. If 28 lbs. 9 oz. of gold be worth £1343. 6s. 10½d., what is the worth of 1 ounce ?

10. A certain number of forks, each weighing 3 oz. 5 dwts. and double that number of spoons, each weighing 3 oz. 10 dwts. are made out of 10 bars of silver, each weighing 3 lbs. 5 oz. ; find the number of spoons.

(*Native Physicians' and Apothecaries' Weights.*)

1. Reduce to *dhans* :—

25 tolas 6 m. 8r. 3 dh. ; 32 tolas 5 m. 9 r. 2 dh.; 8 tolas 7 m. 5r. 2 dh.

2. Reduce to *tolas* :—

1224 ratis ; 13200 dhans ; 426507 dhans ; 2406 ratis.

3. Reduce to *grains* :—

3 lbs. 5 ℥. 19 grs.; 2 lbs. 4 drs. 2 scr. ; 18 lbs. 2 oz. 4 drs. 2 scr. 12 grs.

4. Reduce to *pounds, &c.* :—

270083 grs. ; 26984 ℈ ; 92200 grs. ; 51960 grs. ; 17599 grs.

5. Add together :—

(1)

tolas	m.	r.	dh.
25	7	8	3
32	5	7	2
49	7	5	2
55	3	6	1
60	6	5	3
79	4	6	2

(2)

oz.	drs.	scrs.	grs.
11	4	2	11
10	3	0	4
16	0	1	14
10	0	1	16
6	2	2	18
14	5	1	0

(3)

lbs.	℥	ʒ	℈	grs.
15	3	5	1	17
18	10	6	2	5
20	9	1	2	12
25	7	3	0	18
36	5	4	1	14
26	8	6	2	15

6. Perform the operation of subtraction in the following :—

(1)

tolas	m.	r.	dh.
125	6	3	0
58	7	6	2

(2)

lbs.	oz.	drs.	scrs.	grs.
28	7	1	2	4
12	8	2	1	12

(3)

lbs.	℥	ʒ	℈
75	7	3	0
49	10	6	2

7. Multiply :—

(1) 32 tolas 6 m. 8 r. 3 dh. separately by 132 and 143.
(2) 45 lbs. 7 oz. 3 drs. 2 scrs. 8 grs. ... ... 16 and 64.
(3) 7 lbs. 6 ʒ 14 grs. ... ... 53, 71 and 2500.

8. Divide :—

(1) 65 tolas 7 m. 6 r. 2 dh. separately by 72 and 81.
(2) 120 lbs. 9 oz. 5 drs. 2 scrs. 12 grs. ... ... 120 and 132.
(3) 270 lbs. 5 ℥ 6 ʒ 2 scrs. ... ... 46, 53 and 1000.
(4) 7 lbs. by 2 ʒ 2 ℈ and 1234 lbs. 68 ℈ by 44 lbs. 23 ℈.

9. How many pills, each containing 6℈ 2 grs. can be made out of 2 lbs. 11 ℥ 6 ʒ of rhubarb?

154. *To convert from one system of weights to another.*

(1) To convert Indian weight into Troy, multiply the weight in tolas by 3 and divide by 8; the result will be the weight in *oz.* Troy. Or multiply the weight in tolas by 180; the result will be the weight in *grains* Troy.

Conversely, to convert Troy weight into Indian weight, multiply the weight in *oz.* Troy by 8 and divide by 3; the result will be the weight in *tolas*. Or divide the weight in grs. Troy by 180; the result will be the weight in *tolas*.

(2) To convert Indian weight into Avoir., multiply the weight in chhataks by 9 and divide by 70; the result will be the weight in lbs. Avoir. Or multiply the weight in seers by 72 and divide by 35; the result will be the weight in lbs. Avoir. Or multiply the weight in maunds by 36 and divide by 49; the result will be the weight in *cwts.* Avoir.

Conversely, to convert Avoir. weight into Indian weight, multiply the weight in lbs. Avoir. by 70 and divide by 9: the result will be the weight in *chhataks*. Or multiply the weight in lbs. Avoir. by 35 and divide by 72; the result will be the weight in *seers*. Or multiply the weight in cwts. by 49 and divide by 36; the result will be the weight in *maunds*.

(3) To convert lbs. Avoir. into Troy, multiply the weight in lbs. Avoir. by 7000; the result will be the weight in *grains* Troy. Conversely, to convert Troy weight into Avoir., multiply the weight in lbs. Troy by 144 and divide by 175; the result will be the weight in lbs. Avoir.

(4) As the weight in grains of both Apoth. and Troy weights is the same, therefore the one may be taken for the other.

Ex. 1. Convert 9 cwt. 3 qrs. 6 lbs. into *Indian weight.*

9 cwt. 3 qrs. 6 lbs. = 1098 lbs. = 1098 × 70 ÷ 9 ch. = 8540 ch.
= 13 mds. 13 sr. 12 ch. *Ans.*

Ex. 2. Convert 6 mds. 26 sr. 14 ch. into *cwt. etc.*, (Avoir.)

6 mds. 26 sr. 14 ch. = 4270 ch. = 4270 × 9 ÷ 70 lbs. (Avoir)
= 549 lbs. = 4 cwt. 3 qrs. 17 lbs. *Ans.*

Ex. 3. Reduce 1 cwt. 2 lbs. (Avoir.) to *Troy weight.*

1 cwt. 2 lbs. = 114 lbs = 114 × 7000 grs. Troy = 798000 grs.
= 138 lbs. 6 oz. 10 dwts. *Ans.*

Examples XXXIII.

1. Reduce to *tolas* :—

1440 grs.; 7 lbs. 7 oz. 17 dwts. 12 grs.; 16 lbs. 6 oz.; 2 oz. 5 dwts.

2. Reduce to *grains* (Troy) :—

(1) 16 sr. 8 ch.; 25 sr. 14 ch. 3 tolas; 1 md. 5 sr. 14 ch.; 4 vis. 15 palams 2 tolas.

(2) 8 tons 8 cwt. 98 lbs. 3045 grs.; 425 tons 19 cwt. 100 lbs 15 oz. 200 grs.; 1 cwt. 1 qr. 25 lbs.

3. Reduce 20 lbs. Avoir. to *Troy weight*; 16 dwts. to *Apoth. weight*; 5 drs. Apoth. to *Troy weight*; 525 lbs. Troy to *mds., sr., &c.*

4. Reduce 96 tolas to oz. *Troy*; 37400157 grains Troy to (Madras) *mds., vis, &c.*; 309432159 lbs. Avoir. to *mds., sr., &c.*

5. Reduce to *maunds, sr., &c.* :—

14 cwt. 1 qr. 24 lbs.; 10 cwt. 3 qrs. 20 lbs.; 3 tons 12 cwt. 1 qr. 8 lbs.; 3 tons 19 cwt. 8 lbs.; 4 cwt. 3 qrs. 8 lbs.

6. Reduce to *tons, cwt., &c.* :—

7 mds.; 15 mds. 38 sr. 12 ch.; 9 mds. 7 sr. 8 ch.; 10 mds. 20 sr.; 53 mds. 15 sr.; 21 mds. 35 sr.

7. Reduce 1137 lbs. 6 oz. Troy to *lbs. Avoir.*; 2 cwt. 3 qrs. 17 lbs. and 5 cwt. 18 lbs. 14 oz. to *Troy weight.*

8. Convert 6 tons 10 cwt. 65 lbs. into *Madras maunds*; 8 tons 2 cwt. 9 lbs. into *Bombay maunds*; 368 Bombay maunds and 140 Madras maunds into *tons, cwt., &c.*

9. How many 2 lbs. packets of tea can be made from a chest weighing 7 cwt. 3 qrs. 16 lbs.?

10. Each ton of ore obtained from a gold mine yields on an average 2 oz. 1 dwt. 15 grs. of fine gold. How much fine gold will be obtained from 293 tons ?

11. How many coins each weighing 1 oz. 8 dwt. can be made of 770 lbs. of metal ?

12. A truck is loaded with 120 sacks ; each sack weighs 7 sr. 10 ch., and contains 84 seers of grain. What is the weight of the whole in maunds and seers ?

13. How many pounds Avoir. are equal to 175 lbs. Troy

14. Multiply 88 ka. 12 mds. 16 sr. (Bombay) separately by 99, 66 and 144 ; and 4 ka. 5 mds. 15 sr. by 3268.

15. A train consists of 29 trucks of equal weight ; 9 of them weigh 53 tons 1 cwt. 1 qr. 3 lbs. What do the rest of them weigh ?

16. Convert 2 qrs. 16 lbs. into *seers* , 10 cwt. 1 qr. 13 lbs. into *maunds*, and 15 lbs. 2 oz. 5 dwts. 20 grs. into lbs. *Avoir.*

17. Express 576 lbs. Avoir as lbs. *Troy*, 58 lbs. 4 oz. Troy as lbs. *Avoir.*, and 16 dwts. 16 grs. in *Apoth-weight.*

18. Reduce 9720 grs. Troy to *tolas* and find how many *lbs.* are there in 12288 tolas ?

19. How many times is a weight of 6 tons 7 cwt. 27 lbs. 5 oz. contained in 159 tons 1 cwt. 10 lbs. 13 oz. ?

20. What is the whole weight of 217 waggon loads, each containing 3 tons 13 cwt. 3 qrs. 13 lbs. ?

21. 797 tons 19 cwt. 2 qrs. 14 lbs. is divided among a certain number of people, so that each receives 5 tons 3 cwt. 2 qrs. 15 lbs. How many of them were there ?

22. 84 poor men have distributed equally among them 252 mds. 10 sr. 12 ch. of rice ; what share will each receive ?

23. If 5 ka. 15 mds. 30 sr. of a certain article can be bought for a rupee, what quantity can be bought for 2384 rupees ?

24. 21 tons 3 cwt. 1 qr. 17 lbs. 5 oz. 8 drs. of rice are to be packed in bags of equal size. How many bags will be required if each hold 24 lbs. 6 oz. 8 drs. ?

25. Reduce 2457600 dhans to *maunds.*

26. Multiply 109 ka. 13 mds. 6 sr. (Madras) separately by 72, 35 and 750 ; and 5 ka. 15 mds. 30 sr. by 4503.

27. Divide :—

(1) 6 mds. 6 sr. 27 ta. (Bombay) by 73.

(2) 311 ka. 10 mds. 36 sr. 4 palams (Madras) by 503.

28. Divide 64 ka. 7 mds. 12 sr. by 15 mds. 13 sr. (Madras).

29. Divide 160 ka. 10 mds. 39 sr. by 15 mds. 3 sr. (Bombay).

30. If standard gold contained 12 parts of pure gold to 1 part of copper, and 247 oz. Troy were coined into 960 sovereigns; what would be the weight of pure gold in a sovereign?

31. How many bars of gold each weighing 5 oz. 13 dwts. 21 grs. can be made out of a bar weighing 88 lbs. 8 oz. 14 dwts. 15 grs.?

32. Find the weight of 73 iron bars, each weighing 17 cwt. 2 qrs. 19 lbs. 5 oz.

33. How many bars of iron each weighing 11 lbs. 10 oz. 11 drs. must be taken to make up a weight of 4 tons 8 cwt. 3 lbs. 6 oz. 15drs.?

34. Express in *Troy-weight* the weight of a silver dish weighing 3 sr. 2 poas, and of 6 scruples of soda.

35. Which is the heavier, a pound of gold or a pound of feathers? and by how much?

III. MEASURES OF LENGTH.

155 Indian Lineal Measure.

3	Yabs	make	1 Anguli
4	Angulis	„	1 Mushti
3	Mushtis	„	1 Bighat (*span*)
2	Bighats or 24 angulis	„	1 Hath or **Cubit**
4	Háths	„	1 Danda or Dhanu
2000	Dandas or 8000 háths	„	1 Kros or Kos
4	Kros	„	1 Yo-jan.

156. English Lineal Measure.

3	Barley corns (in length)	make	1 Inch (*in.* or 1′)
12	Inches	„	1 Foot (*ft.*)
3	Feet	„	1 **Yard** (*yd.*)
$5\frac{1}{2}$	Yards	„	1 Rod, Pole (*po.*) or Perch
40	Poles, or 220 yds.	„	1 Furlong (*fur.*)
8	Furlongs, or 1760 yds.	„	1 Mile (*mi.*)
3	Miles	„	1 League (*lea.*)

1 yard = 2 cubits; 1 Ilahi Gaj (N.-W.P.) = 33 in.; 1 Kros = 4000 yds.; 1 Karam (Madras) = 3 cubits; 1 Kathi (Bombay) = 9 4 ft.; 1 half-yard = 1 ft. 6 in.

Cloth Measure.

IN BENGAL.

3	Angulis	make	1 Girah
8	Girahs	„	1 Hath
2	Haths or 16 girahs	„	1 Gaj

IN BOMBAY.

2	Angulis	make	1 Tasu
24	Tasus	„	1 Gaj

ENGLISH.

$2\frac{1}{4}$	Inches	make	1 Nail (*nl.*)
4	Nails	„	1 Quarter (*qr.*)
4	Quarters	„	1 Yard.
3	Quarters	„	1 Flemish ell
5	Quarters	„	1 English ell
6	Quarters	„	1 French ell

1 Nail = 1 Girah ; 1 Bombay gaj (cloth-measure) = 27 in. ; 1 Bengal gaj = 36 in. = 1 yard.

Land Measure.

IN BENGAL.			ENGLISH.		
4 Háths	make	1 Katha	25 Links	make	1 Pole or Rod
20 Kathas	„	1 Bigha	100 Links	„	1 Chain
80 Háths	„	1 Rasi	10 Chains	„	1 Furlong.

In the N.-W. P., 3 Ilahi Gaj = 1 Bans or Ganteh and 20 Bans = 1 Jarib.

The following measures are sometimes used :—

1 Inch = 72 points = 12 lines ; 1 Palm = 3 in. ; 1 Hand = 4 in. (for measuring horses) ; 1 Span = 9 in. ; 1 Cubit = 18 in. ; 1 Pace = $2\frac{1}{2}$ ft. (military) = 5 ft. (geometrical) ; 1 Fathom = 6 ft. ; 1 Cable's length = 120 fathoms ; 1 Knot (nautical) = 6080 ft. ; 1 Degree of Latitude = 60 Knots ; 1 Chain = 4 poles = 22 yds. ; 80 Chains = 1 mile.

157. To reduce poles to yards, we have to multiply by $5\frac{1}{2}$; but since $5\frac{1}{2}$ yds. is 11 half-yards, we multiply the poles by 11, and divide the product by 2 In the converse operation, to divide by $5\frac{1}{2}$, we multiply the yards by 2, and divide the product by 11. The remainder in each case is half-yard, and note that 1 half-yd. is $1\frac{1}{2}$ ft. = 1 ft. 6 in. Also, in reducing miles and furlongs to yards, multiply by 1760 and 220 respectively, unless prevented by the form of the question. To reduce yards to miles, divide by 1760.

Note. 1 half-yd. = 1 ft. 6 in. Also 1 po. = 5 yds. 1 ft. 6 in.

Ex. 1. Reduce 9 mi. 4 fur. 23 po. 4 yds. 2 ft. 9 in to *inches*.

```
 mi.  fur.  po.  yds.  ft.  in.
  9    4    23    4     2    9          or 16846 yds.  1 ft.   6 in.
  8                                          4 yds.  2 ft.   9 in.
 76 fur.                                  16851 yds.  1 ft.   3 in.
 40                                           3
3063 po.                                  50554 ft.
  11                                         12
2)33693 half-yds.                        606651 in.  Ans.
 16846 yds.+1 half-yd.
```

Ex. 2. Reduce 3126749 inches to *miles, &c.*

```
12)3126749 in.
 3) 260562 ft....5 in.   ∴ the result
     86854 yds.          =49 mi. 2 fur. 31 po. 7 half-yds.  5 in.
         2               =49 mi. 2 fur. 31 po. 3 yds. 1 ft. 11 in.
11) 173708 half-yards.
40)  15791 po...7 half-yds. [for 7 half-yds. = 3½ yds. = 3 yds. 1 ft. 6 in.
 8)    394 fur...31 po.
        49 mi...2 fur.
```

Examples XXXIV.

1. Reduce (i) to *haths* or *cubits* and (ii) to *angulis* :—

15 kros 1008 dandas ; 6 yojan 2 kros 1780 dandas ; 20 bi. 4 kat. ; 25 bi. 15 kat. 3 cubits ; 10 kros 875 dandas 3 haths.

2. Reduce to *gaj, &c.* :—

34256 angulis ; 94605 girahs ; 420367 angulis ; 7035 girahs.

3. Reduce to *inches* :—

(1) 3 fur. 135 yds. 4 in.; 5 mi. 200 yds. 3 in.; 512 yds. 2 ft. 9 in.; 4 lea.
(2) 2 mi. 7 fur. 15 po. 1 yd. 1 ft. 6 in. ; 13 lea. 1 mi. 4 fur. 37 po. 1 ft. 8 in.
(3) 31 mi. 4 fur. 115 yds. 1 ft. 8 in. ; 25 mi. 6 fur. 17 po 4 yds. 3 in.
(4) 25 mi. 459 yds. 31 in. ; 25 fur. 39 po. 3 yds. 2 ft. 8 in.

4. Reduce 7 mi. 5 fur. 32 po. 4 yds. to *yards* ; 2 lea. 2 mi. 7 fur. to *yards* ; 5 mi. 3 fur. 208 yds. 1 ft. to *feet* ; 15 mi. 5 fur. 31 po. to *poles*.

5. Reduce to *miles, &c.* :—

(1) 57383 yds. ; 1847638 ft. ; 268543 in. ; 304935 ft. ; 53628 ft.
(2) 1081080 in. ; 231031 yds. ; 517900 in. ; 36090 ft. ; 2000000 in.

6. Reduce 183810 ft. to *leagues* ; 152017634 in. to *miles*.

7. Reduce :—

(1) 20 yds. 3 qrs. 1 nl. to *nails*. (2) 5 miles to *fathoms*.
(3) 35 ells 4 qrs. to *nails*. (4) 16 ells 1 qr. 3 nls. 1 in. to *in*.
(5) 500 fathoms to *yards*. (6) 5 furlongs to *fathoms*.
(7) 35 kros to *cubits*. (8) 5 miles to *links*.
(9) 1 gaj 1 hath 1 girah to *angulis*. (10) 16 haths 9 in. to *feet*.

8. Reduce :—

(1) 2897 in. of cloth to *yards*. (2) 567912 cubits to *bighas, &c.*
(3) 201494 jabs to *dandas*. (4) 74310 tasu to *gaj, &c.*
(5) 25 kros to *miles* and *yards*. (6) 76 miles to *kros* and *haths*.
(7) 1 kros 1999 dandas 1 gaj 1 hath 7 girahs 2 angulis to *angulis*.

9. Add together :—

(1)

yds.	ft.	in.
22	2	7
54	1	9
67	2	10
85	0	11
92	1	3

(2)

po.	yds.	ft.	in.
7	3	1	11
12	2½	2	4
9	4	0	7
2	3½	1	9
10	1	2	8

(3)

mi.	fur.	po.	yds.
14	3	17	2½
23	5	33	4
37	1	24	5
43	7	31	1½
75	6	36	2¾

(4)

mi	po.	yds.	in.
3	84	2½	7
12	113	0	9
6	0	4¼	11
25	44	3	8

(5)

yds.	qrs.	nls.
25	3	2
37	0	3
54	1	1
49	2	3

(6)

ells	qrs.	nls
35	2	3
42	4	5
37	2	2
25	4	3

10. Perform the operation of subtraction in the following :—

(1) mi.	fur.	po.	(2) fur.	po.	yds.	(3) mi.	po.	yds.	(4) yds.	qrs.	nls.
24	0	7	6	37	4	6	0	0	18	2	0
11	5	18	5	18	4½	3	37	2¾	8	3	3

11. Multiply :—

(1) 5 fur. 78 yds. 2 ft. 7 in. by 56; 42 yds. 2 qrs. 2 nls. by 40.
(2) 13 lea. 2 mi. 6 fur. 25 po. separately by 42 and 97.
(3) 25 mi. 6 fur. 23 po. 3 yds. 2 ft. 8 in. ... 56 and 83.
(4) 20 dandas 1 hath 7 girahs ... 4, 5 and 12.

12. Divide :—

(1) 3179 lea. 1 mi. 5 fur. 16 po. by 46; 485 yds. 3 qrs. 3 nls. 2 in. by 11.
(2) 478 mi. 6 fur. 19 po. 2 yds. 1 ft. 10 in. separately by 96 and 4397.
(3) 679 mi. 7 fur. 125 yds. 2 ft. 6 in. ... 11 and 120.
(4) 275 dandas 1 gaj 4 girahs ... 3, 5, 10 & 12.
(5) 1027 mi. 1 fur. 6 po. by 17 mi. 5 fur. 27 po.

13. If 67 pieces of cloth measure 2335 yds. 2 qrs. 7 in., what is the length of 1 piece?

14. If a person complete a journey of 422 mi. 3 fur. 38 po. in 37 days; what distance does he travel per day?

15. Find the aggregate of 4 kros 1 gaj 1 ha. 7 gir.; 5 kros 1978 dan. 4 gir.; 2 kros 150 dan. 1 gaj 1 ha. 2 gir.; 11 kros 25 dan. 1 ha. 5 gir. and 6 kros 1 gaj 3 gir.

16. How many lengths each equal to 9 po. 3 yds. 1 ft. 3 in. will make up 1 mile 6 fur. 26 po. 4 yds. 2 ft. 9 in.?

IV. MEASURES OF SURFACE.

158. **Land Measure in Bengal.**

20 Square cubits or Gandas make 1 Chhatak.
16 Chhataks „ 1 Katha
20 Kathas „ 1 Bigha.

IN N.-W. P.

20 Kachvansi make 1 Bisvansi
20 Bisvansi „ 1 Bisva
20 Bisvas „ 1 Bigha

IN BOMBAY.

39¼ Square cubits make 1 Kathi
20 Kathis „ 1 Pand
20 Pands „ 1 Bigha
6 Bighas „ 1 Rukeh
20 Rukehs „ 1 Chahur

IN THE PUNJAB.

9 Sarsi make 1 Marla
20 Marlas „ 1 Kanal
4 Kanals „ 1 Bigha
2 Bighas „ 1 Ghuma

IN MADRAS.

144 Sq. Inches make 1 Sq. ft.
2400 Sq. feet „ 1 Ground or Manai
24 Grounds „ 1 Cawny
484 Cawnies „ 1 Sq. mile.

1 Bengal Bigha = 1600 sq. yds.; 1 N.-W. P. Bigha = 3025 sq. yds. 1 Punjab Bigha = 1620 sq. yds.; 1 Bombay Bigha = 3927 sq. yds. Also 1 Madras Cawny = 6400 sq. yds. = 4 Bengal Bighas.

159. **English Square Measure.**

144 Square Inches (*sq. in.*)	make	1 Square Foot (*sq. ft.*)
9 Square Feet	„	1 **Square yard** (*sq. yd.*)
$30\frac{1}{4}$ Square Yards	„	1 Square pole (*sq. po.*)
40 Square Poles	„	1 Rood (*ro.*)
4 Roods or 4840 sq. yds.	„	1 **Acre** (*ac.*)
640 Acres	„	1 Square Mile (*sq. mi.*)

10,000 Sq. links make 1 Sq chain	484 Sq. yds. make 1 Sq. chain.
10 Sq. chain „ 1 Acre	100,000 Sq. links...1 Acre.

A Rod of Brickwork = $272\frac{1}{4}$ sq. ft. A Rod of Building = 36 sq. yds. A Square of Flooring, Roofing, &c. = 100 sq. ft. A Yard of Land = 30 ac. A Hide of Land = 100 ac. One sq. chain = 10,000 sq. links.

40 ac. = 121 Bengal Bighas; 5 ac = 8 N.-W. P. Bighas; 81 ac. = 242 Punjab Bighas; 160 ac. = 121 Madras Cawnies. Also 1 sq. mi. = 1936 Bengal Bighas = 1024 N.-W. P. Bighas = 484 Madras Cawnies.

160. To reduce square poles to square yds., we have to multiply by $30\frac{1}{4}$; but since $30\frac{1}{4}$ sq. yds. is 121 qr.-sq. yds., we multiply the sq., poles by 121 and divide by 4. In the converse operation to divide by $30\frac{1}{4}$, we multiply the sq. yds. by 4 and divide by 121. The remainder in each case is qr.-sq. yds. and note that 1 qr.-sq. yd. is $2\frac{1}{4}$ sq. ft. = 2 sq. ft. 36 sq. in.

Also in reducing acres and roods to sq. yards, multiply by 4840 and 1210 respectively, unless prevented by the form of the question. To reduce square yards to acres, divide by 4840.

Note. 1 qr. sq. yd. = 2 sq. ft. 36 sq. in.; 2 qr.-sq. yds. = 4 sq. ft. 72 sq. in.; 3 qr.-sq. yds. = 6 sq. ft. 108 sq. in. Also 1 sq. po. = 30 sq. yds. 2 sq. ft. 36 sq. in.

Ex. 1. Reduce 3 ac. 2 ro. 23 sq. po. 10 sq. yds. 3 sq. ft. 18 sq. in. to *sq. inches.*

```
      3 ac. 2 ro. 23 sq. po. 10 sq. yds. 8 sq. ft. 18 sq. in.
      4
     14 ro.                        17646 sq. yds. 5 sq. ft. 126 sq. in.
     40                                9
    583 sq. po.                   158819 sq. ft.
    121                              144 = 12 × 12.
 4)70543                        22870062 sq. in. Ans.
  17635 sq. yds. + 3 qr.-sq. yds.
 =17635 sq. yds. 6 sq. ft. 108 sq. in.
     10           8          18
  17646 sq. yds. 5 sq. ft. 126 sq. in.
```

Ex. 2. Reduce 9532482 sq. inches to *acres*.

```
144 { 12)9532482 sq. in.
    { 12) 794373...6    }
       9)  66197 sq. ft. 9 }  144 sq. in.        ∴ the result
        7355 sq. yds...2 sq. ft. | = 1 ac. 2 ro. 3 sq. po. 17 qr. sq. yds.
               4                 |                2 sq. ft. 114 sq. in.
121 { 11)29420 qr. sq. yds.      | = 1 ac. 2 ro.  3 sq. po. 4 sq. yds.
    { 11) 2674......6    }       |                2 sq. ft. 36 sq. in.
    4,0)  24,3 sq. po....1 }  17 qr. sq. yds.     + 2 sq. ft. 114 sq. in.
      4)  6 ro....3 sq. po. = 1 ac. 2 ro. 3 sq. po. 4 sq. yds. 5 sq. ft. 6 sq. in.
          1 ac....2 ro.
```

Examples XXXV.

1. Reduce to *gandas* or *square cubits* :—

5 bi. 3 kat. 6 ch.; 45 bi. 9 kat. 7 ch.; 25 bi. 15 kat. 4 ch. 15 ga.; 135 bi. 11 kat.; 425 bi. 17 kat. 13 ch. 17 ga.; 29 bi. 17 kat.

2. Reduce to *bighas* :—

357628 ch.; 10486 ga.; 8326675 sq. cubits; 4675900 ga.; 125720 ch.

3. Reduce to *kachvansi* :—

24 bi. 15 bisv.; 136 bi. 14 bisv. 17 bisvansi; 86 bi. 7 bisv.; 423 bi. 10 bisv. 12 bisvansi 15 kachv.

4. Reduce to *square inches* :—

8 sq. mi. 340 caw; 15 sq. mi. 285 caw. 12 grounds; 25 sq. mi. 375 caw, 20 grounds 1452 sq. ft.; 3 caw. 13 manies 5 sq. ft.

5. Reduce to *sq. karam* or *sarsai* :—

26 ghm. 1 bi.; 42 ghm. 1 bi. 3 ka. 15 marlas; 42 bi. 2 ka. 4 sar.

6. Reduce to *kathis* :—

63 bi. 7 pands 3 ka.; 4 cha. 108 bi. 15 pands; 42 bi. 112 ka.

7. Reduce :—

(1) 246053 kachvansi to *bighas*.
(2) 34512876 kathis to *bighas*.
(3) 43276850 sq. in. to *cawnies*.
(4) 403207654 kathis to *chahurs*.
(5) 1130692 manies to *sq. miles*.
(6) 8740361 sq. sarsai to *ghumas*.

8. Reduce to *sq. inches* :—

(1) 17 sq. yds. 8 sq. ft.; 3 sq. yds. 6 sq. ft. 75 sq. in.; 29 sq. yds.; 54 sq. yds. 8 sq. ft. 104 sq. in.; 3 ro. 17 po. 21 sq. yds. 8 sq. ft.
(2) 17 ac. 14 po.; 1 ac. 2 ro. 3 po. 4 sq. yds.; 3 ro. 22 po. 21 sq. yds. 8 sq. ft. 116 sq. in.; 56 ac. 2 ro 25 po. 37 sq. yds. 5 sq. ft. 73 sq. in.
(3) 38 ac. 2 ro. 35 po.; 324 sq. po.; 3 sq. mi.; 4 ac. 26 po.; 42 ac.

9. Reduce to *acres* :—

(1) 16553 sq. po.; 13678 sq. yds.; 170184 sq. ft; 82973 sq. po.; 895487 sq. yds.; 2709437 sq. ft.

(2) 123456789 sq. in. ; 94501362 sq. in. ; 455462764 sq. in. ; 72013512032 sq. in. ; 355433005 sq. in.

10. Reduce :—

(1) 14 ac. to *sq. links.*
(2) 1803 ac. to *sq. miles.*
(3) 5200000 sq. yds. to *sq. miles.*
(4) 428 sq. chains to *sq. inches.*
(5) 5621 sq. po. to *sq. chains.*
(6) 535 sq. miles to *bighas.*

11. Reduce (*Bengal bighas*) :—

5445 bighas to *acres* ; 2560 ac. to *bighas* ; 9680 bi. to *acres* ; 14400 ac. to *bighas* ; 7260 bi. to *acres* ; 92360 ac. to *bighas*.

12. Reduce : 629200 Bengal bighas to *N.-W. P. bighas* ; 9720 Bengal bighas to *Punjab bighas* ; 320780 Bengal bighas to *Madras cawnies* ; 768000 N.-W. P. bighas to *Bengal bighas* and 28800000 Punjab bighas to *Bengal bighas.*

13. Add together :—

(1)

bi.	ka.	ch.
30	15	10
19	17	12
25	18	13
31	12	15
28	8	9

(2)

sq. yds.	sq. ft.	sq. in.
32	2	98
12	8	120
19	7	47
23	6	135
45	7	85

(3)

ac.	ro.	po.
29	3	28
35	3	35
45	0	25
17	1	20
19	2	16

(4)

ro.	sq. po.	sq. yds.
74	19	15
6	34	$11\frac{1}{4}$
17	0	$27\frac{1}{2}$
23	39	$16\frac{3}{4}$

(5)

ac.	ro.	po.	sq. yds.
35	1	23	$12\frac{1}{2}$
9	2	15	$27\frac{1}{4}$
11	1	24	11
42	0	35	$2\frac{3}{4}$

(6)

ac.	po.	sq. yds.	sq. ft.	sq. in.
25	11	0	8	23
36	39	11	0	136
7	0	27	6	0
18	20	23	7	94

14. Perform the following subtractions :—

(1)

bi.	kat.	ch.
125	8	9
96	12	13

(2)

ac.	ro.	po.
96	1	19
29	3	30

(3)

ac.	ro.	po.	sq. yds.
45	1	29	$25\frac{1}{2}$
39	3	18	$27\frac{1}{2}$

15. Multiply :—

(1) 120 bi. 14 kat. 10 ch. by 49 ; 125 bi. 15 kat. 12 ch. by 154.
(2) 17 ac. 1 ro. 31 po. by 72 ; 2 ro. 27 po. 15 sq. yds. 8 sq. ft. by 6 and by 10.
(3) 37 ac. 3 ro. 19 po. 28 sq. yds. 4 sq. ft. 103 sq. in. by 8 and by 75.

16. Divide :—

(1) 112 bi. 18 kat. 14 ch. by 99 ; 1539 bi. 15 kat. 7 ch. by 102.
(2) 82 bi. 16 kat. 12 ch. by 72 ; 130 ac. 1 ro. 28 po. by 120.
(3) 854 ac. 3 ro. 27 po. 8 sq. yds. 8 sq. ft. 45 sq. in. by 9 and by 246.
(4) 166 ac. 2 ro. 6 po. 30 sq. yds. 5 sq. ft. by 7 ac. 38 po. 17 sq. yds. 1 sq. ft. ; 935 bi. 12 kat. 12 ch. by 55 bi. 12 ch.

17. How many allotments each equal to 2 ro. 5 po. 13 sq. yds. 6 sq. ft. 108 sq. in. can be formed out of 158 ac. 2 ro. 20 po.?

18. A certain district contains 514164 ac. and another 95805 ac. How many sq. miles does the one contain more than the other?

V. MEASURES OF SOLIDITY.

161. Bengal Measure of Solidity.

13824 Cubic Angulis make 1 Cubic Cubit or C. hath
8 Cubic Cubits „ 1 Cubic yard.
8 Cubic yards or 64 cub. cubits „ 1 Chouka.

162. English Measure of Solidity.

1728 Cubic Inches (*cub. in.*) make 1 Cubic Foot (*cub. ft.*)
27 Cubic feet „ 1 **Cubic yard** (*cub. yd.*)

1 Cub. hath = 5832 cub. in. A Load of rough Timber = 40 cub. ft. A Load of squared Timber = 50 cub. ft. A Ton of Shipping = 42 cub. ft. A Stack of wood = 108 cub. ft. A Cord of wood = 128 cub. ft.

Examples XXXVI.

1. Reduce to *cub. cubits* :—

42 choukas 54 cub. cubits; 87 choukas 62 cub. cubits; 146 choukas 32 cub. cubits; 144 choukas.

2. Reduce to *cub. in.* :—

24 cub. yds. 7 cub. ft. 144 cub. in.; 18 cub. yds. 1274 cub. in.; 12 cub. yds. 23 cub. ft.; 23 cub. yds. 1000 cub. in.

3. Reduce to *cub. yds.* :—

200000 cub. in.; 138297 cub. in.; 141721 cub. in.; 863005 cub. in.

4. Reduce to *choukas* :—

36248742 cub. cubits; 4308756 cub. cubits; 862097 cub. cubits.

5. Reduce 1953 choukas 28 cub. cubits to *cubic angulis.*

6. Add together :—

(1) Chouka	cub. yds.	cub. hath.	(2) c. yds.	c. ft.	c. in.	(3) c. yds.	c. ft.	c. in.
18	6	4	53	7	1249	328	15	323
27	5	7	27	23	472	237	19	484
134	4	5	29	16	1384	785	10	1259
49	3	2	45	18	1186	546	0	342
234	3	6	33	9	1324	729	11	1075

7. Perform the following subtractions :—

(1) c. yds.	c. ft.	c. in.	(2) c. yds.	c. ft.	c. in.	(3) c. yds.	c. ft.	c. in.
49	15	542	150	0	0	527	0	1
39	23	736	59	25	1001	279	1	259

8. Multiply :—

(1) 2 cub. yds. 5 cub. ft. 704 cub. in. by 11 and by 23.
(2) 275 cub. yds. 17 cub. ft. 125 cub. in. by 56.

9. Divide :—

(1) 372 cub. yds. 1236 cub. in. by 64.
(2) 6739 cub. yds. 2 cub. ft. 468 cub. in. by 19 and by 509.
(3) 18809 cub. yds. 1 cub.ft. 1156 cub.in. by 723cub. yds. 11c.ft.84c.in.

10. A certain number of bins, each containing 8 cub. yds. 152 cub. in., contain 1512 cub. ft. 1064 cub. in. ; find the number.

VI. MEASURES OF CAPACITY.

163. 1st. Tables of Corn or Dry Measure.

Indian.

BENGAL MEASURE.

5 Chhataks	make	1 Kunka
2 Kunkas	„	1 Khunchi
2 Khunchis	„	1 Rek
2 Reks	„	1 Pali
2 Palis	„	1 Doan
2 Doans	„	1 Kati
8 Katis	„	1 Arhi
20 Arhis	„	1 Bish
16 Bishes	„	1 Kahan
16 pa. or 8 do.	„	1 **Maund** (*md.*)
20 Doans	„	1 Sali

BOMBAY MEASURE.

36 Tanks	make	1 Tipari
2 Tiparis	„	1 Seer
4 Seers	„	1 Payli
16 Paylis	„	1 Phara
8 Pharas	„	1 Kandi
25 Pharas	„	1 Muda

MADRAS MEASURE.

8 Ollaks	make	1 Paddi
8 Paddis	„	1 Markal
5 Markals	„	1 Phara
80 Pharas	„	1 Garce

In Bengal, lime is measured thus: 1 Phara = 27′ × 20′ × 9′; 6 Pharas = 5 cub. hath; 80 Pharas = 100 mds.; 1 markal (Madras) = 750 cub. in.

English.

2 Quarts (*qt.*)	make	1 Pottle (*pot.*)
2 Pottles or 4 qts.	„	1 **Gallon** (*gal.*)
2 Gallons	„	1 Peck (*pk.*)
4 Pecks	„	1 Bushel (*bus.*)
2 Bushels	„	1 Strike (*str.*)
4 Bushels	„	1 Coomb (*co.*)
2 Coombs or 8 bus.	„	1 Quarter (*qr.*)
5 Quarters	„	1 Load (*ld.*)
2 Loads or 10 qrs.	„	1 Last.

COAL MEASURE.

4 Pecks	make	1 Bushel
3 Bushels	„	1 Sack
12 Sacks or 36 bus.	„	1 Chaldron

A gallon (*Imperial*) contains 277·274 cub. in. ; hence a bushel (*Imperial*) consisting of 8 gallons, contains 8 × 277·274 or 2218·192 cub. in.

164. 2nd. Tables of Liquid Measure.

Indian.

4 Chhataks	make	1 Póa
4 Póas	„	1 **Seer**
40 Seers	„	1 Maund

* The weight of a seer for this measure varies in different localities from 40 tolas to 112 tolas.

English.

Wine Measure.

4	Gills (*gil*)	make	1 Pint (*pt.*)
2	Pints	,,	1 Quart (*qt.*)
4	Quarts	,,	1 Gallon (*gal.*)
63	Gallons	,,	1 Hogshead (*hhd.*)
2	Hogsheads or 126 gallons	,,	1 Pipe (*pipe.*)
2	Pipes	,,	1 Tun

10 Gallons = 1 Anker
18 Gallons = 1 Runlet
42 Gallons = 1 Tierce
84 Gallons or 2 Tierces = 1 Puncheon

Ale and Beer Measure.

2	Pints	make	1 Quart
4	Quarts	,,	1 Gallon
36	Gallons	,,	1 Barrel (*bar.*)
1½	Barrels or 54 gallons	,,	1 Hogshead
2	Hogsheads	,,	1 Butt
2	Butts	,,	1 Tun

9 Gallons = 1 Firkin.
18 Gallons = 1 Kilderkin.

A pint of pure water weighs a pound and a quarter, therefore a gallon of distilled water weighs 10lbs. (Avoir.), when the barometer is at 30 in. and the air at a temperature of 62° Fah. thermometer. Hence the weight of a cubic foot of water is very nearly 1000 oz. (Avoir.)

165. English Apothecaries' Measure.

60	Minims (*m.*) or drops	make	1 Fluid Dram (*fl. dr.*)
8	Fluid Drams	,,	1 Fluid Ounce (*fl. oz.*)
20	Fluid Ounces	,,	1 Fluid Pint (*O. Octarius.*)
8	Pints	,,	1 Gallon (*C*; *Congius.*)

A tea-spoonful = 1 fluid dram. A dessert-spoonful = 2½ fluid drams A table-spoonful = 4 fluid drams. 1 Fluid ounce = 1 ounce (Avoir.)

Examples XXXVII.

1. Reduce to *chhataks* ; 2 mds. 3 do. 2 pa. 3 ch. ; 1 md. 3 do. 1 khun. ; 8 kah. 14 bis. 16 arh. ; 125 mds. 6 do. 1 pa 1 rek. ; 14 kah. 10 do. ; 17 salis 58 pa. 2 reks.

2. Reduce : 3842 ch. to *maunds* ; 201372 kunikas to *maunds* ; 48762035 ch. to *maunds* ; 467032000 ch. to *kahans* ; 246780 reks to *maunds* ; 346780 khun. to *doans.*

3. Reduce : 125 pharas to *tanks* ; 416 mudas to *tanks* ; 1 ka. 3 ph. 5 paylis 1 tipari 26 tanks to *tanks* ; 6932843 tiparis to *mudas* ; 54038764 tanks to *kandis.*

4. Reduce : 205 pharas to *ollaks* ; 1 garce 45 pharas 2 markals 3 paddis to *ollaks* ; 28 pharas 4 markals 54 ollaks to *ollaks* ; 256284 ollaks to *garces* ; 123456 ollaks to *pharas* ; 2368 paddis to *pharas* ; 987600 ollaks to *markals.*

5. Reduce to *gallons* : 2 qrs. 7 bus. 2 pks. ; 3 lds. 3 qrs. 3 pks. ; 54 qrs. 7 bus. 6 gal. ; 64 lasts 1 ld. 3 qrs. 7 bus. 1 pk.

6. Reduce to *quarts* : 25 qrs. 2 bus. 2 pks. ; 7 lds. 2 co. 3 pks. ; 17 lasts 1 qr. 7 pks. ; 356 qrs. 7 bus. 2 pks. 1 gal. ; 3 lds. 3 bus.

7. Reduce : 598712 gals. to *quarters* ; 800574 bus. to *lasts* ; 205634 qts. to *coombs* ; 986753 strikes to *quarters*.

8. Reduce to *loads* : 89765 pks. ; 56789 pts. ; 356187 qts. ; 1000000 pks. ; 97324 pts. ; 4357 gals.

9. Reduce to *gills* : 1 hhd. 35 gals. ; 5 pipes ; 2 pipes 17 gals. 1 qt. ; 3 tuns 1 hhd. 57 gals. ; 27 tuns 1 pipe 1 hhd. 54 gals. 1 qt. 1 pt.

10. Reduce to *pints* : 2 qrs. 1 gal. ; 2 qrs. 5 bus. 3 pks. 1 gal. ; 987 bar. 25 gals. 3 qts. 1 pt. ; 21 tuns 3 hhds. 54 gals. 2 qts.

11. Reduce : 8 gals. 2 fl. oz. to *fl. drams* ; 5 C. 7 O. 17 fl. oz. 5 fl. dr. 45 m. to *minims* ; 3 O. 2 fl. oz. 40 m. to *minims*.

12. Reduce : 56321 pts. to *pipes* ; 1000000 qts. to *tuns* ; 5279 pts. to *gallons* ; 62741 gills to *gallons* ; 3720812 gills to *quarters*.

13. Reduce : 84381 pts. to *tuns* ; 24357 gills to *pipes* ; 9000 gals. to *butts* ; 58428092 gills to *lasts* ; 5849206 qts. to *hogsheads*.

14. Reduce to *gallons* : 882743 minims ; 58428092 minims.

15. What is the weight of 14 gals. 3 pts., of water in Avoir. ?

16. What is the weight in *kandis* of 256 pharas of lime ?

17. What is the weight of 12 cub. yds. 12 cub. ft of water in lbs. Avoir. ? In 250 packs of wool, how many tons ?

18. Add together :—

(1)

mds.	do.	reks.
145	6	3
47	5	2
258	4	1
96	7	2
74	0	1

(2)

gals.	qts.	pts.	gils.
57	3	1	3
38	1	1	2
45	2	0	3
26	3	0	3
18	2	1	0

(3)

qrs.	bus.	pks.	gals.
19	6	3	1
38	7	1	1
11	4	3	0
4	7	3	1
32	5	2	[illegible]

(4)

gals.	qts.	pts.
49	3	1
34	1	0
25	0	1
51	3	1
30	1	0

(5)

lds.	qrs.	bus.
13	4	7
24	3	4
37	4	0
43	2	1
58	3	6

(6)

C.	O.	fl.oz	fl.dr.	m.
3	5	18	7	10
	7	13	1	45
1	4	9	3	15
2	0	19	5	20
	3	5	6	30

19. Perform the following subtractions :—

(1)

gals.	qts.	pt.	gils.
57	2	1	2
26	3	1	3

(2)

gals.	qts.	pt.
240	0	0
140	3	1

(3)

tuns	hhds.	gals.	pts.
2	2	0	0
1	3	32	4

(4)

lds.	qrs.	bus.	pks.	gal.
7	3	5	2	0
3	4	7	3	1

(5)

bus.	pks.	gal.
57	1	0
39	3	1

(6)

C.	O.	fl. oz.	dr.	m.
6	3	12	1	15
2	6	17	5	40

20. Multiply :—

(1) 15 qrs. 6 bus. 3 pks. 1 gal. separately by 54 and 111.
(2) 27 gal. 3 qrs. 1 pt. 3 gils. ... ... 36 and 236.

21. Divide :—

(1) 5863 gals. 3 qts. 1 pt. 3 gils. separately by 8 and 75.
(2) 6564 lds. 1 qr. 4 bus. 2 pks. 1 gal. ... 5 and 67.
(3) 739 qrs. 4 bus. 2 pks. 1 gal. by 11 ; 244 qrs. 3 bus. 1 pk. by 3 qrs. 3 pks. ; 7 O. 11 fl. oz. 6 fl. dr. 20 m. by 10.

22. How many sacks of corn can be filled out of a bin containing 52 qrs., if each sack hold 3 bus. 1 pk. ?

23. How long will a butt of beer last a man who drinks 2 qts. 1 pt. daily ?

24. A dishonest inn-keeper buys 2 pipes of wine, and mixes 1 qt. 1 pt. of water with every 3 gallons of wine. How many gallons will he have to sell ?

25. How many jars, each containing 2 gals 3 qts. 1 pt. 3 gils. can be filled out of a cask containing 285 gallons ?

VII. MEASURES OF TIME.

166. **Indian Measure of Time.**

60 Anupals (*anu.*)	make	1 Bipal (*bip.*)
60 Bipals	,,	1 Pal (*pal.*)
60 Pals	,,	1 Danda (*dan.*)
7½ Dandas or 3 hours	,,	1 Prahar (*pr.*)
8 Prahars or 60 dandas	,,	1 Din or day (*da.*)
7 Dins	,,	1 Saptaha (*sap.*)
15 Dins	,,	1 Paksha (*pak.*)
30 Dins or 2 pakshas	,,	1 Mas or month (*ma.*)
12 Masas	,,	1 Batsar or Year (*ba.*)
12 Batsars	,,	1 Yuga.

2½ Dandas = 1 Ghanta ; 1 Danda = 24 minutes. A chandra mas (lunar month) = 29½ days, nearly.

167. **English Measure of Time.**

60 Seconds (*sec.* or 1^s)	make	1 Minute (*min.* or 1^m)
60 Minutes	,,	1 Hour (*hr.*)
24 Hours	,,	1 Day (*da.*)
7 Days	,,	1 Week (*wk.*)
365 Days	,,	1 Year (*yr.*)
100 Years	,,	1 Century.

A month = 30 days. A year = 4 quarters = 12 calendar months = 52 weeks.

A fortnight = 2 weeks. A month = 4 weeks. A Leap-year = 366 days. Each day is considered to commence at midnight.

168. The number of days in the *Calendar* Months are recollected by means of the following lines :—

Thirty days hath September,
April, June and November ;
February has twenty-eight alone,
And all the rest have thirty-one ;
But leap-year coming once in four,
February then has one day more.

Bengali Months.			*English Months.*		
1.	Baisakh	(বৈশাখ)	1.	January	= 31 days.
2.	Jaistha	(জ্যৈষ্ঠ)	2.	February	= 28 ,,
3.	Ashárh	(আষাঢ়)	3.	March	= 31 ,,
4.	Srávan	(শ্রাবণ)	4.	April	= 30 ,,
5.	Bhádra	(ভাদ্র)	5.	May	= 31 ,,
6.	Aswin	(আশ্বিন)	6.	June	= 30 ,,
7.	Kártick	(কার্ত্তিক)	7.	July	= 31 ,,
8.	Agrahâyan	(অগ্রহায়ণ)	8.	August	= 31 ,,
9.	Pous	(পৌষ)	9.	September	= 30 ,,
10.	Magh	(মাঘ)	10.	October	= 31 ,,
11.	Falgoon	(ফাল্গুন)	11.	November	= 30 ,,
12.	Chaitra	(চৈত্র)	12.	December	= 31 ,,

Mahomedan Names : Maharam (মুহরম), Safar (শফর), Raviulayal (রবিয়ল আউয়ল), Raviassani (রবিয়সসানি), Jamadiyal-auyal (জমাদিয়ল আউয়ল), Jamadiyassani (জমাদিয়সসানি), Rajab (রজব), Saban (শাবন), Ramjan (রমজান), Saoyal (শওয়াল), Jelkad (জেলকদ), and Jelhajja (জেলহজ্জ) ৷

A Bengali month is generally supposed to consist of 30 days ; but this is not strictly correct. Some months are 29 days, some 30, some 31 and some 32.

THE HINDU CALENDAR.

169. The Hindu **Chandra Batsar** (Lunar year) consists of 354 days 8 hrs. 48 min. 57 sec. It is therefore shorter than the **Saur Batsar** (Solar year) by 10 days 21 hrs. 23 min. 12 sec. After a period of $32\frac{1}{2}$ months the difference amounts to a month ; consequently to make the Lunar-year system correspond with the Solar year system, a month is intercalated on the occurrence of two conjunctions of the Sun and Moon in the same sign of the Zodiac. The intercalated month and the month preceding it go by the same name. The intercalated month is called **Mala** or **Intercalary Mas**. This is done in those parts of India where the lunar year and lunar month are reckoned. In Bengal, where solar year and solar month are reckoned, a month is rejected in a period of every $32\frac{1}{2}$ lunar months as unfit for any religious festival, in order to make the religious festivals of particular months recur in those months. The rejected month is called *Mala Mas*.

THE ENGLISH CALENDAR.

170. The interval of time between two passages of the Sun across the meridian of any place when taken at its *mean magnitude*, is termed a *day* or a *mean solar day*, which is supposed to be divided into 24 equal portions called *mean solar hours*. It appears from the observations and calculations of Astronomers that the time between the Sun's leaving a certain point (First point of Aries) in his path called the *Ecliptic* and returning to it again, consists of 365·242218 such days or of 365 days, 5 hours, 48 minutes, 47½ seconds, very nearly, which is therefore termed a *Solar Year*.

For the purposes of civil life it would be exceedingly inconvenient that one year should commence at one time of the day and another at a different time; and this circumstance gave rise to the invention of the *civil year*, which will be explained in the next Articles.

171. When the Science of Astronomy was much less perfect than it is at present, the length of the solar year was much less accurately known; and accordingly we find that in the time of *Julius Cæsar* it was supposed to consist of 365 days 6 hours, or of 365¼ days, *exactly*. On this supposition, it is evident that if out of *four* years in succession, any *three* consisted of 365 days each and the remaining one of 366 days, the Sun would have returned at the end of those *four* years to the place in the *Ecliptic* which it occupied at their commencement.

The scheme was called the *Julian Calendar*; and if the hypothesis had been correct, it would have been attended with much convenience; the additional *day* was called **Intercalary**, and the *year* in which it was added or inserted was termed **Bissextile**.

The regulation, applied to the years of the *Christian Era*, was so managed that whenever the number of years was divisible by 4, the corresponding year consisted of 366 days and was called **Leap-year**, the month of *February* having 29 days in that year, and each of the remaining three years 28 days, without interfering at all with their order.

Hence also, the remainder after the division of any other number of years by 4, was the number of years since a leap-year occurred up to that year. Thus, in the year 1893 this remainder is 1; and accordingly it is 1 year since, the last leap-year happened and it is 3 years before the next will occur, according to this scheme. ...

172. Since the true solar year is 365·242218 days and not 365·25 days, it is evident that the reckoning of time according to the Julian Calendar would place the end of the year *after* the time when the Sun had returned to the point of the Ecliptic occupied by it at the beginning of the year and consequently in *advance* of the course of the *Seasons*; but, the error in one year is 365·25—365·242218 = ·007782 of a day. Therefore in 400 years the error would amount to .007782 × 400 or 3·1128 days.

Now, according to the Julian Calendar 400 years would comprise 100 Leap-years; and since we find that this reckoning falls nearly 3 days *after* the true time, if there were only 97 Leap-years in 400 years; the Julian year would very nearly agree with the true solar year; and it is accordingly ordained that whenever the *numbers* expressing the **Centuries** as 16, 17, 18, 19, &c., denoting 1600, 1700, 1800, 1900, &c., are *not* divisible by 4, the corresponding year shall *not* be a Leap-year although according to the Julian Computation it would; as, 1600 would be a Leap-year, but 1700, 1800, 1900 would not.

The Calendar thus corrected though not absolutely accurate, is well adapted to every *practical* purpose, as the error in 1500 *years* will not amount to much more than *twenty-eight* hours. The correction was first promulgated in Europe by *Pope Gregory* in 1582 and the calendar has since been called the *Gregorian Calendar*; but it was not introduced into *Protestant Countries* till a much later period. In *England* it was adopted on the *second* day of September 1752 when the error amounted to 11 days; and it is called the **New Style** to distinguish it from the Julian Calendar which is now termed the **Old Style.**

The New style has not yet been adopted in Russia, so that since 1752 they have had one more leap-year (1800) than we have, and they are now 12 days behind us. Thus Old Michælmas and Old Christmas taking place 12 days after New Michælmas and New Christmas.

173. The **Civil year** thus fixed and determined is then subdivided into twelve Calendar Months, as described in the Table. The word *Month* however is used in different senses; sometimes to denote a *twelfth* part of a year; sometimes as equivalent to 4 weeks or 28 days; and accordingly a year is equivalent to 13 months and 1 day, or to 52 weeks and 1 day, with the addition of another day when it happens to be Leap-year.

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174. To reduce prahars to *dandas*, multiply by 15, and divide the product by 2; the remainder (if any) is a half-danda or 30 pals. Conversely to reduce dandas to *prahars*, multiply by 2 and divide the product by 15; the remainder (if any) is equal to so many half-dandas.

Ex. Reduce 8sap. 5da. 3pr. 4dan. 45 pals to *bipals*; 266330 sec. to *days*, and 2 yrs. 15 da. 6 hrs. to *minutes*.

(1) 8 sap. 5 da. 3 pr. 4 dan. 45 pals.

7	= 3682 dan. 30 pals.
61 da.	4 45
8	3687 dan. 15 pals.
491 pr.	60
15	221235 pals.
2)7365	60
3682 dan. + 1 half-dan.	13274100 bipals. *Ans.*

(2) 6,0)26633,0 sec.
6,0) 443,8 ...50 sec.
24 { 3 73 ...58 min.
8)24 ... 1 } 1 hr.
3 da.
∴ the result
= 3 da. 1 hr. 58 m. 50 sec.

(3) 2 yrs. 15 da. 6 hrs.
365
745 da.
24
17886 hrs.
60
1073160 min. *Ans.*

Examples XXXVIII.

1. Reduce to *anupals* :—

(1) 5 dan. 30 pa. (2) 12 pr. 6 dan. 40 bip. (3) 8 sap. 5 da. 5 pr. (4) 3 ba. 6 m. 5 da. 5 dan. (5) 4 sap. 6 da. 6 pr. 50 pa. 40 bip. (6) 6 pr. (7) 13 sap. (8) 12 days. (9) 10 da. 5 pr. 45 anu. (10) 46 ba. 267 da. 57 dan. 43 pa. 51 bip.

2. Reduce to *seconds* :—

(1) 27 wks. 5 da. 15 hrs. ; 6 hrs. 25 min. 32 sec. ; 5 wks. 3 da.
(2) 3 yrs. 147 da. 15 hrs. ; 76 da. 19 hrs. 43 min. 57 sec.
(3) 2 da. 4 hrs. 51 min. 50 sec. ; 4 mo. 2 wks. 23 hrs. ; 3 leap-years.

3. Reduce : 15 yrs. 26da. 2hrs. 27min. to *minutes* ; 19yrs. 153da. 8 hrs. to *hours* ; 3 yrs. 315 da. to *minutes*.

4. Reduce :—

(1) 563472 pals. to *dins* ; 59018732 anupals to *dins*.
(2) 8463045 bipals to *prahars* ; 74632508 anupals to *dandas*.
(3) 673985643 anupals to *days* ; 364380005 dan. to *batsars*.

5. Reduce :—

(1) 72015 hours to *weeks* ; 2706359 sec. to *weeks* ; 38567 min. to *days*.
(2) 123456 sec. to *hours* ; 3456794 sec. to *days* ; 579574 min. to *years*.

6. Reduce to *years* :—

71871900 sec. ; 1301416510 sec. ; 713969410 sec. ; 413419320 sec.

7. Add together :—

(1)

sap.	da.	pr.
36	5	6
24	4	2
48	6	5
2	3	4
18	6	7

(2)

dan.	pal.	bip.	anu.
41	36	57	51
39	48	39	47
49	55	13	58
59	26	49	38
21	50	28	19

(3)

din.	dan.	pal.	bip.	anu.
60	57	19	21	27
73	40	23	17	13
9	55	19	18	29
37	20	40	19	24
47	30	59	29	34

(4)

hrs.	min.	sec.
15	42	45
57	36	40
32	12	14
16	37	45
5	51	41
24	19	40

(5)

da.	hrs.	min.	sec.
35	14	32	30
47	16	25	27
54	18	52	57
43	21	37	29
62	22	58	57
40	15	20	32

(6)

wks.	da.	hrs.	min.
10	5	14	31
18	4	12	38
25	0	10	14
75	6	23	59
53	4	19	23
40	0	17	25

8. Perform the following subtractions :—

(1) sap.	da.	pr.	(2) dins	dan.	pal.	bip.	(3) da.	hrs.	min.	sec.
527	5	5	80	50	40	20	17	1	0	17
418	6	7	50	55	50	36	7	17	13	22

(4) da.	hrs.	min.	sec.	(5) wks.	da.	hrs.	(6) yrs.	da.	hrs.	min.	sec.
24	14	46	31	7	3	18	7	129	13	26	17
4	21	18	52	4	6	20	3	273	18	34	29

9. Multiply :—

(1) 7 dins 5 dan. 30 pal 15 bip. by 74, and by 140.
(2) 9 ba. 8 ma. 27 da. 45 dan. 56 pal. 38 bip. 52 anu. by 43, 67.
(3) 43 days 18 hrs. 45 min. by 77, and by 147.
(4) 17 wks. 4 da. 13 hrs. 27 min. 36 sec. by 9, and by 79.
(5) 17 years 110 da. 17 hrs. 57 sec. by 144.

10. Divide :—

(1) 694 dins 7 pr. 3 dan. 30 pal. by 32.
(2) 2056 ba. 5 ma. 27 da. 44 dan. 15 pal. by 87.
(3) 17 wks. 5 da. 18 hrs. 25 min. by 49.
(4) 878 wks. 4 da. 15 hrs. 37 min. 36 sec. by 9, and by 56.

11. How many days are there (the last day mentioned in each case being excluded) from
(1) April 5, 1863 to Nov. 3, 1863? (2) Dec. 31, 1863 to Dec. 31, 1864?
(1) Sep. 21, 1863 to March 1, 1864? (4) Nov. 16, 1882 to Sep. 5, 1884?

12. How many bipals are there in a year of 365 days 6 hours ?

13. A solar year = 365 days 5 hrs. 48 min. $47\frac{1}{2}$ sec. : (1) how many more seconds are there in a solar year than in a common year? (2) How many seconds less than in a leap-year ?

14. How many portions of time each equal to 1 day 7 hrs. 45 min. 56 sec. are contained in 346 days 18 hrs. 34 min. 32 sec. ?

15. If the 1st of April is a Monday, on what day of the week will Christmas fall that year ?

VIII. MEASURES OF ANGLES.

175. **English Angular Measure**

60 Seconds (60″)	make	1 Minute (′)
60 Minutes	„	1 Degree (°)
90 Degrees	„	1 **Right Angle** (1 *rt. gle.*)

IX. MEASURES OF NUMBERS.

176. BENGALI TABLE.

4 Units	make	1 Ganda
5 Gandas	„	1 Buri
4 Buris	„	1 Pan
16 Pans	„	1 Kahan

ENGLISH TABLE.

12 Units	make	1 Dozen
12 Dozen	„	1 Gross
12 Gross	„	1 Great Gross
20 Units	„	1 Score (*Kuri*)
120 Units	„	1 Long Hundred.

FOR PAPER.

24 Sheets = 1 Quire ; 20 Quires = 1 Ream ; 10 Reams = 1 Bale.

Examples XXXIX.

1. Reduce to *seconds* :—
(1) 172° 8′ 25.″ (2) 275° 30′ 25.″ (3) 144° 12′ 38.″ (4) 57° 7′ 45.″

2. Reduce to *right angles*, *degrees &c.* :—
(1) 206265.″ (2) 865408.″ (3) 718276.″ (4) 42861.′ (5) 78205.′

3. Add together : 175° 32′ 45,″ 75° 59′ 27″, 114° 28′ 47″, 105° 45″, 144° 12′ 38″, 160° 52′ 58″, and 175° 20′ 46″.

4. Subtract 149° 53′ 56″ from 277° 36′ 47″.

5. Multiply 24° 12′ 16″ by 42 ; 19° 14′ 25″ by 36.

6. Divide 25° 25′ 32″ by 16 ; 144° 44′ 7″ by 22.

7. In 56 reams of paper, how many sheets ?

8. Reduce 67835 kahans 11 pans 18 ga. 3 units to *units*.

9. Reduce 7297865 units to *kahans* ; 9 scores to *dozens*.

10. Multiply 9 kahans 2 pans 17 ga. 2 units by 82, and by 346.

Examples XL.

(*Recapitulatory Exercises.*)

1. In 340 pistoles at 17*s.* 6*d.* each, how many pounds sterling ?

2. How many moidores of 27*s.* each, are equivalent to 198 guineas and to £500638. 1*s.* ?

3. In £453. 16*s.* 8*d.*, how many pieces of coin valued at 3*s.* 4*d.* each ? How many at 11*s.* 8*d.* each ?

4. What number of weights of 14 oz. 13 drs. each, are equivalent to 25 cwt. 2 qrs. 13 lbs. 14 oz. 12 drs. ?

5. If I spend £2. 7*s.* $1\frac{1}{2}$*d.* a day, how much is that in 28 weeks, and also in a year of 365 days ?

6. If each of 114 persons receive £1. 18*s.* $6\frac{3}{4}$*d.*, what is received by them all ?

7. If the clothing of 754 soldiers come to £3178. 11*s.* $7\frac{1}{2}$*d.* ; how much is that for each man ?

8. If a person complete a journey of 422 mi. 3 fur. 38 po. in 37 days ; what distance does he travel each day ?

9. A year being equivalent to 365 days 6 hours, find the number of years, &c., in 295402374 seconds.

10. Multiply 4 dins 3 pr. 2 dan. 25 bip. 15 anu. by 401.

11. Reduce 9367875 angulis to *kros* ; 14978631 gandas to *bighas*.

12. Find how often a rod 2 ft. 10 in. in length, must be applied to measure 10 miles 140 yds.

13. Find the number of yards in 40 pieces of cloth, each containing 42 yds. 2 qrs. 2 nls.

14. If a soldier's pay for a year of 365 days be £9. 2*s.* 6*d.*; how much is that for a day?

15. If a person's yearly income be £65. 12*s.* 6*d.*, and he lay by £20 a year; how much does he spend each day?

16. How many pounds of silver are there in a half-dozen of dishes, each weighing 51 oz. 10 dwts. and a dozen of plates, each weighing 15 oz. 15 dwts. 22 grs.?

17. Express 452 dan. 48 pal. 45 bip. in *English measure.*

18 If 145 sheep cost £169. 3*s.* 4*d.*, what is the price of a score at the same rate?

19. If 8 packages of cloth, each consisting of 4 parcels, each parcel of 10 pieces, and each piece of 26 yards, cost *Rs.*66560; what is the price of a yard?

20 The sum of £263. 8*s.* 11¼*d.* is distributed equally among a number of persons so that the share of each is £37. 12*s.* 8¼*d.*; find the number of persons.

21. A boy's school, to and from which he walks daily, is distant from his home 1 kros 250 dan. 1 gaj 1 hath 7 girahs. How many girahs does he walk every day?

22. Reduce 35 tons 19 cwt. 99 lbs. 12 oz. 135 grs. to *grains.*

23. Reduce 294322493 sq. in. to *acres, &c.*

24. Find the weight of copper coin required to pay a debt of £1000 when 3 pennies weigh 1 oz.

25. Which is the heavier, 1 lb. of gold or 1 lb. of sugar?

26. If 28 lbs. 9 oz. of gold be worth £1343. 6*s.* 10½*d.*, what is the worth of 1 ounce?

27. Among how many boys can I distribute £14. 9*s.* 9*d.* giving to each boy a half-crown, a florin, a four-penny piece, and also a three-penny piece?

28. If a man's net income be £1785. 12*s.* 6*d.*, how much may he spend on an average per day to the nearest farthing, so as not to run into debt?

29. Reduce 5792685 inches to *miles, &c.*

30. Jadu was born at 6 o'clock A.M., 24th June, 1872; how old will he be at 3 o'clock P.M., 10th Jan., 1898?

31. Find the sum of 32 cwt. 2 qrs. 15 lbs. 12 oz.; 47 cwt. 25 lbs. 9 oz.; 5 cwt. 3 qrs. 17 lbs. 10 oz.; 23 cwt. 1 qr. 19 lbs. 15 oz., and 9 cwt. 3 qrs. 14 oz.; divide the sum by 4 cwt. 2 qrs. 18 lbs. 8 oz.

32. If 1*s.* 5½*d.* be the unit of money, what will be the measure of £7. 17*s.* 6*d.* and of £20. 1*s.* 0½*d.*?

33. If 2 ft. 6 in. be the unit of length, what number will represent (i) 10 miles, (ii) 25 miles 760 yds.?

34. If 6 hrs. 32 min. 10 sec. be the unit of time, what will be the measure of 74 days 1 hr. 49 min. 20 sec.?

35. If 2 lbs. 5 oz. be the unit of weight, what number will measure 5 cwt. 6 lbs. 9 oz.?

36. If 5 sec. be the unit of time, what will be the measure of 3 hrs. 5 sec. and of 15 hrs. 20 min.?

37. 22nd September 1897 was Wednesday. What day of the week was 22nd September 1797 and what day of the week will 22nd September 1997 be?

38. 19th September 1897 was Sunday. What day of the week was 23 January 1807 and what day of the week will 23rd January 1907 be?

X. MISCELLANEOUS PROPOSITIONS.

(IN COMPOUND QUANTITIES.)

177. The Unitary Method. (*Simple Cases*)

If the value, weight, length, &c. of any number of units be given, we can by Compound Division find that of one unit of the same kind; and the value, weight, length, &c. of one unit being found, we can by Compound Multiplication find that of any number of units of the same kind. The solution which combines these two processes is called **The Method of Reduction to the Unit or The Unitary Method.**

(1) The value, weight, length, &c. of **one** unit being given, we can by *Compound Multiplication* find the value, weight, length, &c. of any number of units of the same kind.

Ex. The price of a maund of sugar is *Rs.*10. 15*a.* 6*p.*; find the price of 35 maunds.

*Rs.*10. 15*a.* 6*p.*
35
Rs. 383. 14*a.* 6*p.*

The price of 1 maund = *Rs.*10. 15*a.* 6*p.*
∴ the price of 35 mds. = *Rs.*10. 15*a.* 6*p.* × 35
= *Rs.*383. 14*a.* 6*p.*

(2) The value, weight, length, &c. **of any number** of units being given, we can by *Compound Division* find the value, weight, length, &c. of one unit of the same kind.

Ex. If 30 mds. of rice cost *Rs.*134. 1*a.*; what is the price per maund?

```
30) Rs.134. 1a. (4Rs.
    120
     14
     16
    225(7a.
    210
     15
     12
    180(6p.
    180
```

The price of 30 mds. = *Rs* 134. 1*a.*
∴ the price of a md. = *Rs.*134 1*a.* ÷ 30
= *Rs.*4. 7*a.* 6*p.*

(3) **The value, weight, &c. of a certain number of units being given, to find the value, weight, &c. of a certain other number of units of the same kind.**

Proceed as in the following Examples :—

Ex. 1. If 7 yards of cloth cost *Rs.*26. 4*a.*, what will be the cost of 15 yds. of the same?

7)*Rs.*26. 4*a.*
*Rs.*3. 12*a.*
15
*Rs.*56. 4*a.*

7 yards cost *Rs.*26. 4*a.*
∴ 1 yard costs *Rs.*26. 4*a.* ÷ 7 = *Rs.*3. 12*a.*
∴ 15 yards cost *Rs.*3. 12*a.* × 15 = *Rs.*56. 4*a.*

Ex. 2. If 7lbs. of tea cost 15*s.* 9*d.*, what will be the cost of 12lbs.?

7)15*s.* 9*d.*
2*s.* 3*d.*
12
£1. 7*s.*

7lbs. cost 15*s.* 9*d.*
∴ 1 lb costs 15*s.* 9*d.* ÷ 7 = 2*s.* 3*d.*
∴ 12 lbs. cost 2*s.* 3*d.* × 12 = £1. 7*s.*

(4) **The value, weight, &c. of a certain number of units being given, to find the number of units of the same kind corresponding to some other value, weight, &c.**

Proceed as in the following Examples :—

Ex. 1. If 12 maunds of rice cost *Rs.*35, find how many maunds of the same can be bought for *Rs.*20. 6*a.* 8*p.*

12)*Rs.*35
*Rs.*2. 14*a.* 8*p.*
= the price of a maund.

*Rs.*20. 6*a.* 8*p.* = 3920*p.* ; *Rs.*2. 14*a.* 8*p.* = 560*p.*
∴ the no. of mds. required = 3920 ÷ 560
= 7. *Ans.*

Ex. 2. If 25 men finish a piece of work in 16 days, in how many days will 20 men finish it?

25
16
20)400 days
20 days.

25 men finish the work in 16 days,
∴ 1 man will finish in (25 × 16) or 400 days.
∴ 20 men will finish in 400 ÷ 20 or 20 days. *Ans.*

Ex. 3. How many men can perform in 24 days a piece of work which 15 men can perform in 40 days?

15
40
24)600 men
25 men.

In 40 days the work is done by 15 men.
∴ in 1 day the work is done by (15 × 40) or 600 men.
∴ in 24 days, the work is done by 600 ÷ 24 or 25 men. *Ans.*

Note. In questions such as the above, it should be noticed that to a ***diminution*** in the number of men corresponds an ***increase*** in the number of days and ***vice versa.***

Examples XLI.

1. What is the value of 72 reams of paper, at 13*s*. 8*d*. a ream ?

2. Find the cost of 120 ounces of silver, at 5*s*. $3\frac{3}{4}$*d*. an ounce

3. What will be the price of 1 lb., when 1 cwt. costs £137. 18*s*.?

4. If 41 cwt. cost £52. 10*s*. $7\frac{1}{2}$*d*., what is the price of a cwt. ?

5. If 6 chairs cost *Rs*.32. 12*a*., what will 3 dozen cost ?

6. If a workman's wages for 12 days be *Rs*.14. 4*a*. 6*p*., what would it amount to in 18 days ?

7. If 4 yards of flannel cost *Rs*.3. 13*a*. 4*p*., what is the cost of 57 yards of the same ?

8. If 42 bighas of land be rented for *Rs*.640. 8*a*., what would be the rent of 61 bighas ?

9. If a man earn *Rs*.15. 12*a*. in 6 days, in how many days will he earn *Rs*.189 ?

10. If I travel by Railway 85 miles for *Rs*.7. 15*a*. 6*p*., how far may I travel for *Rs*.9. 6*a*. ?

11. If 13 sheep cost *Rs*.175. 8*a*., how many may be purchased for *Rs*.2160 ?

12. If 7 seers of tea cost *Rs*.7. 9*a*. 4*p*., what will be the cost of 1 md. 24 sr. 8 ch. ?

13. A clerk's salary is *Rs*.1916. 4*a*. per annum ; what ought he to receive for 60 days' service ?

14. How much land may be rented for *Rs*.705. 4*a*., if 5 acres are rented for *Rs*.46. 10*a*. 8*p*. ?

15. How many men can perform in 12 days a piece of work, which 15 men can perform in 20 days ?

16. If 3 mds. 12 sr. 8 ch. of sugar cost *Rs*.16. 9*a*., what will 2 mds. 14 sr. 10 ch. cost ?

17. Find the quantity of rice which can be purchased for *Rs*.86. 3*a*. $9\frac{1}{2}$*p*., when 70 mds. 10 sr. cost *Rs*.270. 12*a*. 1*p*.

18. If 3 cwt. 69 lbs. cost £14. 3*s*. 6*d*., how much may be bought for £23. 12*s*. 6*d*. ?

19. If 2 cwt. 3 qrs. 7 lbs. cost £5. 17*s*. $8\frac{1}{4}$*d*., what is the cost of 9 cwt. ?

20. In how many days would 171 men perform a piece of work, which 108 men can perform in 266 days ?

178. Revolution of Wheels.

A **wheel** in making one **revolution** passes over a length of ground exactly equal to its **circumference**. Hence, if we multiply

the circumference by the number of revolutions made, we shall find the distance passed over; and conversely, if we divide the distance passed over by the circumference, we shall find the number of revolutions, or by the number of revolutions we shall find the circumference.

Ex. 1. A carriage wheel is 4 yds. 2 ft. 7 in. in circumference, and makes 1456 revolutions on a journey. What is the length of the journey?

$1456 = 8 \times 13 \times 14$

(220 yds. = 1 furlong).

∴ the distance passed over is 4 mi. 0 fur. 37 yds. 2 ft. 4 in.

mi.	fur.	yds.	ft.	in.
		4	2	7
				8
		38	2	8
				13
	2	65	1	8
				14
4	0	37	2	4

Ex. 2. A wheel makes 131 revolutions in passing over 669 yds. 1 ft. 8 in.; what is its circumference?

The circumference = 669 yds. 1 ft. 8 in. ÷ 131 = 5 yds. 4 in.

Ex. 3. How many revolutions will a carriage-wheel 3 yds. 2 ft. 6 in. in circumference, make in a journey of 7 miles 3 fur. 34 po. 4 yds. 1 ft.?

```
3 yds. 2 ft. 6 in.      7 mi. 3 fur. 34 po. 4 yds. 1 ft.
3                       8
11 ft.                  59 fur.                 138)474168(3436
12                      40                          414
138 in.                 2394 po.                     601
                        11                           552
                        2)26334                       496
                        13167 yds. + 4 yds.           414
                        = 13171 yds.                   828
                        3                              828
                        39514 ft.
                        12          ∴ the number of revolutions
                        474168 in.     required = 3436. Ans.
```

Examples XLII.

1. If a wheel 5 yds. 2 ft. 4 in. in circumference makes 1080 revolutions on a journey, how far will the carriage go?

2. If a wheel 5 yds. 1 ft. 6 in. in circumference makes 64640 revolutions, what space will it pass over?

3. How many revolutions will the wheel of a carriage, 4 ft. 7 in. in circumference, make in 2 mi. 4 fur.?

4. A wheel makes 514 revolutions in passing over 1 mi. 467 yds. 1 ft.; what is its circumference?

5. A boy's hoop is 3 yds. 10 in. round; how many miles of ground will it pass over in 2501 turns?

6. The fore-wheel of a carriage is 4 ft. 6 in. round, and the hind-wheel a foot longer; how many more turns will the former make than the latter in a distance of 30 miles?

7. A wheel makes 1540 revolutions in passing over 2 mi. 458 yds. 1 ft.; what is its circumference?

8. How many revolutions will a wheel 4 yds. 2 ft. in circumference make on a journey of 12 mi. 696 yds. 2 ft.?

9. The circumference of the fore-wheel of a carriage being 8 ft. 3 in., and that of the hind-wheel 11 ft. 11 in., how many more revolutions would be made by the fore-wheel than by the hind-wheel in going a distance of 52 miles?

10. The driving wheel of a locomotive is 5 yds. 2 ft. 9 in. in circumference, and makes on an average 3 revolutions a second; find the rate of the train per hour.

11. The fore-wheel of a carriage which is 2 yds. 2 ft. 6 in. in circumference makes 4350 more revolutions than the hind-wheel in going over a distance of 19 miles 2 fur. 120 yds.; what is the circumference of the hind-wheel?

12. Find the circumference of the wheel of a locomotive which makes on an average 4 revolutions in a second, and which performs a journey of 76 miles in 1 hour 36 min.

13 A wheel revolves 1028 times in going 2 mi. 934 yds. 2 ft. What is its circumference?

14. In going over a distance of 205 miles the fore-wheel turns 98400 times and the hind-wheel 78720 times. How much longer is the circumference of the hind-wheel than that of the fore-wheel?

15. The circumference of the fore-wheel of a carriage is 8 ft. and that of the hind-wheel is 10 ft.; in what distance will the fore-wheel make 100 revolutions more than the hind wheel?

179. Averages.

The **Average** or **Mean** of any number of given quantities of the same kind, is that quantity which when substituted for each of the given quantities makes their sum the same. Hence, to find the *Average* of any number of quantities we divide the sum of them by their number.

Ex. The receipts at a Railway Station are as follow; Jan. *Rs.*2458. 14*a.* 8*p.*; Feb. *Rs.*2019. 6*a.*; March, *Rs.*2857. 4*a.* 8*p.*; April

*Rs.*3051. 1*a.* 4*p.* ; May, *Rs.*3463. 13*a.* 4*p.* ; and June, *Rs.*4007. 10*a.* ; find the average receipts per month.

Rs.	*a.*	*p.*
2458	14	8
2019	6	0
2857	4	8
3051	1	4
3463	13	4
4007	10	0
6)17858	2	0
*Rs.*2976	5	8

The sum of the receipts for the 6 months is found to be *Rs.*17858. 2*a.* ; hence the average month's receipt is found by dividing this sum by 6, and is

*Rs.*2976. 5*a.* 8*p.*

180. Nearest money.

When there is a **remainder** after division, we observe, that if the quotient be multiplied by the divisor the product will be *less* than the dividend ; also that if the quotient be increased by 1 and be then multiplied by the divisor the product will be *greater* than the dividend. Hence, in all cases, a **nearest** sum can be found, which will be exactly divisible by the divisor. Also a quotient correct to the **nearest lowest denomination**. (Art. 149.)

Ex. 1. Find the *nearest* sum of money to £197. 11*s.* 6*d.* that can be divided by 23 without remainder.

```
     £.    s.   d.        £.   s.   d.
23)197    11    6      (8    11   9½
   184
    13
    20
   271 (11s.
    23
    41
    23
    18
    12
   222 (9d.
   207
    15
     4
    60 (2q.
    46
   14q.
```

From the work it appears that if the given sum be diminished by 14*q.*, or 3½*d.* there will be no remainder, or if it be increased by 9*q.* or $2\frac{1}{4}$*d.*, so as to make the last partial dividend 69, there will be no remainder ; hence the *nearest* sum required is £197. 11*s.* 6*d.* + $2\frac{1}{4}$*d.* or

£197. 11*s.* $8\frac{1}{4}$*d.* *Ans.*

Ex. 2. If £197. 11*s.* 6*d.* be given for 23 pieces of cloth, find to the *nearest* penny the price given for each piece.

From the last *Ex.*, it appears that £8. 11*s.* 9*d.* a piece would give 15*d.* too little, and £8. 11*s.* 10*d.* would give 8*d.* too much ; hence, to the *nearest* penny the price would be £8. 11*s.* 10*d.* *Ans.*

Examples XLIII.

1. On Sunday I spent no money, on Monday *Rs.*43. 14*a.*, on Tuesday *Rs* 51. 12*a.* 8*p.*, on Wednesday *Rs.*46. 14*a.* 6*p.*, on Thursday *Rs.*52. 8*a.*, on Friday *Rs.*32. 15*a.* 6*p.*, on Saturday *Rs.*26. 4*a.* ; find my average daily expenditure during the week.

2. The daily receipts of a grocer for the week are as follow :— Monday *Rs.*47. 10*a.* 2*p.* ; Tuesday *Rs.*56. 8*a.* 4*p.* ; Wednesday *Rs.*78. 7*a.* ; Thursday (being a holiday) nothing ; Friday *Rs.*39. 7*a.* 4*p.* ; and Saturday *Rs.*159. 13*a.* 2*p.* ; find his average daily receipts (1) excluding Thursday, and (2) including Thursday.

3. Find the least sum of money that must be subtracted from £663. 14*s.* 8*d.* to make the remainder divisible by 37.

4. Deduct *Rs.*26. 13*a.* 6*p.* from *Rs.*562. 8*a.*, and divide the resulting sum equally among 29 persons to the nearest pie ; how much will each person receive, and how much will remain over ?

5. The average price of a quarter of wheat for 19 years was 56*s.* 8*d.* a quarter ; for the first five years the average price was 61*s.* 3½*d.* a quarter, for the next 4 years 58*s.* 0½*d.*, for the next 7 years 53*s.* 5¼*d.* ; find the average of the last 3 years.

6. Find the nearest sum of money to *Rs.*3339. 10*a.* 10*p.* that can be divided by 29 without remainder.

7. The mean height of 6 mountains is 10357 feet ; find what the height of the seventh mountain must be, in order that the mean height of the seven mountains may be 10643 ft.

8. 120 tons of coal are purchased for £87. 16*s.* 9*d.* ; find to the nearest farthing the price at which they must be retailed per ton, so that no loss may be incurred.

9. Find the least sum of money that must be added to *Rs.*3658. 12*a.* 4*p.* to make the sum divisible by 127.

10. A tradesman's average annual income from 1830 to 1850 was *Rs.*3744. 13*a.* 4*p.* In 1830 his income was *Rs.*3699. 6*a.* 8*p.*, and in 1851 his income was *Rs* 3600. 8*a.* 8*p.* what was his average annual income from 1831 to 1851 (inclusive) ?

181. Gain and Loss.

The price at which an article is bought is called its **cost** price ; that at which it is sold, its **selling** price. If the selling price be greater than the cost price, it is **gain** ; if less, it is **loss**. Hence the difference between the two prices is the *gain* or *loss*.

(1) Given the quantity sold, and also the cost and selling prices, to find the gain or loss.

Ex. 1. A person bought 524 yards of cloth at *Rs*.7. 14*a* 6*p*. per yard and retailed it at *Rs*.8. 2*a*. 4*p*. per yard ; what was his profit ?

Selling price per yard = *Rs*.8. 2*a*. 4*p*.
Cost.............. = *Rs*.7. 14*a*. 6*p*.
∴ gain per yard = 3*a*. 10*p*.
∴ gain on 524 yards = 3*a* 10*p*. × 524 = *Rs*.125. 8*a*. 8*p*. *Ans*.

Ex. 2. A trader bought 1763 yards of cloth at 6*s*. 11*d*. per yard and retailed it at 5*s*. $3\frac{1}{2}d$ per yard ; what was his loss ?

Cost price per yard = 6*s*. 11*d*.
Selling price..... = 5*s*. $3\frac{1}{2}d$.
∴ loss per yard = 1*s*. $7\frac{1}{2}d$.
∴ loss on 1763 yards = 1*s*. $7\frac{1}{2}d$. × 1763 = £143. 4*s*. $10\frac{1}{2}d$ *Ans*

(2) Given the gain or loss, and the cost and selling prices, to find the quantity sold.

Ex. 3. A mercer bought some gloves at 2*s*. $2\frac{1}{2}d$. a pair, and by selling them at 3*s*. 6*d*. per pair, gained £9. 6*s*. ; how many pairs did he buy ?

Selling price per pair = 3*s*. 6*d*.
Cost .... = 2*s*. $2\frac{1}{2}d$.
∴ gain per pair = 1*s*. $3\frac{1}{2}d$. = 62*q*

Now, the whole gain = £9. 6*s*. = 8928*q*.
∴ the number of pairs bought = 8928 ÷ 62 = 144. *Ans*

Examples XLIV.

1. A person bought 500 yds of cloth at *Rs* 7. 14*a*. per yard and retailed it at *Rs*.8. 2*a*. per yard ; what was his profit ?

2. A person gave *Rs*.200 for 48 cwt. of goods ; what does he gain by selling them at *Rs*.5 a cwt. ?

3 A man buys 35 sheep for *Rs*.360 and 30 more for *Rs*.460, what will he gain or lose by selling them at *Rs*.15. 4*a*. each ?

4 A merchant bought 35 pieces of cloth measuring on an average 29 yards each at 3*s*. $10\frac{1}{2}d$. a yard, and sold them at 5*s*. 7*d* a yard ; what profit did he make ?

5 I bought 360 yds. of cloth at *Rs*.2. 10*a*. 8*p*. per yard, of which I sold 210 yds. at *Rs*.3. 9*a*. 4*p*. yard ; but the article advancing in price, sold the remainder at *Rs*.4. 8*a*. per yard ; what did I gain on the whole ?

6. I buy 84 books at *Re*.1. 15*a*. 8*p*. each, and sell them at a profit of *Rs*.70 ; what is the selling price of each ?

7. A shop-keeper purchases 35 reams of scribbling paper at *Rs* 7. 4*a*. per ream ; the carriage of the paper costs *Rs*.4. 12*a*. He

sells it at 8*a*. 8*p*. a quire with the exception of the outside quires of each ream, which he sells at 5*a*. a quire. Find his gain.

8. A grocer gave *Rs*.500 for 16 cwt. 2 qrs. 18 lbs. of sugar, and he lost *Rs*.72. 6*a*. by retailing it ; at what rate did he sell it per lb. ?

9. I buy a number of books at *Re*.1. 6*a*. 4*p*. each and sell them at *Re*.1. 10*a*. each. If I thereby make a profit of *Rs*.22, how many books do I buy ?

10. A person gives *Rs*.556. 8*a*. for a certain number of gallons of wine. He sells it at *Rs*.2. 10*a*. a gallon, and thereby makes a profit of *Rs*.36. 1[illegible] How many gallons does he buy ?

11. Find the cost of 20 dozen bottles of wine at *Rs*.2. 7*a*. 8*p*. per bottle ; and if 3 bottles be spoiled, what will the merchant gain by selling the remainder at *Rs*.2. 10*a*. 8*p*. per bottle ?

12. A cabinet dealer bought chairs at *Rs*.11. 15*a*. a piece, and lost *Rs*.9. 12*a*. by selling each at *Rs*.11. 2*a*. How many chairs did he buy ?

13. A person lays out £43. 9*s*. 4*d*. in spirits at 5*s*. 4*d*. a gallon ; 19 gallons leaked out in the carriage ; he however sold the remainder at 7*s*. 6*d*. a gallon ; what profit did he make ?

14. A merchant bought 7 pieces of cloth, each 27 yards, for £55. 12*s*. ; and sold 56 yards at 5*s*. $3\frac{1}{2}$*d*. per yard and the rest at 6*s*. 8*d*. per yard. Find his whole gain.

15. A merchant laid out *Rs*.693 in spirits which he bought at *Rs*.6. 6*a*. 8*p*. a gallon ; he retailed it at *Rs*.8. 4*a*. a gallon, making a profit of *Rs*.115. 8*a*. How many gallons must he have lost by leakage ?

182. Barter and Exchange.

When we **barter** we give or take one sort of goods in **exchange** for another of a different sort which is regarded as an equivalent. Hence, to find how much of the first sort be given in exchange for a fixed quantity of the second, we must *first* find the money value of the second sort and *then* find what quantity of the first sort is of equal value.

Ex. 1. How many pounds of tea at 3*s*. $2\frac{1}{2}$*d*. a lb. must a grocer give in exchange for 35 yards of cloth at 12*s*. $4\frac{1}{2}$*d*. a yard ?

12*s*. $4\frac{1}{2}$*d*.	594*q*.	3*s*. $2\frac{1}{2}$*d*.	154)20790(135
12	35	12	154
148*d*.	20790*q*.	38*d*.	539
4		4	462
594*q*.		154*q*.	770
			770

∴ the number of lbs. of tea = 135. *Ans.*

Ex. 2. What weight of sugar at 3*a.* a lb. must be given in exchange for a chest of tea weighing 84 lbs. at *Re.*1. 9*a.* a lb.?

*Re.*1. 9*a.*	25*a.*	
16	84	3)2100
25*a.*	2100*a.*	700

∴ the number of lbs. of sugar = 700. *Ans.*

Examples XLV.

1. How many dollars of 4*s.* $1\frac{1}{2}$*d.* each must be given in exchange for 4950 thalers of 2*s.* $11\frac{1}{2}$*d.* each?

2. How many francs of $9\frac{1}{2}$*d.* each will be given in exchange for 475 thalers at 2*s.* $11\frac{1}{2}$*d.* each?

3. How many lbs. of tea at *Re* 1. 9*a.* 8*p.* a lb. must be given in exchange for 46 yards of silk at *Rs.*4. 0*a.* 2*p.* a yard?

4. A man exchanges 45 sheep at *Rs.*22. 14*a.* each and 37 pigs at *Rs.*36. 12*a.* each for 13 oxen at *Rs.*173. 4*a.* each, the difference being paid or received in money; how much does he pay or receive?

5. The Calcutta rupee is worth 1*s.* $11\frac{3}{4}$*d.* each; how many must be given for £9895. 16*s.* 8*d.*?

6. How much coffee at 1*s.* $10\frac{1}{2}$*d.* a lb. should be given in exchange for 72 lbs of tea at 3*s.* 4*d.* per lb.?

7. How many yards of cloth worth 3*s.* $7\frac{1}{4}$*d.* a yard must be given in exchange for 144 yards worth 18*s.* $1\frac{1}{2}$*d.* a yard?

8. How many Rubles at 3*s.* $4\frac{1}{2}$*d.* each are equal in value to 378 Napoleons, at 15*s.* $9\frac{3}{4}$*d.* each?

9. What quantity of tea at *Rs.*2. 6*a.* 6*p.* per lb., must be given in exchange for 5 cwt. 2 qrs. of sugar at *Rs.*3. 15*a.* per stone?

10. A person exchanged 18 dozen of wine for a gold snuff-box weighing 8 oz. 13 dwts. 10 grs. valued at £4. 10*s.* an oz. What did he value his wine at per dozen?

11. *A* gives *B* 98 gallons of brandy worth *Rs.*12. 12*a.* a gallon and gets in return *Rs.*409. 8*a.* and 576 yards of cloth; what is the value of the cloth per yard?

12. A man sold 53 horses at *Rs.*168. 11*a.* 4*p.* each, and with the money he received for them and *Rs.*990 more he bought 355 cows and a certain number of calves; he gave for 198 of the cows *Rs.*22. 2*a.* a head, and for the rest of the cows *Rs.*18. 6*a.* 8*p.* a head, and for the calves *Rs.*14. 6*a.* a head. How many calves did he buy?

183. Allotment.

By **allotment** we divide a given quantity in a certain way into a proposed number of parts and thus ascertain the actual amount of each part.

Ex. 1. How many sovereigns, half-sovereigns, crowns, florins, shillings, six-pences and three-pences, and of each an equal number are there in £67. 16*s.* 3*d.* ?

s.	*d.*
20	0
10	0
5	0
2	0
1	0
	6
	3
38*s.*	9*d.*
12	
465*d.*	

£. *s.* *d.*
67 16 3
20
1356*s.*
12
16275*d.*

465)16275(35
1395
2325
2325

Since every collection of one of each of these coins amounts to 38*s.* 9*d.* ; therefore there will be as many coins of each kind as £67. 16*s.* 3*d.*, contains 38*s.* 9*d.* Hence the number of coins of each kind = 35. *Ans.*

Ex. 2. An equal number of men, women and boys earned *Rs.*556. 8*a.* in 6 weeks ; each man earned *Re.*1. 2*a.* 8*p.* a day, each woman 10*a.* and each boy 6*a.* 8*p.* ; how many were there of each ?

Re.	*a.*	*p.*	
1	2	8	= a man's daily earning.
	10	0	= a woman's..........
	6	8	= a boy's...... ......
*Rs.*2	3	4	= total daily earnings.
		7	
*Rs.*15	7	4	= weekly earnings.
		6	
*Rs.*92	12	0	= earnings of 6 weeks.

Rs.	*a.*
92	12
16	
1484*a.*	

Rs.	*a.*
556	8
16	
8904*a.*	

1484)8904(6
8904

∴ no. of each sort = 6. *Ans.*

Examples XLVI.

1. Divide £39 into four equal number of guineas, half-guineas, crowns and half-crowns respectively.

2. An equal nůmber of gold-mohurs, rupees, eight-anna pieces, four-anna pieces, two-anna pieces and pice amount to *Rs.*447. 4*a.* 1*ps.* ; how many of each sort are there ?

3. An equal number of guineas, pounds, half-guineas, crowns, half-crowns and six-pences amount to £714 ; how many of each are there ?

4. An equal number of rupees, half-rupees, quarter-rupees, two-anna pieces, double-paisas, and piasas amount to *Rs.*803. 5*a.* ; find the number of each.

5. At the end of a week £54. 3*s.* is paid in wages to an equal number of men, women and boys ; a man is paid 4*s.* 6*d.*, a woman 3*s.* 3*d.* and a boy 1*s.* 9*d.* a day ; how many of each class are there ?

6. Tithes of the value of £448. 10*s.* are commuted for an equal number of bushels of wheat, barley and oats ; how many bushels of

each kind will be received when wheat is sold at 7s. 2d. a bushel, barley at 4s. 9d., and oats at 3s. 5d.?

7. Rs.750 is paid in wages at the end of the week to a certain number of men, twice as many women, and three times as many children; each man earns Rs.2. 1a. 4p. a day, each woman Re.1. 6a and each child Re.1. 2a. 8p.; how many children are there?

8. A bag contains a certain number of rupees, twice as many half-rupees, five times as many quarter-rupees, and eight times as many two-anna pieces, and the value of the whole sum in the bag is Rs.272. Find the number of each.

9. One farm produced 111 times as much rice as another; both farms produced 1776 mds. 10 sr.; how much did the smaller farm produce?

10. How many packets of tea of 1 lb. 8 oz. and 1 lb. 12 oz. respectively, an equal number of each, can be made out of a chest of tea, in which the tea weighs 1 cwt. 1 qr. 3 lbs.?

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184. Mixtures.

When several articles of the same kind but of different qualities or value are mixed together to form a **compound**, it is called a **mixture**. The parts forming the compound are called **ingredients** or **components** of the compound.

(1) Given the quantity and price of each of the component parts, to find the price of the mixture.

Ex. 1. A mixture is made of 9 gallons of spirit at Rs.6. 4a per gal., 16 gallons at Rs.9. 6a. and 90 gallons at Rs.11. 2a.; what is the value of a gallon of it?

The cost of 9 gals. = Rs. 6. 4a. × 9 = Rs. 56. 4a.
............16 gals. = Rs. 9. 6a. × 16 = Rs. 150. 0a.
...........90 gals. = Rs.11. 2a. × 90 = Rs.1001. 4a.
∴ the cost of 115 gals. = Rs.1207. 8a.
∴ the cost of 1 gal. = Rs.1207. 8a. ÷ 115 = Rs.10. 8a. *Ans.*

Ex. 2. A man buys 16 lbs. of tea at Rs.2. 2a. per lb., 12 lbs. at Rs.2. 5a. 4p. per lb., and 24 lbs. at Rs.2. 6a. 10p. per lb. At what price per lb. must he sell the mixture, so as to gain Rs.35. 12a. on the whole?

The cost of 16 lbs. = Rs.2. 2a. × 16 = Rs. 34. 0a.
.... 12 lbs. = Rs.2. 5a. 4p. × 12 = Rs. 28. 0a.
.... 24 lbs. = Rs.2. 6a. 10p × 24 = Rs. 58. 4a.
∴ the cost of 52 lbs. = Rs.120. 4a.
Gain = Rs. 35. 12a.
∴ selling price of 52 lbs. = Rs.156. 0a.
∴ selling price per lb. = Rs.156 ÷ 52 = Rs.3. *Ans.*

(2) To find the quantity to be added to a mixture under certain conditions.

Ex. 3. A pipe of wine containing 126 gallons is bought for £112; how much water must be added to it to allow of its being sold at 17*s.* 6*d.* a gallon?

£112 = 112 × 20 × 12*d.* = 26880*d.*; 17*s.* 6*d.* = 210*d.*

Now the quantity sold for £112 at 17*s.* 6*d.* a gal. = (26880 ÷ 210) or 128 gallons.

∴ the quantity of water mixed = (128 − 126) or 2 gallons. *Ans.*

Ex. 4. If a person gives *Rs.*556. 8*a.* for 184 gallons of wine; how much water must be added to it, if he wishes to sell it at *Rs.*2. 10*a.* a gallon and make a profit of *Rs.*36. 12*a.*?

The selling price of the mixture = *Rs.* 556. 8*a.* + *Rs.* 36. 12*a.*
= *Rs.* 593. 4*a* = 9492*a.*

Also the selling price per gal. = *Rs.* 2. 10*a.* = 42*a.*

∴ the quantity sold = (9492 ÷ 42) or 226 gallons.

∴ the quantity of water added = (226 − 184) or 42 gallons. *Ans.*

Examples XLVII.

1. A grocer mixes 40 lbs. of tea at *Re.*1. 3*a.* a lb., 48 lbs. at *Re.*1. 5*a.* 6*p.* a lb. and 64 lbs. at *Re.*1. 9*a.* 10*p.* a lb.; find the value of 1 lb. of the mixture.

2. A grocer mixes 3 cwt. 24 lbs. of sugar at 6¼*d.* per lb. with 2 cwt. 64 lbs. at 4¼*d.*; at what price per lb. must he sell the mixture so as not to lose by the sale?

3. A tea merchant mixes 25 lbs. of tea at 14*a.* a lb., 40 lbs. at *Re.*1. 3*a.* 4*p.*, and 27 lbs. at *Re.*1. 9*a.* 4*p.*; at what rate per lb. must he sell the mixture, so as to gain *Rs.*23. 2*a.* on the transaction?

4. How many lbs. of tea-dust (which cost him nothing) must be put in the above mixture, to enable him to sell the tea at *Re.*1. 3*a.* 4*p.* per lb. and gain at the same time *Rs.*4. 4*a.* on the transaction?

5. A trader buys 756 cwt. of sugar at *Rs.*19. 7*a.* 8*p.* per cwt. with which he mixes 1921 cwt. of sugar which cost him *Rs.*21 per cwt.; at how much per lb. must he sell the mixture in order to make a profit of *Rs.*7396. 1*a.* 4*p.*?

6. A grocer mixes 19 lbs. of tea at 1*s.* 10¾*d.* per lb., 26 lbs. at 2*s.* 3½*d.* per lb., and 27 lbs. at 2*s.* 6¼*d.* per lb.; at how much per lb. must he sell the mixture so as to gain £2. 3*s.* 4*d.* on his outlay?

7. A spirit merchant mixes 26 gallons of wine at 12*s.* 3*d.* a gallon with 39 gallons at 13*s.* 4*d.* a gallon; how many gallons of water must he add to the mixture so as to sell it at 10*s.* 9*d.* a gallon?

8. A man bought 150 eggs at 2 a penny, 150 more at 3 a penny, and mixed them and sold the whole at 5 for 2*d.*, how much does he lose?

9. A grocer buys 4 cwt. of sugar at 6*d.* per lb. ; and 8 cwt. at $4\frac{1}{2}$*d.* per lb. He sells 6 cwt. at $5\frac{1}{2}$*d.* per lb. ; at what rate per lb. must he seil the remainder so as neither to gain nor lose ?

10. A merchant bought 84 gallons of whisky at *Rs.*8. 6*a.* a gallon, and sold it at *Rs.*8. 4*a.* a gallon, making a profit of *Rs* 105. How many gallons of water did he add to the whisky ?

158. Income and Expenditure.

Income including taxes and other rates is called **gross income**, but excluding these, it is **net income**. What a man lays by out of his income after meeting all necessary expenses, is called his **savings**.

Ex. 1. On the reduction of the income-tax from 9*d.* in the pound to 4*d.*, a person saves £29. 15*s.* 10*d.* ; find his gross income.

£29. 15*s.* 10*d.*=7150*d.*=savings.

He saves (9 − 4) or 5*d.* in every £ of his income.

∴ gross income required=£(7150÷5) or £1430. *Ans.*

Ex. 2. A man has a yearly income of *Rs.*4867. 8*a* and sets aside *Rs.*630 for charity, insurance and other purposes. What is the greatest sum he can spend per week, without getting into debt ?

	Rs.	*a.*	*p.*	
	4867	8	0	=yearly income
	630	0	0	=charity, &c.
52 { 4	4237	8	0	=yearly expenditure.
13	1059	6	0	
	*Rs.*81	7	10...8*p.*	

Hence we see that he may spend *Rs.*81. 7*a.* 10*p.* every week, and have 8*p.* over at the end of the year. If he spends *Rs.*81. 8*a.* per week he will run into debt.

Examples XLVIII.

1. A man's annual income is *Rs.*10,000 and his daily expenses are *Rs.*18. 10*a.* 4*p.* ; how much does he save in 9 years ?

2. A man's income in the year 1895 was *Rs.*5250, out of which he saved *Rs.*1691. 4*a.* ; what was his average daily expenditure ?

3. A man spends *Rs.*105. 14*a.* in a week ; how much does he spend in a year of 365 days ?

4. If a person spends in 4 months, as much as he earns in three, how much can he lay by annually, supposing that he earns *Rs.*2505. every 6 months ?

5. What annual income would enable a person to spend 8*s.* 9*d* a day and save £7. 16*s.* $10\frac{1}{4}$*d.*, every calendar month ?

6. If a person has an income of £535. 17*s.* 6*d.* a year, and he spends daily £1. 3*s.* $10\frac{1}{2}$*d.*, how much will he save at the end of the year ?

7. Find the salary of a person who pays £7. 9*s*. 4*d*. income-tax when the tax is 7*d*. in the pound.

8 A person after paying an income-tax of 4*p*. in the rupee, has *Rs*.8567. 11*a*. 4*p*. remaining ; find his gross income.

9. If a person's yearly income be £65. 12*s*. 6*d*. and he lay by £20 a year, how much does he spend per day ?

10. A person has an income of *Rs*.6706. 12*a*. 6*p*., and for the first 7 months he spends on an average *Rs*.588. 6*a*. 6*p*. a month ; how much must he spend during each of the remaining 6 months, so as not to run into debt ?

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186. Division of money.

When a given sum of money is divided among a number of persons in a proposed way, the amounts they severally receive are called their respective **shares**.

Ex 1. Divide £16. 5*s*. 6*d*. among *A*, *B* and *C*, so that *A* may have £1. 2*s*. 6*d*. more than *B*, and *B* 16*s*. 9*d*. more than *C*.

	£	s.	d.	
Here *B* has	0	16	9	more than *C* ;
and *A* ...	1	2	6	more than *B*.
∴ *A* ...	1	19	3	more than *C*.

Now, if we take away these sums, to be subsequently given to *B* and *A* respectively, their shares will be equal to that of *C*.

Hence we have

£.	s.	d.		£.	s.	d.		£.	s.	d.		£.	s.	d.
	16	9		16	5	6		4	9	10		4	9	10
1	19	3		2	16	0			16	9		1	19	3
£2	16	0	3)	13	9	6		£5	6	7		£6	9	1
				£4	9	10								

∴ *A*'s share = £6. 9*s*. 1*d*. ; *B*'s share = £5. 6*s*. 7*d*. ;
and *C*'s share = £4. 9*s*. 10*d*.

Ex. 2. Divide *Rs*.117. 11*a*. among *A*, *B* and *C*, so that *A* may receive twice as much as *B*, and *B* twice as much as *C*.

If *C*'s share is 1, *B*'s share is 2 and *A*'s share is 4.

Now, 1+2+4=7 ; 7)*Rs*.117. 11*a*.
Rs. 16. 13*a*.

∴ *C*'s share = *Rs* 16. 13*a*.
B's share = *Rs*.16. 13*a*. × 2 = *Rs* 33. 10*a*.
and *A*'s share = *Rs*.16. 13*a*. × 4 = *Rs*.67. 4*a*. } *Ans*.

Ex. 3. Divide *Rs.*2415 among *A*, *B* and *C* in such a way that for every *Rs.*20 that *A* gets, *B* gets *Rs.*15, and *C* gets *Rs.*11; how much does each receive?

20+15+11=46; 46)*Rs.*2415
Rs. 52. 8*a.*

∴ *A*'s share = *Rs.*52. 8*a.* × 20 = *Rs.*1050.
B's share = *Rs.*52. 8*a.* × 15 = *Rs.*787. 8*a.*
and *C*'s share = *Rs.*52. 8*a.* × 11 = *Rs.*577. 8*a.* } *Ans.*

Examples XLIX.

1. Divide *Rs.*24. 9*a.* 4*p.* among *A*, *B* and *C*, so that *B* may have *Rs.*3. 5*a.* 4*p.* more than *A*, and *C*'s share may be double of *B*'s.

2. Divide *Rs.*73. 4*a.* 6*p.* between two men so that one may receive as much again as the other.

3. Divide *Rs.*1845. 9*a.* 6*p.* equally among 39 persons; and supposing 15 of them to have received their portions, and of the rest only 21 to appear; how much might be given to each of these?

4. Divide £20. 2*s.* 6*d.* into two sums of money, one of which contains as many half-crowns as the other contains shillings.

5. Divide *Rs.*24515 among *A*, *B* and *C*, so that *A* may have *Rs.*1786. 12*a.* more than *B*, and *C* *Rs.*3257. 5*a.* less than *B*.

6. Divide *Rs.*2509. 14*a.* among *A*, *B* and *C*, so that *B* may receive 3 times, and *C* 5 times, as much as *A*.

7. Divide £189. 5*s.* 7½*d.* among 3 men, so that one of them may have 15 guineas more than either of the other two.

8. A purse and the money it contains are worth *Rs.*19. 4*a.*, and the money is 10 times the value of the purse; how much does the purse contain?

9. Divide *Rs.*690 between *A*, *B* and *C*, so that where *A* receives *Rs.*10, *B* may receive *Rs.*30, and where *B* receives *Rs.*20, *C* may receive *Rs.*50

10. The sum of *Rs.*473. 6*a.* 4*p.* has to be divided among 5 persons, so that the first has 20 shares, the second 17, the third 12, the fourth 8, and the fifth 5; how much will each receive?

11. Divide £119. 16*s.* 3*d.* among 36 persons, in such a way that 17 of them may each receive 18*s.* 9*d.* more than each of the rest.

12. Divide *Rs.*68427. 3*a.* 4*p.* among 3 persons, so that the first shall have *Rs.*5687. 2*a.* 8*p.* more than the second, and the second *Rs.*7289. 1*a.* 4*p.* more than the third.

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187. Men, Women and Boys.

Ex. 1. Divide *Rs.*156. 4*a.* among 7 men, 9 women and 11 boys,

so that each man may receive three times as much as a boy, and each woman twice as much as a boy.

The 7 men will receive as much as 7×3 or 21 boys and the 9 women as much as 9×2 or 18 boys; therefore 7 men, 9 women and 11 boys will receive as much as $21+18+11$ or 50 boys. Thus,

7 men	$=21$ boys		*Rs.*	*a.*
9 women	$=18$ ...	50 {	5) 156	4
11 boys	$=11$ ...		10) 31	4
	50		*Rs.*3	2

Hence a boy's share = *Rs.*3. 2*a.*,

a woman's = *Rs* 6. 4*a*, and a man's = *Rs.*9 6*a.*

Ex. 2. A man and a woman together have *Rs.*40. 6*a.* 8*p.*, a woman and a boy together have *Rs.*30. 8*a*, a man and a boy together have *Rs* 35. 7*a.* 6*p.*; find how much a man, a woman and a boy together have

Here, adding the three given items, we have twice a man's money + twice a woman's money + twice a boy's money = *Rs.*40. 6*a.* 8*p.* + *Rs.*30. 8*a.* + *Rs.*35. 7*a.* 6*p.* = *Rs* 106. 6*a.* 2*p.*

∴ a man + a woman + a boy together have *Rs.*106. 6*a.* 2*p.* ÷ 2 = *Rs.*53. 3*a.* 1*p.* *Ans.*

Examples L.

1. Divide £2. 10*s.* $10\frac{1}{2}$*d.* between 3 men and 2 women, giving to each of the men 3 times as much as to each of the women.

2. A gentleman divided *Rs.*103. 2*a.* among 12 men, 16 women and 30 children; he gave to each man twice as much as to each woman, and to each woman three times as much as to each child. What did each woman receive?

3. Divide *Rs.*3993. 8*a.* among one man, one woman and 15 boys in such a way that the man's share is 10 times and the woman's share 3 times as much as that of each boy; what is the value of the share of each?

4. Divide *Rs.*5501. 9*a.* among 4 men, 6 women and 8 boys, giving to each man double that of a woman and to each woman triple that of a boy.

5. Divide £15. 6*s.* among 12 men, 17 women and 26 children, in such a way that a man shall receive 3 times as much as a child and a woman twice as much as a child; what does a woman receive?

6. Divide *Rs.*1151. 4*a.* among 20 women and 25 men, so that each woman may receive *Rs* 7. 8*a.* more than each man; how much will each woman receive?

7. If 20 men, 40 women and 50 children receive *Rs.*3500 among them for 7 weeks' work, and 2 men receive as much as 3 women or 5 children, what sum does a woman receive per week?

8. The weekly wages at a mill amount to *Rs.*1862. In the mill a certain number of women are employed at *Re.*1. 6*a*. 8*p*. a day, five times as many men, at *Rs* 2. 12*a*. a day, and 6 times as many boys at *Re.*1. 2*a*. 8*p*. a day ; how many men are employed ?

9. *A* and *B* together have *Rs.*48. 14*a*. 9*p*, *B* and *C* together have *Rs* 45 10*a*. 6*p*, *A* and *C* together have *Rs.*54. 8*a*. 11*p* ; how much has *C* ?

10. A goat and a lamb are together worth *Rs.*6. 10*a*, a goat and a calf are together worth *Rs.*10. 4*a*. 8*p*. ; and a calf and a lamb are together worth *Rs* 8 5*a*. 6*p*. , find the price of a goat, of a lamb and of a calf.

Examples worked out.

Ex. 1. A man has a certain number of pice, twice as many two-anna pieces, three times as many four-anna pieces and four times as many rupees If the total amount be *Rs.*501. 9*a*., find the number of coins of each kind.

Here, 1 pice + 2 two-anna pieces + 3 four-anna pieces + 4 rupees = (1 + 16 + 48 + 256) pice = 321 pice ; and *Rs.*501. 9*a*. = 32100 pice.

∴ the number of pice = (32100 ÷ 321) or 100.

Hence, no. of two-anna coins = 2 × 100 = 200 ; the no. of four-anna coins = 3 × 100 = 300, and the no. of rupees = 4 × 100 = 400. *Ans.*

Ex. 2. A man died on June 2 Monday, 1890, having lived 23025 days exclusive of the day of his death. Find the day and date of his birth.

A year = 365 days ; therefore 23025 days ÷ 365 = 63 years 30 days. Now in these 63 years, 16 are leap-years (which = 366 days) ; therefore 23025 days = 63 years + (30 − 16) days or 63 years 14 days.

Again, 1890 − 63 = 1827, and reckoning 14 days backwards from June 1, we come to May 19.

Hence, the man was born on May 19, 1827.

Now 23025 divided by 7 gives a remainder 2 ; therefore he was born on Saturday, reckoning 2 days backwards from Sunday.

Ex. 3. The total expenses of a family when rice is at *Rs.*4. per maund are *Rs.*55 ; when rice is at *Rs.*3. 12*a*. per maund, they are *Rs.*52. 8*a*. (other expenses remaining the same) ; find his total expenses when rice is at *Rs.*4. 4*a*. per maund.

Here, a decrease of (*Rs.*4 − *Rs.*3. 12*a*.) or 4*a*. per md. in the price of rice makes a decrease of (*Rs.*55 − *Rs.*52. 8*a*.) or *Rs.*2. 8*a* = 40*a*. in the family expenses.

Hence, quantity of rice consumed by the family = $\frac{40}{4}$ or 10 mds. Therefore the expenditure on rice = *Rs.*(4 × 10) or *Rs.*40 and the other expenses = *Rs.*(55 − 40) = *Rs.*15.

Now, the price of 10 mds. at *Rs.*4. 4*a.* per md. = *Rs.*4. 4*a.* × 10 = *Rs.*42. 8*a.*

Hence, the required expenses = *Rs.*42. 8*a.* + *Rs.*15 = *Rs.*57. 8*a.*

Ex. 4. A corn-merchant mixed 10 mds. of rice worth *Rs.*4 per md. with a certain quantity worth *Rs* 3. 8*a.* per md., and selling the mixture at *Rs.*3. 12*a.* per md. gained *Rs.*10 on the whole. How many mds. of the second kind did he mix ?

By selling the first sort of rice at *Rs.*3. 12*a.* per md. he incurs a loss of (*Rs.*4 − *Rs.*3. 12*a*) or 4*a.* per maund ; therefore the loss in 10 mds. = 10 × 4*a.* = 40*a.* = *Rs.*2. 8*a.*

Now, gain per md. on the second sort = (*Rs.*3. 12*a.* − *Rs.*3. 8*a.*) = 4*a* and as he shall have to make altogether *Rs.*10. + *Rs.*2. 8*a.* or *Rs.*12. 8*a.* = 200*a.*

∴ the quantity required = $\frac{200}{4}$ or 50 mds. *Ans.*

Ex. 5. A *gowala* mixed milk worth *Rs.*7 per md. with twice as much worth *Rs.*5. 8*a.* per md. and having sold the mixture at *Rs.*6. 4*a.* per md., cleared *Rs.*10. 8*a.* on the whole. How much did he mix of each sort ?

The cost of 1 md. of first + 2 mds. of second = *Rs.*7 × 1 × *Rs.*5. 8*a.* × 2 = *Rs.*18.

∴ the cost of 1 md. of the mixture = *Rs.*18 ÷ 3 = *Rs.*6.

The gain per md. = *Rs.*6. 4*a.* − *Rs.*6. = 4*a.* and the total gain is *Rs.*10. 8*a.* = 168*a.*

∴ the whole mixture contains $\frac{168}{4}$ or 42 mds.

Now, 1 + 2 = 3 ; ∴ quantity of first sort = 42 ÷ 3 = 14 mds.
and ... second... = 14 × 2 = 28 mds. } *Ans.*

Ex. 6. A supply of water suffices for 60 days if 10 maunds leak off every day, but only for 55 days if 15 maunds leak off daily. Find the total quantity of water in the supply.

In the first case 60 × 10 or 600 mds. leak off altogether, while in the second 55 × 15 or 825 mds. leak off.

∴ for (60 − 55) or 5 days' use (825 − 600) or 225 mds. of water are required.

∴ for daily use (225 ÷ 5) or 45 mds. of water are required.

Now, taking the first case, we find that the supply lasts for 60 days ; and in that time (60 × 45) or 2700 mds. are required for use ; and 60 × 10 or 600 mds. leak off.

Hence the total quantity reqd. = (2700 + 600) mds. = 3300 mds. *Ans.*

Ex. 7. On changing 3 four-anna pieces, I received 36 coins in single and double pice. How many did I get of each ?

Here 3 four-anna pieces = 12*a.* = 48 pice.

Now had all been single pice, I would have received 48 ; but as I received (48 − 36) or 12 single pice less, and the difference between a double and a single pice is one pice,

∴ the number of double pice = 12 } *Ans.*
and ... ... single pice = 36 − 12 = 24 }

Ex. 8. A man has three estates, and his incomes from the second and third are respectively twice and thrice as much as from the first. He has to pay an income-tax of 8 pies in the rupee for the first, 1*a.* in the rupee for the second, and 1*a.* 4*p.* in the rupee for the third. If the total income-tax be *Rs* 80, how much income does each estate yield ?

Supposing his income from first to be *Re.*1, his income from second = *Rs.*2, and from third *Rs* 3.

Also, for the first he should have to pay (1 × 8) or 8*p.* in the *Re.*
... second ... ... (2 × 12) or 24*p.* in the *Re.*
... third ... ... (3 × 16) or 48*p.* in the *Re.*

∴ the total tax amounts to (8 + 24 + 48) or 80 pies in the rupee.

Also *Rs.*80 = (80 × 16 × 12) pies, the total tax.

Hence, income from first = *Rs.* (80 × 16 × 12 ÷ 80) = *Rs.*192 }
... ... second = ... 192 × 2 = *Rs.*384 } *Ans.*
... ... third = ... 192 × 3 = *Rs* 576 }

Ex. 9. *A* gave *B* as many sovereigns as is expressed by the sum of all the numbers that can be formed by different arrangements of the digits 2, 4 and 7 taken all together ; and *B* gave *A* as many six-pences as is expressed by the sum of all the numbers that can be formed by different arrangements of the figures 4, 5, 8 and 9 taken all together. Who is the gainer and by how much ?

The sum of all the numbers that can be formed by different arrangements of the digits 2, 4 and 7 taken all together = $2 \times (2 + 4 + 7) \times (10^2 + 10 + 1) = 2 \times 13 \times 111 = 2886$. [See *Ex. 9*, Page 70.]

Similarly, the sum of the numbers formed by the different arrangements of the digits 4, 5, 8 and 9 taken all together = $6 \times (4 + 5 + 8 + 9) + (10^3 + 10^2 + 10 + 1) = 6 \times 26 \times 1111 = 173316$.

Hence *A* gave *B* 2886 sov. or £2886, and *B* gave *A* 173316 six-pences or £4332. 18*s.*

Therefore *A* is the gainer by (£4332. 18*s.* − £2886) or £1446 18*s.* *Ans.*

Miscellaneous Examples II.

1. From 261 times *Rs.*352. 1*a.* 4*p.* take *Rs.*90892. 8*a.* and divide the remainder by 89.

2. How many Napoleons of 15*s.* 9¾*d.* each can be obtained for [illegible] thalers of 2*s.* 11¼*d.* each ?

3. How many Nobles are equivalent to £195. 13*s.* 4*d.* ?

4. 13 rupees, 9 half-crowns and 17 three-penny pieces amount to £2 16*s.* ; find the value of a Rupee. Find the value of a lac of rupees in English money. (1 lac = 1,00,000.)

5. A dealer bought 9 horses at *Rs.*118. 13*a.* 4*p.* each ; one died and the others he sold at a profit on each of *Rs.*21. 1*a.* 8*p.* Find his gain.

6. The value of a mark being 13*s.* 4*d.*, and that of a moidore 27*s.*, shew that there are twice as many farthings in 57 marks and 57 moidores, as there are drams in 1 cwt. 3 qrs. 19lbs. 8 oz. 8 drs. of sugar.

7. To a certain stock-in trade *A* and *B* together contributed *Rs.*22. 10*a.*, *B* and *C* together *Rs.*25. 8*a.* and *A* and *C* together *Rs.*27. 6*a.* ; how much did each contribute ?

8. A boy receiving 4*a.* per week has 2*a.* stopped every third week ; if there are 39 weeks in a school year, how much does he realize in 4 years ?

9. *A* has *Rs.*1002. 7*a.* 8*p.* and *B* 128786 pies ; if *A* receive from *B* 22222 pies and *B* from *A* *Rs.*115. 15*a.* 6*p.*, how much will *A* have more than *B* ?

10. Of 21 people 13 lose *Rs.*1163. 13*a.* 6*p.* each and 8 lose *Rs.*930. 1*a.* 9*p.* each. What is the average loss per man ?

11. *A* and *B* having an equal share in a heap of potatoes containing 86 maunds, *A* takes 24 mds. and *B* the rest, paying *A* *Rs.*27. 11*a.* 4*p.* What is the worth of a maund of potatoes ?

12. A grocer's bill amounts to *Rs.* 1897. 8*a.* It happens to be made up of equal sums for tea at *Re.*1. 14*a.* 8*p.* per seer, sugar at 4*a.* per seer, rice at 3*a.* per seer, and coffee at 11*a.* per seer. How many seers are there of each sort ?

13. A person mixes together 10 lbs. of tea at *Re.*1. 4*a.* per lb., 12 lbs. at *Re.*1. 6*a.* and 14 lbs. at *Re.*1. 8*a.* per lb. He reserves 6 lbs. of the mixture for himself and sells the remainder at *Re.*1. 13*a.* per lb. How much does he gain ?

14. A manufacturer employs 50 men and 35 boys who work respectively 12 and 8 hours a day during 5 days of the week, and half the time the other day ; each man receives 4*a.* and each boy 1*a.* 4*p.* an hour. What is the whole amount of wages for a year ?

15. What quantity of water must I add to a pipe of wine which costs *Rs.*900, to reduce its price to *Rs.*5 a gallon ?

16. The yearly expense of a school is *Rs.*18993. 11*a.* ; there is an endowment yielding *Rs.*4850. 15*a.* and subscriptions *Rs.*743. The rest is to be made up by the fees of the pupils of whom there are 217 ; what must each of them pay on an average ?

17. In what time will a tradesman, who gains 10*a.* 8*p.* a day and spends 5*a.* of it, be able to pay off a debt of *Rs.*208. 9*a.* 8*p.* ?

18. A man's weekly income is *Rs.*18. 7*a.* and his quarterly expenditure is *Rs.*182. How much will he save at the year's end? (A year = 52 weeks.)

19. I buy 80 lbs. of black tea at *Rs.*2. 2*a.* per lb. and 20 lbs. of green at *Rs.*2. 12*a.* per lb. and mix them ; at what rate must I sell the mixture so as to gain 1*a.* 4*p.* in the rupee ?

20. Divide two fields, one of 6 ac. 3 po. 13 sq. yds., the other of 4 ac. 37 po. 27 sq. yds., between *A*, *B* and *C*, so that *A*'s no. of ro. = *B*'s no. of sq. po. = *C*'s no. of sq. yds.

21. September 17, 1893, was Sunday. What day of the week was September 17, 1891 ?

22. A wine merchant bought 2 pipes of wine at £2. 13*s.* 4*d.* per gallon. How much water must he mix with it that by selling a gallon of the mixture for £2. 6*s.* 8*d.*, he may gain on the whole £14 ?

23. A factor bought 25 pieces of cloth for *Rs.*185000 at *Rs.*4. 10*a.* per yard. How many yards are there in each piece ?

24. A house and its furniture are together worth £3367. 2*s.* 6*d.*; if the house is worth 8 times the furniture, what is the house worth ?

25. A man's total expenses are *Rs.*44, when rice sells at *Rs.*2. 8*a.* per maund, and *Rs.*46. 4*a.* when rice sells at *Rs* 2. 11*a.* per md. What are his expenses when rice sells at *Rs.*3. 3*a.* per maund ?

26. Two persons buy mangoes at 16 per rupee ; one sells at 12 per rupee and the other 16 for *Re.*1. 4*a.* How much profit does one make more than the other ?

27. A man spending daily *Rs.*2. 10*a.* 6*p.* lays by *Rs.*150. 2*a.* 11*p.* in the year 1897 ; find his daily income.

28. I received 320 pieces in half-rupees and quarter-rupees in exchange for 100 rupees. How many of each did I get ?

29. *A* and *B* gave equal sums in buying 15 horses and 22 cows. *A* took 5 horses and 17 cows and *B* the rest. If a horse cost *Rs.*56. 8*a.* and a cow *Rs.*35. 10*a.*, how should they settle the account ?

30. A man was born on the 15th of May 1762, and died on the 17th of June 1825. How many days did he live, exclusive of the day of his death ?

31. A goldsmith manufactured 2 lbs. 3 dwts. 8 grs. of gold into rings, each containing 9 dwts. 16 grs ; he sold the rings at *Rs.*25 each ; how much did he receive for them ?

32. A piano, table and carpet cost *Rs.*632. 12*a.* ; the piano and table cost *Rs.*547. 6*a.*, and the table and carpet cost *Rs.*260. 2*a.* 8*p.* Find the price of each.

33. A grocer buys 40 lbs. of tea at *Re.*1. 12*a* per lb. and also some cheaper tea; he mixes the two kinds of tea and by selling all the tea for *Rs.*236 4*a.* at *Re.* 1. 11*a* per lb. gains *Rs* 32. 14*a.* 8*p.* on his outlay; how many lbs. of the cheaper tea does he buy, and at what price per lb.?

34. Twice *A*'s money = 3 times *B*'s money, and the difference of their moneys is *Rs* 12 10*a* How much has each?

35. A bag contains a certain number of rupees, twice as many half-rupees, 4 times as many quarter-rupees and 8 times as many two-anna pieces, and total amount in the bag is *Rs.*100. How many of each are there?

36 *A*, *B* and *C* contributed equal sums in purchasing 22 horses, 28 cows and 56 sheep *A* took 7 horses, 9 cows and 19 sheep, *B* took 8 horses, 8 cows and 17 sheep, and *C* the rest. If the price of a horse be *Rs* 68. 8*a*, of a cow *Rs* 44. 10*a.* and of a sheep *Rs* 7. 6*a*, which of them shall have to pay and which to receive, and how much?

37. A landowner has three estates The first estate yields an income of *Rs* 3000, the second *Rs.*4200 and the third *Rs.*6250. If the rate of tax be 1*a* in the rupee for the first, 1*a.* 4*p* in the rupee for the second and 1*a* 3*p* in the rupee for the third, how much tax has he to pay altogether?

38 Divide *Rs* 7890 among *A*, *B* and *C* in such a way that *A* may receive *Rs.*125 more than twice as much as *B*, and *C Rs.*250 more than thrice as much as *B*.

39. A certain weight of gold worth *Rs* 20. 14*a.* 6*p.* per tola is mixed with an equal weight worth *Rs.*18. 6*a.* 6*p.* per tola. Determine the weight of gold, so that by selling the mixed gold at *Rs.*19. 14*a.* 6*p* per tola, a goldsmith may clear *Rs.*12. 8*a.* on the whole.

40. In making 50 benches, the cost of each for wood is *Re.*1. 2*a.*, for labour 13*a*, for polish 2*a.* and for screws 1*a.* How much is gained on each bench by selling the whole lot for *Rs.*112. 8*a.*?

41. The 15th of May 1890 was Thursday. What day of the week was the 27th April 1790?

42. The cost of maintaining a family is *Rs.*122 8*a.* when milk sells at 2*a.* per seer, and *Rs* 125. 12*a.* when milk sells at 2*a.* 3*p.* per seer. Find the monthly consumption of milk in the family and the amount of other expenses, supposing the latter to be unchanged.

43. A besieged garrison has a supply of water for 50 days. Owing to a leak, however, in the bottom of the reservoir, 5 gallons waste every day, and then the supply suffices for 10 days less. Find for how many days the supply would suffice if 20 gallons leak off every day.

44. A *gowala* mixes 12 mds. 16 sr. of milk at *Rs.*6. 9*a*. per md. with 22 mds. 24 sr. at *Rs.*7. 8*a* per md. He then adds 1 md 20 sr. of water and sells the mixture at 6 seers per rupee. How much does he gain or lose?

45. Divide *Rs.*10256. 12*a*. among three men, so that the first shall get *Rs.*1251. 4*a*. more than the second, and *Rs.*152 less than the third.

46. 8 men, 16 women and 24 boys earned *Rs.*136 in 8 days. A woman earns daily 2*a*. more than a boy, and a man daily earns as much as a woman and a boy together. Find how much a man, a woman and a boy daily earn

47. If 50 pieces of coin consisting of single and double pice make up a rupee, find the number of each coin.

48. A man died on the 7th August, Thursday 1890, having lived 21000 days (exclusive of the day of his death). Find the day and date of his birth.

49. A certain English landowner has three estates, for which he has to pay a total tax of £180. His income from the second and third estates are respectively twice and four times his income from the first The rates of tax for the three are respectively 1*s*. 2*d*.; 1*s*. 3*d*. and 1*s*. 4*d*. in the £. Determine his income from each estate.

50. *A* pays *B* as many rupees as is expressed by the sum of the numbers formed by all the different arrangements of the figures 2, 3 and 4 taken all together, and *B* pays *A* as many double pice as is expressed by the sum of the numbers formed by the figures 1, 2, 3 and 4 taken all together and arranged in all possible ways. Who shall be the gainer and by how much?

51. Divide *Rs.*51. 10*a*. among 8 boys, 4 women and 3 men in such a manner that a woman shall receive 2*a*. more than twice as much as a boy, and a man 4*a*. more than as much as a boy and a woman together.

52. A man died on the 1st of August, Friday morning, 1890. He had lived 10000 days. Find the date and day of his birth.

53. Divide *Rs.*1780. 13*a*. into three such parts that the first part shall be *Rs.*125. 3*a*. more than the sum of the second and third, and the second part *Rs.*17. 12*a*. more than the third.

54. If the monthly expenditure of a family be *Rs.*57. 8*a*., when rice is at *Rs* 4. 6*a*. per maund and *Rs* 58, when rice is at *Rs.*4. 8*a*. per maund; what should the expenditure be when rice would be at *Rs.*4. 12*a*. per maund?

55. What sum of money is that which being multiplied by 16, *Rs.*24 added to the product, the sum divided by 13, and *Rs.*3. 13*a*. added to the quotient, the sum is *Rs* 7. 13*a*.?

56. An equal number of men, women and boys together earned

Rs.62. 8*a*. in 5 days. A boy earns 2*a*., a woman 3*a*. and a man 5*a* daily. Find the number of boys.

57. A goldsmith mixes a certain number of tolas of gold worth *Rs*.20. 8*a*. per tola with twice that quantity worth *Rs*.19. 6*a*. per tola On selling the mixed gold at *Rs*.20 per tola, he gained *Rs* 15. How much of each kind did he mix?

58 Sound travels at the rate of 1142 ft. per second; what is the distance of a thunder-cloud when the sound of thunder follows the flash of lightning after an interval of 9 seconds?

59 *A* gives *B* 112 gallons of brandy at 32*s*. 6*d*. a gallon, and receives in return £40 12*s* 6*d*. and 780 yds. of cloth. What is the price of the cloth per yard?

60. There are 6 presses at work striking off sovereigns, half-sovereigns, florins, shillings, six-pences and four-penny-pieces respectively, and each at the rate of 2500 per hour; find the value of the money struck off in 13 days of 9 hours each.

61. What is the difference in seconds between the Mahomedan year of 354 days 8 hrs. 48 min. and the Hindu year of 365 days 6 hrs. 12 min 30 sec.?

62. If 6 hats cost as much as 25 pairs of gloves, worth *Rs*.1. 10*a*. a pair, how many hats can be bought for *Rs*.616. 2*a*. 4*p*.?

63 If telegraph posts are placed 66 yards apart and a railway train passes one in every three seconds, how many miles an hour is the train running

64. A person observed the flash of a cannon 7 seconds before he heard the report; how far was the cannon distant, supposing that sound moves at the rate of 1142 ft. per second?

65. In how many days of 8 hours each will a person be able to count 10 lacs of rupees at the rate of 80 per minute? How many will remain to be counted on the morning of the 26th day?

66 How much water must be mixed with 30 seers of milk worth 2*a*. per seer, in order to reduce its price to 1*a*. 6*p*. per seer?

67. By the payment of 2*s*. 1*d*. in London a banker will give credit at Calcutta for a rupee; how many rupees may be received in Calcutta for the payment of £5025. 6*s*. 3*d* in London?

68. If 5 oz. of silk can be spun into a thread 2 fur. 20 po. long; what weight of silk would supply a thread sufficient to reach to the Moon if the distance be 240000 miles?

69 A ship's crew of 50 men have a supply of water for 30 days at 2 seers a head; if they lose 125 seers, and find that they will be 50 days at sea, what must be each man's daily allowance?

70. A landowner has four estates, for which he has to pay a tax of *Rs*.760. The second, third and fourth yield respectively twice,

thrice and four times as much income as the first. If the tax be levied at 10, 9, 8 and 6 pies in the rupee respectively, find the amount of his income from each estate.

71. A tradesman in India exchanges with a merchant in China as many maunds of sugar as is expressed by the sum of all the numbers that can be formed by the different arrangements of the digits 7, 8 and 9 taken all together, for as many pounds of tea as is expressed by the sum of all the numbers similarly formed by the digits 3, 0, 5 and 7 taken all together. How much tea does the Indian merchant get in return for 37 mds. of sugar?

72. A man's monthly expenditure consists of 5 mds. of rice, 1 md. 20 sr. of flour, 15 sr. of ghee and 2 mds. 15 sr. of milk. When rice costs *Rs.*3. 10*a.* 6*p.* per md., flour *Rs.*4. 12*a.* per md., ghee *Rs.*37. 8*a.* per md. and milk *Rs.*5 8*a.* per md. the total expenses amount to *Rs.*130. 10*a.* If the prices of other articles remain the same, what would his family expenses amount to, when rice would sell at *Rs.*4. 12*a.*, flour at *Rs.*5. 4*a.*, ghee at *Rs.*41. 8*a.*, and milk at *Rs.*6. 4*a.* per maund?

73. A total weight of 12 mds. 10 sr. consists of a certain number of 10 seer-weights, three times as many of 5 seer-weights, 4 times as many of 2½ seer-weights, 6 times as many of 1 seer-weights, 8 times as many of half seer-weights and 16 times as many of powa weights. Find the number of each kind of weights.

74. A certain number of sovereigns, twice as many crowns, 5 times as many half-crowns, 8 times as many shillings and 12 times as many six-pences together amount to £28. 5*s.*; find the numbers of each coin.

75. A man mixed 3 mds. of milk at *Rs.*4. 8*a.* per md. with a certain quantity worth *Rs.*4. 4*a.* per md. and three times that quantity worth *Rs.*3. 12*a.* per md. He sold the mixture at *Rs.*4. 2*a.* per md. and thus cleared *Rs.*15 on the whole. How much of the second and third sort did he mix?

CHAPTER IV.

Numbers, Measures and Multiples

I. NUMBERS.

188. Numbers which follow a regular order increasing by 1 are called **consecutive** numbers. The consecutive numbers commencing at 1 are called **natural** numbers.

Thus, 4, 5, 6, 7, 8, &c. are *consecutive*, and 1, 2, 3, 4, 5, 6, &c. are *natural* numbers.

189. Numbers are either **even** or **odd.**

Numbers are called **even** when they can be divided by 2 without a remainder, and **odd** when they cannot be so divided.

Thus, 4, 8, 10, 16, &c. are *even* and 3, 5, 7, 13, &c. are *odd* numbers.

190. A **measure** or **factor** of a number is *any* number which divides it without a remainder. It is said to *measure* the number by the **units** contained in the *quotient.*

Thus, 4 is a *measure* or *factor* of 24, because it is contained exactly 6 times in 24. All numbers have 1 for a measure.

191. An **aliquot part** of a number is *any* measure of it.

Thus, 4 is an *aliquot part* of 20, for 4 is a measure of 20.

192. A **multiple** of a number is *any* number which contains t an exact *number of times.*

Thus, 108 is a *multiple* of 12, because 12 is contained exactly 9 times in 108.

193. A measure is sometimes called a **submultiple.**

Thus, 4 is a *submultiple* of 16.

194. Numbers are either **prime** or **composite.**

A **prime number,** or a **prime,** is a number which can be divided exactly only by itself and by unity. A **composite number** is a number which can be separated into *factors* each greater than unity, or which, in other words, arises from the multiplication of *two* or *more* other numbers, termed *factors.*

Thus, 2, 5, 7, 11, &c are *primes*, and 4, 8, 10, 12, &c. are *composite* numbers.

195. Two numbers are **prime** to each other, when their only common measure is 1.

196. One number is **divisible** by another when it can be divided by that other number exactly.

Thus, 20 is *divisible* by 5, for 20 contains 5 exactly 4 times.

197. The following RULES are important, and should be carefully committed to memory.

(1) If a number divide a product of two factors and be prime to one of them, it must divide the other.

Thus, if 4 divide 9×24, and 4 is prime to 9, then 4 must divide 24, for 4 is a measure of 24.

(2) If a number is divisible separately by two others which are prime to each other, it is divisible by their product.

Thus, if 240 be divisible by 3, and by 4, where 3 and 4 are prime to each other, it will be divisible by 3×4, for $240 = (3 \times 4) \times 20$.

(3) If one number is divisible by another, any multiple of the first is also divisible by the second.

Thus, 10 is divisible by 2 and 5; hence any number ending with 0, being a multiple of 10, is divisible by 2 and 5.

100 is divisible by 4 and 25; therefore all numbers ending with two ciphers are divisible by 4 and 25.

1000 is divisible by 8 and 125; hence all numbers ending with three ciphers are divisible by 8 and 125.

Again, $1001 = 7 \times 11 \times 13$, and therefore 1001 is divisible by 7, 11, and 13. Hence all numbers like 7007 or (7×1001), 18018, or (18×1001), 325325 or (325×1001) are all divisible by 7, 11 and 13.

(4) If each of two numbers is divisible by a third, their sum or difference is also divisible by the third.

Thus, $8654 = 8650 + 4$ and is divisible by 2, if 4 is;
$4235 = 4240 - 5$ ... ... 5, if 5 is;
$7336 = 7400 - 64$ ... ... 4, if 64 is;
$78664 = 78000 + 664$ ... ... 8, if 664 is;
$86184 = 86086 + 98$ and is divisible by 7, if 98 or $(184 - 86)$ is;
$429275 = 429429 - 154$ ... . 11, if 154 or $(429 - 275)$ is;
$186459 = 186186 + 273$ ... ... 13, if 273 or $(459 - 186)$ is.

(5) If each of two numbers is divisible by a third then the sum or difference of any multiple of the first and of any multiple of the second is also divisible by the third.

Thus, $627 = 600 + 20 + 7 = 6(99 + 1) + 2(9 + 1) + 7$
$= 6 \times 99 + 2 \times 9 + 6 + 2 + 7$;

$\therefore$ 627 is divisible by 3, if $6 + 2 + 7$ is.

$7362 = 7000 + 300 + 60 + 2 = 7(999 + 1) + 3(99 + 1) + 6(9 + 1) + 2$
$= 7 \times 999 + 3 \times 99 + 6 \times 9 + 7 + 3 + 6 + 2$;

$\therefore$ 7362 is divisible by 9, if $7 + 3 + 6 + 2$ is.

$82654 = 80000 + 2000 + 600 + 50 + 4$
$= 8(9999 + 1) + 2(1001 - 1) + 6(99 + 1) + 5(11 - 1) + 4$
$= 8 \times 9999 + 2 \times 1001 + 6 \times 99 + 5 \times 11 + 8 - 2 + 6 - 5 + 4$;

$\therefore$ 82654 is divisible by 11, if $(8 + 6 + 4) - (2 + 5)$ is.
(for 9999, 1001, 99 and 11 are all divisible by 11.)

198. Criteria of Divisibility.

A number is divisible by

2, if its *last* digit is divisible by 2; as 450, 326.

3, if the *sum* of the digits is divisible by 3; as 267, 531.

4, if its *last two* digits are divisible by 4; as 600, 520, 924.

5, if its *last* digits are 0 or 5; as 370, 865.

6, if it is divisible by both 2 *and* 3; as 318, 588.

8, if its *last three* digits are divisible by 8; as 3000, 5240, 2816.

9, if the *sum* of its digits is divisible by 9; as 648, 702.

10, if its *last* digit is 0; as 4570, 2300.

11, if the difference between the sum of its digits in the *odd* and in the *even* places is 0, or is divisible by 11; as 1067, 2695, 19613.

12, if it is divisible by both 3 *and* 4; as 708, 1164

For **7** and **13**, see Art. 197 (4).

199. There is no direct method for determining primes, and so we give below a list of the prime numbers from 1 to 227.

1	11	29	47	71	97	113	149	173	197
2	13	31	53	73	101	127	151	179	199
3	17	37	59	79	103	131	157	181	211
5	19	41	61	83	107	137	163	191	223
7	23	43	67	89	109	139	167	193	227

200. *To ascertain what numbers are prime.*

(i) Every number whose last digit is 0, 2, 4, 6, or 8 is divisible by 2 (Art. 198), and therefore every such number except 2 itself is not a prime Every number whose last digit is 0 or 5 is divisible by 5, and therefore every such number except 5 itself is not a prime Hence the last digit of every prime number except 2 and 5, must be 1, 3, 7 or 9.

(ii) If then the last digit of the given number be 1, 3, 7 or 9 try as divisors one after another the primes 3, 7, 11, 13, &c.; if there is a remainder in each case the given number is a prime. It is not necessary to try a divisor whose square is greater than the given number.

Ex. Are 689 and 947 primes?

(1) 689 is not divisible by 3 (for $6+8+9=23$), nor by 7 (by trial), nor by 11 (for $6+9-8=7$), but is divisible by 13; therefore 689 is *not* a prime.

(2) 947 is not divisible by 3, 7, 11, 13, 17, 19, 23 or 29: and we need not try the next divisor 31, for the square of 31 is greater than 947. Hence 947 *is* a prime.

201. To **resolve** or **decompose** a composite number into its prime factors is to find those prime numbers which when multiplied together produce the given number.

Thus, $210=2\times3\times5\times7$; $504=2\times2\times2\times3\times3\times7=2^3\times3^2\times7$.

202. When the factors obtained are all primes, the number is said to be resolved or decomposed into its **prime or elementary factors**.

203. No number can be resolved into prime factors in more than **one** way.

204. *To resolve a number into its prime factors.*

RULE. Divide in succession by each of the primes 2, 3, 5, 7, 11, &c., which can be used as divisors, and in each case as often as

possible, until we obtain a quotient which is a prime; these divisors and the last quotient expressed in the form of a product make up the given number.

Ex. 1. Resolve 44856 into prime factors.

$2^3=8)44856$
$3^2=9)5607$
$7)623$
89

The last two digits form 56, which is divisible by 8; the sum of the digits $=4+4+8+5+6=27$. Hence the number is divisible by 8 and 9 or 2^3 and 3^2.

Also $623=7\times 89$, and that 89 is a prime.

$\therefore\ 44856=\underline{2^3\times 3^2\times 7\times 89}.$

Ex. 2. Decompose 8862777 into its prime factors.

$3^2=9)8862777$
$3^2=9)984753$
$11)109417$
$7)9947$
$7)1421$
$7)203$
29

The sum of the digits $=45$, which is divisible by 9 or 3^2; the sum of the digits of the quotient $=36$: also $(8+6+7+7)-(8+2+7)=11$. Hence the number is divisible by 9, 9 and 11.

Again, in 9947, we have $947-9=938$, which is divisible by 7; in like manner, again by 7, and $203=7\times 29$ and 29 is a prime.

$\therefore\ 8862777=9\times 9\times 11\times 7\times 7\times 7\times 29=\underline{3^4\times 11\times 7^3\times 29}.$

Examples LI.

1. Resolve *mentally* the following into elementary factors :—

(1) 6; 10; 14; 21; 35; 28; 45; 64; 81; 96; 72.

(2) 56; 30; 280; 144; 224; 285; 198; 176; 342.

2. Decompose the following numbers into their prime factors :—

(1) 320; 460; 462; 315; 612; 715; 846; 945; 735.

(2) 1188; 1309; 1827; 1331; 1456; 1485; 3675; 4620.

(3) 5250; 55020; 16632; 47089; 53599; 88725; 11025.

(4) 514250; 190463; 259811; 508079; 4149173; 4057690.

(5) 7507500; 73896433; 11176704; 119189070; 125023500.

3. Ascertain which of the following numbers are prime and the prime factors of those which are composite :—

(1) 31; 53; 86; 96; 167; 132; 275; 480; 856; 873.

(2) 397; 289; 461; 727; 667; 851; 953; 971; 997.

(3) 1009; 1517; 1729; 4576; 2501; 4717; 3389.

4. Determine which of the following numbers are divisible by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, and 13 respectively :—

(1) [illegible]165; 216; 324; 425; 639; 936; 868; 512; 795.

(2) 3164 ; 4228 ; 11172 ; 2859 ; 11599 ; 14916 ; 53729.

(3) 1235 ; 6467 ; 38187 ; 123456 ; 777777 ; 601830.

(4) 2709344 ; 50707338 ; 6913580247 ; 726441196.

5. How many prime numbers are there between ?—

(1) 16 and 96. (2) 53 and 100. (3) 140 and 230.

(4) 330 and 350. (5) 556 and 600. (6) 790 and 1008.

6. By what numbers may 179, 313 and 799 be divided that the remainders may be 3, 5 and 7 respectively ?

II. GREATEST COMMON MEASURE.

205. A **common measure** or **common factor** of two or more numbers is *any* number, which will divide each of them without leaving a remainder.

Thus, each of the numbers 2, 3 and 6 is a *common measure* or *common factor* of 18 and 30, for each of the numbers 2, 3 and 6 divides 18 and 30 exactly.

206. The *greatest* number that divides each of two or more numbers exactly is called their **Greatest Common Measure (G. C. M.)** or **Highest Common Factor (H. C. F.)**

Thus, 6 is the *Greatest Common Measure* of 18 and 30, for it is the greatest number capable of dividing each of them exactly.

207. *If one number measure each of two others, it will measure their sum and difference ; also any multiples of each, their sums and differences.*

Thus, 4 is a common measure of 20 and 12 ; and their sum $=20+12=32=4\times8$; their difference $=20-12=8=4\times2$; a multiple of $20=20\times5=100=4\times25$; of $12=12\times7=84=4\times21$;

also, $100+48=148=4\times37$; $100-48=52=4\times13$;

each of which evidently comprises the number 4 as a measure or factor ; and similarly of more numbers.

Examples LII.

Find, by *inspection*, the G. C. M. of :—

1. 4 and 6. **2.** 6 and 9. **3.** 8 and 12. **4.** 9 and 24.

5. 20, 32. **6.** 48, 27. **7.** 42, 28. **8.** 48, 54.

9. 91, 84. **10.** 30, 45. **11.** 45, 57. **12.** 42, 56.

13. 21, 28, 35. **14.** 30, 25, 45. **15.** 32, 40, 48. **16.** 30, 35, 40.

208. The G. C. M. of two or more numbers may often be found by resolving each number into its prime factors and then taking the product of **all** the prime factors common to them.

Ex. Find the G. C. M. of 63 and 168.

$63=7\times9=7\times3\times3$; $168=7\times24=7\times3\times8=7\times3\times2\times2\times2$.

Therefore the factors common to 63 and 168 are 7 and 3 ; hence the G. C. M. $=7\times3=21$. *Ans.*

209. In finding the G C. M. of two or more numbers, it is sufficient to find the prime factors of **one** of the numbers, and then find by trial which of these factors divide each of the remaining numbers exactly ; the product of all these common factors is the required G. C. M.

Ex. Find the G. C. M. of 492, 744 and 1044.

The prime factors of 492 are 2, 2, 3 and 41 ; of these factors 2, 2 and 3 divide 744 and 1044 exactly, but 41 does not divide them.

Hence, the required G. C. M. is $2\times2\times3$ or 12. *Ans.*

Examples LIII.

Find, by *method of factors* the G. C. M. of :—

1. 45 and 72. **2.** 64 and 96. **3.** 48 and 72.
4. 56 and 140. **5.** 81 and 171. **6.** 74 and 259.
7. 205 and 287. **8.** 325 and 425. **9.** 230 and 414.
10. 490 and 546. **11.** 308 and 506. **12.** 247 and 323.
13. 1216 and 424. **14.** 620 and 2108 **15.** 45, 72 and 81.
16. 162, 729 and 4374. **17.** 1326, 3094 and 4420.
18. 372, 994 and 3132. **19.** 504, 5292 and 3040.
20. 102, 612, 476, 816 and 428.

210. When numbers can easily be resolved into their prime factors we have shown in Art. 208, that their G. C. M. is formed by the product of the **least powers** of those factors which are common to all the given numbers, but when the numbers are large and their prime factors cannot be readily determined, we use a different method.

211. To find the G. C. M. of two numbers, whose prime factors cannot be readily ascertained, we use the following Rule.

RULE. Divide the greater of the numbers by the less, then the first divisor by the remainder, then the second divisor by the second remainder, and repeat this operation till there is no remainder ; the **last divisor** will be the G. C. M. required.

Ex. Find the G. C. M. of 9756 and 8496.

```
8496)9756(1
     8496
     1260)8496(6
          7560
           936)1260(1
                936
                324)936(2
                    648
                    288)324(1
                        288
                         36)288(8
                            288
```

The first divisor is 8496 and the first remainder 1260.

The second divisor is 1260 and the second remainder 936.

The third divisor is 936 and the third remainder 324; and so on.

The final divisor is 36.

∴ the required G. C. M. is 36.

213 *To find the* G. C. M. *of three or more numbers.*

RULE Find the G. C. M. of the first two numbers; then the G. C. M. of this G. C. M. and the third number; then the G. C. M. of this last G. C. M. and the fourth number; and continue this process to the last number; the **last G. C. M.** is the required G. C. M. of the given numbers.

Ex. Find the G. C. M. of 741, 1131, 1183 and 1989.

```
741)1131(1          39)1183(30          13)1989(153
    741                117                  13
    390)741(1           13)39(3              68
        390                39                65
        351)390(1                             39
            351                               39
             39)351(9
                351
```

∴ the required G. C. M. is 13. *Ans.*

Examples LIV.

Find the G. C. M. of :—

1.	126 and 444.	2.	646 and 950.	3.	54 and 258.
4.	366, 128.	5.	3556, 3444.	6.	5187, 5850.
7.	4833, 6237.	8.	9367, 14501.	9.	3252, 4248.
10.	2145, 3471.	11.	4081, 5141.	12.	1441, 1572.
13.	6441, 10283.	14.	13667, 14186.	15.*	43365, 44688.
16.	12925, 63305.	17.	11050, 35581.	18.	109056, 179712.
19	125075, 225025.	20.	105945, 945105.	21.	428571, 999999.
22.	143278, 1278142.	23.	385629, 7855323.		

24. 1257214, 11215246.

25. 703037, 5134083.

26. 3876519, 3101729671.

27. 41615795893, 877267019106.

28. 6186, 10310, 15465.

29. 12018, 20030, 30045.

30. 1617, 2871, 4213.

31. 13338, 14136, 15903.

32. 16442, 24663, 41105.

33. 2697, 3441, 1271.

34. 204, 1190, 1445, 2006.

35. 12558, 20769, 47403, 12581

36. 5040, 23940, 28350, 31773.

37. 11573, 19397, 28036.

38. 70843288, 852706430 and 686138242.

39. 1070784, 1180608, 1455168 and 1520376.

40. 22680, 49140, 154980, 429660 and 925932.

---

213. Numbers which have no common measure greater than unity, are said to be **prime to each other.**

Thus, 15 and 29 are prime to each other.

Ex. Are 1726 and 1623 prime to each other?

```
1623)1726(1                    78)103(1
     1623                          78
      103)1623(15                  25)78(3
           103                        75
            593                        3)25(8
            515                                24
             78                                 1
```

∴ 1726 and 1623 are prime to each other.

214. Every common measure of two numbers is a measure of their G. C. M.

Thus, 2 and 3 being common measures of 18 and 30, is a measure of 6, the G. C. M. of 18 and 30.

215. The numbers of which the G. C. M. is required must refer to the same unit, and the G. C. M. refers to that unit.

Thus, the G. C. M. of *Rs.*429 and *Rs.*715 is *Rs.*143; the G. C. M. of 224 feet and 336 feet is 112 feet.

Examples LV.

1. Are the following numbers prime to each other?—

(1) 5789 and 7337. (2) 3375 and 5836. (3) 49561 and 97073.

(4) 58573 and 84329. (5) 9367 and 14501. (6) 19001 and 46253.

(7) [illegible]698705 and 54987262. (8) 18432, 21952 and 42895.

2. Find the G. C. M of :—

(1) 8029 and 23791. (2) 441441 and 844272. (3) 181896 and 147576.
(4) 39835 and 162424. (5) 218707, 526769 and 695822.

Examples worked out.

Ex. 1. Find the greatest number that will divide 2293, 4245 and 5348 leaving the remainders 18, 20 and 23 respectively.

$2293-18=2275$; $4245-20=4225$; $5348-23=5325$.
The reqd. no. is the G. C. M. of 2275, 4225 and 5325 = 25. *Ans.*

Ex. 2 Two bills, one amounting to *Rs.*78. 12*a.* and the other to *Rs.*420 are to be paid in coins of one kind ; what is the largest coin that can be used ?

*Rs.*78. 12*a.* = 1260*a.* ; *Rs.*420 = 6720*a.*
∴ the largest coin required is the G. C. M. of 1260*a.* and 6720*a.*
= 420*a.* = Rs.26. 4a. *Ans.*

Ex. 3. The sum of two numbers is 1144, and their G. C. M. is 143 ; how many pairs of such numbers can be formed ? Form them.

$1144 \div 143 = 8$.

Now $8 = 1+7 = 2+6 = 3+5 = 4+4$, and no more.
Of these parts the only pairs of numbers that are prime to each other are 1, 7 and 3, 5. Hence *only two* pairs of numbers can be formed.

Thus, the first pair = 1×143 and 7×143, or 143 and 1001 ; and the 2nd pair = 3×143 and 5×143, or 429 and 715. } *Ans.*

As regards the other pairs that can be formed, 143 will be a common measure, but not the G. C. M.

Ex. 4. The product of two numbers is 3240, and their G. C. M. is 18 ; how many pairs of such numbers can be formed ? Form them.

$3240 \div 18^2 = 10$; and $10 = 1 \times 10$ or 2×5.

Hence *only two* pairs of numbers can be formed.

Thus, the 1st pair = 18×1 and 18×10, or 18 and 180 ; and the 2nd pair = 18×2 and 18×5 or 36 and 90. } *Ans.*

Ex. 5. What number is that which, when divided by 6, the quotient again by 6, and that quotient again by 6, will give the G. C. M. of 35 and 135 ?

The G. C. M. of 35 and 135 is 5.

Now the question is, what number is that which, when divided by 6, the quotient again by 6, and that quotient again by 6, will give 5 ?

Since, 6, 6 and 6 are the three divisors and 5 the last quotient,

∴ the first dividend or the required number is $5 \times (6 \times 6 \times 6)$
= 1080. *Ans.*

Examples LVI.

1. What is the greatest sum of money contained exactly in *Rs.*34. 7*a.* 6*p.* and *Rs.*70. 12*a.* 6*p.* ?

2. Find the greatest number that will divide 35 and 61, leaving remainders 3 and 5 respectively.

3. What number is that which, when divided by 12, the quotient again by 12, and that quotient again by 12, will give the G. C. M. of 148 and 772 ?

4. Find the greatest weight in grains, that will measure both pounds Avoir. and pounds Troy.

5. The sum of two numbers is 928, and their G. C. M. is 58 ; form as many pairs of numbers as convenient.

6. What is the greatest unit of time with which 15 hrs. 12 min. and 1 day 3 hrs. 33 min. can be both represented by integers ?

7. Find the greatest number that will divide 1624, 2878 and 4220 leaving 7 as remainder after each division.

8. The product of two numbers is 5700, and their G. C. M. is 5 ; find as many pairs of numbers as convenient.

9. In working out a question in the G. C. M. of two numbers, I found the different remainders were 2388, 180, 48, 36 and 12, and the first two quotients 1 and 9 ; find the numbers, and the last three quotients.

10. In solving a question in the G. C. M. of two numbers, the quotients are 5, 1, 18, 1, 3, 1 and 2. The last divisor is 15. Find the numbers.

11. The sum of two numbers is 1394, and their G. C. M. is 34 ; how many pairs of numbers can be formed ?

12. The product of two numbers is 4608, and their G. C. M. is 16 ; how many pairs of numbers can be formed ?

13. What highest number will divide 287, 480 and 599 leaving the remainders 2, 5 and 10 respectively ?

14. What is the greatest number by which, when 399, 695, 548, 1003 are divided, the respective remainders are 3, 2, 8 and 4 ?

15. Two bills, one of £4. 13*s.* 8*d.* and the other of £6. 9*s.* 4*d.* are to be paid in the same coin. Find the largest coin that can be used.

16. *A* has *Rs.*679, *B Rs.*5901 and *C Rs.*6734 ; they agree to lay it out for sheep, at the highest price per head that will allow each exactly to invest his money ; how much can they pay a head and how many can each purchase ?

17. Find the two numbers nearest to 10000 that have 169 for their G. C. M.

18. A national school-master divided his scholars, consisting of 221 boys and 143 girls, into the largest possible equal classes so

that each class of boys should contain the same number as each class of girls. Find the number of classes.

19. A person wishes to distribute 805 mangoes, 1311 guavas and 1978 plantains, equally among a number of beggars. Find the greatest number receiving the charity in this way.

20. A labourer was engaged for a certain number of days for *Rs.*10. 15*a.* 8*p.*, but being absent on some of these days he was paid only *Rs.*3. 3*a.* 8*p.*; shew that his daily wages cannot exceed 10*a.* 4*p.*

21. Find the greatest number of 4 digits and the least number of 5 digits that have 124 for their G. C. M.

22. Find the greatest and the least number of 6 digits that have 251 for their common measure. What is their G. C. M.?

III. LEAST COMMON MULTIPLE.

216. **A common multiple** of two or more numbers is *any* number which is divisible by each of them separately.

Thus, 96 is a *common multiple* of 2, 3, 4, 6, 8 and 12, because it is divisible by each of them.

217. The **Least or Lowest Common Multiple** (L. C. M.) of two or more numbers is the *least* number that can be divided by each of them without a remainder.

Thus, 24 is the *Least Common Multiple* of 2, 3, 4, 6, 8 and 12, for it is the *least* number that the above numbers can divide without leaving a remainder.

218. The L. C. M. of two or more numbers may be obtained by resolving them into their prime factors, and taking the product of the **highest powers** of all the factors that are found in the given numbers.

Ex. Find the L. C. M. of 8, 12, 16, 20, 25 and 30.

$8=2\times2\times2=2^3$; $12=2\times2\times3=2^2\times3$;

$16=2\times2\times2\times2=2^4$; $20=2\times2\times5=2^2\times5$;

$25=5\times5=5^2$; $30=2\times3\times5=2\times3\times5$.

Here the factors that occur in the given numbers are 2, 3 and 5, of which the highest power of 2 is 2^4, and that of 5 is 5^2; therefore the L. C. M. is $2^4\times3\times5^2=16\times3\times25=\underline{1200}$. *Ans.*

Examples LVII.

1. Find *mentally* the L. C. M. of :—

(1) 6, 8.	(2) 8, 16.	(3) 10, 15.	(4) 18, 30.
(5) 12, 27.	(6) 10, 18.	(7) 16, 24.	(8) 12, 15.
(9) 3, 4, 5.	(10) 2, 5, 7.	(11) 3, 4, 16.	(12) 5, 8, 20.
(13) 16, 12, 24.	(14) 7, 10, 24.	(15) 5, 12, 15.	(16) 20, 40, 60.

2. Find, by *resolving into factors*, the L. C. M. of :—

(1) 12, 16, 18. (2) 16, 24, 30. (3) 24, 56, 84.
(4) 15, 35, 16, 56. (5) 25, 60, 84, 15. (6) 81, 27, 45, 18.
(7) 756, 6435. (8) 729, 1681. (9) 1008, 2064.
(10) 756, 350, 9075. (11) 735, 1575, 2205. (12) 225, 336, 360.
(13) 196, 350, 728, 924. (14) 11573, 19397, 28036.
(15) 72, 96, 144, 180, 450, 540 (16) 44, 126, 280, 198, 330.

---

219. To find the L. C. M. of two large numbers which cannot easily be resolved into prime factors, we use the following Rule.

RULE. Find the G. C. M. of two numbers, and then multiply *either* of the numbers by the quotient arising from dividing the *other* by the G. C. M. The **product** will be the L. C. M. of the numbers.

Ex. Find the L. C. M. of 209 and 304.

Here, the G. C. M. is 19. Also $209 \div 19 = 11$.

$\therefore$ the L. C. M. $= 11 \times 304 = \underline{3344}$. *Ans.*

220. To find the L. C. M. of three or more numbers which cannot be readily resolved into factors, use the following Rule.

RULE. First find the L. C. M. of two of the numbers as in Art. 219; then the L. C. M. of this and another and so on, until all are taken. The last **L. C. M.** is the L. C. M. required.

Ex. Find the L. C. M. of 64, 250 and 432.

The G. C. M. of 64 and 250 is 2, and their L. C. M. is 8000.

The G. C. M. of 8000 and 432 is 16, and the L. C. M. is 216000.

Hence, the L. C. M. required $= \underline{216000}$. *Ans.*

Examples LVIII.

Find the L. C. M. of :—

1. 289, 323. **2.** 849, 1132. **3.** 508, 889.
4. 420, 798. **5.** 1287, 6281. **6.** 7247, 9365.
7. 12432, 36075. **8.** 15863, 21489. **9.** 24, 39, 376.
10. 84, 672, 472. **11.** 629, 851, 253. **12.** 64, 720, 960.
13. 1003, 2301, 4017. **14.** 14491, 16641, 3707.
15. 2523, 5887, 203, 8631. **16.** 1175, 4747, 5875, 9447.

---

22. When the L. C. M. of several small numbers is required, **the easiest** method is that given by the following Rule.

RULE. Arrange the given numbers in a horizontal line from left to right, with a comma placed between every two. Divide by any one of the prime numbers 2, 3, 5, 7, 11...which will divide any two

at least of the given numbers exactly; set down the quotient so obtained and the undivided numbers in a line below, separated as before. Proceed in the same way with the numbers in the second, and each succeeding line, till we come to a line where no two numbers have a common divisor. The **product** of the numbers in the last line and of the several divisors is the L. C. M. of the given numbers.

Note. The work may often be shortened by *striking out* in the same line every number which exactly measures any other number in that line.

Ex. Find the L. C. M. of 2, 3, 8, 9, 15, 21 and 35.

```
2) 2, 3, 8, 9, 15, 21, 35
3) 1, 3, 4, 9, 15, 21, 35
5) 1, 1, 4, 3,  5,  7, 35
7) 1, 1, 4, 3,  1,  7,  7
   1, 1, 4, 3,  1,  1,  1
```

∴ the L. C. M. $= 2\times3\times5\times7\times4\times3\times$
$= 2520$. *Ans.*

```
3) 2̸, 3̸, 8, 9, 15, 21, 35
          8, 3,  5̸,  7̸, 35
```

∴ the L. C. M. $= 3\times8\times3\times35$
$= 2520$ *Ans.*

In the first line 2 is contained in 8, and 3 in 9, and ∴ struck off. In the second line 5 and 7 are both contained in 35, and ∴ struck off.

Examples LIX.

Find the L. C. M. of :-

1. 12, 15, 16
2. 8, 16, 20
3. 15, 25, 105.
4. 9, 15, 18, 20
5. 8, 12, 15, 20
6. 34, 68, 17, 2.
7. 16, 9, 12, 18
8. 36, 56, 75, 72.
9. 81, 27, 45, 18.
10. 15, 35, 16, 56
11. 15, 20, 24, 21, 35.
12. 24, 28, 36, 22, 16.
13. 3, 9, 7, 15, 28, 42.
14. 8, 18, 28, 36, 54, 72, 90
15. 9, 12, 15, 18, 21, 24, 27, 30
16. 32, 63, 25, 36, 42, 49, 84.
17. 12, 18, 28, 35, 60, 84, 100
18. 15, 16, 18, 20, 24, 25, 27, 30.
19. 48, 64, 27, 81, 33, 110, 165.
20. 48, 64, 27, 33, 110, 165, 240.
21. 35, 52, 63, 77, 132, 117, 143.
22. 27, 91, 42, 39, 63, 156, 234.
23. 27, 36, 54, 72, 84, 96, 215, 248, 324.
24. 18, 24, 35, 48, 56, 60, 72, 90, 120.
25. 7, 11, 21, 63, 91, 99, 117, 143.
26. 24, 35, 52, 60, 91, 108, 126, 156, 315.
27. 26, 30, 34, 39, 51, 65, 78, 85, 102, 195, 255.
28. 27, 87, 189, 126, 145, 210, 203, 261, 385.
29. 8, 9, 10, 11, 12, 14, 15, 18, 21, 24, 28, 35, 36, 40, 42, 44, 45, 50.
30. The first 12 numbers; the even numbers from 10 to 28 inclusive.

---

222. Every common multiple of two numbers is a multiple of their L. C. M.

Thus, 48 a common multiple of 8 and 12 is a multiple of 24, the L. C. M. of 8 and 12.

223. If two numbers are prime to each other, their L. C. M. is their product.

Thus, the L. C. M. of 13 and 15 is $13 \times 15 = 195$.

224. Since the L. C. M. of two numbers is their produc divided by their G. C. M. (Art. 219), therefore the L. C. M. × the G. C. M. of two numbers is equal to their product. Hence, if the G. C. M., the L. C. M., and one of the two numbers be given, we can find the other number by multiplying the G. C. M. and the L. C. M. and dividing the product by the given number.

Ex. The G. C. M. and the L. C. M. of two numbers are 11 and 11803 respectively, and one of them is 319; what is the other?

Here, the G. C. M. × the L. C. M. $= 11 \times 11803 = 129833$.

∴ the required number $= 129833 \div 319 = \underline{407}$. *Ans.*

225. (1) To find the *least* number that will contain each of two or more given numbers exactly.

RULE. The required *least* number is the L. C. M. of the given numbers.

Ex. 1. Find the least number that is divisible by 40, 63, 112.

The required number = the L. C. M. of 40, 63, $112 = \underline{5040}$. *Ans.*

Ex. 2. Five bells toll at intervals of 5, 8, 9, 10 and 12 seconds respectively; what interval will elapse between two of their successive tollings together?

The L. C. M. of 5, 8, 9, 10, 12 is 360.

∴ the required time = 360 sec. or 6 min. *Ans.*

(2) To find the *least* number which, when divided by each of several given numbers, leaves the same remainder.

RULE. Find the L. C. M. of the several given numbers and to it add the given remainder. The sum is the required *least* number.

Ex. Find the least number which, when divided by 4, 18, 21 and 20, leaves in each case a remainder 3.

The L. C. M. of 4, 18, 21 and 20 is 1260.

∴ the required number $= 1260 + 3 = \underline{1263}$. *Ans.*

Examples LX.

1. Find the least number which, when divided by 6, 8 and 9, gives in every case the remainder 5.

2. What is the smallest sum that can be paid either in guineas, or in half-crowns, or in florins or in half-sovereigns?

3. Five bells begin to toll simultaneously and they toll at intervals of 4, 6, 8, 9 and 10 seconds. After what time will they again toll simultaneously?

4. Find the least number which, when divided by 675, 1050 and 4368, will leave the same remainder 32.

5. Find the least weight that can be weighed by either pounds Avoir. or pounds Troy.

6. Six men fire at a target at intervals of 2, 5, 7, 10, 12 and 14 minutes respectively. After what time will they all *first* fire simultaneously, and how many times will each man have fired?

7. Seven bells are tolling, and they toll at intervals of 3, 5, 7, 8, 9, 10 and 12 seconds respectively. What interval will elapse between their once tolling together and tolling together again?

8. *A* can go round a circular course in 6 minutes, *B* in 8, *C* in 12, *D* in 15, and *E* in 18; if they all start together from the same place at the same time (7h. 13m. A. M.), when will they be together again?

9. Find the least sum of money that can be paid in pence, shillings, florins, half-crowns, crowns, sovereigns or half-sovereigns.

10. The G. C. M. and the L. C. M. of two numbers are 124 and 10540 respectively, and one of the numbers is 620; find the other.

11. A heap of pebbles can be made up exactly into groups of 25; but when made up into groups of 18, 27 and 32, there is always a remainder of 11; find the least number of pebbles such a heap can contain.

12. A basket contains a number of oranges ascertained to be between 500 and 900. If 2 fruits are taken away, the remainder may be distributed equally among 3, 4, 5, 6 or 7 boys. Find the number of oranges in the basket.

13 A book is divided into four parts, each part being divided into chapters. The number of pages in each part is the same. Each chapter in the first part contains 20 pages, each chapter in the second 40, each chapter in the third 60, and each chapter in the fourth 80. Find the number of pages and chapters in the book, the number of pages in the book is known to be between 900 and 1000.

14. Three horses are running round a race course of 5280 yards; the first horse runs 440 yards a minute, the second 352 yards, and the third 264 yards; find the time between their once coming all together, and their coming all together again.

15. What is the least number which, when increased by 17, is divisible by 22, 25, 33, 44 and 45 separately?

16. The G. C. M. and the L. C. M of two numbers are 19 and 49077 respectively, and one of them is 779; find the other.

17. What is the least number which, when diminished by 145, is exactly divisible by 24, 27, 28, 32, 36 and 56?

18 What is the least number which, when divided by all the digits except the first, leaves the remainder 1?

19. The G. C. M. of two numbers of 4 digits is 221, and their L. C. M. is 46189; determine the numbers.

20. Find all the numbers between 250 and 600 that have 1728 or their L. C. M.

21. Find the least sum of money that can be paid in coins worth either 8 pies, half-rupees, rupees, 5 *sikis*, 10 *sikis*, 14 *sikis*, *Rs.*5. 4*a.*, *Rs.*10. 8*a.*

22. There is an island 48 miles in circumference. Four persons *A*, *B*, *C* and *D* begin to walk continually round it starting from the same place at the same time They walk 3, 4, 6 and 8 miles per hour respectively. How soon will they all be again together at the starting point ?

23. Five men run round a circular park in 4, 5, 6, 7 and 8 hours respectively. If they all start at the same time from the same point, find the least number of hours in which they will again be at that point together.

24. Three round pillars are 10 ft. 5 in., 14 ft. 7 in. and 6 yds. 9 in. respectively in circumference ; find the length of the shortest rope that can be wrapped round each an exact number of times.

25. The circumferences of the wheels of a carriage are 7ft. 4in. and 11 ft. ; what is the least distance in which both the wheels will make an exact number of revolutions ?

26. A cask is required to be exactly filled by any one of the following measures, 1 seer, 2 seers, 3 seers, 5 seers, 6 seers, or 9 seers ; find the smallest cask for this purpose.

27. I have travelled between 700 and 760 miles ; had I travelled 20 miles less, I could have completed my journey in a train which goes at the rate of 40 miles an hour, or in a carriage which goes at the rate of 16 miles an hour, or on foot at the rate of 6 miles an hour in an exact number of hours. Find the distance I have travelled.

28. Find the least number of 8 digits that is divisible by 15, 18, 25, 35, 40 and 55. Also the greatest number of 5 digits that is divisible by 14, 20, 35, 45 and 75.

CHAPTER V.

The Doctrine of Fractions.

(USUALLY TERMED VULGAR FRACTIONS.)

226. When a magnitude contains its unit a number of times exactly, the resulting number is called an **integer** or **whole number** (Art. 7). Hence all *whole numbers* or *integers*, being supposed to be formed by the *repetition* of the unit, may therefore be regarded as the result of the *multiplication* of that element ; but if the unit be considered capable of *division* into any number of *equal* portions, the quantities thence arising must be viewed in the light of *broken* magnitudes ; and these are therefore termed **Fractions** or more generally, **Vulgar Fractions**, in order to distinguish them from fractions of a different *form*, whose nature will be discussed in the next chapter.

I. NOTATION AND NUMERATION OF FRACTIONS.

227. A **Fraction** denotes a part or parts of a unit; it is expressed in figures by two numbers placed one above the other with a bar or line between them.

228. If we suppose the *unit* to be divided into 2, 3, 4, 5, &c., equal portions, *one* of the portions in each case is represented by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c., which may be regarded as the **primitive fractions** of their respective denominations and are called the **reciprocals** of the natural numbers 2, 3, 4, 5, &c.; also the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, &c., are read, *one-half*, *one-third*, *one-fourth*, *one-fifth*, &c.

229. If *two or more* of these equal portions be taken together, the *aggregates* thence arising are expressed by repeating the unit as *often* as such portions are repeated, in the *form* of their sum, the number below the line remaining the same.

Thus, if the primitive fraction $\frac{1}{3}$ be taken *twice* there will arise a new fraction expressed by $\frac{2}{3}$; if $\frac{1}{4}$ be repeated *thrice* there results a new fraction expressed by $\frac{3}{4}$; again, if $\frac{1}{5}$ be taken *four times*, the new fraction will be $\frac{4}{5}$; and similarly of all the other primitive fractions; also, the fractions $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, &c, are read *two-thirds*, *three-fourths*, *four fifths*, &c., and all quantities of this *form* are called **Simple Fractions**.

230. Hence, the number *below* the line denotes the number of equal portions into which the unit is supposed to be divided, and is therefore called the **denominator**; and the number *above* the line expressing the number of such equal portions intended to be taken, is therefore termed the **numerator**. The numerator and denominator are called the **terms** of a fraction.

Thus, of the fraction $\frac{5}{7}$, whose *terms* are 5 and 7, the denominator 7 implies that the unit is supposed to be divided into *seven* equal portions; and the numerator 5 shews that *five* of such equal portions are here the object of our consideration.

231. The sum of a whole number and a fraction is called a **Mixed number**; as, $4+\frac{5}{7}$ or rather $4\frac{5}{7}$; for the addition sign is almost always omitted.

232. From what has been said above, it appears, that a fraction expressed in figures is read by first reading the numerator and then the denominator with the termination "ths"; thus, $\frac{5}{7}$ is read five-sevenths. The exceptions are that fractions with denominator 2 or 3 are read as so many *halves* or *thirds*, and with denominator 4 as so many *quarters* as well as *fourths*. A mixed number is read by connecting the integer and the fraction by "*and*"; thus, $4\frac{5}{7}$ is read four *and* five-sevenths.

233. From Art. 230, it follows, that if the numerator be less than the denominator, the value of the fraction is less than the unit: if the numerator be equal to the denominator, the value of the frac-

tion is the unit ; and if the numerator be greater than the denominator, the value of the fraction is greater than unit.

234. Every whole number or integer may be expressed as a fraction whose denominator is 1.

Thus, $7=\frac{7}{1}$, for the unit is divided into 1 part, comprising the whole unit, and 7 of such parts, that is 7 units, are taken.

235. A fraction also expresses the quotient of the numerator by the denominator.

Thus, $\frac{5}{7}=5\div7$; since 1 unit is 7-sevenths, therefore 5 units is 35-sevenths, and therefore 5 divided by 7 is 35-sevenths divided by 7, and is therefore 5-sevenths ; that is $5\div7=\frac{5}{7}$. Hence $\frac{5}{7}$ is not only read 5 *sevenths* but also 5 *by* 7.

Similarly, $\frac{8}{4}=8\div4=2$; $\frac{7}{7}=7\div7=1$; and so on.

236. From the last Art. it follows, that if we multiply a fraction by its denominator we get its numerator.

Thus, since $\frac{5}{7}$ is the seventh part of 5, $\frac{5}{7}$ repeated 7 times gives 5, or $\frac{5}{7}\times7=5$; and 5 may therefore be expressed in a *Fractional Form* by $\frac{5}{1}$.

237. If we take a fractional magnitude and considering it as a new unit, divide it into any number of equal parts and take one or more of these parts, we shall obtain a **fraction of a fraction** ; as, $\frac{5}{7}$ of $\frac{3}{8}$.

238. When fractions are represented in the manner above explained, they are called **Vulgar fractions**, (*i. e.*) *common* or *ordinary* fractions.

239. We make the following distinctions in fractions :—

(1) A **proper fraction** is one in which the numerator is less than the denominator ; thus, $\frac{3}{4}$, $\frac{5}{7}$, $\frac{3}{8}$ are *proper* fractions.

(2) An **improper fraction** is one in which the numerator is either equal to or greater than the denominator ; thus $\frac{7}{7}$, $\frac{9}{8}$, $\frac{13}{4}$ are *improper* fractions.

(3) A **simple fraction** is one in which numerator or denominator are both whole numbers ; thus, $\frac{3}{4}$, $\frac{10}{7}$ are *simple* fractions.

(4) A **compound fraction** is a fraction of a fraction ; thus $\frac{3}{4}$ of $\frac{5}{6}$, $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{10}{9}$ are *compound* fractions.

(5) A **complex fraction** is one in which numerator or denominator or both are not whole numbers ; thus, $\frac{\frac{2}{3}}{\frac{4}{7}}$, $\frac{2\frac{1}{4}}{8}$, $\frac{3}{4\frac{1}{2}}$, $\frac{2\frac{1}{7}}{3\frac{1}{3}}$, $\frac{2\frac{1}{3}+1\frac{4}{5}}{2\frac{1}{3}-1\frac{4}{5}}$ are *complex* fractions.

240. The **reciprocal** of a fraction is the fraction formed by interchanging its terms ; thus the *reciprocal* of $\frac{5}{7}$ is $\frac{7}{5}$; of 5 or $\frac{5}{1}$ is $\frac{1}{5}$.

241. We are hence enabled to find the results of the multiplication and division of a fraction by an integer, and these may be integers or fractions.

(1) To *multiply* a fraction by a whole number, only multiply the *numerator* by it.

Thus, $\frac{4}{13}\times 3=\frac{4\times 3}{13}=\frac{12}{13}$; because in $\frac{12}{13}$, *three times* as many parts of the unit are implied, as there are in $\frac{4}{13}$.

(2) To *divide* a fraction by a whole number, only multiply the *denominator* by it.

Thus, $\frac{2}{7}\div 5=\frac{2}{7\times 5}=\frac{2}{35}$; because the same number of parts are indicated in $\frac{2}{7}$ and $\frac{2}{35}$, and each part in the former is *five times* as great as each part in the latter, by Art. 230.

Examples LXI.

1. What fraction do we form in dividing a unit into 13 equal parts, and taking 11 of them; into 1000 equal parts, and taking 101?

2. Express in figures :—

One-seventh; one quarter; seven-halves; thirty-four thirds; forty-five seventy-ninths; seven-eighths; seven, and a half; nine, and seven-ninths; sixteen, and four twenty-oneths; two hundred, and three-elevenths; ninety-four, and five-seventeenths.

3. Express in words :—

$\frac{2}{3}$, $\frac{3}{4}$, $\frac{8}{9}$, $\frac{15}{17}$, $\frac{87}{100}$, $3\frac{4}{7}$, $8\frac{11}{18}$, $24\frac{27}{44}$ and $125\frac{98}{1000}$.

4. Multiply :—

(1) $\frac{4}{5}$ and $\frac{24}{87}$ each separately by 2, 3, 5, 7, 9, 11, 12, 13, and 18.
(2) $\frac{15}{28}$ and $\frac{87}{109}$ ... ... 36, 68, 80, 95, 112 and 157.

5. Divide :—

(1) $\frac{6}{7}$ and $\frac{8}{13}$ each separately by 2, 3, 5, 7, 9, 11, 12, 13, and 18.
(2) $\frac{19}{87}$ and $\frac{28}{189}$ ... ... 36, 68, 80, 95, 112 and 157.

II. TRANSFORMATION OF FRACTIONS.

242. *If the numerator and denominator of a fraction be both multiplied or both divided by the same number, the value of the fraction will not be altered.*

For, if the fraction $\frac{3}{7}$ be multiplied by 5, the product is $\frac{15}{7}$; and again if this be divided by 5, the quotient is $\frac{15}{35}$, by Art. 241; but since these two operations are the *reverse* of and therefore *neutralize* each other, it follows that—

$$\frac{3}{7}=\frac{15}{35}=\frac{3\times 5}{7\times 5};\ \text{and also, that}\ \frac{15}{35}=\frac{3}{7}=\frac{15\div 5}{35\div 5}.$$

243. It is clear from the above, that a whole number may be expressed in the form of a fraction with *any* denominator we please

$$\text{Thus, } 5=\frac{5}{1}=\frac{5\times 2}{1\times 2}=\frac{10}{2}=\frac{20}{4}=\frac{35}{7}=\&c.$$

Also, a fraction may be transformed into another with a *given* denominator or numerator, provided it be a *multiple* or *sub-multiple* of the denominator or numerator of the proposed fraction.

Ex. 1. Convert $\frac{7}{8}$ into a fraction with 96 for its denominator and reduce $\frac{24}{40}$ to a fraction with denominatior 5.

$$(1)\ \frac{7}{8}=\frac{7\times 12}{8\times 12}=\frac{84}{96}; \qquad (2)\ \frac{24}{40}=\frac{24\div 8}{40+8}=\frac{3}{5}.$$

Ex. 2. Convert $\frac{5}{6}$ into a fraction with numerator 55, and $\frac{56}{64}$ into a fraction with numerator 7.

$$(1)\ \frac{5}{6}=\frac{5\times 11}{6\times 11}=\frac{55}{66}; \qquad (2)\ \frac{56}{64}=\frac{56+8}{64+8}=\frac{7}{8}.$$

244. Since $\frac{5}{8}\times 4=\frac{20}{8}=\frac{5\times 4}{2\times 4}=\frac{5}{2}$;

therefore, *to multiply* a fraction by an integer, it appears to be immaterial whether the numerator be multiplied, or the denominator be divided, by it ; and since

$$\frac{8}{9}+4=\frac{8}{36}=\frac{2\times 4}{9\times 4}=\frac{2}{9};$$

therefore, *to divide* a fraction by a whole number, it amounts to the same thing whether we multiply the denominator, or divide the numerator by it.

245. Now referring to Art. 241, we see that we have a choice of two methods both in the multiplication and division of a fraction by an integer, and we prefer the latter in accordance with the direction : "*Divide when you can, multiply when you are obliged.*"

Examples LXII.

1. Reduce each of the whole numbers 3, 5, 7, 8, 15, 18, 20, 25 to fraction with the denominator 13.

2. Convert 26, 117 and 125 into fractions with denuminators 13, 25 and 35 respectively.

3. Convert $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{8}$ and $\frac{7}{15}$ into fractions having 120 for their denominator.

4. Express $\frac{14}{17}$, $\frac{18}{23}$, $\frac{21}{32}$ and $\frac{27}{35}$ as fractions having 756 for their common numerator.

5. Express $\frac{20}{45}$, $\frac{28}{63}$, $\frac{36}{81}$, $\frac{48}{108}$ and $\frac{60}{135}$ each as a fraction with denominator 9. Also express each as a fraction with numerator 5040.

6. Convert $\frac{48}{80}$, $\frac{81}{120}$, $\frac{98}{210}$, [illegible] and [illegible] into equivalent fractions with denominators 5, 40, 15, 14 and 20 respectively.

246. *To express a mixed number as an improper fraction.*

RULE. Multiply the integer by the denominator of the fraction; to the product add the numerator and the result will be the new numerator, which placed over the given denominator will form the *improper* fraction required.

Ex. Represent $3\frac{4}{5}$ as an improper fraction.

$3\frac{4}{5}=\frac{3\times5+4}{5}=\frac{15+4}{5}$
$=\frac{19}{5}$. *Ans.*

For $3\frac{4}{5}=\frac{3}{1}+\frac{4}{5}=\frac{3\times5}{1\times5}+\frac{4}{5}$ (Art. 242.)
$=\frac{15}{5}+\frac{4}{5}=\frac{19}{5}$. (Art. 229.)

247. *To represent an improper fraction as a whole or mixed number.*

RULE. Divide the numerator by the denominator, and the quotient will be the integral part; and the fractional part will be formed by placing the remainder over the given denominator. If there be no remainder, the fraction is equivalent to the integer thus found.

Ex. Reduce $\frac{32}{8}$ and $\frac{327}{11}$ to whole or mixed numbers.

(1) $8)\underline{32}$
$\quad 4$
$\therefore\ \frac{32}{8}=4$. *Ans.*

For $\frac{32}{8}=\frac{8\times4}{8\times1}=\frac{4}{1}$ (Art. 242) $=4$. (Art. 234.)

(2) $11)\underline{327}$
$\quad 29\ldots8$
$\therefore\ \frac{327}{11}=29\frac{8}{11}$. *Ans.*

For $\frac{327}{11}=\frac{319+8}{11}=\frac{319}{11}+\frac{8}{11}$ (Art. 229.)
$=29+\frac{8}{11}=29\frac{8}{11}$. (Art. 231.)

248. The **complete quotient** of one number divided by another is the mixed number obtained by the above Rule.

Thus, the *complete quotient* of 79 divided by 15 is the mixed number $5\frac{4}{15}$, for $79\div15=\frac{79}{15}=5\frac{4}{15}$.

Examples LXIII.

1. Express *orally* the following as improper fractions :—

(1) $1\frac{1}{3}$; $2\frac{1}{12}$; $3\frac{1}{15}$; $8\frac{3}{5}$; $9\frac{4}{7}$; $6\frac{4}{9}$; $5\frac{3}{8}$; $7\frac{5}{12}$; $4\frac{3}{14}$; $9\frac{7}{13}$.

(2) $13\frac{13}{14}$; $15\frac{9}{17}$; $16\frac{13}{15}$; $19\frac{7}{15}$; $14\frac{13}{16}$; $20\frac{11}{18}$; $17\frac{9}{17}$.

2. Convert into improper fractions :—

(1) $121\frac{5}{11}$; $54\frac{8}{11}$; $41\frac{7}{12}$; $123\frac{4}{17}$; $156\frac{5}{18}$; $95\frac{15}{16}$; $22\frac{7}{18}$.

(2) $275\frac{14}{15}$; $374\frac{54}{103}$; $344\frac{24}{49}$; $101\frac{16}{17}$; $49\frac{74}{87}$; $19\frac{38}{39}$; $44\frac{9}{39}$.

(3) $704\frac{19}{126}$; $5\frac{7}{550}$; $148\frac{227}{463}$; $25\frac{389}{2400}$; $685\frac{219}{999}$; $9879\frac{1457}{8579}$.

3. Express *orally* as mixed or whole numbers :—

(1) $\frac{7}{3}$; $\frac{8}{2}$; $\frac{16}{4}$; $\frac{13}{3}$; $\frac{47}{7}$; $\frac{19}{5}$; $\frac{38}{9}$; $\frac{76}{8}$; $\frac{50}{9}$; $\frac{21}{8}$; $\frac{24}{11}$; $\frac{19}{7}$.

(2) $\frac{140}{11}$; $\frac{199}{12}$; $\frac{91}{13}$; $\frac{281}{14}$; $\frac{132}{11}$; $\frac{143}{12}$; $\frac{187}{17}$; $\frac{112}{14}$; $\frac{81}{18}$.

4. Represent the following as mixed or whole numbers :—

(1) $\frac{440}{13}$; $\frac{2417}{19}$; $\frac{3797}{29}$; $\frac{9999}{31}$; $\frac{30471}{37}$; $\frac{523}{23}$; $\frac{747}{45}$; $\frac{775}{31}$.

(2) $\frac{3003}{217}$; $\frac{4521}{171}$; $\frac{6984}{481}$; $\frac{52504}{572}$; $\frac{51637}{152}$; $\frac{9999}{347}$; $\frac{19585}{144}$.

(3) $\frac{76845}{999}$; $\frac{830526}{9891}$; $\frac{907133}{7816}$; $\frac{4003187}{99999}$; $\frac{1516461}{30125}$; $\frac{1001010111}{100001}$.

5. Express the reciprocals of the following fractions, as mixed numbers :—

$$\frac{7}{15};\ \frac{15}{49};\ \frac{17}{65};\ \frac{100}{6874};\ \frac{87}{3415};\ \frac{99}{4567};\ \frac{152}{51847};\ \frac{1251}{59956}.$$

6. Express $41\frac{7}{8}$, $25\frac{11}{12}$, $9\frac{15}{16}$, and $10\frac{9}{20}$ as fractions, with denominators 240 and 720.

7. Find the respective values of :—

(1) $\frac{17}{24}\times 8$; $\frac{12}{17}\times 17$; $6\frac{13}{35}\times 7$; $10\frac{8}{17}\times 17$; $6\frac{10}{11}\times 11$; $2\frac{11}{65}\times 13$.

(2) $\frac{45}{49}\div 9$; $\frac{14}{15}\div 7$; $\frac{117}{120}\div 13$; $\frac{143}{205}\div 11$; $6\frac{6}{7}\div 12$; $9\frac{12}{17}\div 15$.

249. *To express a compound fraction as a simple one.*

A **Compound Fraction** is made up of two or more simple fractions connected by the word **of** ; as, $\frac{1}{2}$ of $1\frac{2}{3}$ of $3\frac{4}{5}$.

RULE. Multiply all the numerators together for the numerator of the simple fraction, and all the denominators together for its denominator.

Ex. 1. Convert $\frac{1}{3}$ of $\frac{4}{5}$ of $\frac{6}{7}$ into a simple fraction.

$$\tfrac{1}{3} \text{ of } \tfrac{4}{5} \text{ of } \tfrac{6}{7} = \frac{1\times 4\times 6}{3\times 5\times 7} = \tfrac{24}{105}. \quad \textit{Ans.}$$

For $\frac{1}{5}$ of $\frac{6}{7}=\frac{6}{7}\div 5=\frac{6}{35}$
and $\frac{4}{5}$ of $\frac{6}{7}=\frac{6}{35}\times 4=\frac{24}{35}$ (Art. 241.)
whence, $\frac{1}{3}$ of $\frac{4}{5}$ of $\frac{6}{7}=\frac{1}{3}$ of $\frac{24}{35}=\frac{24}{35}\div 3=\frac{24}{105}$.

$$\therefore \tfrac{1}{3} \text{ of } \tfrac{4}{5} \text{ of } \tfrac{6}{7} = \frac{1\times 4\times 6}{3\times 5\times 7} = \frac{24}{105}.$$

Note 1. Before applying the above Rule mixed numbers must be expressed as improper fractions.

Note 2. If there are factors common to both numerator and denominator, they may be *cancelled* or struck out, before obtaining the final result ; for this is in fact simply dividing the numerator and denominator of a fraction by the same number. (Art. 242.)

Ex. 2. Reduce $\frac{3}{5}$ of $2\frac{1}{12}$ of $5\frac{1}{15}$ to a simple fraction.

$$\tfrac{3}{5} \text{ of } 2\tfrac{1}{12} \text{ of } 5\tfrac{1}{15} = \tfrac{3}{5} \text{ of } \tfrac{25}{12} \text{ of } \tfrac{76}{15} = \frac{3\times(5\times 5)\times(4\times 19)}{5\times(3\times 4)\times(3\times 5)}$$

$$= \frac{3\times 5\times 5\times 4\times 19}{5\times 3\times 4\times 3\times 5} = \tfrac{19}{3} = 6\tfrac{1}{3}. \quad \textit{Ans.}$$

(dividing numerator and denominator by the factors 3, 5, 5, 4 common to both).

Examples LXIV.

Reduce the following compound fractions to simple ones :—

1. $\frac{3}{4}$ of $\frac{4}{5}$; $\frac{4}{5}$ of $\frac{10}{12}$; $\frac{3}{10}$ of $\frac{8}{15}$; $\frac{5}{6}$ of $\frac{21}{8}$; $\frac{2}{3}$ of $\frac{5}{4}$; $\frac{5}{7}$ of $\frac{10}{3}$.
2. $\frac{22}{5}$ of $\frac{8}{9}$; $\frac{8}{3}$ of $\frac{20}{14}$; $\frac{5}{6}$ of $\frac{10}{3}$ of 9 ; $\frac{4}{7}$ of $\frac{15}{4}$ of $\frac{7}{3}$; $\frac{3}{4}$ of $\frac{4}{9}$ of $\frac{25}{2}$.
3. $\frac{3}{7}$ of $\frac{5}{9}$ of $\frac{2}{3}$ of $\frac{3}{5}$; $\frac{3}{4}$ of $\frac{4}{7}$ of $\frac{7}{11}$ of $\frac{11}{5}$ of $\frac{5}{7}$; $\frac{4}{7}$ of $\frac{14}{12}$ of $\frac{6}{5}$ of $\frac{10}{12}$.
4. $\frac{1}{2}$ of $\frac{5}{6}$ of 4 of $\frac{2}{3}$ of $\frac{5}{7}$ of 6 ; $\frac{2}{5}$ of $\frac{5}{8}$ of $\frac{8}{10}$ of $\frac{2}{7}$ of $\frac{5}{8}$ of $\frac{7}{2}$.
5. $\frac{3}{8}$ of $\frac{3}{2}$ of $\frac{5}{8}$ of $\frac{2}{5}$ of $\frac{34}{11}$ of $\frac{55}{6}$. 6. $\frac{21}{5}$ of $\frac{15}{4}$ of 10 of $\frac{5}{2}$ of $\frac{3}{5}$ of $\frac{22}{3}$ of $\frac{2}{15}$.
7. $\frac{7}{31}$ of $\frac{63}{9}$ of $\frac{35}{2}$ of $\frac{98}{9}$ of $\frac{268}{29}$. 8. $\frac{19}{8}$ of $\frac{14}{15}$ of $\frac{99}{10}$ of $\frac{24}{7}$ of $\frac{25}{42}$ of $\frac{98}{171}$.
9. $\frac{5}{7}$ of $\frac{11}{12}$ of 9 of $\frac{49}{8}$ of $\frac{12}{18}$ of $\frac{8}{3}$ of $\frac{26}{25}$ of $\frac{75}{84}$ of $\frac{8}{21}$ of $\frac{2}{11}$.
10. $\frac{2}{5}$ of $\frac{8}{11}$ of $\frac{29}{3}$ of $\frac{3}{7}$ of $\frac{10}{9}$ of $\frac{42}{19}$ of $\frac{3}{2}$ of $\frac{5}{3}$ of $\frac{16}{3}$ of $\frac{1}{20}$ of $\frac{11}{8}$.
11. $15\frac{7}{11}$ of $8\frac{4}{5}$ of $13\frac{3}{7}$. 12. $\frac{3}{14}$ of $4\frac{5}{9}$ of $\frac{6}{37}$ of $6\frac{8}{11}$ of $\frac{7}{82}$.
13. $5\frac{1}{5}$ of $\frac{20}{33}$ of $7\frac{1}{3}$ of $\frac{15}{29}$ of $7\frac{1}{4}$. 14. $5\frac{1}{2}$ of $\frac{7}{22}$ of $1\frac{1}{14}$ of $3\frac{1}{5}$.
15. $4\frac{1}{3}$ of $\frac{3}{10}$ of $3\frac{1}{13}$ of $\frac{3}{4}$ of $\frac{5}{7}$ of $5\frac{1}{3}$ of $\frac{9}{25}$ of $\frac{4}{5}$ of $2\frac{3}{14}$.
16. $5\frac{1}{4}$ of $\frac{16}{25}$ of $\frac{4}{7}$ of $3\frac{4}{7}$ of $\frac{1}{5}$ of $\frac{5}{10}$ of $\frac{5}{8}$ of $\frac{7}{10}$ of $\frac{7}{11}$ of $1\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{7}$.

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250. A fraction is in its **lowest terms**, or in its **simplest form** when there are no factors common to both numerator and denominator. This will be the case when the numerator and denominator are **prime** to each other.

251. *To reduce a fraction to its lowest terms.*

RULE. Divide the numerator and denominator by their G. C. M.

Ex. Express the fraction $\frac{825}{960}$ in its lowest terms.

The G. C. M. of 825 and 960 is 15.

$$\therefore \frac{825}{960}=\frac{825\div 15}{960\div 15}=\frac{55}{64}. \quad Ans.$$

252. In many instances, it is unnecessary to find the G. C. M. at *first*, the fractions being reducible to lower terms by successive divisions of the numerators and denominators by common factors discovered by *inspection*, or by employing the tests of divisibility given in Art. 198.

Ex. Reduce $\frac{4968}{5904}$ to its lowest terms.

$$\frac{4968}{5904}=\frac{2484}{2952}=\frac{1242}{1476}=\frac{621}{738}=\frac{207}{246}=\frac{69}{82}. \quad Ans.$$

from *three* successive divisions of the numerator and denominator by 2, and then from *two* successive divisions by 3 ; and these are the terms which would have been obtained from dividing *at once* by 72 which is their G. C. M.

253. In examples like the following, it is convenient to break

up both the numerator and denominator into factors, and then *cancel* those which are common to both.

Note. It should be remarked that when a factor is *cancelled*, it is to be replaced by 1 and not by 0.

Ex. Reduce $\frac{35\times 63}{60\times 77}$ to its lowest terms.

$$\frac{35\times 63}{60\times 77}=\frac{(5\times 7)\times(3\times 3\times 7)}{(3\times 5\times 4)\times(7\times 11)}=\frac{5\times 7\times 3\times 3\times 7}{3\times 5\times 7\times 4\times 11}=\frac{3\times 7}{4\times 11}=\frac{21}{44}.\quad \textit{Ans.}$$

Examples LXV.

1. Reduce to their lowest terms (by *inspection*) :—

(1) $\frac{3}{9}$; $\frac{4}{12}$; $\frac{5}{15}$; $\frac{7}{28}$; $\frac{4}{6}$; $\frac{10}{15}$; $\frac{14}{21}$; $\frac{9}{24}$; $\frac{9}{27}$; $\frac{24}{87}$; $\frac{60}{96}$; $\frac{27}{45}$; $\frac{48}{56}$.

(2) $\frac{28}{63}$; $\frac{64}{84}$; $\frac{18}{24}$; $\frac{24}{48}$; $\frac{26}{39}$; $\frac{42}{48}$; $\frac{21}{28}$; $\frac{32}{48}$; $\frac{48}{56}$; $\frac{56}{91}$; $\frac{108}{180}$; $\frac{96}{144}$.

(3) $\frac{77}{121}$; $\frac{25}{100}$; $\frac{75}{100}$; $\frac{63}{144}$; $\frac{72}{108}$; $\frac{35}{189}$; $\frac{39}{52}$; $\frac{46}{115}$; $\frac{76}{114}$; $\frac{75}{185}$.

2. Reduce the following fractions to their lowest terms :—

(1) $\frac{435}{957}$; $\frac{203}{315}$; $\frac{455}{1645}$; $\frac{256}{1024}$; $\frac{444}{703}$; $\frac{925}{1025}$; $\frac{768}{2592}$; $\frac{1476}{1764}$.

(2) $\frac{3094}{3042}$; $\frac{3444}{3556}$; $\frac{5239}{6076}$; $\frac{5565}{8533}$; $\frac{7568}{9504}$; $\frac{1775}{2350}$; $\frac{3565}{4930}$; $\frac{9050}{17919}$.

(3) $\frac{1261}{1649}$; $\frac{6435}{7293}$; $\frac{1236}{4764}$; $\frac{6006}{8008}$; $\frac{9504}{10692}$; $\frac{7497}{15729}$; $\frac{48510}{49005}$.

(4) $\frac{8991}{10989}$; $\frac{12540}{21945}$; $\frac{13478}{10701}$; $\frac{8398}{29393}$; $\frac{43365}{44688}$; $\frac{13667}{14186}$; $\frac{217800}{245025}$.

(5) $\frac{11050}{35581}$; $\frac{20301}{33633}$; $\frac{714285}{999999}$; $\frac{109375}{1000000}$; $\frac{135795}{222210}$; $\frac{99715}{113960}$.

(6) $\frac{95469}{359784}$; $\frac{180194}{1973594}$; $\frac{256417}{7006987}$; $\frac{1854432}{3471276}$; $\frac{1832051}{2592525}$; $\frac{496606401}{1006110363}$.

3. Reduce (by *cancelling*) to their lowest terms :—

(1) $\frac{18\times 32}{27\times 52}$; $\frac{16\times 45}{24\times 75}$; $\frac{21\times 24}{56\times 84}$; $\frac{45\times 70\times 15}{81\times 90\times 100}$; $\frac{48\times 64\times 49}{96\times 88\times 63}$.

(2) $\frac{51\times 39\times 42}{68\times 52\times 70}$; $\frac{19\times 23\times 26\times 56}{57\times 92\times 78\times 98}$; $\frac{85\times 84\times 38}{102\times 154\times 95}$; $\frac{76\times 87\times 65}{114\times 145\times 143}$.

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254. *To reduce two or more fractions having different denominators to equivalent fractions having a common denominator.*

RULE. Multiply each numerator by all the denominators *except* the one placed under it, for the new numerator ; and multiply all the denominators together for the common denominator.

Ex. Express $\frac{1}{2}$, $\frac{2}{5}$ and $\frac{3}{7}$ as equivalent fractions with a common denominator.

Here, first $\left.\begin{array}{l} 1\times5\times7=35 \\ 2\times2\times7=28 \\ 3\times2\times5=30 \end{array}\right\}$ the new numerators :

and $2\times5\times7=70$, the com. denr. :

For $\frac{1}{2}=\frac{1\times5\times7}{2\times5\times7}=\frac{35}{70}$,

$\frac{2}{5}=\frac{2\times2\times7}{5\times2\times7}=\frac{28}{70}$,

and $\frac{3}{7}=\frac{3\times2\times5}{7\times2\times5}=\frac{30}{70}$.

∴ the equivalent fractions are $\frac{35}{70}$, $\frac{28}{70}$ and $\frac{30}{70}$. *Ans.*

255. If two or more of the denominators have a common measure, the equivalent fractions may be expressed in simpler terms than obtainable by the above Rule, and having a least common denominator (L. C. D.) by the following Rule.

RULE. Find the L. C. M. of the denominators ; this will be the least common denominator. Then divide the L. C. M. so found by the denominator of each fraction, and multiply each quotient so found into the numerator of the fraction which belongs to it for the new numerator of that fraction.

Note. Before applying the above Rules, reduce mixed numbers to improper fractions, and compound fractions to simple ones ; moreover, if the L. C. D. be required, the given fractions should be reduced to their lowest terms.

Ex. Reduce $\frac{4}{5}$, $\frac{11}{12}$ and $\frac{3}{20}$ to equivalent fractions having the least common denominator.

The L. C. M. of 5, 12 and 20 is 60, which is here the L. C. D.

$$60\div5=12\ ;\ 60\div12=5\ ;\ 60\div20=3.$$

$$\therefore\ \frac{4}{5}=\frac{4\times12}{5\times12}=\frac{48}{60};\ \frac{11}{12}=\frac{11\times5}{12\times5}=\frac{55}{60};\ \frac{3}{20}=\frac{3\times3}{20\times3}=\frac{9}{60}.$$

Hence, the equivalent fractions are $\frac{48}{60}$, $\frac{55}{60}$ and $\frac{9}{60}$. *Ans.*

256. Similarly we can reduce fractions to equivalent ones having a least common numerator (L. C. N.).

Ex. Reduce $\frac{5}{6}$, $\frac{4}{9}$, $\frac{8}{9}$ and $\frac{16}{17}$ to fractions having a least common numerator.

The L. C. M. of 5, 4, 8 and 16 = 80, which is here the L. C. N.

$$80\div5=16\ ;\ 80\div4=20\ ;\ 80\div8=10\ ;\ 80\div16=5.$$

$$\therefore\ \frac{5}{6}=\frac{5\times16}{6\times16}=\frac{80}{96};\ \frac{4}{9}=\frac{4\times20}{9\times20}=\frac{80}{180};$$

$$\frac{8}{9}=\frac{8\times10}{9\times10}=\frac{80}{90};\ \frac{16}{17}=\frac{16\times5}{17\times5}=\frac{80}{85}.$$

∴ the fractions with a L. C. N. are $\frac{80}{96}$, $\frac{80}{180}$, $\frac{80}{90}$, $\frac{80}{85}$. *Ans.*

Examples LXVI.

1. Reduce to equivalent fractions with a common denominator :—

(1) $\frac{2}{3}$, $\frac{4}{5}$. (2) $\frac{1}{7}$, $\frac{3}{8}$. (3) $\frac{3}{4}$, $\frac{9}{11}$. (4) $\frac{1}{3}$, $\frac{4}{5}$, $\frac{6}{7}$. (5) $\frac{3}{5}$, $\frac{2}{7}$, $\frac{4}{11}$.
(6) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{5}$, $\frac{9}{13}$. (7) $\frac{1}{3}$, $\frac{2}{5}$, $\frac{3}{7}$, $\frac{4}{9}$; $\frac{5}{11}$. (8) $1\frac{1}{3}$, $2\frac{1}{5}$, $3\frac{1}{7}$.
(9) $\frac{1}{3}$, $2\frac{5}{9}$, $3\frac{2}{11}$. (10) 7, $\frac{5}{8}$, $10\frac{9}{12}$, $26\frac{1}{7}$. (11) $\frac{2}{3}$ of $\frac{9}{16}$, $\frac{1}{18}$ of $5\frac{1}{4}$, $\frac{1}{2}$ of $1\frac{4}{5}$.

2. Reduce the fractions in each of the following sets to equivalent fractions, having the least common denominator :—

(1) $\frac{1}{2}$, $\frac{2}{3}$, $\frac{3}{4}$. (2) $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{8}$. (3) $\frac{19}{21}$, $\frac{22}{35}$. (4) $\frac{3}{16}$, $\frac{8}{21}$, $\frac{5}{9}$.
(5) $\frac{7}{15}$, $\frac{11}{21}$, $\frac{23}{30}$. (6) $\frac{5}{6}$, $\frac{7}{9}$, $\frac{8}{15}$, $\frac{9}{10}$. (7) $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{9}$, $\frac{11}{12}$. (8) $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{8}{9}$.
(9) $\frac{7}{12}$, $\frac{17}{30}$, $\frac{13}{18}$, $\frac{15}{20}$, $\frac{7}{15}$. (10) $\frac{14}{15}$, $\frac{15}{16}$, $\frac{17}{18}$, $\frac{19}{20}$, $\frac{23}{24}$, $\frac{26}{27}$
(11) $\frac{1}{3}$, $\frac{4}{5}$ of $\frac{1}{3}$ of $\frac{5}{8}$, $\frac{1}{3}$ of $\frac{3}{8}$ of $2\frac{1}{3}$, $\frac{4}{5}$, $\frac{5}{6}$. (12) $1\frac{1}{2}$, $3\frac{1}{3}$, $4\frac{1}{4}$, $6\frac{1}{6}$.
(13) $3\frac{2}{15}$, $\frac{27}{40}$, $1\frac{13}{35}$, $\frac{16}{27}$, $13\frac{11}{12}$. (14) $\frac{3}{7}$ of $\frac{21}{10}$, $\frac{5}{6}$ of $2\frac{5}{8}$, $\frac{4}{7}$ of $3\frac{3}{4}$ of $3\frac{1}{2}$.

3. Reduce the following fractions to equivalent ones with the least common numerator :—

(1) $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{2}{7}$. (2) $\frac{4}{5}$, $\frac{8}{9}$, $\frac{11}{12}$, $\frac{13}{15}$. (3) $\frac{7}{12}$, $\frac{9}{20}$, $\frac{14}{15}$, $\frac{7}{18}$.
(4) $\frac{3}{7}$, $\frac{7}{13}$, $\frac{9}{22}$, $\frac{21}{26}$. (5) $\frac{2}{27}$, $\frac{12}{24}$, $\frac{9}{17}$, $\frac{24}{31}$. (6) $\frac{4}{9}$, $\frac{2}{3}$, $\frac{16}{19}$, $\frac{18}{19}$, $\frac{5}{13}$.

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257. *To compare the magnitudes of different fractions.*

(1) RULE. Reduce the fractions to equivalent ones with the least common denominator (L. C. D.), and then compare the numerators so obtained. That fraction which has the greatest numerator is the *greatest*, and that which has the least is the *least.*

Ex. 1. Find the *greatest* and *least* of the fractions $\frac{7}{9}$, $\frac{5}{8}$ and $\frac{11}{14}$.

The L. C. M. of the denominators $= 504$.

$504 \div 9 = 56$; $504 \div 8 = 63$; $504 \div 14 = 36$.

$$\therefore \frac{7}{9} = \frac{7 \times 56}{9 \times 56} = \frac{392}{504} ; \frac{5}{8} = \frac{5 \times 63}{8 \times 63} = \frac{315}{504} ; \frac{11}{14} = \frac{11 \times 36}{14 \times 36} = \frac{396}{504}.$$

Hence $\frac{11}{14}$ is the *greatest* and $\frac{5}{8}$ is the *least.* *Ans.*

Ex. 2. Arrange $\frac{4}{5}$, $\frac{11}{12}$, $\frac{13}{15}$ and $\frac{8}{9}$ in order of magnitude.

The L. C. M. of the denominators $= 180$.

$180 \div 5 = 36$; $180 \div 12 = 15$; $180 \div 15 = 12$; $180 \div 9 = 20$.

$$\therefore \frac{4}{5} = \frac{4 \times 36}{5 \times 36} = \frac{144}{180} ; \frac{11}{12} = \frac{11 \times 15}{12 \times 15} = \frac{165}{180} ;$$

$$\frac{13}{15} = \frac{13 \times 12}{15 \times 12} = \frac{156}{180} ; \frac{8}{9} = \frac{8 \times 20}{9 \times 20} = \frac{160}{180}.$$

Hence the fractions arranged in order of magnitude stand thus :—

$\frac{11}{12}$, $\frac{8}{9}$, $\frac{13}{15}$ and $\frac{4}{5}$. *Ans.*

(2) Fractions may also be compared by reducing them to a least common numerator (L. C. N.). In this case, the new fraction that has the lest denominator is the *greatest*, and that which has the greatest denominator is the *least.*

Ex. Find the *greatest* and the *least* of $\frac{2}{5}$, $\frac{3}{10}$, $\frac{7}{15}$ and $\frac{9}{16}$.

The L. C. M. of the numerators = 126.

$126 \div 2 = 63$; $126 \div 3 = 42$; $126 \div 7 = 18$; $126 \div 9 = 14$.

$$\therefore \frac{2}{5} = \frac{2 \times 63}{5 \times 63} = \frac{126}{315};\ \frac{3}{10} = \frac{3 \times 42}{10 \times 42} = \frac{126}{420};$$

$$\frac{7}{15} = \frac{7 \times 18}{15 \times 18} = \frac{126}{270};\ \frac{9}{16} = \frac{9 \times 14}{16 \times 14} = \frac{126}{224}.$$

Hence $\frac{9}{16}$ is the *greatest* and $\frac{3}{10}$ is the *least.* *Ans.*

258. The defect of a fraction from 1 is called its **complement.**

Thus, $\frac{1}{4}$ and $\frac{3}{8}$ are respectively the *complements* of $\frac{3}{4}$ and $\frac{5}{8}$.

(3) Fractions may also be compared by taking their complements, provided that each of the complements has 1 for its numerator.

The *greatest* and *least* fractions will be those that have the least and the greatest complement.

Ex. Find the *greatest* and the *least* of the fractions $\frac{4}{5}$, $\frac{23}{24}$, and $\frac{5}{6}$.

The complements of these fractions are $\frac{1}{5}$, $\frac{1}{24}$ and $\frac{1}{6}$ respectively.

Now, of these complements $\frac{1}{24}$ is the least and $\frac{1}{5}$ the greatest ;

$\therefore$ $\frac{23}{24}$ is the *greatest* and $\frac{4}{5}$ is the *least.* *Ans.*

(4) Fractions may also be compared by the method illustrated by the following example.

Ex. Arrange in order of magnitude $\frac{3}{7}$, $\frac{5}{21}$ and $\frac{6}{19}$.

$$\frac{3}{7} = \frac{3+3}{7+3} = \frac{1}{2\frac{1}{3}};\ \frac{5}{21} = \frac{5+5}{21+5} = \frac{1}{4\frac{1}{5}};\ \frac{6}{19} = \frac{6+6}{19+6} = \frac{1}{3\frac{1}{6}}.\quad \text{(Art. 242).}$$

The given fractions $= \frac{1}{2\frac{1}{3}}$; $\frac{1}{4\frac{1}{5}}$; $\frac{1}{3\frac{1}{6}}$ respectively. Of these $\frac{1}{2\frac{1}{3}}$ is the greatest and $\frac{1}{4\frac{1}{5}}$ is the least, for they have respectively the least and greatest denominators.

$\therefore$ the order of magnitude is $\frac{3}{7}$, $\frac{6}{19}$ and $\frac{5}{21}$. *Ans.*

Examples LXVII.

1. Which is the greater ? (by the *first method*).

[illegible] or [illegible] ; [illegible] or [illegible] ; [illegible] or [illegible] ; [illegible] or [illegible] ; [illegible] or [illegible] ; $\frac{15}{19}$ or $\frac{15+8}{19+8}$.

2. Which is the less? (by the *second method.*)

$\frac{7}{18}$ or $\frac{17}{45}$; $\frac{4}{9}$ or $\frac{5}{12}$; $\frac{64}{81}$ or $\frac{85}{108}$; $\frac{81}{88}$ or $\frac{11}{14}$; $\frac{3}{7}$ of $\frac{5}{9}$ or $\frac{1}{2}$ of $\frac{8}{9}$; $\frac{15}{19}$ or $\frac{15-8}{19-8}$

3. Which is the greatest, and which is the least of the following?

(1) $\frac{5}{6}$, $\frac{11}{12}$, $\frac{1}{18}$; $\frac{4}{9}$, $\frac{2}{3}$, $\frac{8}{15}$; $\frac{15}{42}$, $\frac{22}{49}$, $\frac{48}{63}$; $\frac{5}{18}$, $\frac{4}{9}$, $\frac{7}{27}$; $\frac{7}{9}$, $\frac{11}{13}$, $\frac{11}{19}$.

(2) $\frac{4}{5}$, $\frac{5}{6}$, $\frac{9}{8}$, $\frac{15}{14}$; $\frac{2}{3}$, $\frac{3}{4}$, $\frac{7}{9}$, $\frac{9}{10}$; $\frac{4}{7}$, $\frac{26}{51}$, $\frac{65}{102}$, $\frac{15}{17}$; $\frac{2}{9}$, $\frac{5}{18}$, $\frac{11}{24}$, $\frac{5}{27}$.

(3) $\frac{1}{2}$ of $\frac{5}{6}$ of 4, $\frac{2}{3}$ of $\frac{5}{7}$ of 6, $\frac{3}{4}$ of $\frac{4}{5}$ of 3 ; $\frac{5}{32}$, $\frac{3}{28}$, $\frac{1}{24}$, $\frac{13}{48}$.

(4) $\frac{2}{3}$ of $\frac{5}{8}$ of $\frac{3}{10}$, $\frac{2}{7}$ of $\frac{5}{8}$ of $3\frac{1}{2}$, $4\frac{1}{5}$ of $3\frac{3}{4}$ of 10 ; $\frac{3}{7}$ of $\frac{5}{9}$, $\frac{1}{2}$ of $\frac{3}{7}$, $7\frac{3}{8}$.

(5) $\frac{17}{24}$, $\frac{11}{18}$, $\frac{19}{36}$, $\frac{25}{48}$, $\frac{7}{16}$; $\frac{4}{11}$, $\frac{9}{25}$, $\frac{18}{55}$, $\frac{9}{22}$; $\frac{4}{5}$, $\frac{5}{6}$, $\frac{4+5}{5+6}$.

4. Arrange in order of magnitude the following :—

(1) $\frac{2}{3}$, $\frac{7}{8}$, $\frac{11}{12}$, $\frac{19}{20}$. (2) $\frac{3}{25}$, $\frac{7}{10}$, $\frac{3}{50}$, $\frac{7}{55}$. (3) $\frac{7}{15}$, $\frac{10}{21}$, $\frac{18}{35}$.

(4) $1\frac{1}{7}$ of $\frac{1}{4}$, $\frac{5}{21}$ of $3\frac{1}{2}$, $\frac{1}{3}$ of $2\frac{5}{8}$. (5) $\frac{5}{8}$ of $2\frac{2}{7}$, $1\frac{3}{8}$, $3\frac{1}{2}$ of $\frac{5}{14}$ of [illegible].

(6) $\frac{13}{14}$, $\frac{15}{16}$, $\frac{13+15}{14+16}$. (7) $\frac{5}{6}$, $\frac{6}{7}$, $\frac{7}{8}$, $\frac{8}{10}$, $\frac{5+6+7+9}{6+7+8+10}$.

5. Arrange in order of magnitude :—(by the *third method.*)

(1) $\frac{2}{3}$, $\frac{4}{5}$, $\frac{1}{2}$, $\frac{3}{4}$. (2) $\frac{9}{10}$, $\frac{11}{12}$, $\frac{8}{9}$, $\frac{6}{7}$. (3) $\frac{13}{14}$, $\frac{7}{8}$, $\frac{5}{6}$, $\frac{15}{16}$.

(4) $\frac{89}{90}$, $\frac{99}{100}$, $\frac{11}{12}$, $\frac{15}{16}$. (5) $\frac{49}{50}$, $\frac{47}{48}$, $\frac{61}{62}$, $\frac{74}{75}$. (6) $\frac{15}{16}$, $\frac{17}{18}$, $\frac{19}{20}$, $\frac{24}{25}$.

6. Arrange in order of magnitude :—(by the *fourth method*).

(1) $\frac{1}{4}$, $\frac{2}{9}$, $\frac{7}{8}$, $\frac{2}{3}$. (2) $\frac{21}{22}$, $\frac{15}{16}$, $\frac{5}{48}$, $\frac{14}{35}$. (3) $\frac{5}{9}$, $\frac{3}{8}$, $\frac{4}{17}$, $\frac{1}{18}$.

(4) $\frac{9}{90}$, $\frac{15}{16}$, $\frac{19}{20}$, $\frac{23}{30}$. (5) $\frac{4}{9}$, $\frac{2}{3}$, $\frac{16}{19}$, $\frac{13}{15}$, $\frac{8}{15}$. (6) $\frac{2}{3}$, $\frac{5}{9}$, $\frac{6}{13}$, $\frac{13}{15}$.

7. Find a fraction intermediate in value to $\frac{5}{6}$ and $\frac{6}{7}$ whose denominator is 84 ; to $\frac{3}{5}$ and $\frac{7}{12}$ whose denominator is 720.

III. ADDITION OF FRACTIONS.

259. *To find the sum of two or more given fractions.*

(1) If the given fractions have the *same* denominator.

RULE. Add the numerators of the given fractions together for the numerator of the **sum**, and take their denominator for its denominator.

Ex. Find the sum of $\frac{4}{7}$, $\frac{5}{7}$ and $\frac{8}{7}$.

Here, $\frac{4}{7}+\frac{5}{7}+\frac{8}{7}=\frac{4+5+8}{7}$ $=\frac{17}{7}=2\frac{3}{7}$.

For 4 *sevenths* + 5 *sevenths* + 8 *sevenths* = (4 + 5 + 8) *sevenths*. = 17 *sevenths* = $\frac{17}{7}$.

(2) If the given fractions have *different* denominators.

RULE. Express the fractions with a least common denominator; add together the new numerators for the numerator of their sum, and take the least common denominator for its denominator.

The sum should always be expressed in its lowest terms ; and, if an improper fraction, should be reduced to a mixed number.

Ex. Find the sum of $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$ and $\frac{7}{8}$.

The L. C. M. of the denominators $=24$.

$$\frac{2}{3}=\frac{2\times8}{3\times8}=\frac{16}{24};\ \frac{3}{4}=\frac{3\times6}{4\times6}=\frac{18}{24};$$

$$\frac{5}{6}=\frac{5\times4}{6\times4}=\frac{20}{24};\ \frac{7}{8}=\frac{7\times3}{8\times3}=\frac{21}{24}.$$

$$\therefore \text{ the sum}=\frac{16}{24}+\frac{18}{24}+\frac{20}{24}+\frac{21}{24}=\frac{16+18+20+21}{24}$$

$$=\frac{75}{24}=\frac{25\times3}{8\times3}=\frac{25}{8}=3\tfrac{1}{8}.\quad \textit{Ans.}$$

260. All fractions should be reduced to their lowest terms, improper fractions to whole or mixed numbers, and compound fractions to simple ones, before the application of the Rule.

261. If any one of the given numbers be whole or mixed numbers, add together the whole numbers as in simple addition and the fractional parts by the Rule given above.

Ex. Add together $5\frac{5}{12}$, $\frac{19}{6}$, $2\frac{21}{48}$ and $\frac{3}{4}$ of $3\frac{2}{3}$.

Here, $\frac{19}{6}=3\frac{1}{6}$; $2\frac{21}{48}=2\frac{7}{16}$; $\frac{3}{4}$ of $3\frac{2}{3}=\frac{3}{4}$ of $\frac{11}{3}=\frac{11}{4}=2\frac{3}{4}$.

$$\therefore \text{ sum of the fractions}=5\tfrac{5}{12}+3\tfrac{1}{6}+2\tfrac{7}{16}+2\tfrac{3}{4}$$

$$=(5+3+2+2)+(\tfrac{5}{12}+\tfrac{1}{6}+\tfrac{7}{16}+\tfrac{3}{4})$$

$$=12+\frac{20+8+21+36}{48}$$

$$=12+\tfrac{85}{48}=12+1\tfrac{37}{48}=13\tfrac{37}{48}.\quad \textit{Ans.}$$

Examples LXVIII.

1. Add together *orally* the following fractions :—

(1) $\frac{2}{7}+\frac{3}{7}$; $\frac{1}{5}+\frac{7}{5}$; $\frac{4}{5}+\frac{3}{5}$; $\frac{2}{9}+\frac{5}{9}+\frac{7}{9}$; $\frac{5}{6}+\frac{1}{6}+\frac{7}{6}$; $\frac{3}{10}+\frac{7}{10}$; $\frac{7}{15}+\frac{11}{15}$.

(2) $\frac{1}{2}+\frac{1}{3}$; $\frac{1}{3}+\frac{1}{4}$; $\frac{1}{4}+\frac{1}{5}$; $\frac{1}{9}+\frac{1}{6}$; $\frac{1}{7}+\frac{1}{14}$; $\frac{7}{10}+\frac{3}{15}$; $\frac{2}{5}+\frac{3}{10}+\frac{7}{15}$.

(3) $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}$; $\frac{1}{3}+\frac{1}{4}+\frac{1}{12}$; $\frac{1}{2}+\frac{3}{4}+\frac{5}{6}$; $\frac{1}{3}+\frac{5}{6}+\frac{7}{12}$: $\frac{1}{4}+\frac{5}{12}+\frac{8}{15}+\frac{7}{30}$.

(4) $3\frac{1}{2}+2\frac{1}{3}$; $4\frac{2}{3}+3\frac{1}{2}$; $1\frac{1}{4}+3\frac{3}{4}+2\frac{1}{2}$. $3\frac{1}{6}+4\frac{1}{2}+7\frac{7}{12}$.

2. Find the values of the following :—

(1) $\frac{11}{30}+\frac{2}{45}$; $\frac{2}{5}+\frac{4}{7}$; $\frac{3}{4}+\frac{5}{9}$; $\frac{3}{8}+\frac{7}{9}$; [illegible] ; $\frac{3}{8}+\frac{5}{14}$; $\frac{3}{7}+\frac{9}{10}+\frac{15}{28}$.

(2) $1\frac{1}{2}+7\frac{1}{3}$; $2\frac{3}{7}+13\frac{3}{10}$; $5\frac{1}{6}+12\frac{4}{5}$; $37\frac{2}{11}+24\frac{13}{33}$; $7\frac{8}{9}+4\frac{7}{8}$.

(3) $\frac{31}{7}+\frac{47}{8}$; $\frac{41}{18}+\frac{21}{15}$; $\frac{57}{8}+\frac{38}{15}$; $\frac{81}{9}+\frac{49}{8}$; $\frac{27}{16}+\frac{71}{32}$; $\frac{17}{35}+\frac{22}{45}$.

(4) $\frac{2}{3}+\frac{3}{4}+\frac{5}{6}$; $\frac{1}{7}+\frac{2}{5}+\frac{3}{11}$; $\frac{4}{5}+\frac{6}{7}+\frac{9}{10}$; $\frac{5}{6}+\frac{4}{9}+\frac{14}{21}$; $\frac{3}{4}+\frac{5}{6}+\frac{7}{8}+\frac{9}{10}$.

(5) $\frac{11}{16}+\frac{45}{8}+\frac{97}{9}$; $2\frac{1}{3}+3\frac{3}{5}+5\frac{2}{3}$; $8\frac{4}{7}+13\frac{5}{6}+27\frac{8}{11}$; $\frac{15}{4}+3\frac{1}{3}+11+\frac{14}{5}$.

(6) $\frac{3}{7}+\frac{5}{14}+\frac{11}{42}+\frac{20}{21}$; [illegible] ; [illegible].

(7) $\frac{8}{15}$ of $9\frac{8}{15}+\frac{4}{17}$ of $8\frac{1}{2}$; $14\frac{3}{4}+\frac{2}{3}$ of $\frac{5}{6}$ of 8 ; $\frac{2}{7}+4\frac{1}{3}+\frac{2}{3}$ of 2.

(8) $1\frac{2}{7}+\frac{5}{8}+\frac{8}{11}+3\frac{1}{6}$. (9) $3\frac{5}{8}+2\frac{2}{3}+\frac{7}{11}+7\frac{1}{6}$.

(10) $2\frac{7}{12}+3\frac{5}{16}+4\frac{1}{21}+5\frac{11}{60}$. (11) $3\frac{5}{12}+7\frac{1}{9}+8\frac{2}{3}+4\frac{1}{6}$.

(12) $1\frac{1}{12}+2\frac{2}{15}+3\frac{3}{20}+4\frac{4}{27}$. (13) $4\frac{2}{7}+8\frac{2}{3}+3\frac{5}{14}+8\frac{5}{6}$.

(14) $\frac{4}{11}$ of $\frac{2}{3}$ of $\frac{1}{2}+\frac{7}{8}$ of $\frac{4}{5}$ of $\frac{10}{33}+\frac{2}{3}$ of $1\frac{2}{3}+2\frac{5}{7}$.

(15) $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{9}{20}$ of $11\frac{1}{9}$; $\frac{2}{3}+2\frac{1}{2}$ of $\frac{7}{25}+3\frac{1}{3}$ of $\frac{9}{20}$.

(16) $387\frac{1}{2}+285\frac{1}{4}+394\frac{3}{8}+\frac{2}{3}$ of 3704 ; $1\frac{5}{18}+2\frac{6}{7}+4\frac{11}{18}+6\frac{1}{14}$.

(17) $275\frac{1}{3}+62\frac{11}{120}+1031\frac{1}{5}+\frac{7}{8}$ of $4150\frac{1}{7}$.

(18) $\frac{1}{3}+\frac{3}{4}+\frac{5}{6}+\frac{4}{9}+\frac{7}{12}+\frac{11}{18}+\frac{19}{24}$. (19) $\frac{1}{2}+\frac{2}{3}+\frac{2}{5}+\frac{7}{15}+\frac{23}{30}+\frac{11}{40}+\frac{23}{60}+\frac{41}{120}$.

(20) $\frac{3}{4}$ of $\frac{5}{7}$ of $52\frac{4}{15}+\frac{2}{3}$ of $\frac{2}{9}$ of $506\frac{1}{4}+\frac{2}{3}$ of $\frac{3}{8}$ of 1864.

(21) $\frac{3}{7}$ of $\frac{5}{7}+\frac{4}{5}$ of $\frac{1}{10}+\frac{2}{3}$ of $(\frac{1}{2}+\frac{1}{14})+\frac{3}{70}$ of $(\frac{2}{7}+\frac{4}{5})$.

IV. SUBTRACTION OF FRACTIONS.

262. *To subtract one fraction from another fraction.*

(1) When the given fractions have the *same* denominator.

RULE. Find the difference of the numerators of the given fractions for the numerator of the **remainder**, and take their denominator for its denominator.

Ex. Subtract $\frac{4}{17}$ from $\frac{10}{17}$.

Here, $\frac{10}{17}-\frac{4}{17}=\frac{10-4}{17}=\frac{6}{17}$. For, 10 *seventeenths* − 4 *seventeenths* = (10 − 4) *seventeenths* = 6 *seventeenths* = $\frac{6}{17}$.

(2) When the given fractions have *different* denominators.

RULE. Ruduce the fractions to a least common denominator; subtract the less numerator from the greater; under the **remainder** place the least common denominator, and the result, properly reduced will be the required difference.

Note. Before applying the Rule, reduce fractions to their lowest terms, improper fractions to whole or mixed numbers, and compound fractions to simple ones.

Ex. Subtract $\frac{13}{18}$ from $\frac{19}{24}$, and $\frac{2}{3}$ of $\frac{5}{16}$ from $\frac{7}{8}$ of $\frac{3}{4}$.

(1) The L. C. M. of 18 and 24 = 72.

$\therefore \frac{19}{24}-\frac{13}{18}=\frac{57}{72}-\frac{52}{72}=\frac{57-52}{72}=\frac{5}{72}$. *Ans.*

(2) Here, $\frac{7}{8}$ of $\frac{3}{4}=\frac{21}{32}$, and $\frac{2}{3}$ of $\frac{5}{16}=\frac{5}{24}$. Also L. C. D. = 96

$\therefore$ their difference $=\frac{21}{32}-\frac{5}{24}=\frac{63-20}{96}=\frac{43}{96}$. *Ans.*

263. *Additions and subtractions of fractions may be performed in any order.*

Thus, $7\frac{3}{4}-4\frac{5}{7}=(7+\frac{3}{4})-(4+\frac{5}{7})=7+\frac{3}{4}-4-\frac{5}{7}$, (Art. 107)
$=(7-4)+(\frac{3}{4}-\frac{5}{7})=3+\frac{3}{4}-\frac{5}{7}$.

Hence, if either of the given fractions be a whole or mixed number, it is most convenient to take separately the difference of the integral parts and that of the fractional parts, and then add the two results together.

Ex. 1. From $3\frac{2}{3}$ take $2\frac{1}{6}$.

Here, $3\frac{2}{3}-2\frac{1}{6}=(3+\frac{2}{3})-(2+\frac{1}{6})=(3-2)+(\frac{2}{3}-\frac{1}{6})$
$=1+(\frac{4}{6}-\frac{1}{6})=1\frac{3}{6}=1\frac{1}{2}$. *Ans.*

Ex. 2. Find the difference between $2\frac{6}{7}$ and $5\frac{1}{3}$.

Here, $5\frac{1}{3}-2\frac{6}{7}=(4+1+\frac{1}{3})-(2+\frac{6}{7})=(4-2)+(1+\frac{1}{3}-\frac{6}{7})$
$=2+(\frac{4}{3}-\frac{6}{7})=2+(\frac{28}{21}-\frac{18}{21})=2+\frac{10}{21}=2\frac{10}{21}$. *Ans.*

264. The following peculiarities in *Subtraction of Fractions* should be carefully noticed.

(1) *When both the fractions have a common numerator.*

RULE. Multiply the difference of the denominators by the common numerator for the *new numerator*, and take the product of the denominators for the *new denominator*. The resulting fraction is the required difference.

Ex. 1. $\frac{8}{11}-\frac{8}{13}=\frac{(13-11)\times 8}{13\times 11}=\frac{2\times 8}{13\times 11}=\frac{16}{143}$. *Ans.*

(2) *To subtract fractions when both have 1 for numerator.*

RULE. Find the difference between the denominators for a *new numerator* and multiply the denominators for a *new denominator*. The resulting fraction is the required difference.

Ex. 2. $\frac{1}{8}-\frac{1}{9}=\frac{9-8}{8\times 9}=\frac{1}{72}$.

(3) *To subtract a proper fraction from unity.*

RULE. Subtract the numerator from the denominator for the *new numerator*, and underneath place the given denominator. The resulting fraction is the required difference.

Ex. 3. $1-\frac{5}{11}=\frac{11-5}{11}=\frac{6}{11}$. *Ans.*

(4) *To subtract a mixed number from an integer.*

RULE. Subtract the fractional part from unity as in (3) and the integral part from the integer diminished by unity.

Ex. 4. $7-3\frac{4}{7}=(6-3)+(1-\frac{4}{7})=3\frac{3}{7}$. *Ans.*

(5) *To subtract a mixed number from another, when the fractional part of the subtrahend is greater than that of the minuend.*

RULE. Subtract the subtrahend (composed of the integral and fractional part) from the integral part of the minuend as in (4) and to this difference add the fractional part of the minuend.

Ex. 5. $15\frac{3}{7}-7\frac{2}{3}=(15-7\frac{2}{3})+\frac{3}{7}=7\frac{1}{3}+\frac{3}{7}=7\frac{16}{21}$. *Ans.*

265. *An expression made up of additions and subtractions of fractions may be made equal to the difference of two sums.*

Thus, $5\frac{3}{4}-1\frac{4}{7}+2\frac{5}{6}+\frac{2}{3}-\frac{3}{8}=(5\frac{3}{4}+2\frac{5}{6}+\frac{2}{3})-(1\frac{4}{7}+\frac{3}{8})$. (Art. 107.)

Examples LXIX.

1. Perform *orally* the following subtractions :—

(1) $\frac{5}{6}-\frac{1}{6}$; $\frac{7}{9}-\frac{2}{9}$; $\frac{4}{5}-\frac{2}{5}$; $\frac{1}{2}-\frac{1}{8}$; $\frac{1}{3}-\frac{1}{5}$; $\frac{1}{2}-\frac{1}{6}$; $\frac{2}{3}-\frac{1}{2}$; $\frac{1}{4}-\frac{1}{8}$.

(2) $1-\frac{7}{8}$; $2-\frac{1}{3}$; $2-\frac{5}{18}$; $1-\frac{2}{5}$; $1-\frac{3}{4}$; $\frac{5}{8}-\frac{1}{2}$; $\frac{3}{4}-\frac{2}{5}$; $\frac{4}{5}-\frac{1}{4}$.

(3) $1-\frac{7}{9}$; $3-\frac{4}{7}$; $2\frac{1}{3}-1\frac{1}{6}$; $4\frac{2}{3}-3\frac{1}{2}$; $6\frac{1}{2}-4\frac{1}{3}$; $7\frac{7}{8}-4\frac{1}{4}$; $4\frac{9}{10}-2\frac{9}{20}$.

2. Perform the following subtractions :—

(1) $\frac{3}{5}-\frac{1}{6}$; $\frac{7}{9}-\frac{4}{7}$; $\frac{8}{11}-\frac{2}{9}$; $\frac{5}{6}-\frac{7}{18}$; $\frac{12}{13}-\frac{5}{18}$; $\frac{14}{15}-\frac{11}{12}$.

(2) $1\frac{8}{25}-1\frac{2}{21}$; $\frac{81}{56}-\frac{13}{98}$; $8\frac{4}{9}-\frac{8}{9}$; $9\frac{8}{15}-2\frac{7}{18}$; $3\frac{17}{22}-2\frac{19}{24}$; $8\frac{7}{8}-5\frac{4}{9}$.

(3) $19\frac{2}{7}-13\frac{2}{11}$; $18\frac{12}{25}-17\frac{13}{27}$; $1000-384\frac{7}{39}$; $279\frac{2}{7}-168\frac{3}{8}$.

(4) $2\frac{7}{9}$ of $16\frac{2}{7}-1\frac{2}{3}$ of $3\frac{5}{8}$; $\frac{2}{7}$ of $\frac{2}{11}$ of $25-\frac{3}{20}$ of $\frac{5}{9}$; $7\frac{1}{7}$ of $10\frac{1}{9}-2\frac{1}{2}$ of $4\frac{1}{4}$.

3. Find the values of :—

(1) $\frac{5}{6}-\frac{3}{4}+\frac{2}{3}-\frac{1}{2}$.

(2) $\frac{15}{16}-\frac{14}{15}+\frac{13}{14}-\frac{11}{12}$.

(3) $\frac{1}{2}+\frac{2}{3}-\frac{1}{6}+\frac{3}{8}-\frac{1}{12}$.

(4) $\frac{1}{2}-\frac{3}{4}+\frac{7}{15}-\frac{11}{40}+\frac{5}{6}+\frac{3}{8}-\frac{5}{12}+\frac{3}{5}$.

(5) $13\frac{1}{3}-9\frac{2}{9}-1\frac{8}{45}$.

(6) $3\frac{5}{18}$ $\frac{8}{23}-1\frac{3}{26}+\frac{5}{48}$.

(7) $7\frac{2}{3}+6\frac{3}{4}-3\frac{4}{15}-2\frac{17}{25}+\frac{11}{20}$.

(8) $3\frac{1}{3}+2\frac{4}{7}-(5\frac{1}{2}+1\frac{2}{3})+2\frac{8}{35}$.

(9) $10\frac{1}{2}-(4\frac{6}{7}+6\frac{3}{11})+7\frac{5}{9}+(8\frac{2}{3}-6\frac{23}{42})$.

(10) $6\frac{7}{8}$ of $2\frac{2}{11}-(6\frac{7}{8}-2\frac{2}{11})$.

(11) $2\frac{1}{6}-(4\frac{7}{8}+10\frac{5}{12}+3\frac{9}{20})+3\frac{1}{9}+20\frac{2}{5}$.

(12) $\frac{3}{8}$ of $\frac{4}{7}-\frac{2}{11}$ of $3\frac{1}{7}+\frac{5}{9}$ of $3\frac{3}{5}$.

(13) $22\frac{11}{17}-(9\frac{3}{4}-7\frac{16}{17}+\frac{1}{3}$ of $\frac{16}{17})+\frac{2}{5}$ of $3\frac{3}{4}$.

(14) $8\frac{3}{4}-3\frac{11}{12}+2\frac{7}{9}$ of $\frac{3}{14}$ of $4\frac{4}{5}-(5\frac{11}{12}-2\frac{7}{8})$.

(15) $47\frac{5}{27}-(3\frac{2}{3}+31\frac{5}{54}+2\frac{5}{8})+6\frac{5}{8}-(2\frac{8}{25}-1\frac{9}{104})$.

V. MULTIPLICATION OF FRACTIONS.

266. *To multiply a fraction by a whole number.*

[We have already given an outline of this method in Arts. 241 and 244. Now, we propose to treat it at length.]

RULE. Multiply the numerator by the whole number for the *new numerator*, and leave the denominator unchanged. The resulting fraction should always be expressed in its lowest terms, by cancelling those factors that are common to the multiplier and to the denominator of the fraction.

Thus, $8\times\frac{5}{9}=\frac{8\times5}{9}=\frac{40}{9}=4\frac{4}{9}$. | For $8\times\frac{5}{9}=8\times5$ *ninths* $=40$ *ninths* $=\frac{40}{9}=\frac{8\times5}{9}$.

Also, $9\times\frac{4}{15}=\frac{3\times3\times4}{3\times5}=\frac{3\times4}{5}=\frac{12}{5}=2\frac{2}{5}$.

267. *To multiply a mixed number by an integer.*

RULE. Either reduce the mixed number to an improper fraction and multiply as above, or multiply the integral part and the fractional part separately, and add the two products.

Thus, (1) $6\frac{4}{5}\times 3=\frac{34}{5}\times 3=\frac{102}{5}=20\frac{2}{5}$.

(2) $6\frac{4}{5}\times 3=6\times 3+\frac{4}{5}\times 3=18+\frac{12}{5}=18+2\frac{2}{5}=20\frac{2}{5}$.

268. To multiply a proper fraction differing very little from 1, or a mixed number differing very little from the next superior integer by a whole number, we have recourse to such artifices as is explained in Art. 264.

Thus. (1) $\frac{99}{100}\times 35=(1-\frac{1}{100})\times 35=35-\frac{35}{100}=35-\frac{7}{20}=34\frac{13}{20}$.

(2) $15\frac{77}{78}\times 12=(16-\frac{1}{78})\times 12=192-\frac{12}{78}=192-\frac{4}{26}=191\frac{22}{26}$.

(3) $99\frac{13}{14}\times 46=(100-\frac{1}{14})\times 46=4600-\frac{46}{14}=4600-3\frac{2}{7}=4596\frac{5}{7}$.

Examples LXX.

1. Multiply *orally* :—

(1) $\frac{1}{4}$ by 3 ; $\frac{2}{11}$ by 2 ; $\frac{5}{18}$ by 3 ; $\frac{4}{9}$ by 4 ; $\frac{5}{28}$ by 5 ; $\frac{5}{12}$ by 7 ; $\frac{3}{4}$ by 3 ; $\frac{8}{9}$ by 2.

(2) $\frac{17}{63}$ by 7 ; $\frac{13}{84}$ by 21 ; $\frac{23}{150}$ by 100 ; $\frac{88}{75}$ by 25 ; $\frac{29}{40}$ by 28 ; $\frac{8}{75}$ by 30.

2. Multiply :—

(1) $\frac{13}{33}$ separately by 55, 88, 90. (2) $\frac{5}{21}$ separately by 12, 36, 48, 49.

(3) $\frac{41}{112}$ ... ... 32, 128, 168 (4) $3\frac{17}{22}$ ... ... 11, 15, 21, 132.

(5) $3\frac{14}{33}$ ... ... 55, 77, 110. (6) $2\frac{9}{52}$ ... ... 13, 39, 42, 117.

(7) $159\frac{3}{8}$ by 12 ; $1625\frac{7}{11}$ by 23 ; $4\frac{18}{115}$ by 23 ; $1727\frac{2}{7}$ by 34 ; $3589\frac{11}{12}$ by 47.

3. Find the product of :—

(1) $99\frac{9}{10}$ separately by 6, 8, 15, 18, 25. (2) $999\frac{17}{18}$ by 99, 550.

(3) $499\frac{324}{325}$ ... ... 25, 50, 75, 100, 150, 200, 250.

(4) $74\frac{86}{87}$ by 43 ; $99\frac{323}{324}$ by 324 ; $999\frac{998}{999}$ by 999.

269. The meaning of Multiplication as given in Art. 59 is not applicable when the multiplier is a fraction. Hence, to suit our purpose we make the following definition.

"To multiply by a fraction is to take that fraction of the multiplicand."

Thus, to multiply $\frac{5}{7}$ by $\frac{3}{8}$, we take $\frac{3}{8}$ of $\frac{5}{7}$ by the new definition.

But $\frac{3}{8}$ of $\frac{5}{7}=\frac{3\times 5}{8\times 7}$ by Art. 249 ; therefore $\frac{5}{7}\times\frac{3}{8}=\frac{3\times 5}{8\times 7}$.

Hence the Rule.

270. *To multiply a fraction by a fraction.*

RULE. Multiply together the respective numerators and denominators, reduced to fractional forms if necessary; and the fraction thence arising will be the product, which may be simplified by striking out any factor common to numerator and denominator.

Ex. 1. Multiply $\frac{2}{9}$ by $\frac{7}{8}$.

Here, $\frac{2}{9}\times\frac{7}{8}=\frac{2\times 7}{9\times 8}=\frac{2\times 7}{9\times 2\times 4}$
$=\frac{7}{9\times 4}=\frac{7}{36}$. *Ans.*

For, if $\frac{2}{9}$ be multiplied by 7, the product will be $\frac{14}{9}$ (Art. 241); but 7 being 8 times as great as $\frac{7}{8}$, the multiplier above used is 8 times too *large*, and the product $\frac{14}{9}$ will therefore be 8 times too *large* also; whence the product required must be $\frac{14}{9}\div 8=\frac{14}{72}$ (Art. 241) $=\frac{7}{36}$.

Ex. 2. Multiply $3\frac{24}{25}$ by $2\frac{37}{54}$; and $5\frac{4}{9}$ by $2\frac{3}{7}$ of $\frac{15}{17}$.

(1) Product $=\frac{99}{25}\times\frac{145}{54}=\frac{99\times 145}{25\times 54}=\frac{11\times 9\times 5\times 29}{5\times 5\times 6\times 9}$

$=\frac{11\times 29}{5\times 6}=\frac{319}{30}=10\frac{19}{30}$. *Ans.*

(2) Product $=\frac{49}{9}\times(\frac{17}{7}\times\frac{15}{17})=\frac{49\times 17\times 15}{9\times 7\times 17}=\frac{49\times 15}{9\times 7}$

$=\frac{7\times 7\times 3\times 5}{3\times 3\times 7}=\frac{7\times 5}{3}=\frac{35}{3}=11\frac{2}{3}$. *Ans.*

271. *To find the continued product of three or more fractions.*

RULE. Multiply all the numerators together for the numerator of the continued product, and all the denominators for its denominator; cancelling all the factors common to numerator and denominator before obtaining the final result.

Ex. 1. Find the continued product of $\frac{3}{4}$, $\frac{5}{7}$ and $\frac{8}{15}$.

Here, Product $=\frac{3\times 5\times 8}{4\times 7\times 15}=\frac{3\times 5\times 2\times 4}{4\times 7\times 3\times 5}=\frac{2}{7}$. *Ans.*

Ex. 2. Multiply $\frac{5}{6}$, $3\frac{2}{11}$, $19\frac{1}{5}$ and $\frac{11}{56}$ together.

Product $=\frac{5}{6}\times\frac{35}{11}\times\frac{96}{5}\times\frac{11}{56}=\frac{5\times(5\times 7)\times(2\times 2\times 2\times 2\times 2\times 3)\times 11}{(3\times 2)\times 11\times 5\times(2\times 2\times 2\times 7)}$

$=\frac{5\times 2}{1}=\frac{10}{1}=10$. *Ans.*

Examples LXXI.

1. Multiply *orally* :—

(1) $\frac{3}{4}$ separately by $\frac{1}{2}$, $\frac{1}{3}$, $\frac{8}{9}$, $\frac{7}{8}$. (2) $\frac{1}{4}$ separately by $\frac{1}{5}$, $\frac{1}{2}$, $\frac{5}{8}$, $\frac{8}{9}$.

(3) $\frac{3}{7}$ ... ... $\frac{4}{7}$, $\frac{5}{8}$, $\frac{2}{3}$, $\frac{1}{6}$. (4) $\frac{4}{11}$ ... ... $\frac{7}{13}$, $\frac{4}{7}$, $\frac{6}{11}$, $\frac{12}{13}$.

2. Multiply :—

(1) $\frac{2}{3}$ by $\frac{3}{7}$; $\frac{3}{8}$ by $\frac{5}{9}$; $\frac{7}{9}$ by $\frac{5}{8}$; $\frac{4}{15}$ by $\frac{5}{8}$; $\frac{6}{25}$ by $\frac{7}{18}$; $\frac{18}{35}$ by $\frac{30}{56}$.
(2) $\frac{14}{27}$ by $\frac{12}{35}$; $\frac{6}{25}$ by $\frac{25}{24}$; $\frac{11}{19}$ by $\frac{15}{48}$; $\frac{26}{65}$ by $\frac{52}{99}$; $\frac{35}{72}$ by $\frac{27}{45}$.
(3) $2\frac{3}{5}$ by $7\frac{3}{9}$; $8\frac{3}{7}$ by $10\frac{5}{11}$; $6\frac{3}{8}$ by $14\frac{6}{17}$; $15\frac{4}{5}$ by $3\frac{3}{8}$; $6\frac{2}{4}$ by $2\frac{1}{37}$.
(4) $2\frac{3}{8}$ by $\frac{1}{3}$ of $\frac{2}{5}$ of $\frac{7}{9}$; $13\frac{5}{9}$ of $7\frac{1}{5}$ by $\frac{3}{4}$ of $\frac{4}{9}$ of $12\frac{1}{2}$; $\frac{8}{25}$ of $4\frac{3}{8}$ by $2\frac{18}{25}$.
(5) $\frac{8}{9}$ of $15\frac{6}{7}$ by $\frac{7}{12}$ of $3\frac{6}{27}$; $3\frac{5}{21}$ of $5\frac{5}{8}$ by $6\frac{2}{3}$ of $10\frac{11}{25}$; $\frac{3}{5}$ of $19\frac{5}{7}$ by $\frac{10}{23}$.

3. Find the values of :—

(1) $\frac{2}{5}\times\frac{3}{8}\times\frac{7}{12}$; $\frac{3}{8}\times\frac{11}{7}\times\frac{16}{11}$; $\frac{3}{4}\times\frac{8}{7}\times\frac{8}{9}$; $\frac{9}{11}\times16\frac{1}{2}\times\frac{5}{81}$; $4\frac{1}{2}\times\frac{1}{3}\times\frac{4}{9}$.
(2) $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}$; $\frac{3}{4}\times\frac{6}{7}\times\frac{8}{9}\times\frac{10}{11}$; $\frac{8}{15}\times2\frac{2}{3}\times1\frac{1}{35}\times1\frac{11}{64}$; $\frac{7}{12}$ of $1\frac{1}{7}\times3\frac{8}{20}\times\frac{13}{84}$.
(3) $\frac{5}{6}\times2\frac{3}{4}\times3\frac{5}{11}\times5\frac{2}{15}\times6\frac{1}{104}$; $12\frac{17}{19}\times8\frac{13}{15}\times\frac{61}{148}\times6\frac{1}{13}$ of $\frac{1}{17}\times2\frac{1}{7}$.
(4) $1\frac{3}{8}$ of $\frac{5}{8}\times\frac{5}{18}$ of $2\frac{1}{29}$ of $8\times12\frac{2}{5}$ of $6\frac{3}{8}$ of $1\frac{1}{17}\times\frac{6}{81}$.
(5) $4\frac{1}{2}$ of $3\frac{1}{8}$ of $7\frac{3}{20}\times\frac{2}{3}$ of $1\frac{1}{7}\times2\frac{2}{5}$ of $4\frac{3}{8}$ of $2\frac{2}{9}\times\frac{1}{8000}$.

4. Find the continued product of :—

(1) $\frac{49}{133}$, $\frac{76}{75}$ and $\frac{28}{98}$. (2) $\frac{428}{515}$, $\frac{5253}{1819}$ and $\frac{615}{492}$.

(3) $\frac{17}{24}$, $\frac{384}{391}$, $\frac{851}{864}$ and $\frac{1584}{1591}$. (4) $\frac{5687}{319}$, $\frac{667}{22011}$, $\frac{221}{629}$ and $\frac{72816}{8528}$

(5) $\frac{324}{361}$, $\frac{1444}{1296}$, $\frac{441}{529}$ and $\frac{2116}{1764}$. (6) $\frac{36}{65}$, $\frac{35}{132}$, $\frac{39}{108}$ and $\frac{75}{144}$.

5. Simplify :—

(1) $(3\frac{5}{8}+2\frac{7}{8})\times10\frac{3}{7}$; $3\frac{5}{8}+2\frac{7}{8}\times10\frac{3}{7}$; $(\frac{1}{2}+\frac{1}{3}+\frac{1}{4})\times(\frac{1}{3}$ of $\frac{2}{7})$.
(2) $\frac{1}{2}\times\frac{2}{3}+\frac{3}{7}\times\frac{3}{8})-(\frac{2}{7}\times\frac{5}{8}-\frac{1}{8}\times\frac{4}{11})$; $\frac{1}{2}$ of $(6\frac{1}{2}+2\frac{2}{3}-3)$; $(\frac{1}{2}+\frac{1}{3})\times(\frac{1}{2}-\frac{1}{3})$.
(3) $(19\frac{1}{5}-3\frac{3}{4})\times(3\frac{1}{5}-2\frac{3}{7})$; $19\frac{1}{5}-3\frac{3}{4}\times3\frac{1}{5}-2\frac{3}{7}$; $19\frac{1}{5}-3\frac{3}{4}\times(3\frac{1}{5}-2\frac{3}{7})$.
(4) $\{(\frac{1}{2}+\frac{1}{3})$ of $(1\frac{1}{3}+2\frac{2}{3})\}\times\{(2\frac{1}{14}-1\frac{1}{2})$ of $(3\frac{1}{10}-\frac{3}{7})\}$.
(5) $\{1\frac{2}{7}$ of $26\frac{1}{3}$ of $(1-\frac{2}{3})\}\times\{2\frac{5}{8}$ of $(4\frac{1}{3}-3\frac{2}{3})$ of $\frac{45}{106}\}$.
(6) $(1\frac{3}{5}$ of $2\frac{7}{9}-3\frac{11}{18})\times(5\frac{1}{3}$ of $4\frac{1}{2}-3\frac{1}{4}$ of $3\frac{1}{2})\times\frac{27}{37}$ of $\frac{3}{17}\times\frac{1}{9}$.

VI. DIVISION OF FRACTIONS.

272. *To divide a fraction by a whole number.*

[We have already given an outline of this method in Arts. 241 and 244. Now, we propose to treat it at length.]

RULE. Multiply the denominator by the whole number, and leave the numerator unaltered. The resulting fraction should always be reduced to its lowest terms by removing all factors common to numerator and denominator.

Thus, $\frac{35}{36}\div28=\frac{35}{36\times28}=\frac{7\times5}{36\times7\times4}=\frac{5}{36\times4}=\frac{5}{144}$.

273. The meaning of Division as given in Art. 87 is not applicable when the divisor is a fraction. Hence, *Division* may be extended to express the finding of the fraction, the product of which and the divisor is the dividend ; and the *quotient* shows what *part* or *parts* the dividend is of the divisor.

Thus, to divide $\frac{3}{8}$ by $\frac{5}{7}$, we have, by definition,

$$\text{quotient} \times \tfrac{5}{7} = \tfrac{3}{8} \;;$$

multiply each term of this equality by $\frac{7}{5}$,

$$\text{therefore quotient} \times \tfrac{5}{7} \times \tfrac{7}{5} = \tfrac{3}{8} \times \tfrac{7}{5}$$

$$\text{or quotient} \qquad = \tfrac{3}{8} \times \tfrac{7}{5},$$

that is, $\frac{3}{8} \div \frac{5}{7} = \frac{3}{8} \times \frac{7}{5}$. Hence the rule.

274. *To divide a fraction by a fraction.*

RULE. Multiply the dividend by the divisor *inverted*, and the result will be the quotient which may be reduced to its lowest terms by cancelling any factors common to numerator and denominator ; or, which is the same thing, *invert* the divisor, and then proceed by the Rule for the Multiplication of Fractions.

Ex. Divide $\frac{3}{7}$ by $\frac{4}{5}$.

Here, $\frac{3}{7} \div \frac{4}{5} = \frac{3}{7} \times \frac{5}{4}$
$= \frac{15}{28}$. *Ans.*

For, if $\frac{3}{7}$ be divided by 4, the quotient is $\frac{3}{28}$ (Art 241) : but this quotient is 5 times too *small*, because the divisor has been taken 5 times too *great* ; whence the quotient will be $\frac{3}{28} \times 5 = \frac{15}{28}$. (Art. 241.)

275. If the dividend be a whole number, or if dividend or divisor or both be mixed numbers, reduce them to improper fractions and compound fractions to simple ones before the application of the Rule.

Ex. Divide $1\frac{1}{14}$ by $5\frac{5}{7}$; and $7\frac{7}{8}$ by $3\frac{3}{14}$ of $2\frac{1}{10}$.

(1) $1\frac{1}{14} \div 5\frac{5}{7} = \frac{15}{14} \div \frac{40}{7} = \frac{15}{14} \times \frac{7}{40} = \frac{3 \times 5 \times 7}{7 \times 2 \times 5 \times 8} = \frac{3}{2 \times 8} = \frac{3}{16}$. *Ans.*

(2) $7\frac{7}{8} \div 3\frac{3}{14} \text{ of } 2\frac{1}{10} = \frac{63}{8} \div \frac{45}{14} \text{ of } \frac{21}{10} = \frac{63}{8} \div \frac{5 \times 9 \times 7 \times 3}{7 \times 2 \times 5 \times 2}$

$$= \frac{63}{8} \div \frac{9 \times 3}{2 \times 2} = \frac{63}{8} \times \frac{2 \times 2}{9 \times 3} = \frac{7 \times 9 \times 2 \times 2}{2 \times 2 \times 2 \times 9 \times 3}$$

$$= \frac{7}{2 \times 3} = \frac{7}{6} = 1\frac{1}{6}. \textit{ Ans.}$$

276. Numbers connected by *of* are considered a **single number**. The student should carefully notice the difference in meaning between $2\frac{1}{3} \div 1\frac{1}{2} \times \frac{3}{4}$ and $2\frac{1}{3} \div 1\frac{1}{2}$ of $\frac{3}{4}$. In the former, the sign ÷ applies only to the next number $1\frac{1}{2}$; but in the latter, $1\frac{1}{2}$ of $\frac{3}{4}$ is a single number.

Thus, the former $= \frac{7}{3} \times \frac{2}{3} \times \frac{3}{4} = \frac{7}{6} = 1\frac{1}{6}$; **the latter** $= \frac{7}{3} \times \frac{2}{3} \times \frac{4}{3} = \frac{56}{27} = 2\frac{2}{27}$.

Examples LXXII.

1. Divide *orally*:—

(1) $\frac{1}{9}$ separately by 2, 3, 4, 5, 6. (2) $\frac{3}{5}$ separately by 4, 5, 7, 10, 12.

(3) $\frac{24}{37}$ ... ... 12, 14, 15, 18, 20. (4) $\frac{15}{28}$ ... ... 3, 5, 30, 45.

2. Divide :—

(1) $\frac{40}{67}$ separately by 8, 16, 24, 36. (2) $\frac{145}{349}$ separately by 25, 75, 87.

(3) $2\frac{1}{6}$ ... ... 13, 65, 117. (4) $16\frac{2}{7}$ ... ... 19, 25, 32.

3. Divide *orally* :—

(1) $\frac{1}{2}$ separately by $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, $\frac{1}{9}$. (2) $\frac{8}{5}$ separately by $\frac{8}{7}$, $\frac{8}{9}$, $\frac{4}{7}$, $\frac{1}{8}$.

(3) $\frac{7}{9}$ ... ... $\frac{6}{4}$, $\frac{2}{3}$, $\frac{4}{5}$, $\frac{1}{8}$. (4) $\frac{1}{2}$ ... ... $\frac{2}{3}$, $\frac{3}{8}$, $\frac{4}{7}$, $\frac{5}{9}$.

4. Divide :—

(1) $\frac{7}{8}$ separately by $\frac{4}{7}$, $\frac{3}{8}$, $\frac{5}{16}$, $\frac{14}{15}$. (2) $1\frac{1}{4}$ separately by $\frac{5}{8}$, $\frac{5}{7}$, $\frac{12}{13}$, $\frac{3}{14}$.

(3) $3\frac{3}{5}$ ... ... $5\frac{1}{2}$, $2\frac{1}{3}$, $4\frac{1}{5}$, $6\frac{2}{3}$. (4) $4\frac{8}{19}$ ... ... $1\frac{7}{38}$, $5\frac{2}{5}$, $4\frac{1}{2}$, $1\frac{8}{15}$.

Examples LXXIII.

1. Divide :—

(1) $\frac{2}{7}$ by $\frac{3}{8}$; $\frac{3}{5}$ by $\frac{4}{9}$; $\frac{4}{11}$ by $\frac{6}{14}$; [illegible] by $\frac{88}{15}$; $\frac{26}{5}$ by $\frac{91}{34}$; $\frac{11}{12}$ by $\frac{47}{56}$; 143 by $\frac{11}{12}$

(2) $2\frac{1}{8}$ by $3\frac{7}{9}$; $10\frac{4}{5}$ by $13\frac{3}{7}$; $17\frac{2}{3}$ by $7\frac{13}{14}$; $24\frac{3}{4}$ by $5\frac{5}{18}$; $2\frac{7}{8}$ by $\frac{5}{8}$ of $4\frac{1}{3}$

(3) $3\frac{4}{5}$ by $\frac{1}{2}$ of $\frac{4}{7}$ of $\frac{5}{6}$; $3\frac{1}{25}$ of $3\frac{4}{7}$ by $\frac{47}{81}$ of 9; $15\frac{7}{11}$ of $8\frac{4}{5}$ by $\frac{4}{5}$ of $\frac{6}{11}$ of $15\frac{5}{8}$

(4) $\frac{4}{5}$ of $1\frac{1}{7} \times 2\frac{2}{5}$ of $4\frac{3}{8}$ of $2\frac{2}{9}$ by $1\frac{3}{5}$ of $\frac{5}{8} \times \frac{5}{18}$ of $2\frac{1}{28}$ of 8.

2. Find the values of :—

(1) $(\frac{5}{7} \times \frac{8}{9} \times 13\frac{1}{2}) \div (\frac{1}{9} \times \frac{3}{7} + 54)$. (2) $(\frac{2}{7} + \frac{3}{4}$ of $\frac{128}{135}) \div (\frac{4}{11}$ of $2\frac{5}{12} - \frac{14}{33})$.

(3) $6\frac{7}{8} + (1\frac{5}{6} + 3\frac{8}{15}) + 6\frac{3}{7}$. (4) $6\frac{7}{8} + 1\frac{5}{6} + 3\frac{8}{15} + 6\frac{3}{7}$.

(5) $10\frac{2}{3} \times 3\frac{3}{4} \div 4\frac{1}{6} - (2\frac{2}{3} + 3\frac{3}{4}) + 4\frac{2}{5} + 7\frac{1}{8} \div (3\frac{3}{4} + 4\frac{1}{8})$.

(6) $2\frac{1}{4} \times 1\frac{1}{2} + 1\frac{1}{8}$ of $2\frac{2}{5}$. (7) $2\frac{1}{4}$ of $1\frac{1}{2} + 1\frac{1}{8} \times 2\frac{2}{5}$. (8) $2\frac{1}{4}$ of $1\frac{1}{2} + 1\frac{1}{8}$ of $2\frac{2}{5}$.

(9) $\frac{2247}{1017} + \frac{908}{1107} \times \frac{774}{615} + \frac{1026}{562}$. (10) $1 \div [4 - 1 \div \{2 - 1 \div (1 - \frac{5}{18})\}]$.

(11) $(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \frac{1}{8} + \frac{1}{10}) \div (\frac{1}{3} + \frac{1}{5} + \frac{1}{9} + \frac{1}{15})$.

(12) $(2 + \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \frac{1}{15} + \frac{1}{18}) \div (1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5})$

VII. COMPLEX AND CONTINUED FRACTIONS.

277. A fraction having a fraction or mixed number in the numerator or denominator or in both, is a **Complex Fraction.**

Thus, $\frac{\frac{3}{5}}{\frac{4}{7}}$, $\frac{8\frac{4}{5}}{9}$, $\frac{7}{9\frac{5}{8}}$, $\frac{4\frac{2}{5}}{5\frac{1}{3}}$, $\frac{1\frac{3}{5}+\frac{2}{3}}{4\frac{1}{2} \text{ of } 3\frac{1}{3}}$ are *Complex Fractions.*

278. A complex fraction is read by inserting the word *by* for *divided by*, between the readings of numerator and denominator.

Thus, $\frac{3\frac{1}{2}}{4\frac{1}{3}}$ is read $3\frac{1}{2}$ *by* $4\frac{1}{3}$.

279. In the *sum* of a whole number and a fraction, when the fraction is either complex or simple (Art. 231), the sign is sometimes omitted, as in $5\frac{4\frac{2}{3}}{7}$ which means $5+\frac{4\frac{2}{3}}{7}$; and in a *product* when one of the factors is enclosed in a bracket the sign is often omitted, as in $\frac{2}{3}(\frac{5}{6}-\frac{3}{4})$, which means $\frac{2}{3}\times(\frac{5}{6}-\frac{3}{4})$.

280. Complex fractions are subject to the same rules as simple fractions, and can always be reduced to simple ones by treating them as the *quotient* of the numerator by the denominator. (Art. 235.)

Thus, $\frac{2\frac{1}{5}}{3\frac{2}{9}}=2\frac{1}{5}\div3\frac{2}{9}=\frac{11}{5}\div\frac{29}{9}=\frac{11}{5}\times\frac{9}{29}=\frac{99}{145}$.

281. *To reduce a complex fraction to a simple fraction.*

RULE. Express the numerator and denominator of the complex fraction in the form of proper or improper fractions, and multiply the numerator by the denominator *inverted*; or more simply, multiply the numerator and denominator of the complex fraction by the L. C. M. of the denominators of the simple fractions.

Ex. 1. Reduce $\frac{5\frac{7}{9}}{9\frac{5}{11}}$ and $\frac{13\frac{1}{3}}{20}$ to simple fractions.

(1) $\frac{5\frac{7}{9}}{9\frac{5}{11}}=5\frac{7}{9}\div9\frac{5}{11}=\frac{52}{9}\div\frac{104}{11}=\frac{52}{9}\times\frac{11}{104}=\frac{52\times11}{9\times2\times52}=\frac{11}{18}$. *Ans.*

Or thus, $\frac{5\frac{7}{9}}{9\frac{5}{11}}=\frac{5\frac{7}{9}\times99}{9\frac{5}{11}\times99}=\frac{495+77}{891+45}=\frac{572}{936}=\frac{11\times52}{18\times52}=\frac{11}{18}$. *Ans.*

(2) $\frac{13\frac{1}{3}}{20}=13\frac{1}{3}\div20=\frac{40}{3}\div20=\frac{40}{3}\times\frac{1}{20}=\frac{20\times2}{3\times20}=\frac{2}{3}$. *Ans.*

Or thus, $\frac{13\frac{1}{3}}{20}=\frac{13\frac{1}{3}\times3}{20\times3}=\frac{40}{20\times3}=\frac{20\times2}{20\times3}=\frac{2}{3}$. *Ans.*

Ex. 2. Simplify $\frac{12\frac{2}{3}\text{ of }1\frac{8}{19}}{1\frac{5}{9}\text{ of }3\frac{3}{7}}$ and $\frac{8\frac{5}{6}-4\frac{2}{3}}{3\frac{3}{4}+7\frac{5}{12}}$.

(1) $\frac{12\frac{2}{3}\text{ of }1\frac{8}{19}}{1\frac{5}{9}\text{ of }3\frac{3}{7}}=12\frac{2}{3}\text{ of }1\frac{8}{19}\div1\frac{5}{9}\text{ of }3\frac{3}{7}=\frac{38}{3}\text{ of }\frac{27}{19}\div\frac{14}{9}\text{ of }\frac{24}{7}$

$=\frac{19\times2\times3\times9}{3\times19}\div\frac{7\times2\times8\times3}{3\times3\times7}=2\times9\div\frac{2\times8}{3}$

$=\frac{2\times9\times3}{2\times8}=\frac{27}{8}=3\frac{3}{8}$. *Ans.*

(2) $\frac{8\frac{5}{6}-4\frac{2}{3}}{3\frac{3}{4}+7\frac{5}{12}}=\frac{(96+10)-(48+8)}{(36+9)-(84+5)}$ { Multiplying Numr. and Denr. by 12, the L.C.M. of the Denrs.

$=\frac{106-56}{45+89}=\frac{50}{134}=\frac{2\times25}{2\times67}=\frac{25}{67}$. *Ans.*

Examples LXXIV.

1. Reduce to their simplest forms :—

(1) $\frac{4\frac{2}{3}}{5\frac{3}{5}}$; $\frac{8\frac{3}{11}}{14\frac{6}{7}}$; $\frac{3\frac{3}{8}}{6}$; $\frac{18}{5\frac{1}{7}}$; $\frac{9\frac{7}{9}}{12\frac{6}{9}}$; $\frac{25\frac{7}{8}}{34\frac{11}{16}}$; $\frac{\frac{8}{27}}{\frac{16}{32}}$; $\frac{\frac{7}{18}}{4\frac{1}{36}}$; $\frac{2\frac{1}{3}}{3\frac{1}{3}+2\frac{1}{3}}$.

(2) $\frac{1\frac{2}{3}\text{ of }1\frac{1}{7}}{1\frac{3}{5}\text{ of }\frac{6}{11}}$; $\frac{2\frac{1}{5}\text{ of }8\frac{1}{3}}{2\frac{7}{9}\text{ of }11\frac{2}{3}}$; $\frac{3\frac{1}{3}\text{ of }\frac{9}{14}}{3\frac{1}{3}+\frac{9}{14}}$; $\frac{3\frac{1}{7}-2\frac{2}{3}}{8\frac{3}{5}\text{ of }17\frac{1}{2}}$; $\frac{5\frac{1}{7}+3\frac{1}{3}}{7\frac{1}{12}-1\frac{1}{4}\text{ of }1\frac{8}{21}}$.

(3) $\frac{7\frac{2}{3}}{9\frac{7}{8}}$ of $\frac{14\frac{4}{11}}{13\frac{2}{7}}$; $\frac{6\frac{4}{9}}{13\frac{3}{12}}$ of $\frac{4\frac{5}{7}}{23\frac{1}{3}}$; $5\frac{1}{7}$ of $\frac{6\frac{2}{5}}{10\frac{6}{7}}$ of $\frac{15}{12\frac{4}{7}}$ of $\frac{8\frac{1}{3}}{42}$; $\frac{5}{6}$ of $\frac{13\frac{2}{5}}{4\frac{7}{9}}$.

(4) $\frac{2\frac{4}{11}}{2\frac{3}{8}}\div\frac{2\frac{7}{11}}{8\frac{7}{10}}$; $\frac{7\frac{1}{7}-3\frac{1}{4}}{6+4\frac{1}{4}}\div\frac{5\frac{1}{3}+1\frac{1}{7}}{6\frac{1}{7}-2\frac{1}{3}}$; $\frac{5\frac{1}{3}+4\frac{2}{4}}{3\frac{5}{8}+2\frac{7}{8}}\times\frac{5\frac{1}{3}-4\frac{2}{4}}{3\frac{5}{8}-2\frac{7}{8}}\div\frac{28\frac{4}{9}-22\frac{9}{18}}{14\frac{8}{9}-8\frac{1}{4}}$

(5) $\frac{5\frac{5}{7}+7\frac{2}{3}}{2\frac{3}{8}-1\frac{4}{7}}$ of $\frac{2\frac{1}{4}\times8\frac{1}{4}}{4\frac{1}{3}+(\frac{1}{2}-\frac{1}{3})}$; $\frac{5\frac{4}{5}-2\frac{1}{5}}{3\frac{3}{4}+\frac{9}{10}}$ of $\frac{4\frac{1}{3}+5\frac{19}{33}}{4\frac{1}{20}}$; $\frac{3\frac{7}{11}-2\frac{5}{8}}{3\frac{7}{11}\times2\frac{5}{8}}\times\frac{3\frac{7}{11}+2\frac{5}{8}}{3\frac{7}{11}\text{ of }2\frac{5}{8}}$.

2. Reduce $\frac{2\frac{1}{3}}{7}$, $8\frac{1}{4}$, $\frac{9+\frac{1}{11}}{9\times\frac{1}{11}}$ and $16\frac{17}{18}$ to equivalent fractions with the least common denominator, also reduce $\frac{3}{4}$ to a complex fraction having the denominator 5 and $\frac{7}{8}$ to a complex fraction having the numerator 10.

3. Compare the quantities $2\frac{1}{3}$, $\frac{4}{9}$ of $9\frac{2}{3}$ and $\frac{7\frac{1}{4}}{2\frac{5}{6}}$.

4. Find the values of :—

(1) $\frac{2}{3}$ of $3\frac{3}{10}+\frac{1\frac{2}{3}}{2\frac{5}{8}}$ of $17+\frac{2}{3}$ of $5\frac{3}{4}$ of $\frac{2}{3}\frac{4}{1}$. (2) $\frac{2}{3}$ of $\frac{5}{7}+9+\frac{2\frac{4}{5}}{7}+\frac{1\frac{2}{3}}{2\frac{1}{3}}$.

(3) $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{8}+\frac{1}{3}$ of $\frac{2}{3}$ of $1\frac{7}{8}+\frac{3}{10}$ of $\frac{1}{7}$ of $\frac{28\frac{1}{3}}{2}$. (4) $1\frac{1}{3}+\frac{8}{9}$ of $\frac{4}{5}\frac{1}{4}+\frac{4}{5\frac{1}{10}}$.

5. Find the difference between :—

(1) $\frac{2}{3}$ of $\frac{4\frac{1}{4}}{5\frac{1}{7}}$ and $\frac{2}{3}$ of $7\frac{1}{3}$. (2) $\frac{3\frac{2}{7}}{4\frac{7}{9}}$ and $\frac{6\frac{2}{5}}{12\frac{2}{7}}$. (3) $2\frac{2}{3}$ of $\frac{5\frac{1}{4}}{4\frac{1}{3}}$ and $\frac{7\frac{1}{3}}{11}$ of $15\frac{2}{3}$.

6. Find the values of :—

(1) $2\frac{2}{3}$ of $\frac{6\frac{1}{4}}{1\frac{2}{4}}\times\frac{3\frac{2}{5}}{5\frac{1}{7}}$. (2) $6\frac{2}{3}$ of $9\frac{5}{8}\times12\frac{8}{11}$ of $\frac{1\frac{1}{3}}{112}$. (3) $\frac{7\frac{2}{7}}{40\frac{2}{3}}+\frac{17\frac{1}{5}}{73}$.

(4) $\frac{2}{3\frac{8}{15}}$ of $\frac{6\frac{7}{12}}{8}\times\frac{2}{3}$ of $8\frac{1}{4}$ of $\frac{8}{8}\frac{8}{6}$. (5) $\frac{2\frac{2}{4}}{5\frac{5}{8}}$ of $\frac{1}{3}\times\frac{1}{2}$ of $\frac{4\frac{2}{3}}{7\frac{1}{3}}\times\frac{7\frac{2}{7}}{5\frac{1}{3}}$.

(6) $2\frac{19}{21}$ of $5\frac{1}{17}$ of $1331+3\frac{1}{13}$ of $\frac{44}{13\frac{2}{7}}$ of $202\frac{1}{3}$.

(7) $\frac{11\frac{2}{3}}{29}$ of $\frac{4\frac{1}{3}}{13}$ of $\frac{3\frac{1}{4}}{10\frac{2}{3}}\times5\frac{1}{3}$ of $6\times20\frac{2}{3}$. (8) $8\frac{1}{3}$ of $7\frac{1}{2}+3\frac{1}{3}$ of $\frac{4\frac{1}{3}}{14\frac{1}{3}}-8\frac{1}{4}$.

(9) $\dfrac{\frac{2}{11}+\frac{1}{37}+\frac{8}{77}-\frac{1}{8}}{\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}}$ of $\dfrac{\frac{4}{1\frac{1}{7}} \text{ of } \frac{3}{1\frac{1}{9}} - \frac{4}{3\frac{1}{9}} \text{ of } \frac{2\frac{1}{2}}{1\frac{1}{3}}}{\frac{8}{505} \text{ of } \frac{11}{7} \text{ of } 3\frac{1}{2}+2\frac{1}{55} \text{ of } 1\frac{5}{6} \text{ of } \frac{1}{19}}$.

282. *To find the complete quotient in dividing a number by an integer.*

RULE. Divide in the usual way, and to the integral quotient add the fraction whose numerator is the remainder and denominator the divisor.

Ex. Divide 4148 by 117, and $3136\frac{7}{12}$ by 95, giving the complete quotient in each case.

(1) 117)4148(35
351
638
585
53

∴ the complete quotient $= 35\frac{53}{117}$.

(2) 95)$3136\frac{7}{12}$(33
285
286
285
1

In dividing by 95, the integral remainder is 1 and the full remainder is $1\frac{7}{12}$.

But $1\frac{7}{12} \div 95 = \frac{19}{12} \times \frac{1}{95} = \frac{1}{60}$.

∴ the complete quotient $= 33\frac{1}{60}$.

283. Fractions of the nature given below are **Continued Fractions**, and can be simplified by beginning at the bottom and working upwards.

Ex. $\dfrac{1}{4-\dfrac{1}{2-\dfrac{1}{1-\frac{5}{13}}}} = \dfrac{1}{4-\dfrac{1}{2-\dfrac{13}{13-5}}} = \dfrac{1}{4-\dfrac{1}{2-\dfrac{13}{8}}} = \dfrac{1}{4-\dfrac{8}{16-13}}$

$= \dfrac{1}{4-\frac{8}{3}} = \dfrac{3}{12-8} = \dfrac{3}{4}$. *Ans.*

Examples LXXV.

1. Find the complete quotient in dividing :—

(1) 3127 by 43. (2) 6556 by 401. (3) 2221 by 87.
(4) 8768 by 45. (5) $845\frac{14}{15}$ by 12. (6) $6739\frac{13}{18}$ by 37 and by 73.
(7) $4164\frac{7}{8}$ by 11, and by 132. (8) $56941\frac{8}{81}$ by 27.

2. Simplify :—

(1) $\dfrac{3}{7+\dfrac{2}{5+\frac{7}{8}}}$. (2) $\frac{1}{18}$ of $\dfrac{1}{1+\dfrac{\frac{1}{3}}{3+\frac{1}{4}}}$. (3) $\dfrac{2}{5+\dfrac{3}{4+\frac{1}{3}}}$. (4) $2+\dfrac{1}{3+\dfrac{4}{5+\frac{1}{6}}}$

(5) $3+\cfrac{1}{2+\cfrac{3}{5+\frac{1}{2}}}$.

(6) $2+\cfrac{3}{1+\cfrac{3}{6+\cfrac{4}{5+\frac{1}{3}}}}$.

(7) $6\frac{7}{8}+\cfrac{3\frac{2}{7}}{\cfrac{2}{3\frac{1}{2}+\cfrac{3}{4\frac{4}{5}}}}$.

(8) $2\frac{3}{4}+\cfrac{4}{5+\cfrac{6}{7+\frac{8}{9}}}$.

(9) $\cfrac{3}{4+\cfrac{3}{5+\cfrac{3}{6+\frac{2}{7}}}}+\cfrac{1}{2+\cfrac{1}{3+\cfrac{1}{4+\frac{1}{5}}}}$.

(10) $2\frac{1}{2}\times\cfrac{1}{3\frac{1}{3}+4\frac{1}{4}}+\cfrac{\frac{6}{7}-\frac{1}{3}\times(\frac{1}{2}+\frac{1}{7})}{1+\cfrac{1}{2-\frac{1}{4}}}$.

(11) $1+\cfrac{2}{3+\cfrac{1}{4+\cfrac{1}{5\frac{2}{3}}}}+\frac{18}{23}$.

(12) $\left(3-\cfrac{1}{2-\cfrac{1}{6+\frac{1}{5}}}\right)+\left(1+\cfrac{1}{4+\cfrac{1}{3-\cfrac{1}{3+\frac{1}{2}}}}\right)$.

(13) $3\frac{4}{5}+\cfrac{5\frac{5}{6}}{7\frac{7}{8}+\cfrac{8\frac{9}{10}}{10\frac{11}{12}+\cfrac{13}{9\frac{3}{4}}}}$.

(14) $\cfrac{1}{3+\cfrac{1}{5+\frac{1}{7}}}+\cfrac{3}{8-\cfrac{7}{2-\frac{3}{4}}}+\cfrac{5}{6-\cfrac{5}{2-\frac{5}{6}}}$.

(15) $\cfrac{\cfrac{1}{1-\cfrac{1}{2-\frac{1}{2}}}+\frac{1}{3}+(1+\frac{1}{2})}{3\left(1+\cfrac{2}{3\frac{1}{2}}\right)-4}$.

(16) $1\frac{1}{4}\times 12\frac{1}{3}$ of $\frac{4}{5}-\cfrac{6\frac{1}{4}+7\frac{1}{15}}{12-6\frac{2}{3}}-\cfrac{4+\cfrac{1}{4+\cfrac{1}{4-\frac{1}{4}}}}{4}$.

(17) $\cfrac{2}{4+\cfrac{2}{3+\frac{1}{4}}}+\cfrac{3}{3-\cfrac{2}{3-\frac{1}{4}}}+\cfrac{4}{4+\cfrac{2}{4\div\frac{1}{4}}}+\cfrac{5}{3+\cfrac{\frac{1}{7}}{2+\frac{1}{3}}}$.

(18) $\cfrac{4\frac{2}{7}}{4\frac{2}{7}+\cfrac{4\frac{2}{7}}{4\frac{2}{5}+\cfrac{5}{4\frac{2}{5}+2\frac{1}{5}}}}$.

(19) $\cfrac{2}{2+\cfrac{2}{2+\cfrac{1}{2+\cfrac{2}{5+\frac{1}{4}}}}}+\cfrac{2}{5+\cfrac{1}{2+\cfrac{1}{3+\frac{1}{3}}}}+\cfrac{1\frac{1}{3}\times\frac{2\frac{1}{3}}{3\frac{3}{4}}\left(3-\frac{1}{4}\right)+1\frac{1}{2}}{2+\cfrac{1}{3+\cfrac{1}{1+\cfrac{4}{5+\frac{1}{3}}}}}$.

(20) $1-\cfrac{1}{2+\cfrac{1}{1-\cfrac{1}{2+\cfrac{1}{1-\cfrac{1}{\frac{4}{5}+\frac{5}{8}}}}}}\times 1-\cfrac{1}{2+\cfrac{1}{2\times\cfrac{1}{1-\cfrac{1}{1\frac{2}{3}}}}}\times 2+\cfrac{1}{1-\cfrac{1}{1+\cfrac{1}{1-\cfrac{1}{1+\frac{1}{4}}}}}$.

3. Simplify :—

(1) $15\frac{1}{4}+1\frac{2}{3}\div(7\frac{3}{4}-6\frac{2}{3})$. (2) $(15\frac{1}{4}+1\frac{2}{3})\div\frac{2}{13}(7\frac{3}{4}-6\frac{2}{3})$.

(3) $16-\{2\frac{1}{2}-(4\frac{1}{3}+1\frac{1}{4})\}$. (4) $16+\{2\frac{1}{2}-(4\frac{1}{2}-1\frac{1}{4})\}$.

(5) $5\frac{2}{3}-\{5\frac{3}{4}-\frac{2}{3}(3\frac{2}{3}+2\frac{1}{3})\}$. (6) $3\frac{2}{3}\times\frac{10}{11}\div3\frac{1}{7}$ of $3\frac{2}{7}$ of $\frac{5}{24}$ of $1\frac{13}{33}$.

(7) $(\frac{1}{2}+\frac{1}{3})$ of $(1\frac{1}{3}+2\frac{3}{4})$ of $(2\frac{1}{14}-1\frac{1}{2})$ of $(3\frac{1}{10}-\frac{6}{7})$.

(8) $(3\frac{7}{11}\div4\frac{2}{7})$ of $(10\frac{5}{7}+7\frac{1}{2})$ of $\frac{77}{840}$. (9) $3\frac{7}{11}\div(4\frac{2}{7}+10\frac{5}{7})+(7\frac{1}{2}+\frac{77}{840})$.

(10) $1\frac{1}{6}$ of $5\frac{1}{4}+(4\frac{1}{2}-\frac{3}{10})\div1\frac{1}{18}-6\frac{3}{4}\div\frac{1}{2}$ of $2\frac{2}{3}$.

4. Shew that the simple fraction equivalent to the value of $\frac{1}{3}$ of $\frac{\frac{2}{3}}{\frac{5}{4}}+\frac{1}{2}$ of $\frac{\frac{3}{4}}{\frac{9}{16}+1}+\frac{1}{6}$ of $\frac{\frac{1}{5}}{4}$, is of the same magnitude as that expressed by $\frac{3}{10}+\frac{1}{2}$ of $\frac{7}{18}$ of $\frac{7}{20}$.

5. Prove that $\frac{1}{3}$ of $(1-\frac{2}{7})+\frac{4}{5}$ of $\frac{1}{10}+\frac{2}{5}$ of $(\frac{1}{2}+1\frac{1}{4})+\frac{9}{70}$ of $(\frac{2}{7}+\frac{4}{5})=1$.

VIII. SIMPLIFICATION OF FRACTIONS.

284. What has been proved in the adaptation of the fundamental operations to fractions, will furnish the means of simplifying arithmetical expressions formed by their combinations; and, in general, only very slight *mental* exertion will be required, if the attention of the *eye* be directed to the *composition* of the *terms* of the fractions concerned, and their *resolution* into the *factors* of which they are made up.

Ex. 1. Simplify $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{2\frac{1}{2}}+\frac{1}{3\frac{1}{2}}+\frac{1}{4\frac{1}{2}}}-\frac{13}{24}$ of $\frac{576}{264}$.

$$\text{The expression}=\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{2}{5}+\frac{2}{7}+\frac{2}{9}}-\frac{13}{24}\text{ of }\frac{24\times24}{264}=\frac{\frac{6+4+3}{12}}{\frac{126+90+70}{315}}-\frac{13\times24}{264}$$

$$=\frac{\frac{13}{12}}{\frac{286}{315}}-\frac{13\times24}{11\times24}=\frac{13}{12}\times\frac{315}{286}-\frac{13}{11}$$

$$=\frac{13}{4\times3}\times\frac{105\times3}{13\times22}-\frac{13}{11}=\frac{105}{88}-\frac{13}{11}=\frac{105-104}{88}=\frac{1}{88}.\quad\textit{Ans.}$$

Ex. 2. Simplify $\left\{2\frac{3}{4}+\frac{5}{2}\text{ of }\frac{7}{3\frac{4}{5}}-\frac{1\frac{2}{3}}{2\frac{1}{2}}\right\}\div1\frac{77}{228}$.

$$\text{The expression}=\left\{\frac{11}{4}+\frac{5}{2}\text{ of }\frac{7\times5}{19}-\frac{\frac{5}{3}}{\frac{5}{2}}\right\}\div\frac{305}{228}$$

$$=\left\{\frac{11}{4}+\frac{5\times7\times5}{2\times19}-\frac{5}{3}\times\frac{2}{5}\right\}\times\frac{228}{305}$$

$$=\left\{\frac{11}{4}+\frac{175}{38}-\frac{2}{3}\right\}\times\frac{228}{305}=\frac{627+1050-152}{228}\times\frac{228}{305}$$

$$=\frac{1677-152}{228}\times\frac{228}{305}=\frac{1525}{305}=\underline{5}.\quad \textit{Ans.}$$

Examples LXXVI.

Simplify the following :—

1. $\frac{3}{8}$ of $\frac{4}{7}-\frac{3}{11}$ of $3\frac{1}{7}+\frac{5}{9}$ of $3\frac{3}{8}$. 2. $(\frac{2}{7}+\frac{3}{4}$ of $\frac{128}{133})+2\frac{4}{11}$ of $(1\frac{5}{14}-\frac{14}{33})$.

3. $\dfrac{4\frac{1}{3}\times4\frac{1}{3}\times4\frac{1}{3}-1}{4\frac{1}{3}\times4\frac{1}{3}-1}$. 4. $\dfrac{4\frac{2}{3}\times4\frac{2}{3}-3\frac{1}{4}\times3\frac{1}{4}}{4\frac{2}{3}-3\frac{1}{4}}$.

5. $\dfrac{1+6\frac{7}{8}\times(1+6\frac{7}{8})}{1+5\frac{4}{5}\times(1+5\frac{4}{5})}$. 6. $\dfrac{7\frac{1}{2}}{6\frac{1}{2}}+\dfrac{11\frac{1}{2}-2\frac{2}{5}}{11\frac{1}{2}+2\frac{2}{5}}\times10\frac{9}{13}-6\frac{43}{273}$.

7. $\dfrac{14\frac{1}{4}-6\frac{1}{2}}{3\frac{1}{8}+6\frac{1}{18}}\div\dfrac{4\frac{1}{12}+6\frac{1}{3}}{9\frac{1}{3}-3\frac{1}{4}}+(30\frac{7}{18}-22\frac{5}{9})$. 8. $\dfrac{1+2\times\frac{4}{5}+\frac{4}{5}\times\frac{4}{5}}{1-\frac{4}{5}\times\frac{4}{5}}$.

9. $\dfrac{1}{3\frac{1}{3}}-\dfrac{2\frac{1}{4}}{9}+\dfrac{3\frac{5}{8}}{2}+\dfrac{\frac{1}{7}}{4\frac{4}{7}}$. 10. $(\frac{1}{26}$ of $3\frac{1}{4})+(\frac{3}{8}+\frac{27}{68})-\left(\dfrac{1}{1\frac{1}{7}}-\dfrac{7\frac{1}{8}}{9}\right)$.

11. $\dfrac{\dfrac{3\frac{1}{3}}{7\frac{1}{3}}-\dfrac{1}{2\frac{2}{7}}}{4\frac{1}{3}\text{ of }\dfrac{2\frac{2}{3}}{6\frac{2}{3}}}$ of $\dfrac{11\frac{1}{3}}{17}\times52\frac{4}{11}$. 12. $\dfrac{3+\frac{5}{3}\text{ of }\dfrac{21}{7\frac{7}{8}}-\dfrac{1}{4}-\dfrac{1\frac{2}{3}}{2\frac{1}{3}}}{10-\dfrac{151}{228}\text{ of }5}$.

13. $\dfrac{1\frac{2}{3}}{3+\dfrac{1}{3\frac{1}{3}}}+\dfrac{1\frac{2}{3}\text{ of }4\frac{2}{7}}{1\frac{2}{3}\text{ of }3\frac{6}{7}}+\dfrac{5\frac{2}{3}\text{ of }7\frac{2}{3}}{8\frac{7}{24}-3\frac{5}{12}}$. 14. $\dfrac{2\frac{1}{2}}{3\frac{1}{4}}+\dfrac{1\frac{1}{2}-\frac{5}{6}}{1\frac{1}{2}+\frac{5}{6}}-1\frac{3}{35}$.

15. $\dfrac{2\frac{1}{3}-\frac{5}{6}}{2\frac{1}{3}+\frac{5}{6}}+\dfrac{7}{12}$ of $\dfrac{9\times10}{14\times3}-\dfrac{22\frac{1}{2}}{30}$. 16. $\dfrac{1\frac{2}{3}}{3\frac{1}{2}}-\dfrac{5\frac{5}{8}}{6\frac{1}{4}}$ of $\left(\dfrac{1}{5}-\dfrac{\frac{1}{2}-\frac{1}{3}}{4\frac{3}{4}-3\frac{2}{3}}\right)$.

17. $\dfrac{1}{26}\left(5\dfrac{1}{2}-2\dfrac{1}{4}\right)+\left(5\dfrac{1}{3}\text{ of }\dfrac{9}{128}+\dfrac{9}{8}\text{ of }\dfrac{3}{8}\right)-\left(\dfrac{1}{1\frac{1}{7}}-\dfrac{1\frac{1}{3}}{3}\right)\div\dfrac{8}{7}\left(2-\dfrac{4}{9}\right)$.

18. $\dfrac{\frac{3}{7}-\frac{2}{5}}{\frac{3}{7}+\frac{2}{5}}$ of $2\dfrac{11}{26}\div\dfrac{4}{13-3\frac{5}{8}}+3\dfrac{11}{16}-\dfrac{3}{3-1\frac{10}{13}}$.

19. $\dfrac{1+2\frac{1}{3}+3\frac{1}{3}}{\dfrac{1}{1\frac{1}{2}}+\dfrac{2}{2\frac{1}{3}}+\dfrac{3}{3\frac{1}{4}}}\times\dfrac{55\frac{2}{3}\div11}{1\dfrac{2}{11}\text{ of }13\dfrac{2}{3}}$. 20. $\dfrac{1-\frac{4}{5}}{3\frac{1}{4}+1\frac{1}{2}+\dfrac{1\frac{1}{8}}{6\frac{2}{3}}}\times\left(\dfrac{1+\frac{1}{5}}{1-\frac{1}{5}}\right)^2$.

21. $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\dfrac{1}{2\frac{1}{2}}+\dfrac{1}{3\frac{1}{3}}+\dfrac{1}{4\frac{1}{2}}}$ of $\dfrac{1}{13}$ of $\dfrac{1}{1+\dfrac{\frac{1}{3}}{3+\frac{1}{4}}}$.

22 $\left\{\dfrac{11\frac{3}{4}-10\frac{1}{3}}{11\frac{3}{4}+10\frac{1}{3}}\div\dfrac{10\frac{3}{8}+11\frac{1}{6}}{10\frac{3}{8}-9\frac{1}{6}}\right\}\times\dfrac{\frac{4}{7}+\frac{3}{11}}{\frac{4}{7}-\frac{3}{11}}\times\dfrac{8\frac{1}{11}}{2\frac{3}{11}}$.

23. $\frac{2\frac{1}{4}}{2\frac{2}{5}} + \frac{2\frac{1}{2}+5\frac{1}{3}}{3\frac{1}{3}+9\frac{1}{2}} + \frac{5}{9} \text{ of } \frac{9}{10} + \frac{3}{8} \text{ of } \frac{3}{20}$.

24. $\left(6\frac{3}{7} \text{ of } \frac{5\frac{1}{3}-4\frac{7}{12}}{12\frac{2}{3}-7\frac{5}{12}}\right) + \frac{1+\frac{1}{2\frac{1}{3}}}{2}$. / **25.** $\frac{4\frac{1}{5}}{\frac{7}{10} \text{ of } 3\frac{1}{3}} - \left(\frac{2}{7} \text{ of } \frac{2\frac{2}{3}+1\frac{1}{6}}{2-\frac{4}{25}}\right)$.

26. $\frac{1}{3} \times \frac{17}{3} \text{ of } \frac{27}{85} + \left(2\frac{1}{3} + \frac{1}{3+\frac{1}{4}}\right) \times \frac{39}{103}$.

27. $1\frac{1}{11} - \frac{1-\frac{7}{22}}{2-\frac{1}{3}} + \frac{1\frac{2}{5}}{3\frac{1}{2}} - \frac{5\frac{5}{8}}{6\frac{1}{4}} \text{ of } \frac{2}{3}\left\{\frac{1}{3} - \frac{\frac{1}{2}-\frac{1}{3}}{4\frac{3}{4}-3\frac{2}{9}}\right\}$.

28. $\left\{\frac{7}{5-\frac{3}{8}} + \frac{3-\frac{2}{3-1\frac{1}{2}}}{4-1\frac{1}{2}}\right\} - \frac{5}{7} \text{ of } \frac{3}{4}\left\{\frac{1}{1\frac{3}{7}} + \frac{6}{5} \text{ of } \frac{3\frac{1}{3}-2\frac{1}{2}}{\frac{47}{21}-2}\right\}$.

29. $\frac{1+5\frac{4}{5}(1+5\frac{4}{5})}{1+2\frac{1}{3}(1+2\frac{1}{3})} \times \frac{4\frac{1}{12}+2\frac{2}{3}}{13\frac{5}{12}-3\frac{1}{3}} \text{ of } \frac{11}{111}$. **30.** $\frac{1\frac{3}{8}+7\frac{1}{3}\times 1\frac{1}{11}-9\frac{4}{7}}{\frac{4\frac{3}{4}}{3\frac{1}{8}} \text{ of } \left(5\frac{5}{7}-3\frac{7}{10}\right)}$.

31. $\frac{5\frac{5}{8}+\frac{2}{3}}{1\frac{1}{5} \text{ of } \frac{5}{6}+10\frac{1}{3}} \times \frac{2}{5} \text{ of } \frac{1\frac{1}{2} \text{ of } 4\frac{1}{9}}{3\frac{7}{8} \text{ of } 5\frac{1}{3}}$. **32.** $\frac{1}{4\frac{2}{3}} \text{ of } \frac{4\frac{3}{4}}{2\frac{1}{4}} + \frac{2}{4\frac{1}{2}} \text{ of } \frac{1\frac{4}{5}}{1\frac{2}{5}} - \frac{2\frac{3}{7}}{8}$.

33. $\frac{\frac{2}{5} \text{ of } (\frac{6}{7}-\frac{3}{4})}{\frac{7}{9} \text{ of } (\frac{3}{4}-\frac{5}{7})} \text{ of } \left\{\frac{\frac{8}{13}-\frac{6}{13}}{\frac{5}{13}-\frac{3}{11}} - \frac{9\frac{1}{3}+3\frac{4}{9}}{5\frac{1}{14} \text{ of } 7\frac{1}{9}} + \frac{7\frac{5}{8}-4\frac{2}{3}}{2\frac{1}{3}-1\frac{2}{9}}\right\}$.

34. $7\frac{3}{13} \text{ of } \frac{1}{10+\frac{1}{3+\frac{1}{30}}}$. **35.** $3\frac{1}{2} + \frac{2\frac{1}{4}}{3\frac{1}{4}+\frac{2}{5\frac{1}{6}+\frac{1}{4\frac{1}{2}}}}$.

36. $11 + \frac{1}{1-\frac{1}{1+\frac{1}{8+\frac{1}{11}}}}$. **37.** $\left(2+\frac{1}{3-\frac{1}{5+\frac{1}{6}}}\right) + \{1\frac{5}{8} + (1\frac{3}{4} \times 14\frac{1}{2})\}$.

38. $\frac{3\frac{3}{7}}{1\frac{1}{5} \text{ of } 2\frac{2}{3}} + \frac{8\frac{1}{4} \text{ of } 2\frac{2}{7}}{11} - \frac{9\frac{3}{4} \text{ of } 1\frac{1}{8}}{4(\frac{1}{2}+\frac{1}{24})} + \frac{2-\frac{4}{7}}{1-\frac{4}{9}}$.

39. $\frac{4\frac{1}{3} \text{ of } 3 - 3\frac{1}{4} \text{ of } 3\frac{1}{2}}{4\frac{1}{2}-3\frac{1}{3}} + \left\{6\frac{1}{2} \text{ of } \frac{\frac{1}{3\frac{1}{3}}+\frac{1}{2\frac{1}{4}}}{3\frac{1}{3}-2\frac{1}{4}}\right\} \times 6\frac{35}{117}$.

40. $\frac{5\frac{1}{4}-2\frac{1}{3}}{3\frac{3}{4}+\frac{9}{20}} \text{ of } \frac{4\frac{1}{3}+5\frac{11}{23}}{4\frac{1}{20}} \text{ of } \frac{2\frac{2}{3}+1\frac{2}{3}}{7\frac{19}{24}-2\frac{1}{4}} \text{ of } \frac{6\frac{1}{4}}{7\frac{1}{9}}$.

41. $\left\{\left(3\frac{8}{63} - \frac{2}{3} \text{ of } \frac{8\frac{1}{2}}{7} + \frac{2\frac{2}{3}}{5\frac{1}{3}}\right) + \frac{2\frac{7}{9}+9\frac{2}{7}}{6\frac{1}{3}-4\frac{3}{4}}\right\} \text{ of } 34\frac{18}{23}$.

42. $\frac{6\frac{3}{4}-1\frac{5}{12}}{2\frac{1}{6}+1\frac{4}{7}} \text{ of } \frac{(3\frac{1}{8}+5\frac{1}{9}-\frac{7}{18})(4\frac{1}{3}-3\frac{1}{4})}{1\frac{6}{11}+2\frac{1}{3}-(2\frac{9}{10}-\frac{1}{8}-\frac{1}{12})}$. **43.** $8-8\times\frac{2\frac{1}{3}-1\frac{4}{7}}{2-\frac{1}{6-\frac{1}{6}}}$.

44. $\left\{\frac{3\frac{1}{2}+2\frac{1}{3}}{\frac{3}{4} \text{ of } 9\frac{1}{6}} \div \frac{2\frac{4}{5}-1\frac{1}{2}+9\frac{1}{11}}{4\frac{1}{5}-2\frac{1}{4}+13\frac{7}{11}}\right\} \text{ of } \frac{5\frac{1}{2}}{2\frac{1}{3}}$.

45. $\frac{1\frac{1}{5}-\frac{2}{3} \text{ of } 1\frac{5}{7}+1\frac{1}{9}}{1\frac{4}{5}-\frac{3}{2} \text{ of } 1\frac{1}{7}+1\frac{8}{9}} \text{ of } 1\frac{1}{2} \text{ of } \frac{6\frac{3}{4}-1\frac{5}{14}}{2\frac{1}{6}+1\frac{3}{7}}$.

46. $\left\{\frac{2}{3-\frac{1}{1-\frac{1}{2}}}-\frac{1}{3} \text{ of } \left(5-\frac{2}{\frac{3}{2}-\frac{1}{6}}\right)\right\} \div \frac{\frac{1}{2}+\frac{3}{4}}{1\frac{1}{2}}$.

47. $\frac{17}{7+\frac{3}{4-2\frac{3}{4}}} \times \frac{2021}{2193} \div \left(1\frac{37}{48}-\frac{15}{16}\right)$.

48. $\left(\frac{1+\frac{1}{3}}{1-\frac{1}{3}}\right)^2 \div \left(\frac{1+\frac{5}{13}}{1-\frac{5}{13}}\right)^2$.

49. $\frac{8\frac{7}{8}-7\frac{6}{7}+5\frac{5}{6}-4\frac{4}{5}}{9\frac{9}{10}-8\frac{13}{18}+7\frac{7}{8}-6\frac{6}{7}} \div \left\{\frac{6}{7} \text{ of } 2\frac{1}{17} \div \frac{1\frac{2}{3}}{2\frac{3}{7}}\right\}$.

50. $\left(\frac{2}{3-\frac{4}{5}}+\frac{3}{4-\frac{5}{6}}\right) \div \left(\frac{3}{2-\frac{1}{4}}-\frac{1}{3-\frac{4}{5}}\right) \times \left(\frac{1}{\frac{2}{3}-\frac{5}{6}}-\frac{1}{1\frac{1}{2}-\frac{19}{22}}\right)$

$\div \left(\frac{1}{1\frac{1}{7}+\frac{1}{5}}-\frac{2}{6\frac{5}{8}-2\frac{5}{11}}\right)$; $\frac{\frac{1}{4}+\frac{5}{8}}{4-\frac{5}{8} \text{ of } 5\frac{1}{2}}+\frac{\frac{5}{8}+\frac{1}{4}}{\frac{7}{10} \text{ of } 4\frac{1}{2}-2\frac{1}{2}}$.

51. $\left\{\left(\frac{1}{\frac{2}{3}-\frac{5}{6}}-\frac{1}{1\frac{1}{2}-\frac{19}{22}}\right) \div \left(\frac{1}{1\frac{1}{7}-\frac{1}{5}}-\frac{2}{6\frac{5}{8}-2\frac{5}{11}}\right)\right\} \text{ of } \left\{\left(\frac{2}{3-\frac{4}{5}}+\frac{3}{4-\frac{5}{6}}\right)\right.$

$\left.\div \left(\frac{3}{2-\frac{1}{4}}-\frac{1}{3-\frac{4}{5}}\right)\right\}$; $\frac{3\frac{6}{7} \div 2\frac{4}{11}}{\frac{6}{7} \text{ of } (11\frac{3}{4}-2\frac{1}{5})} \div \frac{3\frac{3}{5} \text{ of } 5\frac{5}{8}}{5\frac{1}{16}+\frac{1}{2} \text{ of } 4\frac{3}{5}}$.

52. $1\frac{5}{8} \text{ of } \frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{2\frac{1}{2}-3\frac{1}{3}+4\frac{1}{4}} \times \left(\frac{2\frac{1}{2}}{3\frac{1}{7}}+\frac{1\frac{3}{16}}{1\frac{3}{5}}\right) \div \left(\frac{3}{4\frac{1}{2}}+\frac{4\frac{1}{2}}{3}\right)$.

53. $\frac{11\frac{1}{6}-2\frac{1}{4}}{6\frac{1}{4}-3\frac{1}{16}} \div \frac{3\frac{1}{3}+1\frac{1}{2}}{2\frac{1}{2}+1\frac{3}{4}} \times \frac{3\frac{1}{3}-1\frac{1}{2}}{2\frac{1}{2}-1\frac{3}{4}}$.

54. $\frac{7\frac{1}{2}+1\frac{17}{22}}{8\frac{2}{3}+3\frac{8}{33}}-\frac{3\frac{4}{5}+\frac{2}{45}}{3\frac{3}{4}+14\frac{7}{36}}$.

55. $\left(\frac{\frac{1}{3}}{5\frac{3}{4}}+\frac{4}{4\frac{5}{8}}+\frac{3\frac{1}{4}-8\frac{1}{6}}{\frac{1}{2\frac{1}{2}}+\frac{1}{1\frac{1}{3}}} \text{ of } \frac{1}{6}\right) \times \left(\frac{\frac{1}{4}+\frac{1}{5}}{1\frac{3}{4}}-\frac{1}{3}+\frac{1}{2\frac{4}{5}}-\frac{\frac{1}{3}}{2\frac{1}{12}}+7\right) \text{ of } \frac{\frac{1}{3}+\frac{1}{9}}{\frac{1}{3}+\frac{5}{7}}$;

$3-\left(\frac{3\frac{1}{3}-\frac{1}{6}}{3\frac{1}{3}+\frac{1}{6}}-2\frac{5}{7} \text{ of } \frac{4}{19}\right)$.

56. $\left\{\frac{\frac{2}{3} \text{ of } \frac{7}{8} \text{ of } 6\frac{2}{7}+7\frac{7}{11}+19\frac{13}{20}+8\frac{13}{22}}{3\frac{5}{6}+\frac{2}{7}+4\frac{1}{10}-1\frac{1}{5} \text{ of } 1\frac{6}{7}}-\frac{221}{680}\right\} \div \left\{\frac{1\frac{39}{112}+\frac{6}{35}}{\frac{9}{28}}+39\frac{11}{12}-24\frac{5}{8}\right\}$.

IX. G. C. M. AND L. C. M. OF FRACTIONS.

285. The definitions that we have already given of the G. C. M. and L. C. M. of two or more whole numbers will also be applicable when the given numbers are fractions, provided that we understand by *exactly*, that the complete quotients must be *integers*.

286. *To find the* G. C. M. *of two or more fractions.*

RULE. Express the fractions in their lowest terms, if they be not already so. Then take the G. C. M. of the numerators for numerator and the L. C. M. of the denominators for denominator. The fraction so formed is the G. C. M. of the given fractions.

Ex. Find the G. C. M. of $\frac{8}{9}$, $\frac{12}{14}$, $\frac{48}{63}$.

Here, the fractions reduced to their lowest terms are $\frac{8}{9}$, $\frac{6}{7}$, $\frac{16}{21}$.

The G. C. M. of the numerators 8, 6, 16 is 2; and the L. C. M. of the denominators 9, 7, 21 is 63.

Thus the required G. C. M. $= \frac{2}{63}$. *Ans.*

287. *To find the* L. C. M. *of two or more fractions.*

RULE. Express the fractions in their lowest terms. Then take the L. C. M. of the numerators as numerator and the G. C. M. of the denominators as denominator. The fraction so formed is the L. C. M. of the given fractions.

Ex. Find the L. C. M. of $\frac{8}{25}$, $\frac{4}{14}$, $\frac{9}{20}$.

Here, the fractions reduced to their lowest terms are $\frac{8}{25}$, $\frac{2}{7}$, $\frac{9}{20}$.

The L. C. M. of the numerators 8, 2, 9 is 72; and the G. C. M. of the denominators 25, 7, 20 is 1.

Thus, the required L. C. M. $=$ 72. *Ans.*

Note. Before applying the rules given above, reduce mixed numbers to improper fractions and compound fractions to simple ones.

Examples LXXVII.

1. Find the G. C. M. and the L. C. M. of :—

(1) $\frac{5}{6}$, $\frac{8}{9}$. (2) $\frac{11}{12}$, $\frac{14}{15}$. (3) $\frac{1}{9}$, $\frac{1}{12}$, $\frac{1}{15}$. (4) $\frac{7}{9}$, $6\frac{5}{12}$, $\frac{1}{3}$.

(5) $\frac{2}{3}$, $\frac{4}{5}$, $\frac{6}{7}$, $\frac{8}{9}$. (6) $\frac{3}{4}$, $\frac{4}{5}$, $\frac{5}{6}$, $\frac{6}{7}$. (7) $1\frac{6}{25}$, $2\frac{2}{21}$, $3\frac{1}{7}$. (8) $\frac{15}{16}$, $2\frac{1}{4}$, 4, $5\frac{1}{3}$.

(9) $1\frac{14}{25}$, $5\frac{17}{20}$, $14\frac{3}{10}$, $6\frac{1}{15}$. (10) $1\frac{23}{55}$, $2\frac{2}{77}$, $3\frac{31}{33}$, $8\frac{19}{22}$.

(11) $7\frac{1}{2}$, $1\frac{13}{14}$, $1\frac{1}{3}$, $4\frac{1}{3}$, $\frac{3}{4}$. (12) $3\frac{3}{7}$, $1\frac{2}{3}$, $1\frac{3}{5}$, $2\frac{2}{3}$, $3\frac{10}{12}$.

2. What is the greatest length that is contained a whole number of times exactly in $26\frac{1}{4}$ ft. $28\frac{7}{8}$ ft., and $29\frac{1}{6}$ ft.?

3. A man gives away to each of five people $\frac{1}{12}$, $\frac{2}{15}$, $\frac{1}{10}$, $\frac{5}{18}$, $\frac{3}{20}$ of a basket of apples; how many has he left, supposing he has only just enough apples to do the above operation without dividing an apple?

4. What is the least number which when divided by each of the fractions $\frac{2}{3}$, $\frac{7}{10}$, $\frac{14}{15}$, and $\frac{8}{25}$, gives a whole number as quotient in each case?

5. Three lines of paling run side by side for a distance of 150 yds. The upright posts are respectively $2\frac{1}{4}$, $3\frac{1}{3}$, $4\frac{1}{6}$ ft. apart.

How often will a person walking outside be able, on looking across, to see 3 posts in a line?

6. Eight bells commence to toll simultaneously. They toll at intervals of $1\frac{1}{3}$, $2\frac{1}{3}$, 3, $4\frac{2}{3}$, 5, 6, 8 and 9 seconds respectively; after what interval will they again toll together?

7. Three wheels are respectively $10\frac{1}{2}$ ft., $6\frac{7}{8}$ ft. and $4\frac{9}{10}$ ft. round. Find the least distance travelled when they will make complete revolutions.

8. A man gives away to each of four people $\frac{1}{18}$, $\frac{7}{30}$, $\frac{5}{18}$ and $\frac{7}{45}$ of a basket of apples, and has only just enough apples to be able to do this without dividing an apple; how many apples had he?

X. MISCELLANEOUS EXAMPLES IN FRACTIONS.

288. The following Solutions, we hope, will be of service to students in acquiring a thorough knowledge of the principles of Vulgar Fractions.

Examples worked out.

Ex. 1. What fraction added to the sum of $\frac{5}{7}$, $\frac{2}{3}$, $\frac{4}{21}$ and $2\frac{1}{2}$ will make the sum equal to 5?

Here, $\frac{5}{7}+\frac{2}{3}+\frac{4}{21}+2\frac{1}{2}=2+2\frac{1}{14}=4\frac{1}{14}$.

$\therefore$ the required fraction $=5-4\frac{1}{14}=\frac{13}{14}$. *Ans.*

Ex. 2. What fraction is that from which if there be taken $\frac{2}{7}$ of $\frac{3}{8}$ and to the remainder be added $\frac{8}{15}$ of $\frac{5}{16}$, the sum will be 10?

Here, $\frac{2}{7}$ of $\frac{3}{8}=\frac{3}{28}$ and $\frac{8}{15}$ of $\frac{5}{16}=\frac{1}{6}$.

$\therefore$ the required number $=10-\frac{1}{6}+\frac{3}{28}=9\frac{5}{6}+\frac{3}{28}=9\frac{79}{84}$. *Ans.*

Ex. 3. Find what fraction multiplied by the sum of $2\frac{3}{5}$, $1\frac{1}{10}$ and $\frac{4}{15}$ will make the product equal to 17.

Here, $2\frac{3}{5}+1\frac{1}{10}+\frac{4}{15}=3+\frac{29}{30}=3\frac{29}{30}$.

$\therefore$ the required fraction $=17\div3\frac{29}{30}=17\times\frac{30}{119}=\frac{30}{7}=4\frac{2}{7}$.

Ex. 4. Find what least fraction added to the sum of $\frac{3}{4}$, $1\frac{7}{8}$ and $2\frac{5}{6}$ will make the result an integer.

Here, $\frac{3}{4}+1\frac{7}{8}+2\frac{5}{6}=3+2\frac{11}{24}=5\frac{11}{24}$.

$\therefore$ the required fraction $=1-\frac{11}{24}=\frac{13}{24}$. *Ans.*

Ex. 5. What number divided by $2\frac{1}{16}$ will produce $\frac{4}{15}$?

The required number $=\frac{4}{15}\times2\frac{1}{16}=\frac{4}{15}\times\frac{33}{16}=\frac{11}{20}$. *Ans.*

Ex. 6. A man has $\frac{3}{5}$ of an estate, he gives his son $\frac{1}{4}$ of his share; what portion of the estate has he then left?

$\frac{1}{4}$ of his share being given away, there remains $(1-\frac{1}{4})$ or $\frac{3}{4}$.

But his share $=\frac{3}{5}$ of the estate; $\therefore$ he remains $\frac{3}{4}$ of $\frac{3}{5}=\frac{9}{20}$. *Ans.*

Examples LXXVIII.

1. What number added to $\frac{34}{37}$ makes $1\frac{2}{5}$? and what taken from $1\frac{22}{27}$ leaves $\frac{11}{18}$?

2. What number added to $\frac{10}{11}$, $\frac{11}{12}$, $\frac{2}{33}$, $\frac{41}{42}$, will make the sum total equal to 3 ?

3. Multiply the sum of $3\frac{2}{5}$, $4\frac{3}{4}$ and $4\frac{4}{5}$ by the difference of $7\frac{5}{7}$ and $5\frac{5}{6}$; and divide the product by the sum of $94\frac{1}{8}$ and $93\frac{1}{8}$.

4. Prove that the sum of $5\frac{1}{3}$ and $3\frac{1}{5}$ is equal to four times their difference.

5. Compare the product and quotient of $\frac{7}{9}$ by $\frac{10}{11}$.

6. Find what quantity multiplied by $\frac{2}{3}$ of $\frac{4}{5}$ of $3\frac{1}{2}$, gives a result equal to $\frac{4}{5}$ of $\frac{7}{32}$ of $6\frac{1}{4}$.

7. What number is that, whereof the part expressed by $\frac{1}{3}+\frac{1}{4}+\frac{1}{6}$ is 45 ? What number must be added to $\frac{1}{3}$ of $2\frac{1}{7}$ to give $3\frac{3}{8}$?

8. Find the least fraction which, added to the sum of $\frac{7}{8}$, $\frac{3}{10}$ and $\frac{22}{15}$, will make the result an integer.

9. To 479 add $1\frac{808}{905}$ and repeat the addition 6 times.

10. From $11\frac{9}{10}$ take the sum of $2\frac{1}{2}$, $3\frac{2}{3}$ and $4\frac{3}{4}$, and multiply the difference by $2\frac{1}{3}$ of $\frac{57}{85}$ of $6\frac{1}{2}$.

11. Multiply $49\frac{17}{18}$ by $50\frac{1}{18}$ and add $\frac{1}{324}$ to the result.

12. How many times does $\frac{2}{5}+\frac{2}{3}-\frac{1}{15}$ contain $\frac{2}{7}+\frac{1}{3}-\frac{1}{21}$?

13. Multiply the sum of 1, $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$ by the difference of $\frac{4}{15}$ and $\frac{8}{35}$ and divide the product by the double of $21\frac{7}{9}$.

14. Of the fractions $\frac{3}{16}$, $\frac{5}{24}$, $\frac{7}{36}$, $\frac{11}{54}$, find how much the sum of the greatest and least exceeds the difference of the other two.

15. From 1 take its half, third, and twenty-fourth parts : add the product of those parts to the remainder ; and multiply this sum by $7\frac{11}{13}$. What must $\frac{3}{4}$ be divided by to produce 2 ?

16. To $\frac{4}{15}$ of a dozen add $\frac{12}{25}$ of three hundred, and divide this sum by the difference of $3\frac{3}{4}$ of a hundred and $43\frac{3}{8}$.

17. Find the sum of the greatest and least of the fractions $\frac{3}{8}$, $\frac{5}{12}$, $\frac{4}{9}$ and $\frac{7}{20}$, the sum of the other two, and the difference of these sums.

18. Multiply the sum of $\frac{1}{3}$, $1\frac{6}{7}$ and $\frac{5}{6}$ by the difference of $\frac{4}{15}$ and $\frac{3}{20}$, and divide the product by $\frac{11}{18}$ of $1\frac{14}{15}$.

19. What fraction is that from which if $\frac{3}{8}$ of $\dfrac{3-1\frac{1}{2}}{2\frac{3}{8}}$ be subtracted and the remainder be divided by $5\frac{1}{3}+16\frac{2}{3}$, the result will be $\frac{1}{3}$?

20. Divide the sum of $2\frac{2}{3}$, $3\frac{3}{4}$ and $5\frac{5}{6}$ by the sum of $4\frac{1}{3}$ and $8\frac{2}{3}$, and to the quotient add the difference of $10\frac{9}{16}$ and $5\frac{5}{8}$.

21. To the sum of $3\frac{3}{4}$ and $4\frac{4}{5}$ add the difference between $4\frac{4}{5}$ and $5\frac{5}{8}$ and multiply the result by $11\frac{5}{11}$.

22. A merchant owned $\frac{5}{16}$ of a ship and sold $\frac{3}{8}$ of his share; what share has he remaining?

23. If I pay away $\frac{1}{2}$ of my money, then $\frac{1}{3}$ of what remains, then $\frac{1}{4}$ of what then remains and then $\frac{1}{5}$ of what still remains, what fraction of the whole will be left?

24. What is the least fraction which must be added to the sum of 4 and $\frac{1}{4}$ divided by their difference to make the result an integer?

25. The difference of two numbers is $15\frac{4}{35}$; the smaller number is $5\frac{18}{21}$; find the greater number.

26. Multiply $3\frac{3}{20}$ by $15\frac{5}{7}$, and divide $\dfrac{2}{3\frac{3}{4}}$ by $\dfrac{2\frac{3}{4}}{3}$; and add together the sum and difference of these results.

27. Divide 2 by the sum of $2\frac{3}{8}$, $\frac{4}{5}$ and 4; add $1\frac{2}{3}-\frac{7}{8}$ to the quotient; and multiply the result by the difference of $5\frac{1}{3}$ and $4\frac{1}{2}$.

28. If I pay away $\frac{1}{5}$ of my money, then $\frac{2}{3}$ of the remainder then $\frac{1}{2}$ of what then remains and then $\frac{1}{20}$ of the original sum; what fractional part of my money have I left after the second, and also after the final payment?

29. What must be taken from

$$8\frac{4}{7} \text{ of } \frac{5\frac{4}{5}-2\frac{3}{8}}{3\frac{3}{4}+\frac{9}{20}} + \frac{\dfrac{5\frac{5}{8}+\frac{2}{3}}{1\frac{1}{5}\text{ of }\frac{5}{6}}}{1\frac{1}{5}} \text{ of } \frac{\frac{\frac{4}{5}}{3}}{1-\frac{8}{9}}$$ to reduce its value to $\frac{16}{21}$?

30. *A* has a certain sum of money in his pocket of which he loses $\frac{3}{8}$ths; he gives $\frac{1}{13}$th of what remains to *B*, and then $\frac{1}{4}$th of $(\frac{1}{2}-\frac{1}{3})$ of what then remains to *C*; find what fractional part of *A*'s original money *B* and *C* respectively receive; and compare these sums with the amount *A* has after his loss.

31. A man having $$\frac{17\frac{5}{6}-\left(\dfrac{3\frac{1}{7}}{4\frac{2}{5}}+\dfrac{\frac{1}{9}}{8-5\frac{4}{7}}\right)}{7\frac{1}{4}\text{ of }\dfrac{\frac{4}{5}}{7}+\dfrac{\frac{4}{7}}{\frac{5}{9}}}+\frac{6\frac{2}{3}\text{ of }7}{5\frac{5}{6}}$$ of an estate, gives $\frac{1}{3}$ of his share to his son, and $\frac{8}{23}$ of the remainder to his daughter; what fraction of the estate has he still remaining?

32. If I cut half a cake into 5 equal parts, and the remainder into 7 equal parts, and then cut one of the 5 equal parts into 6 equal parts, and one of the 7 equal parts into 4 equal parts and then give 2 children each one of each of these small slices, what fractional part of the whole cake will they receive, and what part of the cake will be left?

XI. APPLICATION OF FRACTIONS TO COMPOUND QUANTITIES.

289. In the Fundamental Operations of Compound Quantities, if the lowest denominations of the given compound quantities be mixed numbers, we shall treat separately, first the fractional parts by the ordinary method for Fractions and then the integral parts.

Ex. 1. Add together £16. 2*s.* $1\frac{4}{9}$*d.*, £4. 18*s.* $1\frac{4}{5}$*d.* and £1. 0*s.* $9\frac{4}{15}$*d.*

£.	s.	d.
16	2	$1\frac{4}{9}$
4	18	$1\frac{4}{5}$
1	0	$9\frac{4}{15}$
22	1	$0\frac{23}{45}$

Now $(\frac{4}{9}+\frac{4}{5}+\frac{4}{15})d. = \frac{20+36+12}{45}d. = \frac{68}{45}d. = 1\frac{23}{45}d.$; we therefore put down $\frac{23}{45}d.$, carry on 1*d.* to the column of pence, and proceed in the usual way.

Ex. 2. Subtract *Rs.*32. 14*a.* $9\frac{11}{12}$*p.* from *Rs.*87. 8*a.* $6\frac{3}{8}$*p.*

Rs.	a.	p.
87	8	$6\frac{3}{8}$
32	14	$9\frac{11}{12}$
54	9	$8\frac{11}{24}$

Here $\frac{11}{12}$ is greater than $\frac{3}{8}$, therefore we add 1 to $\frac{3}{8}$, which makes it $\frac{11}{8}$.

Now $\frac{11}{8}-\frac{11}{12}=\frac{33-22}{24}=\frac{11}{24}$. We must add 1*p.* to 9*p.*, and proceed in the usual way.

Ex. 3. Multiply £6. 12*s.* $8\frac{5}{6}$*d.* by 57, and divide *Rs.*21. 14*a.* $5\frac{1}{4}$*p.* by 21.

(1)

£.	s.	d.	
6	12	$8\frac{5}{6}$	$57 = 11\times5+2$
		11	
73	0	$1\frac{1}{6}$,	for $\frac{5}{6}d.\times11=\frac{55}{6}d. = 9\frac{1}{6}d.$
		5	
365	0	$5\frac{5}{6}$,	for $\frac{1}{6}d.\times5=\frac{5}{6}d.$
13	5	$5\frac{2}{3}$	for $\frac{5}{6}d.\times2=\frac{5}{3}d.$
378	5	$11\frac{1}{2}$	$=1\frac{2}{3}d.$

(2)

		Rs.	a.	p.	
21	3	21	14	$5\frac{1}{4}$	$21=3\times7$
	7	7	4	$9\frac{3}{4}$	for $2\frac{1}{4}\div3=\frac{3}{4}$
		1	0	$8\frac{1}{4}$	for $1\frac{3}{4}\div7=\frac{1}{4}$

Examples LXXIX.

1. Add together :—

(1)

Rs.	a.	p.
3	15	$7\frac{1}{2}$
5	14	$2\frac{2}{3}$
7	6	$10\frac{5}{8}$
8	1	$11\frac{11}{15}$
2	4	$6\frac{2}{3}$
1	4	$5\frac{9}{10}$

(2)

Rs.	a.	p.
17	13	$5\frac{1}{3}$
32	6	$11\frac{1}{4}$
12	10	$9\frac{5}{6}$
7	0	$8\frac{1}{2}$
11	5	$4\frac{1}{8}$
6	10	$5\frac{7}{12}$

(3)

£.	s.	d.
7	13	$1\frac{1}{3}$
2	17	$4\frac{2}{7}$
5	2	$8\frac{1}{6}$
6	11	$2\frac{11}{18}$
4	5	$0\frac{2}{7}$
6	3	$4\frac{1}{3}$

(4)

£.	s.	d.
23	2	$6\frac{2}{3}$
14	1	$5\frac{1}{6}$
7	8	$11\frac{1}{3}$
4	9	$5\frac{11}{12}$
16	4	$2\frac{7}{15}$
5	4	$3\frac{3}{4}$

(5)

oz.	dwts.	grs.
5	16	$15\frac{2}{3}$
1	14	$23\frac{5}{8}$
	17	$0\frac{7}{18}$
2	4	$21\frac{5}{6}$
3	19	$8\frac{3}{4}$
6	18	$20\frac{1}{2}$

(6)

cwt.	qrs.	lbs.	oz.
13	0	21	$13\frac{5}{6}$
	3	18	$9\frac{7}{16}$
		25	$15\frac{5}{8}$
	1	13	$3\frac{13}{16}$
	2	12	$12\frac{3}{4}$
4	0	8	$15\frac{9}{16}$

(7)

poles	yds.	ft.	in.
25	4	2	$8\frac{3}{8}$
17	2	0	$6\frac{6}{7}$
2	0	1	$7\frac{7}{16}$
15	5	1	$11\frac{9}{14}$
6	4	2	$10\frac{5}{6}$
20	3	0	$9\frac{11}{12}$

2. Perform the following subtractions :—

(1) *Rs.*	*a.*	*p.*	(2) *Rs.*	*a.*	*p.*	(3) *£.*	*s.*	*d.*	(4) *£.*	*s.*	*d.*
15	0	$3\frac{5}{8}$	17	15	$7\frac{5}{12}$	48	13	$6\frac{3}{8}$	163	1	$7\frac{5}{12}$
9	14	$9\frac{11}{16}$	6	15	$9\frac{11}{12}$	34	19	$9\frac{7}{8}$	64	2	$5\frac{11}{16}$

(5) cwt.	qrs.	lbs.	(6) cwt.	qrs.	lbs.	(7) fur.	po.	yds.	in.	(8) hrs.	min.	sec.
15	1	$16\frac{3}{7}$	23	1	$7\frac{7}{11}$	5	15	0	0	23	45	$35\frac{5}{8}$
8	3	$25\frac{4}{5}$	14	0	$24\frac{5}{8}$	2	4	3	$8\frac{7}{18}$	15	50	$48\frac{11}{16}$

3. Multiply :—

(1) *Rs.*9. 4*a.* $2\frac{3}{32}$*p.* separately by 8, 11, 45 and 139.

(2) £75. 13*s.* $9\frac{1}{8}$*d.* separately by 4, 15, 88 and 96.

(3) 14 cwt. 3 qrs. 25 lbs. $13\frac{8}{21}$ oz. separately by 12, 24 and 96.

(4) 45 mds. 14 sr. $7\frac{7}{18}$ ch. separately by 9, 24 and 35.

(5) 3 fur. 34 po. 4 yds. 1 ft. $8\frac{17}{36}$ in. separately by 45 and 99.

4. Divide :—

(1) *Rs.*246. 13*a.* $8\frac{1}{4}$*p.* separately by 12, 14, 26 and 58.

(2) £997. 18*s.* $10\frac{2}{3}\frac{1}{3}$*d.* separately by 26, 53, 84 and 145.

(3) 789 lbs. 12 oz. $14\frac{3}{8}\frac{1}{2}$ drs. separately by 7, 15 and 67.

(4) 1994 mds. $20\frac{3}{8}$ sr. separately by 729 and 1521.

(5) *Rs.*7. 8*a.* $11\frac{5}{8}$*p.* separately by *Rs.*3. 2*a.* $7\frac{3}{8}$*p.*, and 15*a.* $9\frac{3}{8}$*p.*

(6) £282. 18*s.* $7\frac{1}{2}\frac{4}{7}$*d.* separately by £6. 18*s.* $0\frac{5}{27}$*d.*, and £27. 15*s.* $9\frac{3}{8}$*d.*

XII. REDUCTION OF FRACTIONS.

290. Our attention has hitherto been confined to fractions considered *generally*, without regard to the particular value of their **units**; and it remains to apply what has been said to such *concrete* quantities as constitute the principal subjects of practical computation.

291. We shall notice here, that while **times** denotes the multiplication of a quantity by an integer, **of** denotes its multiplication by a fraction, and either **times** or **of** its multiplication by a mixed number.

Thus, each of the expressions 5 *times Rs.*7, $\frac{4}{5}$ *of Rs.*7, and either $3\frac{1}{4}$ *times Rs.*7 or $3\frac{1}{4}$ *of Rs.*7 denotes the multiplication of *Rs.*7 by 5, by $\frac{4}{5}$ and by $3\frac{1}{4}$ respectively. Also the notation for 5 *times Rs.*7 is either 5 × *Rs.*7 or *Rs.* (5 × 7).

292. Reduction of Fractions can conveniently be divided into the two following cases :—

(1) To reduce a **fraction** of one denomination to a lower denomination, and conversely.

(2) To reduce a quantity of one denomination to a **fraction** of a higher denomination.

293. Case I. *To reduce a fraction of one denomination to a lower denomination.* **(Descending Reduction).**

RULE. Multiply the fraction of the given denomination by the number which connects the lower denomination with one (or unit) of the given denomination.

Ex. Reduce £$\frac{2}{7}$ to *pence*, and $\frac{8}{27}$ of a day to *seconds.*

(1) £$\frac{2}{7} = \frac{2}{7} \times (20 \times 12)d. = \frac{2 \times 20 \times 12}{7}d. = \frac{480}{7}d. = 68\frac{4}{7}d.$ *Ans.*

(2) $\frac{8}{27}$ of a day $= \frac{8}{27} \times (24 \times 60 \times 60)$ sec. $= \frac{8 \times 24 \times 60 \times 60}{27}$ sec.

$= 25600$ sec. *Ans.*

294. Case II. *To reduce a quantity of one denomination to a fraction of a higher denomination.* **(Ascending Reduction).**

RULE. Divide the number of the given denomination by the number which connects that denomination with one (or unit) of the higher denomination.

Ex. Reduce $5\frac{1}{4}d.$ to the fraction of a *pound*, and $18\frac{3}{4}$ grs. to the fraction of an *oz. Troy.*

(1) $5\frac{1}{4}d. = £\frac{5\frac{1}{4}}{12 \times 20} = £\frac{21}{4} \times \frac{1}{12 \times 20} = £\frac{7}{320}.$ *Ans.*

(2) $18\frac{3}{4}$ grs. $= \frac{18\frac{3}{4}}{24 \times 20}$ oz. $= \frac{75}{4} \times \frac{1}{24 \times 20}$ oz. $= \frac{5}{128}$ oz. *Ans.*

295. Sometimes we employ both the *descending* and the *ascending* process in reducing a fraction of one denomination to a fraction of another denomination.

Ex. Reduce $\frac{3}{5}$ of a guinea to the fraction of £1.

$\frac{3}{5}$ of a guinea $= \frac{3 \times 21}{5}$ *s.* $= \frac{63}{5}$ *s.* $= £\frac{63}{5 \times 20} = £\frac{63}{100}.$

Examples LXXX.

1. Reduce $\frac{1}{3}$, $\frac{2}{7}$, $\frac{5}{8}$, $\frac{4}{15}$ and $\frac{1}{32}$ of a rupee to *annas*; and $\frac{3}{64}$ of *Re.*1 to *gandas.*

2. Reduce $\frac{1}{7}$, $\frac{4}{9}$, $\frac{3}{84}$, and $\frac{544}{675}$ of a pound to *pence.*

3. Express $\frac{3}{8}$ of a shilling, $\frac{5}{7}$ of a penny, and $\frac{160}{11}$ of a farthing as fractions of a *pound.*

4. Reduce $\frac{3}{5}$ of a guinea, $\frac{3}{4}$ of a half-guinea, and $6\frac{3}{7}$ of a crown to fractions of £1.

5. Reduce $\frac{3}{32}$ of a cwt. to the fraction of 1 lb.; $\frac{4}{7}$ of an ounce to that of 1 *cwt.*; and $\frac{13}{35}$ of an ounce (Avoir.) to that of 1 *grain.*

6. Express $\frac{7}{324}$ of a yard as the fraction of an *inch*, and $\frac{108}{125}$ of an inch as that of a *pole*.

7. Find the fraction of a *yard* which expresses $\frac{2}{3}$ of an ell of 5 quarters; and that of a *day* which is equal to $\frac{4}{146}$ of a year of 365 days.

8. Reduce $\frac{7}{80}$ of a maund to the fraction of a *seer*; $\frac{84}{875}$ lb. to the fraction of 1 lb. Troy, and $\frac{8}{21}$ of a maund to *chhataks*.

9. Reduce $\frac{4}{9}$ of a barrel of beer to the fraction of a *quart*; and $\frac{4}{11}$ of a pint of wine to the fraction of a *hogshead*.

10. Reduce $\frac{487}{1056}$ of a mile to *poles*; $\frac{887}{704}$ of an acre to *sq. yards*.

11. Express $\frac{4}{7}$ of a guinea, $\frac{3}{8}$ of a shilling and $\frac{16}{19}$ of a farthing as fractions of £10.

12. Reduce $492\frac{1}{3}\frac{2}{3}$ hours to the fraction of a *year* of $365\frac{1}{4}$ days.

13. Express $\frac{1}{3}$ of $2151\frac{1}{9}$ sq. yards in *acres*; $3\frac{487}{1056}$ miles in *yards*; and $\frac{1}{8}$ cubit as the fraction of an *anguli*.

14. What fraction expresses $\frac{5\frac{1}{7}}{7\frac{1}{14}}$ of 5940 seconds in *weeks*?

296. The preceding two cases in Art. 292 enable us

(1) To find the value of a given fraction of any **concrete** quantity in terms of its own or lower denominations; and

(2) To reduce a **compound** quantity to a fraction of a higher denomination.

297. Case I. *To find the value of a given fraction of any concrete quantity in terms of its own or lower denominations.*

(1) When the quantity is *simple* or can be easily reduced to a *simple quantity*.

RULE. Multiply the given quantity by the numerator of the fraction, and divide the product (if possible) by the denominator; the quotient (if any) is the required number of parts of that denomination. If there be a remainder, multiply the numerator of the fraction which remains by the number of units of the next inferior denomination which are equivalent in value to the given denomination and divide the product by the denominator; the quotient will be the number of parts of that denomination. Proceed in the same way with the remainder (if any), and the parts of the next denomination will be found; repeat this process till the next denomination, to which the given quantity is capable of being reduced, is obtained.

Ex. 1. Find the value of $\frac{3}{8}$ of £1.

$\frac{3}{8}$ of £1 $= \frac{3 \times 20}{8}s. = \frac{3 \times 5}{2}s. = \frac{15}{2}s. = 7\frac{1}{2}s.$; $\frac{1}{2}s. = \frac{1 \times 12}{2}d. = 6d.$

∴ the required value $= 7s.\ 6d.$

Ex. 2. Find the value of $\frac{5}{6}$ of *Rs.*4.

$\frac{5}{6}$ of $Rs.4 = Rs.\frac{5 \times 4}{6} = Rs.\frac{10}{3} = Rs.3\frac{1}{3}$; $Re.\frac{1}{3} = \frac{1 \times 16}{3}a. = \frac{16}{3}a. = 5\frac{1}{3}a.$

$\frac{1}{3}a. = \frac{1 \times 12}{3}p. = 4p.$ ∴ the required value $=$ Rs.3 5a. 4p.

Ex. 3. Find the value of $\frac{3}{5}$ of 13*s.* 4*d.*

$\frac{3}{5}$ of 13*s.* 4*d.* $= \frac{3}{5}$ of $160d. = \frac{3 \times 160}{5}d. = (3 \times 32)d. = 96d. = 8s.$ *Ans.*

298. When the given fraction is a mixed number,—(1) multiply separately by the integer and by the fraction and add the products so obtained; or (2) reduce the mixed number to a fraction and proceed as in Art. 297, Case 1.

Ex. Find the value of $3\frac{1}{12}$ of *Re.*1. 4*a.*

The required value $= Re.1.\ 4a. \times 3 + Re.1.\ 4a. \times \frac{1}{12}$

$= Rs.3.\ 12a. + 20a. \times \frac{1}{12} = Rs.3.\ 12a. + 1\frac{2}{3}a$

$= Rs.3.\ 12a. + 1a.\ 8p. =$ Rs.3. 13a. 8p.

299. Before applying the Rule, reduce compound and complex fractions to simple ones.

Ex. 1. Find the value of $2\frac{2}{3}$ of $\frac{5}{4}$ of 10*a.* 9*p.*

Here, $2\frac{2}{3}$ of $\frac{5}{4} = \frac{8}{3}$ of $\frac{5}{4} = \frac{10}{3}$.

∴ the required value $= \frac{10}{3}$ of $129p. = \frac{10 \times 129}{3}p. = (10 \times 43)p.$

$= 430p. =$ Rs.2. 3a. 10p.

Ex. 2. Find the value of $\frac{7}{8}$ of $7\frac{1}{3}$ of $\frac{8\frac{1}{2}}{4}$ of 3 maunds.

Here, $\frac{7}{8}$ of $7\frac{1}{3}$ of $\frac{8\frac{1}{2}}{4} = \frac{7}{8}$ of $\frac{22}{3}$ of $\frac{17}{2} \times \frac{1}{4} = \frac{1309}{96}$.

∴ $\frac{1309}{96}$ of 3 mds. $= \frac{1309}{32}$ mds. $= 40\frac{29}{32}$ mds. ;

$\frac{29}{32}$ mds. $= \frac{29}{32} \times 40$ sr. $= \frac{29 \times 40}{32}$ sr. $= \frac{145}{4}$ sr. $= 36\frac{1}{4}$ sr. ;

$\frac{1}{4}$ sr. $= \frac{1}{4} \times 16$ ch. $= \frac{16}{4}$ ch. $= 4$ ch.

∴ the required value $=$ 40 mds 36 sr. 4 ch.

300. The preceding Articles enable us to find the value of the sum or difference of fractional parts of magnitudes of the same kind.

Ex. 1. Find the value of $\frac{2}{3}$ of £1 + $\frac{4}{9}$ of a guinea − $\frac{3}{7}$ of 3*s.* 6*d.*

$\frac{2}{3}$ of £1 $= \frac{2}{3} \times 20s. = \frac{40}{3}s. = 13s.\ 4d.$ }
$\frac{4}{9}$ of a gui. $= \frac{4}{9} \times 21s. = \frac{28}{3}s. = 9s.\ 4d.$ } = £1. 2*s.* 8*d.*

$\frac{3}{7}$ of 3*s.* 6*d.* $= \frac{3}{7}$ of $42d. = (3 \times 6)d. = 18d. =$ 1*s.* 6*d.*

∴ the required value = £1. 1*s.* 2*d.*

301 The following table, if carefully committed to memory, will greatly help a student in his calculations:

Re. $\frac{1}{16}$ = 1*a.* ;	*Re.* $\frac{1}{3}$ = 5*a.* 4*p.*	£ $\frac{1}{20}$ = 1*s.* ;	£ $\frac{1}{4}$ = 5*s.*
Re. $\frac{1}{12}$ = 1*a.* 4*p.* ;	*Re.* $\frac{1}{2}$ = 8*a.*	£ $\frac{1}{12}$ = 1*s.* 8*d.* ;	£ $\frac{1}{3}$ = 6*s.* 8*d.*
Re $\frac{1}{8}$ = 2*a.* ;	*Re.* $\frac{2}{3}$ = 10*a.* 8*p.*	£ $\frac{1}{10}$ = 2*s.* ;	£ $\frac{1}{2}$ = 10*s.*
Re. $\frac{1}{6}$ = 2*a.* 8*p.* ;	*Re.* $\frac{3}{4}$ = 12*a.*	£ $\frac{1}{8}$ = 2*s.* 6*d.* ;	£ $\frac{2}{3}$ = 13*s.* 4*d.*
Re. $\frac{1}{4}$ = 4*a.*		£ $\frac{1}{5}$ = 4*s.* ;	£ $\frac{3}{4}$ = 15*s.*

Examples LXXXI.

1. Find the respective values of :—

(1) $\frac{5}{8}$ of *Re.*1 ; $\frac{7}{12}$ of *Re.*1 ; $\frac{9}{16}$ of *Rs.*30 ; $\frac{11}{18}$ of *Rs.*9 ; $\frac{17}{20}$ of *Rs.*8.

(2) $7\frac{7}{25}$ of *Rs.*50 ; $\frac{2}{3}$ of *Rs.*2 ; $8\frac{2}{3}$ of $\frac{1}{2}$ of 10*a.* 9*p.* ; $\frac{1}{12}$ of $\frac{3}{4}$ of 5*a.*

(3) $\frac{3}{5}$ of £1 ; $\frac{5}{8}$ of 1*s.* ; $\frac{5}{18}$ of a guinea ; $\frac{5}{6}$ of £1 ; $\frac{1}{15}$ of £1 ; $\frac{17}{36}$ of £1.

(4) $\frac{13}{20}$ of £5 ; $\frac{3}{8}$ of 6*s.* 8*d.* ; $3\frac{3}{4}$ of 2*s.* 6*d.* ; $2\frac{5}{9}$ of a guinea ; £$3\frac{5}{8}$.

(5) $\frac{3}{8}$ of $\frac{7}{9}$ of $\frac{20}{21}$ of 5 guineas ; $\frac{8}{21}$ of a moidore ; $\frac{3}{8}$ of 13*s.* 4*d.* ; $\frac{13}{24}$ of £9.

(6) $\frac{5}{7}$ of a cwt. ; $\frac{5}{14}$ of 1 qr. ; $\frac{3}{8}$ of 1 lb. ; $\frac{47}{84}$ of a cwt. ; $2\frac{5}{8}$ of 8 cwt.

(7) $\frac{8}{35}$ of a ton ; $2\frac{1}{3}$ of 6*s.* 8*d.* ; $\frac{7}{24}$ of 5*s.* 3*d.* ; $\frac{7}{18}$ of a mile.

(8) $\frac{4}{7}$ of a lb. Troy ; $\frac{4}{7}$ of a lb. Avoir. ; $\frac{4}{7}$ of a lb. Apoth. ; $2\frac{17}{36}$ lbs. Troy.

(9) $\frac{13}{20}$ of $2\frac{1}{2}$ yds. ; $\frac{3}{8}$ of $\frac{1}{20}$ of $2\frac{6}{7}$ fur. ; $\frac{3}{5}$ of an acre ; $\frac{39}{182}$ of an acre.

(10) $2\frac{13}{16}$ of $\frac{4}{15}$ of a cwt. ; $\frac{5}{18}$ of a week ; $\frac{3}{16}$ of 1 mo. of 28 days.

(11) $\frac{1}{32}$ of 1 qr. ; $\frac{3}{8}$ of a bus. ; $\frac{5}{7}$ of a peck ; $\frac{8}{15}$ of $\frac{5}{6}$ of $2\frac{7}{8}$ of $2\frac{1}{2}$ loads.

(12) $8\frac{11}{18}$ of 17 cub. yds. ; $\frac{7}{8}$ of $3\frac{3}{8}$ of $4\frac{1}{2}$ of 2 mds. ; $\frac{5}{6}$ of 175 tons.

(13) $\frac{7}{8}$ of $\frac{8}{9}$ of $10\frac{3}{8}$ hrs. ; $\frac{3}{16}$ of a day ; $2\frac{11}{18}$ of a pipe of wine.

(14) $\frac{6\frac{5}{19}}{3\frac{3}{8}}$ of £4 ; $\frac{3\frac{13}{18}}{12\frac{13}{17}}$ of 365 days ; $\frac{3\frac{7}{11}}{4\frac{2}{7}}$ of $\frac{10\frac{5}{7}}{7\frac{1}{2}}$ of $\frac{77}{540}$ of a moidore.

(15) $\frac{3\frac{1\frac{3}{14}}{4}}{20}$ of a ton ; $\frac{6\frac{11\frac{13}{20}}{12}}{14}$ of a week ; $\frac{2\frac{7\frac{1}{4}}{20}}{25}$ of £50.

2. Find the respective values of :—

(1) $\frac{3}{8}$ of 5 guineas + $\frac{3}{4}$ of $\frac{2}{3}$ of £1 ; $\frac{3}{8}$ of 5 guineas − $\frac{3}{4}$ of $\frac{7}{9}$ of £1.

(2) $\frac{5}{6}$ of a guinea + $\frac{5}{16}$ of £1 + $\frac{7}{10}$ of a crown + $\frac{5}{8}$ of 1*s.*

(3) $\frac{2}{3}$ of a guinea + $\frac{3}{8}$ of a crown + $\frac{5}{8}$ of 7*s.* 6*d.* − $\frac{3}{4}$ of 2*d.*

(4) $\frac{3}{8}$ of a ton + $\frac{5}{8}$ of a cwt. + $\frac{3}{8}$ of a lb. ; $\frac{4}{7}$ cwt. + $8\frac{5}{8}$ lbs. + $3\frac{9}{10}$ oz.

(5) $\frac{2}{3}$ of a week + $\frac{3}{4}$ of a day + $\frac{5}{7}$ of an hour + $\frac{3}{8}$ of a minute.

(6) $3\frac{7}{11}$ miles − $7\frac{7}{16}$ fur. + $35\frac{35}{88}$ po. ; $\frac{5}{8}$ of 28mds. + $\frac{5}{8}$ of $1\frac{3}{8}$ mds. + $\frac{3}{8}$ of 8ch.

(7) $\frac{3}{5}$ of *Rs.*$10\frac{1}{2}$ + $\frac{1}{3}$ of $\frac{3}{4}$ of *Rs.*10 − $\frac{1}{6}$ of $\frac{2}{3}$ of *Rs.*$2\frac{1}{2}$ + $\frac{1}{3}$ of $\frac{5}{8}$ of 8*a.*

(8) $\frac{15\frac{3}{8}}{7\frac{4}{5}}$ of £1 + $\frac{1}{3}$ of £$140\frac{3}{16}$ + $1\frac{13}{126}$ guineas.

(2) When the quantity is a *compound* one.

RULE. Multiply the quantity by the numerator and divide the product by the denominator of the fraction.

Ex. Find the value of $\frac{5}{8}$ of *Rs.*3. 9*a.* 4*p.*

The required value $=$ (*Rs.*3. 9*a.* 4*p.* $\times 5) \div 8$

$=$ *Rs.*17. 14*a.* 8*p.* $\div 8 =$ *Rs.*2. 3*a.* 10*p.*

302. To multiply a compound quantity by a mixed number, multiply separately by the integer and by the fraction and add the two products thus obtained.

Ex. Multiply £13. 15*s.* 4*d.* by $4\frac{5}{8}$.

	£	*s.*	*d.*	£	*s.*	*d.*
	13	15	4	13	15	4
			5			4
8)	68	16	8	55	1	4
	8	12	1			

∴ the required value
$=$ £55. 1*s.* 4*d.* + £8. 12*s.* 1*d.*
$=$ £63. 13*s.* 5*d.*

303. To divide a compound quantity by a fraction, multiply by the denominator and divide the product by the numerator.

Ex. Divide *Rs.*600. 13*a.* 4*p.* by $\frac{7}{9}$.

The required value $=$ (*Rs.*600. 13*a.* 4*p.* $\times 9) \div 7$

$=$ *Rs.*5407. 8*a.* $\div 7 =$ *Rs.*772. 8*a.*

304. To divide a compound quantity by a mixed number, reduce the mixed number to an improper fraction and then proceed as in Art. 303.

Ex. Divide £5. 4*s.* $6\frac{1}{4}$*d.* by $1\frac{2}{3}$.

	£	*s.*	*d.*	
	5	4	$6\frac{1}{4}$	
			3	for $1\frac{2}{3} = \frac{5}{3}$.
5)	15	13	$6\frac{3}{4}$	
	3	2	$8\frac{11}{20}$	for $2\frac{3}{4} \div 5 = \frac{11}{20}$.

Hence the required value
$=$ £3. 2*s.* $8\frac{11}{20}$*d.*

Note. Before applying the above Rules of both Multiplication and Division, the compound and complex fractions must first be reduced to simple ones.

Examples LXXXII.

1. Multiply :—

(1) £3. 16*s.* $8\frac{3}{4}$*d.* by $\frac{3}{7}$; £6. 18*s.* $7\frac{5}{16}$*d.* by $\frac{9}{13}$; £10. 11*s.* $2\frac{3}{4}$*d.* by $3\frac{3}{4}$.
(2) *Rs.*50. 5*a.* 6*p.* separately by $9\frac{2}{3}$, $18\frac{9}{10}$, $53\frac{5}{13}$ and $156\frac{8}{9}$.
(3) £12. 5*s.* $7\frac{5}{8}$*d.* by $6\frac{3}{4}$; £13. 5*s.* $7\frac{1}{4}$*d.* by $7\frac{5}{11}$; £34. 12*s.* $5\frac{3}{4}$*d.* by $11\frac{7}{11}$.
(4) 5 tons 3 cwt. 6 lbs. separately by $4\frac{3}{8}$, $20\frac{7}{11}$, $46\frac{19}{20}$ and $213\frac{4}{7}$.
(5) 19 hrs. 43m. $56\frac{5}{13}$ sec. by $12\frac{7}{15}$; 10ac. 3ro. 37 po. $15\frac{5}{8}$yds. by $10\frac{4}{5}$.

2. Divide :—

(1) *Rs.*307. 4*a.* 4*p.* by $\frac{4}{25}$; *Rs.*76. 10*a.* 8*p.* by $\frac{12}{77}$; £5. 4*s.* 6¼*d.* by $3\frac{3}{5}$.
(2) £25 8*s.* 4¾*d.* by $\frac{3}{5}$; £4. 7*s.* 3¾*d.* by $\frac{12}{127}$; £34. 16*s.* 9¼*d.* by $9\frac{5}{8}$.
(3) *Rs.*173. 5*a.* 4*p.* separately by $2\frac{2}{3}$, $4\frac{3}{4}$, $8\frac{7}{8}$ and $20\frac{4}{5}$.
(4) 13 cwt. 3 qrs. 26 lbs. $15\frac{7}{8}$ oz. by $3\frac{4}{7}$; 15 ac. 3 ro. 25 po. by $\frac{15}{18}$.
(5) 8 days 15hrs. 48m. $57\frac{13}{18}$sec. by $4\frac{7}{9}$; 12 cub. yds. 20c.ft. 100in. by $1\frac{6}{7}$.
(6) 1 mi. 5 fur. 91 yds. 2 ft. by $2\frac{7}{8}$ of $1\frac{9}{11}$; 7 mds. 35 sr. by $1\frac{2}{3}$.

3. Find the respective values of :—

(1) $\frac{5}{16}$ of *Rs.*10. 8*a.* ; $\frac{3}{4}$ of *Rs.*2. 6*a.* ; $\frac{5}{8}$ of *Rs.*31. 8*a.* ; $\frac{3}{7}$ of *Re.*1. 12*a.*
(2) $3\frac{5}{11}$ of *Rs.*7 5*a.* 4*p.* ; $2\frac{3}{5}$ of *Rs.*51. 4*a.* ; $2\frac{4}{9}$ of $3\frac{2}{5}$ of *Rs.*173. 12*a.*
(3) $11\frac{7}{9}$ of 6*s.* 11¼*d.* ; $\frac{1}{3}$ of $5\frac{1}{3}$ of 2*s.* 9¾*d.* ; $\frac{2}{3}$ of $\frac{4}{11}$ of 16*s.* 6*d.*
(4) $\dfrac{3\frac{1}{7}}{1\frac{2}{9}}$ of £4. 14*s.* 6*d.* ; $\dfrac{1\frac{8}{11}}{1\frac{2}{3}}$ of £8. 8*s.* 5¼*d.* ; $\dfrac{3\frac{1}{7}}{1\frac{2}{9}}$ of *Rs.*15. 12*a.*
(5) $3\frac{1}{2}\frac{1}{4}$ of 3 mds. 10 sr. 8 ch. ; $2\frac{3}{5}\frac{3}{5}$ of 3 cwt. 3 qrs. 20 lbs.
(6) $\dfrac{3\frac{1}{3}}{7\frac{1}{2}}$ of 10 ft. $6\frac{1}{2}$ in. ; $\frac{5}{6}$ of $\dfrac{\frac{4}{7}}{4\frac{2}{7}}$ of $3\frac{3}{8}$ sq. yds. ; $\dfrac{3\frac{5}{6}}{7\frac{3}{5}}$ of $\dfrac{4}{7\frac{1}{4}}$ of $4\frac{3}{8}\frac{4}{7}$ cub.ft.
(7) $4\frac{1}{12}$ of $\dfrac{8}{1\frac{3}{4}}$ of 5 mi. 3 fur. 37 po. $4\frac{1}{3}$ yds. ; $\frac{3}{7}$ of $\dfrac{1}{3\frac{1}{3}}$ of $\frac{1}{12}$ of £$6304\frac{2}{7}$.
(8) $\dfrac{5\frac{9}{11}}{3\frac{5}{7}+\frac{2}{5}}$ of $(3\frac{2}{7}-\frac{3}{5})$ of 5 cwt. 2 qrs. 10 lbs. 7¾ oz. ; $\frac{3}{20}$ of $\dfrac{7}{9\frac{1}{3}}$ of 1 mile
(9) $\dfrac{3\frac{1}{7}}{3\frac{1}{3}}$ of $(3\frac{2}{7}+1\frac{2}{3})$ of 5 days $17\frac{2}{3}$ hrs. ; $\dfrac{4}{3\frac{1}{2}}$ of $\dfrac{4}{\frac{5}{7}-\frac{1}{9}}$ of *Rs.*10. 8*a.*
(10) $\dfrac{7\frac{2}{3}-3\frac{11}{12}}{18\frac{1}{2}+\frac{6}{7}}$ of 3 ac. 1 ro. 35 po. ; $\dfrac{\frac{1}{8}+\frac{1}{3}(\frac{3}{5}-\frac{1}{2})-\frac{6}{7}(\frac{4}{5}+\frac{1}{6})}{\frac{3}{4}(\frac{7}{8}-\frac{1}{2})-\frac{1}{3}(\frac{1}{7}-\frac{1}{10})}$ of £44. 17*s.*

4. Find the values of :—

(1) $\frac{3}{5}$ of *Rs.*3. 5*a.* 4*p* + $\frac{5}{7}$ of *Rs.*21. 14*a.* + $\frac{9}{11}$ of *Rs.*47. 3*a.* 4*p.*
(2) $1\frac{3}{4}$ of $3\frac{1}{3}$ of *Rs.*13. 8*a.* + $\frac{5}{8}$ of *Rs.*6. 10*a.* 8*p.* − $\frac{3}{7}$ of $1\frac{5}{9}$ of *Rs* 3. 5*a.* 4*p.*
(3) $\frac{3}{5}$ of 6*s.* 8*d.* + $\frac{4}{7}$ of £2. 3*s.* 9*d.* + $\frac{9}{11}$ of £4. 14*s.* 5*d.*
(4) $\frac{3}{7}$ of £15 + $\frac{1}{4}$ of $\dfrac{1}{2\frac{4}{9}}$ of $\frac{4}{5}$ of £1. 2*s.* + $\frac{4}{7}$ of 3*d.*
(5) $\dfrac{15\frac{5}{9}}{7\frac{4}{5}}$ of £1 + $\frac{1}{3}$ of £140. 10*s.* 6*d.* + $2\frac{1}{8}\frac{3}{5}$ of half-a-guinea.
(6) $\frac{7}{8}$ of £5 10*s.* 6*d.* − $\frac{4}{7}$ of 2 guineas + $\dfrac{2}{3\frac{1}{2}}$ of $\dfrac{4}{\frac{6}{7}-\frac{2}{9}}$ of £$\frac{5}{8}$.
(7) $\dfrac{1\frac{5}{11}}{1\frac{2}{5}}$ of *Rs.*84. 3*a.* 6*p.* − $\dfrac{3\frac{7}{11}}{4\frac{8}{9}}$ of $\dfrac{10\frac{5}{7}}{7\frac{1}{2}}$ of *Rs.*20. 4*a.*
(8) $\frac{3}{4}$ of 3 mds. 34 sr. + $\frac{3}{7}$ of 8 mds. 9 sr. + $\frac{2}{5}$ of 3 sr. 12 ch.
(9) $7\frac{3}{5}$ of a year of 365¼ days + $3\frac{9}{10}$ of $\frac{3}{8}$ of a week + $\frac{3}{4}$ of $5\frac{3}{5}$ hrs.
(10) 5 yds. 2 ft. $5\frac{4}{5}$ in. × $7\frac{3}{5}$ − 9 yds. 2 ft. $7\frac{3}{8}\frac{4}{5}$ in. − 15 yds. 1 ft. $9\frac{5}{8}$ in. ÷ $3\frac{5}{8}$ + 3 po. 3 yds. 2 ft. 3 in. ÷ $2\frac{1}{10}$.

305. Case II. *To reduce a compound quantity to a fraction of a higher denomination.*

Proceed as in the following Examples.

Ex. 1. Reduce 8*p*., 6*a*. 10*p*. and 14*a*. 4*p*. to the fraction of a rupee.

$8p. = \frac{8}{12}a. = \frac{2}{3} \times Re.\frac{1}{16} = Re.\frac{1}{24}.$

$6a.\ 10p. = 6a. + \frac{10}{12}a. = 6\frac{5}{6}a. = \frac{41}{6} \times Re.\frac{1}{16} = Re.\frac{41}{96}.$

$14a.\ 4p. = 14a. + \frac{4}{12}a. = 14\frac{1}{3}a = \frac{43}{3} \times Re.\frac{1}{16} = Re.\frac{43}{48}.$

Ex. 2. Express 9*d*., 2*s*. 4*d*. and 18*s*. $11\frac{3}{4}d$. in pounds.

$9d. = \frac{9}{12}s. = \frac{3}{4}s. = \frac{3}{4} \times £\frac{1}{20} = £\frac{3}{80}.$

$2s.\ 4d. = 2s. + \frac{4}{12}s. = 2\frac{1}{3}s. = \frac{7}{3} \times £\frac{1}{20} = £\frac{7}{60}.$

$18s.\ 11\frac{3}{4}d. = 18s. + \frac{47}{48}s. = \frac{911}{48}s. = \frac{911}{48} \times £\frac{1}{20} = £\frac{911}{960}.$

Ex. 3. Reduce £4. 9*s*. $2\frac{1}{4}d$. to pounds.

$9s.\ 2\frac{1}{4}d. = 9s. + \frac{9}{48}s. = 9\frac{3}{16}s. = \frac{147}{16} \times £\frac{1}{20} = £\frac{147}{320}.$

$\therefore £4.\ 9s\ 2\frac{1}{4}d. = £4\frac{147}{320}.$ *Ans.*

Ex. 4. Reduce 5 cwt. 3 qrs. 24 lbs. to the fraction of a ton

3 qrs. 24 lbs. $= 3$ qrs. $+ \frac{24}{28}$ qrs. $= 3\frac{6}{7}$ qrs. $= \frac{1}{4} \times 3\frac{6}{7}$ cwt.

$= \frac{1}{4} \times \frac{27}{7}$ cwt. $= \frac{27}{28}$ cwt

$\therefore$ 5 cwt. $+ \frac{27}{28}$ cwt. $= 5\frac{27}{28}$ cwt. $= \frac{1}{20} \times 5\frac{27}{28}$ ton.

Hence 5 cwt. 3 qrs. 24 lbs $= \frac{1}{20} \times \frac{167}{28}$ ton $= \frac{167}{560}$ ton. *Ans.*

Examples LXXXIII.

1. Reduce 3*a*. 6*p*. ; 5*a*. ; 5*a*. 10*p*. ; 6*a*. 10*p*. ; 7*a*. 8*p*. ; 13*a*. $7\frac{1}{5}p$. ; 15*a*. $7\frac{3}{5}p$. ; each to the fraction of a rupee.

2. Express 4*s*. 11*d*. : 17*s*. $11\frac{1}{3}d$. ; 19*s*. $10\frac{4}{7}d$. ; 6*s*. $11\frac{3}{4}d$. ; 14*s*. $4\frac{5}{7}d$. ; 16*s*. $9\frac{3}{4}d$. ; each as the fraction of a pound.

3. Express £1. 13*s*. $11\frac{1}{2}d$. $\frac{10}{11}q$. ; £3. 19*s*. $8\frac{1}{4}d$. ; £37. 16*s* $6\frac{3}{4}d$. ; £5. 16*s*. $11\frac{3}{4}d$. $\frac{5}{8}q$. ; each in pounds.

4. Reduce *Rs*.3. 10*a*. 8*p*. ; *Rs*.8. 5*a*. 4*p*. ; *Rs*.15. 10*a*. 7*p*. ; *Rs*.81. 7*a*. $3\frac{1}{2}ps$. to rupees.

5. Reduce 2 cwt. 1 qr. 16 lbs. to the fraction of a ton ; 3 qrs. 27 lbs. 9 oz. $12\frac{4}{9}$ drs to the fraction of a cwt. ; 2 sr. 15 ch. 2 kan. to the fraction of a maund.

6. What fraction is 2 ft. 9 in. of a pole ; 23 po. 4 yds. of a mile ; and 3 ro. 26 po. of an acre ?

7. Reduce 3 *fur*. 29 po. 4 yds. 1 ft. 9 in. to the fraction of a mile, and 1 sq. ft. $\frac{7}{8}$ sq. in. to the fraction of a sq. yd.

8. Reduce 4 mds. 37 sr. 8 ch to maunds ; 2 bi. 15 kat. 5 ch. to bighas ; and 2 qts. $1\frac{1}{2}$ pt. to the fraction of a barrel.

9. Express 5 bus. 3 pks. 1 gal. as the fraction of a quarter.

10. Express 2 wks. 5 days 18 hrs. as the fraction of a year of 365 days, and 3 ro. $27\frac{1}{2}$ po. as the fraction of an acre.

11. Reduce 72 days 6 hrs. 56 m. 15 sec. to the fraction of a year of $365\frac{1}{4}$ days, and 1 sc. 13 grs. to the fraction of a ℔.

12. Express 1 day 1 hr. 4 dan. 30 pals and 3 mo. 12 days as fractions of a year.

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306. *To find what fraction one concrete quantity is of any other of the same kind.*

RULE. Reduce both the quantities to the same denomination; then the fraction whose numerator is the first and denominator the second of these results, will be the one required.

Ex. 1. Reduce *Rs.*2. 8*a.* 2*p.* to the fraction of *Rs.*3. 12*a.*

*Rs.*2. 8*a.* 2*p.* = 482*p.*; and *Rs.*3. 12*a.* = 720*p.*

$\therefore$ the required fraction $= \frac{482p.}{720p.} = \frac{241}{360}$. *Ans.*

Ex. 2. What part of $4\frac{3}{8}$ of £1 is $3\frac{3}{4}$ of a guinea?

$3\frac{3}{4}$ of a gui. $= \frac{15}{4} \times 21s. = \frac{315}{4}s.$, and $4\frac{3}{8}$ of £1 $= \frac{35}{8}$ of $20s. = \frac{175}{2}s.$

$\therefore$ the required fraction $= \frac{315}{4}s. \div \frac{175}{2}s. = \frac{9}{10}$. *Ans.*

Ex. 3. What fraction is 1 md. 4 sr. of 2 mds. 32 sr.?

1 md. 4 sr. = 44 sr.; and 2 mds. 32 sr. = 112 sr.

$\therefore$ the required fraction $= \frac{44 \text{ sr.}}{112 \text{ sr.}} = \frac{11}{28}$. *Ans.*

307. By means of the preceding Articles, magnitudes of the same kind, consisting of fractions of simple or compound quantities, and connected by the operations of Addition or Subtraction, may be reduced to simple fractions of a given denomination.

Ex. 1. Express $\frac{2}{3}$ of a guinea $-\frac{3}{4}$ of a shilling $-\frac{4}{9}$ of 7*s.* 6*d.* as the fraction of £2. 19*s.* 6*d.*

Here, $\frac{2}{3}$ of a guinea $= \frac{2}{3} \times £\frac{21}{20} = £\frac{7}{10}$; $\frac{3}{4}s. = £\frac{3}{4} \times \frac{1}{20} = £\frac{3}{80}$;

and $\frac{4}{9}$ of 7*s.* 6*d* $= \frac{4}{9}$ of $7\frac{1}{2}s. = £\frac{4}{9} \times \frac{15}{2} \times \frac{1}{20} = £\frac{1}{6}$.

$\therefore$ the exp. $= £(\frac{7}{10} - \frac{3}{80} - \frac{1}{6}) = £\frac{119}{240}$; also £2. 19*s.* 6*d.* $= £2\frac{39}{40} = £\frac{119}{40}$.

$\therefore$ the required fraction $= \frac{119}{240} \div £\frac{119}{40} = \frac{119}{240} \times \frac{40}{119} = \frac{1}{6}$. *Ans.*

Ex. 2. Reduce $\frac{7}{9}$ of *Rs.*10 $-\frac{2}{5}$ of *Rs.*10. 8*a.* to the fraction of *Rs.*5. 4*a.*

$\frac{7}{9}$ of *Rs.*10 = *Rs.*$\frac{70}{9}$; $\frac{2}{5}$ of *Rs.*10. 8*a.* $= \frac{2}{5}$ of *Rs.*$10\frac{1}{2}$ = *Rs.*$\frac{21}{5}$.

$\therefore$ the difference = *Rs.*$(\frac{70}{9} - \frac{21}{5})$ = *Rs.*$\frac{161}{45}$; also *Rs.*5. 4*a.* = *Rs.*$\frac{21}{4}$.

$\therefore$ the required fraction = *Rs.*$\frac{161}{45} \div$ *Rs.*$\frac{21}{4} = \frac{161}{45} \times \frac{4}{21} = \frac{92}{135}$. *Ans.*

Ex. 3. What fraction of *Rs.*10. 8*a.* together with *Rs.*2. 4*a.* is equivalent to *Rs.*7. 8*a.* ?

Here, *Rs.*7. 8*a.* − *Rs.*2. 4*a.* = *Rs.*5. 4*a.*

Now, the question reduces itself to finding—What fraction of *Rs.*10. 8*a.* is *Rs.*5. 4*a.* ?

*Rs.*5. 4*a.* = 84*a.* ; and *Rs.*10. 8*a.* = 168*a.*

$\therefore$ the fraction required $= \frac{84a.}{168a.} = \frac{1}{2}$. *Ans.*

Ex. 4. What fraction of £2. 10*s.* is the sum which being diminished by 10*s.* 6*d.* is equal to £2. 2*s.* ?

The meaning is—What fraction of £2. 10*s.* is £2. 2*s.* + 10*s.* 6*d.* ?

£2. 2*s.* + 10*s.* 6*d.* = £2. 12*s.* 6*d.* = £$2\frac{5}{8}$ = £$\frac{21}{8}$;

also £2. 10*s.* = £$2\frac{1}{2}$ = £$\frac{5}{2}$.

$\therefore$ the required fraction = £$\frac{21}{8}$ ÷ £$\frac{5}{2}$ = $\frac{21}{8} \times \frac{2}{5} = \frac{21}{20} = 1\frac{1}{20}$. *Ans*

Ex. 5. Compare the values of $\frac{7}{8}$ of £1, $\frac{5}{9}$ of a guinea, and $\frac{2}{3}$ of 15*s.* 7½*d.*

$\frac{7}{8}$ of £1 = £$\frac{7}{8}$; $\frac{5}{9}$ of a guinea = $\frac{5}{9}$ × £$\frac{21}{20}$ = £$\frac{7}{12}$; $\frac{2}{3}$ of 15*s.* 7½*d.* = £$\frac{25}{48}$.

Now, to compare £$\frac{7}{8}$, £$\frac{7}{12}$, £$\frac{25}{48}$, reduce them to equivalent fractions with the same denominator and proceed as in Art. 257.

The L. C. M. of the denominators 8, 12, 48 is 48 ;

$\therefore$ £$\frac{7}{8}$ = £$\frac{7 \times 6}{8 \times 6}$ = £$\frac{42}{48}$; £$\frac{7}{12}$ = £$\frac{7 \times 4}{12 \times 4}$ = £$\frac{28}{48}$; £$\frac{25}{48}$ = £$\frac{25 \times 1}{48 \times 1}$ = £$\frac{25}{48}$.

$\therefore$ $\frac{7}{8}$ of £1 is the *greatest* and $\frac{2}{3}$ of 15*s.* 7½*d.* is the *least.* *Ans.*

Ex. 6. What sum is that $\frac{2}{5}$ of which is *Rs.*2. 10*a.* 8*p.* ?

*Rs.*2. 10*a.* 8*p.* = *Rs.*$2\frac{2}{3}$ = *Rs.*$\frac{8}{3}$.

$\therefore$ the required sum = *Rs.*$\frac{8}{3}$ ÷ $\frac{2}{5}$ = *Rs.*$\frac{8}{3} \times \frac{5}{2}$ = *Rs* 6. 10*a.* 8*p.* *Ans.*

Examples LXXXIV.

1. Express :—

(1) *Rs.*4. 15*a.* 4*p.* as the fraction of *Rs.*6. 11*a.* 4*p.* ; and *Rs.*7. 7*a.* 2*p.* as the fraction of *Rs.*31. 4*a.* 6*p.*

(2) *Rs.*25. 0*a.* 6*p.* as the fraction of *Rs.*29. 8*a.* 6*p.*

(3) £1. 7*s.* 8¼*d.* as the fraction of £2. 7*s.* 6*d.* ; and £3. 4*s.* 0¾*d.* as the fraction of £7. 7*s.* 11*d.*

(4) 13*s.* 10½*d.* ⅓*q.* as the fraction of £2. 9*s.* 7*d.*

(5) $\frac{5}{11}$ of £5. 17*s.* 4*d.* and $\frac{3}{7}$ of $\frac{6\frac{2}{3}}{9\frac{1}{10}}$ of £1. 12*s.* 1½*d.* as fractions of £10.

(6) 2 fur. 29 po. 2 ft. 10 in. as the fraction of 1 ml. 5 fur. 16½ po.

(7) £22. 13*s.* 8¾*d.* as the fraction of 3½ guineas.

(8) $\dfrac{7\frac{3}{4}-3\frac{11}{12}}{18\frac{1}{2}+\frac{6}{7}}$ of £33. 14*s.* $5\frac{3}{4}$*d.* as the fraction of £157. 17*s.* $8\frac{1}{2}$*d.*

2. Reduce :—

(1) $\frac{3}{8}$ of 2*s.* $4\frac{1}{2}$*d.* to the fraction of a half-crown ; and 9*s.* $10\frac{1}{2}$*d.* to the fraction of 13*s.* $2\frac{1}{4}$*d.*

(2) $6\frac{3}{7}\frac{3}{7}$ of *Rs.*15. 3*a.* 10*p.* to the fraction of *Rs.*31. 8*a.* 2*p.*

(3) $33\frac{1}{3}$ of 1 md. 2 sr. to the fraction of $3\frac{3}{4}$ of 28 mds. ; and 32 seers to the fraction of 3 mds. 22 sr. 2 ch.

(4) $2\frac{2}{3}$ of 2 bi. 7 kat. 4 ch. to the fraction of 4 bi. 11 kat.

(5) 1 md. 11 sr. 8 ch. to the fraction of 28 mds. ; and $12\frac{3}{8}$ of 15 sr. 12 ch. to the fraction of 30 mds. 32 sr.

(6) 3 qrs. 1 pt. $2\frac{2}{3}$ gills to the fraction of 5 gals. 2 qts. 1 pt.

(7) 2 sq. yds. 2 ft. 120 in. to the fraction of 3 sq. po. $13\frac{1}{4}$ yds. 1ft. 72in

(8) 12 oz. $12\frac{4}{5}$ drs. Avoir. to the fraction of 1 lb. Troy ; and 35 lbs. $8\frac{3}{4}$ oz. Troy to the fraction of a cwt.

(9) $2\frac{2}{3}$ half-guineas to the fraction of 10*s.* $11\frac{1}{4}$*d.*

(10) $7\frac{18}{21}$ of 10 oz. 18 dwts. 11 grs. to the fraction of 8 lbs. $8\frac{8}{15}$ oz. Avoir.

3. What part of $\frac{3}{5}$ of $\frac{6}{7}$ of 3 guineas is $\frac{3}{4}$ of $\frac{5}{8}$ of 15*s.* 9*d.* ?

4. What part of 13 cwt. 2 qrs. 21 lbs. is 11cwt. 1qr. 14lbs. 15oz. ?

5. What part 6 ft. $3\frac{3}{8}$ in. of 13 ft. $8\frac{9}{16}$ in. ?

6 What part of a maund is 10 sr. 13 ch. 2 kan. ?

7. What fraction of $\frac{11}{13}$ of *Rs.*2. 5*a.* 8*p.* is $3\frac{1}{2}$ of $3\frac{1}{3}$ of *Rs.*12. 9*a.* 3*p.*, and of 7 guineas is $\frac{3}{8}$ of a moidore ?

8 What fraction of 3 cwt. 2 qrs. 14 lbs. is 3 cwt. 19 lbs. 2 oz. ?

9. What fraction of a year of $365\frac{1}{4}$ days is 27 days 16 hrs. 29 min. 4 sec., and of 1 oz. Avoir. is 1 oz. Troy ?

10. What fraction of $19\frac{8}{8}\frac{1}{3}$ of 4 cub. yds. 18 ft. 1127 in. is $\frac{7}{15}$ of 200 cub. yds., and of $2\frac{7}{8}$ miles is $3\frac{7}{8}$ furlongs ?

11. What fraction of $\dfrac{7\frac{4}{5}}{4\frac{3}{8}}$ of *Rs.*306. 9*a.* 10*p.* is $(8\frac{4}{7}-3\frac{5}{8})$ of *Rs.*54. 15*a.* 8*p.*, and of $2\frac{1}{2}$ tons is $\frac{5}{8}$ of 2 lbs. ?

12. What fraction of 8 lbs. $12\frac{3}{8}$ oz. is 3 lbs. 9 oz. $62\frac{1}{2}$ grs. ?

13. How many times is :—

(1) *Rs.*9. 12*a.* $4\frac{1}{2}$*p.* contained in *Rs* 7. 9*a.* $7\frac{1}{2}$*p.* ?

(2) £24. 16*s.* $4\frac{1}{2}$*d.* contained in £335. 1*s.* $0\frac{3}{4}$*d.* ?

(3) 2 tons 2 cwt. 2 qrs. contained in 3 cwt. 14 lbs. ?

(4) 7 kathas 9 ch. contained in a bigha ?

14. Express $\dfrac{3\frac{3}{8}}{7\frac{4}{5}}$ of $\dfrac{4}{7\frac{1}{2}}$ of *Rs.*33. 11*a.* 6*p.*, and $\frac{4}{5}$ of $\dfrac{7}{3\frac{4}{5}}$ of *Rs.*19. 1*a.* 7*p.* in terms of *Rs.*70. 5*a.* 10*p.* as unit.

15. What is the measure of $7\frac{3}{8}$ of $\frac{3\frac{3}{4}}{3\frac{17}{19}}$ of 5 cwt. 3 qrs. $3\frac{1}{2}$ lbs., when the unit is $(5\frac{3}{8}-3\frac{1}{4})$ of 3 tons 16 cwt. 3 qrs. $22\frac{2}{5}$ lbs. ?

16. Express :—

(1) $\frac{1}{3}$ of $\frac{7}{10}$ of 13*s*. 4*d*.+$\frac{3}{4}$ of $\frac{5}{7}$ of 10*s*. 6*d*. as the fraction of £1.

(2) $\frac{1}{3}$ of a guinea+$\frac{3}{8}$ of £1+$\frac{5}{24}$ of 1*s*.+$\frac{1}{4}$ of 1*d*. as the fraction of a guinea, and of £24. 3*s*.

(3) $\frac{3}{8}$ of *Rs*.2. 8*a*.+$\frac{1}{8}$ of 8*a*. as the fraction of *Rs*.10. 8*a*.

(4) $\frac{2}{5}$ of *Rs*.3. 8*a*.+$\frac{3}{7}$ of *Rs*.5. 4*a*.−$\frac{3}{8}$ of *Rs*.10. 8*a*. as the fraction of *Rs*.13. 8*a*., and of *Rs*.39. 8*a*.

(5) $\frac{2}{11}$ of £13. 10*s*. $10\frac{1}{2}$*d*.−$\frac{5}{7}$ of £1. 2*s*. 9*d*. as the fraction of £6.

(6) *Rs*.$7\frac{3}{7}$−$\frac{3}{7}$ of *Rs*.7 as the fraction of *Rs*.103. 5*a*. 4*p*.

(7) $\frac{3\frac{1}{6}}{1\frac{1}{18}}$ of $\left\{\frac{19}{120}\text{ of £}1-\frac{17}{48}\text{ of }1s.\right\}$ as the fraction of 27*s*.

17. Compare the values of :—

(1) $\frac{1}{19}$ of £1, $\frac{1}{20}$ of a guinea and $\frac{8}{85}$ of a crown.

(2) $\frac{1}{21}$ of £1, $\frac{1}{22}$ of £1. 1*s*. and $\frac{1}{4}$ of 3*s*. $9\frac{1}{4}$*d*.

(3) $\frac{7}{8}$ of *Rs*.10, $\frac{5}{6}$ of *Rs*.10. 8*a*. and $\frac{2}{3}$ of *Rs*.7. 13*a*.

(4) $\frac{1}{5}$ of a maund, $\frac{1}{6}$ of 14 sr. and $\frac{5}{6}$ of 3 sr. 6 ch.

(5) $\frac{2}{5}$ of 5 days, $\frac{6}{7}$ of 20 hours, and $\frac{4}{5}$ of 59 min.

18. What fraction of *Rs*.100 together with *Rs*.36. 12*a*. is equivalent to *Rs*.52. 8*a*. ?

19. What fraction of 3 mds. 20 sr. together with 1 md. 9 sr. will give 42 mds. ?

20. What fraction of a ton added to $\frac{11}{18}$ of 2 cwt. will make it equal to 1 cwt. 2 qrs. 11 lbs. ?

21. What fraction of 2 tons 12 lbs. is the weight which being diminished by 1 cwt. 20 lbs. is equal to 1 cwt. 1 qr. 8 lbs. ?

22. What fraction of *Rs*.29. 12*a*. must be added to $\frac{3\frac{1}{3}}{4\frac{1}{3}}$ of $(3\frac{5}{7}+4\ 1\frac{2}{3})$ of *Rs*.6. 9*a*. to make the sum equal to *Rs*.32. 8*a*. ?

23. What fraction of a mile diminished by 39 yds. 1 ft. 9 in. is equal to 87 yds. 9 in. ?

24. What fraction of 2 lbs. 10 oz. Avoir. must be added to 1 lb. 8 oz. Troy to give 3 lbs. 7 oz. 10 dwts. ?

25. What sum is that $\frac{2}{3}$ of $\frac{1}{3}$ of which is $\frac{1}{7}$ of $\frac{2}{3}$ of *Rs*.5. 10*a*. ?

26. What length is that $\frac{27}{80}$ of which is $\frac{3}{8}$ of $7\frac{1}{3}$ of $16\frac{1}{2}$ yards ?

27. What is the sum $\frac{1\frac{1}{3}}{150}$ of which is $(4\frac{1}{3}\frac{4}{7}-10\frac{4}{5}+9\frac{3}{4}-\frac{62}{117})$ of [illegible], and what fraction is it of $\frac{8}{75}$ of *Rs*.6. 8*a*. ?

28. What weight is the same fraction of 15 cwt. 2 qrs. 13 lbs. that £1. 11*s*. $10\frac{1}{2}$*d*. is of £3. 10*s*. $10\frac{1}{2}$*d*. ?

XIII. SIMPLIFICATION OF CONCRETE FRACTIONS.

308. It should be borne in mind what has already been said that when a concrete number is divided by another concrete number of the same kind, the quotient is an abstract number.

Ex. 1. Simplify $\dfrac{\frac{6}{7} \text{ of } 1\frac{3}{4} \text{ of } Rs.37.\ 6a.}{Rs.65.\ 6a.\ 6p.} \times \dfrac{1\frac{9}{16} \text{ of } 16 \text{ cwt. } 3 \text{ qrs.}}{41 \text{ cwt. } 3\frac{1}{2} \text{ qrs.}}$

The result $= \dfrac{\frac{6}{7} \text{ of } \frac{7}{4} \text{ of } Rs.37\frac{3}{8}}{Rs.65\frac{13}{32}} \times \dfrac{1\frac{9}{16} \text{ of } 67 \text{ qrs.}}{167\frac{1}{2} \text{ qrs.}} = \dfrac{\frac{6}{7} \text{ of } \frac{7}{4} \text{ of } \frac{299}{8}}{\frac{2093}{32}}$

$\times \dfrac{\frac{25}{16} \text{ of } 67}{\frac{335}{2}} = \left(\dfrac{6 \times 7 \times 299}{7 \times 4 \times 8} \times \dfrac{32}{2093}\right) \times \left(\dfrac{25 \times 67}{16} \times \dfrac{2}{335}\right)$

$= \frac{6}{7} \times \frac{5}{8} = \frac{15}{28}$. *Ans.*

Ex. 2. Simplify $\dfrac{£1.\ 11s.\ 8d.}{£2.\ 17s.\ 0d.}$ of $\dfrac{142 \text{ yds. } 0\frac{4}{5} \text{ ft.}}{2 \text{ yds. } 1\frac{7}{10} \text{ ft.}}$ of 13 days 3 hrs.

The result $= \dfrac{31\frac{2}{3}s.}{57s.}$ of $\dfrac{426\frac{4}{5} \text{ ft.}}{7\frac{7}{10} \text{ ft.}}$ of $13\frac{1}{8}$ days $= \left(\dfrac{95}{3} \times \dfrac{1}{57}\right)$ of

$\left(\dfrac{2134}{5} \times \dfrac{10}{77}\right)$ of $\dfrac{105}{8}$ days $= \dfrac{95}{3 \times 57} \times \dfrac{2134 \times 10}{5 \times 77} \times \dfrac{105}{8}$ days

$= \dfrac{5}{9} \times \dfrac{388}{7} \times \dfrac{105}{8}$ days $= \dfrac{2425}{6}$ days $= 404\frac{1}{6}$ days $=$ 404 days 4 hrs. *Ans.*

Examples LXXXV.

Simplify the following :—

1. $1\frac{1}{2}$ of $\dfrac{4\frac{1}{5}}{5\frac{1}{4}}$ of $\dfrac{18s.\ 6\frac{3}{4}d.}{£1}$ of 3 days 2 hrs. **2.** $\dfrac{£3\frac{9\frac{1}{2}}{14}}{8s.}$ of 2 mds. 32 sr.

3. $\dfrac{14 \text{ lbs. } 8 \text{ oz. } 18 \text{ dwts.}}{1 \text{ lb. } 0 \text{ oz. } 10 \text{ dwts.}}$ of 5*s*. $2\frac{1}{2}$*d*. **4.** $\dfrac{7 \text{ lbs. } 3 \text{ oz.}}{11 \text{ lbs. } 8 \text{ oz.}}$ of 8 po. 1 ft. 4 in.

5. $\dfrac{15s.\ 10d.}{£1.\ 18s.\ 6d.}$ of $\dfrac{71 \text{ yds. } 0\frac{3}{8} \text{ ft.}}{5 \text{ yds. } 0\frac{3}{8} \text{ ft.}}$ of $2021\frac{1}{4}$ days.

6. $\dfrac{\frac{1}{2} - \frac{3}{8}}{6\frac{1}{8} - 1\frac{3}{7} \text{ of } 4\frac{1}{5}}$ of $\dfrac{£1.\ 11s.\ 8d.}{£2.\ 17s.}$ of $\dfrac{142 \text{ yds. } 0\frac{4}{5} \text{ ft.}}{2 \text{ yds. } 1\frac{7}{10} \text{ ft.}}$ of 13 days 3 hrs.

7. $\dfrac{7 \text{ tons } 13 \text{ cwt. } 1\frac{1}{3} \text{ qrs. } 14 \text{ lbs.}}{15 \text{ cwt. } 3 \text{ qrs. } 7 \text{ lbs. } 12 \text{ oz.}}$ of $\dfrac{£15.\ 10s.\ 6\frac{3}{4}d.}{8s.\ 9d.}$.

8. $\dfrac{£10.\ 17s.\ 6d.}{£19.\ 6s.\ 8d.} + \dfrac{5 \text{ years } 73 \text{ days}}{18 \text{ hrs. } 40 \text{ min.}} - \dfrac{Rs.4.\ 6a.\ 8p.}{Rs.5.\ 8a.}$.

9. $\left(\frac{3\frac{1}{2} \text{ of } 5\frac{5}{8}}{2\frac{5}{8} \text{ of } 3\frac{1}{3}} + \frac{2\frac{2}{11} \text{ of } \frac{11}{12}}{3\frac{1}{9} \text{ of } 7\frac{5}{7}}\right)$ of $\frac{1s.\ 5d.}{4s.\ 7d.}$ of $\frac{2 \text{ ft. } 3 \text{ in.}}{5 \text{ ft. } 5 \text{ in.}}$ of 24 weeks 4 days 19 hrs.

10. Reduce $\frac{£2.\ 3s.\ 4d.}{£5.\ 6s\ 8d.}$ of $\frac{2 \text{ tons } 4 \text{ cwt.}}{5 \text{ tons } 10 \text{ cwt.}}$ to a complex fraction having $12\frac{1}{2}$ for its numerator, and also to a complex fraction having $5\frac{1}{3}$ for its denominator.

XIV. MISCELLANEOUS PROPOSITIONS.

(ON VULGAR FRACTIONS.)

309. The Unitary Method. We have in Art. 171 given an outline of this method and treated it in the case of *integers*. We now propose to extend the method to fractional quantities. The following solutions, we hope, will serve as a guide to the students.

If the value, weight, length, &c. of **one** thing be given, the value, weight, length, &c. of **any number** of them (whether *integral* or *fractional* or *mixed*) may always be found by Multiplication; and *conversely*, if the value, weight, length, &c. of any **number** of things (whether *integral* or *fractional* or *mixed*) be given, the value, weight, length, &c. of **one** of them may always be found by Division.

Ex. 1. If a yard of lace cost *Re.*1. 6*a.* 6*p.* what will 7 yds. 4 in. cost?

Here, *Re.*1. 6*a.* 6*p.* $=$ *Rs.*$1\frac{13}{32}$; and 7 yds 4 in. $= 7\frac{1}{9}$ yds.

The cost of 1 yard $=$ *Rs.*$1\frac{13}{32}$;

$\therefore$ the cost of $7\frac{1}{9}$ yds $=$ *Rs.*$1\frac{13}{32} \times 7\frac{1}{9} =$ *Rs.*$\frac{45}{32} \times \frac{64}{9} =$ *Rs.*10. *Ans.*

Ex. 2. If the cost of $20\frac{4}{5}$ yds. of cloth be *Rs.*173. 5*a.* 4*p.*, find the cost per yard of the same quality.

The cost per yard $=$ *Rs.*173. 5*a.* 4*p.* $\div 20\frac{4}{5} = \frac{Rs.173.\ 5a.\ 4p. \times 5}{104}$

$=$ *Re.*1. 10*a.* 8*p.* $\times 5 =$ *Rs.*8. 5*a.* 4*p.* *Ans.*

Ex. 3. If $3\frac{2}{5}$ lbs. of tea cost *Rs.*7. 10*a.*, how much can I buy for *Rs.*41. 15*a.*?

Here, *Rs.*7. 10*a.* $=$ *Rs.*$7\frac{5}{8}$; and *Rs.*41. 15*a.* $=$ *Rs.*$41\frac{15}{16}$.

The cost of $3\frac{2}{5}$ lbs. $=$ *Rs.*$7\frac{5}{8}$;

$\therefore$ the cost of 1 lb. $=$ *Rs* $7\frac{5}{8} \div 3\frac{2}{5} =$ *Rs.*$(\frac{61}{8} \times \frac{5}{17})$;

$\therefore$ the reqd. no. of lbs. $=$ *Rs.*$41\frac{15}{16} \div$ *Rs.*$(\frac{61}{8} \times \frac{5}{17})$

$= \frac{671}{16} \times \frac{8}{61} \times \frac{17}{5} = \frac{187}{10} = 18\frac{7}{10}$ *Ans.*

Ex. 4. If $\frac{2}{3}$ of an estate be worth *Rs.*2200, find the value of $\frac{8}{11}$ of it.

$\frac{2}{3}$ of the estate is worth *Rs.*2200;

∴ the whole estate is worth $Rs.2200 \div \frac{2}{3} = Rs\ 3300$;

∴ $\frac{3}{11}$ of the estate is worth $Rs.3300 \times \frac{3}{11} = Rs.900.$ *Ans.*

Ex. 5. A person, possessed of $\frac{2}{5}$ths of a coal mine, sells $\frac{3}{4}$ths of his share for £2000 ; what is the whole mine worth ?

Here, the part sold $= \frac{3}{4}$ of $\frac{2}{5}$ of the whole mine $= \frac{3}{10}$ of the mine.

The cost of $\frac{3}{10}$ of the mine = £2000 ;

∴ the cost of the whole mine = £2000 × $\frac{10}{3}$ = $\frac{1}{3}$ × £20000

= £6666. 13*s.* 4*d.* *Ans.*

Ex. 6. Express $\frac{2}{3}$ of $1\frac{5}{9}$ of a mile in terms of a metre, supposing 32 metres = 35 yards.

35 yards = 32 metres ; ∴ 1 yard = (32 ÷ 35) metres ;

∴ $\frac{2}{3}$ of $1\frac{5}{9}$ of a mile $= (\frac{2}{3} \times \frac{14}{9} \times 1760)$ yds.

$= (\frac{2}{3} \times \frac{14}{9} \times 1760 \times 32 \div 35)$ metres

$= \frac{45056}{27}$ metrese $1668\frac{20}{27}$ metres.

Ex. 7. If 5 men or 7 women can do a piece of work in 37 days ; in what time will 7 men and 5 women do the same piece of work ?

The work of 5 men = that of 7 women ;

∴ the work of 1 man = that of $\frac{7}{5}$ women,

∴ the work of 7 men = that of $\frac{49}{5}$ women ;

∴ the work of 7 men + 5 women = that of $(\frac{49}{5} + 5)$ women,

= that of $\frac{74}{5}$ women.

Now, 7 women do the work in 37 days,

∴ 1 woman does the work in (37 × 7) days,

∴ $\frac{74}{5}$ women do ....in $(37 \times 7 \div \frac{74}{5})$ days.

Hence the required time $= 37 \times 7 \times \frac{5}{74}$ days $= 17\frac{1}{2}$ days. *Ans.*

Ex. 8. If the six-penny loaf weigh $4\frac{2}{3}$ lbs. when wheat is 6*s.* 9*d.* a bushel, what is the price of wheat per bushel when the same loaf weighs $3\frac{1}{2}$ lbs. ?

The loaf weighs $4\frac{2}{3}$ lbs. when wheat is $6\frac{3}{4}$*s.* a bus. ;

∴ ... ... 1 lb. ... ... $(6\frac{3}{4} \times 4\frac{2}{3})$*s.* a bus. ;

∴ ... ... $3\frac{1}{2}$ lbs. ... ... $(6\frac{3}{4} \times 4\frac{2}{3} \div 3\frac{1}{2})$ *s.* a bus.

Hence the required price $= \frac{27}{4} \times \frac{14}{3} \times \frac{2}{7}$*s.* = 9*s.* *Ans.*

Ex. 9. If 1000 men have provisions for 85 days, and if after 17 days, 150 of the men go away, find how long the remaining provisions will serve the number left.

Here 85 − 17 = 68 ; and 1000 − 150 = 850.

After 17 days, 1000 men have provisions for 68 days.

∴ 10 men ... ... for (68 × 100) days.

∴ 850 men ... ... for (68 × 100 ÷ 85) days,

or 80 days. *Ans.*

Ex. 10. If the cost of maintaining a family be *Rs.*50 a month, when rice is 12 seers a rupee, and *Rs.*48 when rice is 14 seers a rupee ; what will be the cost when rice is 16 seers a rupee ?

The price of 1 sr. is first reduced from *Re.*$\frac{1}{12}$ to *Re.*$\frac{1}{14}$ and lastly to *Re.*$\frac{1}{16}$.

Now, $\frac{1}{12}-\frac{1}{14}=\frac{1}{84}$ and $\frac{1}{14}-\frac{1}{16}=\frac{1}{48}$; also $50-48=2$.
Since a reduction of *Re.*$\frac{1}{84}$ in price causes a diff. of *Rs.*2 in expenses

∴ ... of *Re.*1 ... ... *Rs.*(2 × 84)..............

∴ ... of *Re.*$\frac{1}{48}$ ... ... *Rs.*$\frac{2 \times 84}{48}$..............

or *Rs.*3. 8*a*......... .....

Hence the required expenses = (*Rs.*50 − *Rs.*3. 8*a.*) = *Rs.*46. 8*a.* *Ans.*

Examples LXXXVI.

1. Find the value of $5\frac{2}{7}$ yds. of silk, when $3\frac{4}{7}$yds. cost *Rs.*21. 14*a.*

2. If $12\frac{1}{3}$ articles cost *Rs.*26. 3*a.* 4*p.*, how many can be bought for *Rs.*117. 3*a.* 8*p.* ?

3. If 3 cwt. 3 qrs. 21 lbs. $12\frac{4}{5}$ oz. cost £4. 8*s.* 9*d.*, what is the price per cwt. ?

4. If a silver cup weighing 20 oz. 19 dwts. $2\frac{3}{11}$ grs. cost *Rs.*57. 10*a.*, what is the price per oz. ?

5. If $4\frac{5}{8}$ oz. of tea cost $8\frac{2}{5}$*s.*, what will $30\frac{3}{7}$ lbs. cost ?

6. If $\frac{3}{16}$ of a lottery ticket cost £4. 10*s*, what is the price of $\frac{1}{3}$ of a ticket ?

7. The owner of $\frac{4}{17}$ of a ship sold $\frac{8}{11}$ of $\frac{3}{5}$ of his share for £$12\frac{4}{33}$; what would $\frac{2\frac{1}{2}}{4\frac{1}{4}}$ of $\frac{2}{5}$ of it cost, at the same rate ?

8. Express a degree of $69\frac{1}{2}$ miles in metres, where 32 metres are equal to 35 yards.

9. If the sum paid for 247 bottles of wine amount, together with the duty, to *Rs.*774. 7*a.* 2*p.* ; and the duty on each bottle be $\frac{1}{7}$th part of its original cost ; what is the duty per bottle ?

10. If the rent of 39 ac. 2 ro. 20 po. be *Rs.*1485. 15*a.*, what is the rent of 6 acres ?

11. If $\frac{1}{8}$ of a ship be worth *Rs.*365. 5*a.*, what share of it will cost *Rs.*1252. 8*a.* ?

12. A ship is worth *Rs.*160000 and a person possessed of $\frac{4}{15}$ of it, sells $\frac{3}{4}$ of his share ; what share has he remaining, and what is it worth ?

13. A party having bill to pay of *Rs.*123. 9*a.*, one of them

pays for himself and three friends the sum of *Rs.*54. 14*a.* 8*p.*; how many were there?

14. If 7 men or 11 women can finish a piece of work in 17 days how many days will it take 11 men and 7 women to finish it?

15. If 74 men had provisions for 35 days, and if after 5 days, 20 men were sent away; how long will the provisions last the remaining men?

16. If 6 men or 10 women can do a piece of work in 12 days, in what time will 5 men and 7 women do a piece of work twice as great?

17. If $3\frac{3}{4}$ tons of goods are carried 49 miles for *Rs.*19. 6*a.*, how far ought 26 tons 5 cwt. to be carried for the same money?

18. If $22\frac{1}{2}$ cwt. be carried 20 miles for *Rs.*5. 7*a.*; what weight can be carried the same distance for *Rs.*14. 8*a.*?

19. The four-penny loaf weighs 1 lb. $15\frac{5}{8}$ oz., when wheat is at 7*s.* 11*d.* per bushel; find what its weight should be when wheat is at 7*s.* $1\frac{1}{2}$*d.* per bushel.

20. A fortress is provisioned for 3 weeks at the rate of 15 ch. a day for each man; if only $10\frac{5}{8}$ ch. be served out daily to each man, how long can the place hold out?

21. *A* borrowed of *B Rs.*1752. 8*a.* for 102 days, and afterwards would return the favour by lending *B* the sum of *Rs.*2103; for how long should he lend it?

22. A besieged town, containing 22400 inhabitants, has provisions to last 3 weeks; how many must be sent away that they may be able to hold out 7 weeks?

23. If the two-anna loaf weighs 4 ch., when wheat is *Rs.*3. 6*a.* a maund; what would be the price of wheat per maund when the same loaf weighs 3 ch.?

24. When rice is *Rs.*3 a maund, how many people can be fed for the same sum that would feed 90 people when rice is *Rs* 2. 8*a.* a maund?

25. If 2000 men have provisions for 95 days, and if after 15 days 400 men go away, find how long the remaining provisions will serve the number left.

26. The monthly expenditure of a shop in oil is *Rs.*40. 8*a.* when oil is sold at $3\frac{5}{8}$ seers a rupee; what will it amount to when the price of oil has risen to 4*a.* 10*p.* per seer?

27. A piece of cloth, measured with a yard measure which is $\frac{3}{8}$ of an inch too short, appears to be $10\frac{1}{4}$ yards long; what is its true length?

28. The expenses of a family when rice is sold at 20 seers a rupee are *Rs.*50 a month; when rice is sold at 25 seers a rupee the

expenses are *Rs.*48 a month; what will they be when rice is sold at 30 seers a rupee?

310. Bankruptcy or Insolvency.

A tradesman becomes **bankrupt** or **insolvent**, when the money that he owes is more than that which he has in his possession. What he owes is called his **liabilities** or **debts**; his property or what he possesses is called his **effects** or **assets**. He is the **debtor**; those to whom he owes anything are his **creditors**. The amount paid by bankrupts is generally reckoned at so much in the rupee or pound, called **a dividend**, and each creditor receives the same fraction of the assets that the money due to him is of the bankrupt's whole debts.

Thus, if the assets amount to $\frac{2}{3}$ of the debts, each creditor receives $\frac{2}{3}$ of a rupee for each rupee due to him; and the bankrupt is said to pay *a dividend* of 10*a.* 8*p.* in the rupee.

Book debts are moneys which other men owe to the bankrupt; they are, therefore, considered a part of his assets. Book debts may be **good** or **bad**, as the whole or part can be recovered or realized.

Ex. 1. A bankrupt's estates amount to *Rs.*3780 and his debts to *Rs.*5040; how much can he pay in the rupee?

On *Rs.*5040 he can pay *Rs.*3780;

∴ in one rupee he can pay *Re.*$\frac{3780}{5040}$ or *Re.*$\frac{3}{4}$.

Hence he can pay *Re.*$\frac{3}{4}$ or 12*a.* *Ans.*

Ex. 2. A bankrupt's debts amount to *Rs.*3240, and he can pay 5*a.* 4*p.* in the rupee; find the amount of his assets.

On every rupee of his debts he can pay $5\frac{1}{3}$*a.* or *Re.*$\frac{1}{3}$;

∴ on *Rs.*3240 of debts .... $\frac{1}{3}\times$ *Rs.*3240;

Hence assets $=\frac{1}{3}\times$ *Rs.*3240 = *Rs.*1080. *Ans.*

Ex. 3. A bankrupt can pay 10*a.* 8*p.* in the rupee; had he *Rs* 4250 more he could have paid 12*a.* in the rupee. Find the amount of his debts and assets.

Here, 12*a.* − 10*a.* 8*p.* = 1*a.* 4*p.* = *Re.*$\frac{1}{12}$.

He could have paid *Re.*$\frac{1}{12}$ more on *Re.*1 of his debts;

∴ he could have paid *Re.*1 more on *Rs.*12 of his debts.

∴ ....*Rs.*4250......*Rs.*12 × 4250 of his debts.

Hence his debts = *Rs.*12 × 4250 = *Rs.*51000;
also his assets = 10*a.* 8*p.* × 51000 = *Rs.*34000. } *Ans.*

Ex. 4. A creditor receives on a debt of £296 a dividend of 12*s.* 4*d.* in the £, and he receives a further dividend of 3*s.* 9*d.* in the £ upon the deficiency; find how much the creditor receives in all.

The first payment $=$ £$(12\frac{1}{3} \div 20)$ or £$\frac{37}{60}$ on £1 of debt;

∴ the deficiency $=$ £$(1 - \frac{37}{60})$ or £$\frac{23}{60}$ on £1 of debt.

Also the second payment $=$ £$(3\frac{3}{4} \div 20)$ or £$\frac{3}{16}$ on £1 of deficiency;

∴ the second payment $=$ £$\frac{3}{16} \times \frac{23}{60}$ or £$\frac{23}{320}$ on £1 of debt;

∴ first payment + second payment $=$ £$(\frac{37}{60} + \frac{23}{320})$ on £1 of debt

$= \frac{661}{960}$ on £1 of debt.

Now, in £1 of debt the creditor receives £$\frac{661}{960}$;

∴ in £296 of debt.............. ...£$\frac{661}{960} \times 296$.

Hence the creditor receives £$\frac{661}{960} \times 296 =$ £203. 16*s.* 2*d.* *Ans.*

Ex. 5. A bankrupt has book-debts equal in amount to his liabilities; but on £3000 of them he can only recover 6*s.* 8*d.* in the £, and the expenses of the bankruptcy are £5 for every £100 of the book-debts; if he pay 15*s.* in the £, what is the amount of his liabilities?

As he can recover 6*s.* 8*d.* or £$\frac{1}{3}$ in the £, he recovers £$\frac{1}{3} \times 3000$ or £1000 out of £3000; therefore his *loss* amounts to £2000. Again, he pays £5 for £100, or 1*s.* in the £ for expenses. Therefore he recovers $(15+1)$*s.* or 16*s.* in the £, and his *loss* per £ $=$ 4*s.* or £$\frac{1}{5}$.

Now, £$\frac{1}{5}$ is the loss on £1 of liabilities.

∴ £1..........................£5............

∴ £2000...................£5 × 2000......

Hence liabilities $=$ £5 × 2000 $=$ £10000. *Ans.*

Examples LXXXVII.

1. A bankrupt's estates amount to *Rs.*950 and his debts to *Rs.*1200; how much can he pay in the rupee?

2. A bankrupt's debts amount to £5069. 10*s.*, and he can pay 14*s.* 11$\frac{1}{4}$*d.* in the £; find the value of his assets.

3. A bankrupt's debts amount to *Rs.*35000, and his assets to *Rs.*13708. 5*a.* 4*p.*; find how much his estate will pay in the rupee.

4. A bankrupt's effects amount to *Rs.*1980, and he pays his creditors 13*a.* 4*p.* in the rupee; what do his debts amount to?

5. A bankrupt's debts amount to *Rs.*53422. 8*a.* and his creditors lose *Rs.*17362. 5*a.*; find how much in the rupee the bankrupt pays.

6. A bankrupt owes *A* *Rs.*5156. 4*a.*, *B* *Rs.*4070 and *C* *Rs.*2933. 5*a.* 4*p.*; his estate is worth *Rs.*9119. 11*a.*; how much can he pay in the rupee, and what will *A*, *B* and *C* each receive?

7. A bankrupt owes *Rs.*9000 to his three creditors; and his whole property amounts to *Rs.*6750; the claims of two of his creditors are *Rs.*1250 and *Rs.*3750 respectively; what sum will the remaining creditor receive for his dividend?

8. A creditor received 16*s*. 3*d*. in the £, and thereby lost £135. 10*s*.; how much was due to him?

9. A bankrupt's debts amount to £1700, and his assets to £900. 15*s*.; after paying costs his cerditors receive 5*s*. 9*d*. in the £; find the amount of the costs.

10. A bankrupt has good debts to the amount of £456. 18*s*. 1*d*., and the following bad debts, £360. 7*s*. 10*d*., £120. 13*s*. and £19. 18*s*. for which he receives respectively 4, 5 and 9 shillings in the £; his own liabilities amount to £3408. 12*s*.; how much can he pay in the £?

11. A creditor received on a debt of *Rs*.3600 a dividend of 9*a*. 10*p*. in the rupee; and a further dividend of 6*a*. 8*p*. upon the remainder. What did he receive altogether?

12. A bankrupt can pay 12*s*. 4*d*. in the £; if his assets were £4205 more, he could pay 15*s*. 8*p*. in the £. Find his debts and assets.

13. A bankrupt has book debts equal in amount to his liabilities; but on *Rs*.8640 of such debts he can recover only 8*a*. 6*p*. in the rupee, and on *Rs*.6300 only 5*a*. 3*p*. in the rupee. After allowing *Rs*.1054. 11*a*. for the expenses of bankruptcy, he finds he can pay his creditors 12*a*. in the rupee. Find the total amount of his debts.

14. A bankrupt pays £5850 on the whole liabilities, at the rate of 13*s*. 6*d*. in the £ on half his debts and 15*s*. 9*d*. in the £ on the other half; find the amount of his debts.

15. A bankrupt can pay 11*a*. in the rupee; had he *Rs*.2550 more, he could have paid 14*a*. in the rupee. Find the amount of his debts and assets.

16. A bankrupt has book debts equal in amount to his liabilities; but on £6000 of them he can only recover 13*s*. 4*d*. in the pound, and the expenses of the bankruptcy are £5 on every £100 on the book debts; if he pay 13*s*. in the pound, what is the amount of his liabilities?

311. Incomes, Taxes and Rates.

Proceed as in the following Examples.

Ex. 1. If the income-tax be at the rate of 4*p*. in the rupee, and a man has to pay *Rs*.13. 6*a*. 8*p*., what is the amount of his income?

Here *Rs*.13. 6*a*. 8*p*. $=$ *Rs*.$13\frac{5}{12}$.

He pays *Re*.$\frac{4}{192}$ or *Re* $\frac{1}{48}$ income-tax on every *Re*.1 of income;

$\therefore$ he pays *Re*.1 income-tax on every *Rs*.48 of income;

$\therefore$ ....*Rs*.$13\frac{5}{12}$..........................*Rs*.$48 \times 13\frac{5}{12}$......

Hence income required $=$ *Rs*.$48 \times \frac{161}{12} =$ *Rs*.644. *Ans.*

Ex. 2. After paying an income-tax of 8*p.* in the rupee, a man has *Rs.*7283. 5*a.* 4*p.* left; find his gross income.

Here, *Re.*1 − 8*p.* = 15*a.* 4*p.* = *Re.*$\frac{23}{24}$; and *Rs.*7283. 5*a.* 4*p.* = *Rs.*7283$\frac{1}{3}$.

Since *Re.*$\frac{23}{24}$ is left out of *Re.*1 of income;

∴ *Re.*1 ....*Re.*$\frac{24}{23}$..........

∴ *Rs.*7283$\frac{1}{3}$ ....*Re.*$\frac{24}{23}$ × 7283$\frac{1}{3}$.....

Hence income required = *Re.*$\frac{24}{23} \times \frac{21850}{3}$ = *Rs.*7600. *Ans.*

Ex. 3. Find a man's gross rental, if after paying an income-tax of 6*d.* in the £ on the whole, and 3*s.* 6*d.* in the £ on $\frac{3}{4}$ of his rental, his net income is £2700.

Tax on £$\frac{3}{4}$ at 3*s.* 6*d.* = $\frac{3}{4}$ × 42*d.* = 31$\frac{1}{2}$*d.*

∴ total amount paid in taxes = (6 + 31$\frac{1}{2}$)*d.* or 37$\frac{1}{2}$*d.* in the £.

∴ he has (240 − 37$\frac{1}{2}$) or 202$\frac{1}{2}$*d.*, or £$\frac{27}{32}$ left out of £1.

Since £$\frac{27}{32}$ is left out of £1 of gross income;

∴ £1 ....£$\frac{32}{27}$........ .....

∴ £2700 ....£$\frac{32}{27}$ × 2700 .... .

Hence gross rental = £$\frac{32}{27}$ × 2700 = £3200 *Ans.*

Ex. 4. When the income-tax is 7*d.* in the £, a person has to pay £63 less than when the tax was 11*d.* in the £; find his income.

On the diminution of tax from 11*d.* to 7*d.* in the £, the man has to pay 4*d.* or £$\frac{4}{240}$ or £$\frac{1}{60}$ less on £1.

In every £$\frac{1}{60}$ less of income-tax the man has £1;

∴ ....£1........................ ......£60.

∴ ......£63.... ....£60 × 63 = £3780. *Ans.*

Ex. 5. The rent of a man's house is £120 per annum. It is assessed to the rates at $\frac{2}{3}$ of this; the poor-rate is 7*s.* 6*d.* in the £, the paving rate is 1*s.* 9*d.*, and the church rate 4*d.*; how much does he pay altogether for his residence?

Assessed value = $\frac{2}{3}$ of £120 = £80.

Amount of rates on £1 is (7*s.* 6*d.* + 1*s.* 9*d.* + 4*d.*) = 9*s.* 7*d.*

∴ rates on £80 is 9*s.* 7*d.* × 80 = £38. 6*s.* 8*d.*

Hence the annual cost of the house = £120 + £38. 6*s.* 8*d.*

= £158 6*s.* 8*d.* *Ans.*

Examples LXXXVIII.

1. A man pays an income-tax of *Rs.*63. 14*a.* 5$\frac{1}{4}$*p.* at the rate of 7*p.* in the rupee; find his income.

2. How much will a poor-rate of 2*s*. 8*d*. in the £ produce in a parish in which the whole property is rated at £4736. 5*s*. ?

3. A person after paying 7*p*. in the rupee for income tax has *Rs*.346. 14*a*. left. What was his gross income ?

4. After paying an income-tax of 3*d*. in the £, a person has a net income of £590. 10*s*. 6*d*. ; find his gross income.

5. Find a man's gross rental if after paying an income-tax of 8*d*. in the £ on the whole, and 2*s*. 6*d*. in the £ on two-thirds of his rental, he has a net income of £398. 16*s*. 6*d*.

6. After deducting 4*p*. in the rupee for income-tax and $\frac{1}{25}$ of the value of the whole estate for collecting expenses, the value of the remainder is *Rs*.11270 ; what is the value of the whole estate ?

7. The net rental of an estate, after deducting 7*d*. in the £ for income-tax and $\frac{1}{20}$ of the remainder for cost of collecting, is £959. 3*s*. 8*d*. ; find the gross rental.

8. A reduction in the income-tax diminishes a tax which is *Rs*.15 when the tax is 8 pies in the rupee by *Rs*.3. 12*a* ; what is the diminished rate of tax in the rupee ?

9. I hire a house at £90 a year, which is assessed in the rate-book at $\frac{4}{5}$ths of its rent ; I agree to pay the rates upon it, *viz*., 3 poor-rates of 9*d*, 10*d*. and 1*s*. 2*d*. respectively in the £, a church rate of 8*d*. in the £. and a paving rate of 1*s*. 7*d*. in the £ ; what is the whole annual cost of the house ?

10. A man allows his agent $\frac{4}{5}$ of one anna in the rupee on his gross income for the expense of collecting his rents. He spends $\frac{1}{7}$ of his net income in assuring his own life, and this part of his income is in consequence exempt from income-tax. The income-tax being 8*p*. in the rupee, and his income-tax amounting to *Rs*.389 8*a*, find his gross income.

11. A man pays a house-rate of 1*s*. 6*d*. per £ on his rental ; a water-rate of 1*s*. per £ ; a poor-rate of 1*s*. 10½*d*. per £. If the rent and rates amount to £85. 6*s*. 3*d*., what is the rent ?

12. An occupier pays house-rate of 3*a*., police-rate of 9*p*., water-rate of 2*a*. 6*p*. and a lighting-rate of 1*a*. 9*p*. in the rupee. If the rent and rates amount to *Rs*.1440, what is the assessed annual value of the house ?

13. The income tax having been raised to 10*d*. in the pound a man has to pay £45. 10*s*. 6*d*. more than when it was 7*d*. in the pound. Find his income.

14. If a person's net income after paying an income tax of 7*d*. in the £ be £291. 5*s*., find his net income after paying an income-tax of 1*s*. 4*d*. in the £.

312. Division into Parts and Shares.

Proceed as follows :—

Ex. 1. A post has $\frac{1}{4}$ of its length in the mud, $\frac{1}{3}$ in water and 10 ft. above the water. Find its whole length.

Let unity or 1 represent the length of the post.

Then the part in the mud $=\frac{1}{4}$, and .... water $=\frac{1}{3}$; } Now, $\frac{1}{4}+\frac{1}{3}=\frac{7}{12}$. ∴ the part above water $=1-\frac{7}{12}=\frac{5}{12}$

Hence, by question, $\frac{5}{12}$ of the post = 10 ft.

∴ the length of the post = 10 ft. $\times \frac{12}{5}$ = 24 ft. *Ans.*

Ex. 2. One-half of the trees in an orchard are apple trees, one-fourth are pear trees, one-sixth plum trees, and there are 50 cherry trees ; what number of trees does it contain ?

Representing the number of trees in the orchard by the unit or 1, we have

$\frac{1}{2}$ = number of apple trees ;
$\frac{1}{4}$ = number of pear trees ;
$\frac{1}{6}$ = number of plum trees. } Now, $\frac{1}{2}+\frac{1}{4}+\frac{1}{6}=\frac{11}{12}$; ∴ the no. of other trees $=1-\frac{11}{12}=\frac{1}{12}$.

Hence, by question, $\frac{1}{12}$ of the whole no. of trees = 50,
∴ the whole no. of trees $=50\times12=$ 600. *Ans.*

Ex. 3. After paying away one-half of a sum of money, and then $\frac{3}{5}$ of what was left, *Rs.*5. 4*a.* remained ; what was the sum ?

Let 1 represent the sum of money.

Then $\frac{1}{2}$ of the sum $=\frac{1}{2}$, the first paid-up part ;
∴ $1-\frac{1}{2}=\frac{1}{2}$, ...remaining......... ;
Again $\frac{3}{5}$ of $\frac{1}{2}=\frac{3}{10}$, the second paid-up......... ;
∴ $\frac{1}{2}-\frac{3}{10}=\frac{1}{5}$,...............remaining...........

Hence, by question, $\frac{1}{5}$ of the sum = *Rs.*5. 4*a.*
∴ the whole sum = *Rs.*5. 4*a.* × 5 = *Rs.*26. 4*a.* *Ans.*

Ex. 4. *A* met two beggars, *B* and *C* ; and having $\frac{3\frac{7}{11}}{4\frac{2}{7}}$ of $\frac{10\frac{5}{7}}{7\frac{1}{2}}$ of $\frac{77}{540}$ of a moidore of 27*s.* in his pocket, gave $\frac{1}{7}$ of $\frac{3}{4}$ of it to *B*; and $\frac{3}{5}$ of the remainder to *C* ; what did each receive ?

$$A \text{ had at first } \frac{\frac{40}{11}}{\frac{30}{7}} \text{ of } \frac{\frac{75}{7}}{\frac{15}{2}} \text{ of } \frac{77}{540} \text{ of } 27s.$$

$$= \frac{40\times7}{30\times11} \text{ of } \frac{75\times2}{7\times15} \text{ of } \frac{77\times27}{540}s. = \frac{14}{3}s.$$

∴ *B* received $\frac{1}{7}$ of $\frac{3}{4}$ of $\frac{14}{3}s.=\frac{1}{2}s.=$ 6*d.*,

and *A* had afterwards left $(\frac{14}{3}-\frac{1}{2})s.=\frac{25}{6}s.$

∴ *C* received $\frac{3}{5}$ of $\frac{25}{6}s.=\frac{5}{2}s.=$ 2*s.* 6*d.*

Ex. 5. A person left $\frac{7}{18}$ of his property to his eldest son and $\frac{7}{18}$ of the remainder to his younger son and the rest to his widow. The elder son received £1029. 16*s*. 4*d*. more than the younger; how much did the widow receive?

Let 1 represent the whole property.

Then the elder son received $\frac{7}{18}$, and the part left is $(1-\frac{7}{18})=\frac{11}{18}$.

The younger son received $\frac{7}{18}$ of $\frac{11}{18}=\frac{77}{324}$, and the part left is $\frac{11}{18}-\frac{77}{324}=\frac{121}{324}$.

∴ the widow's share is $\frac{121}{324}$ of the property.

The sons' shares differ by $\frac{7}{18}-\frac{77}{324}=\frac{49}{324}$ of the whole.

Hence, by question, $\frac{49}{324}$ of the whole estate = £1029. 16*s*. 4*d*.

∴ the whole estate = £1029. 16*s*. 4*d*. $\times\frac{324}{49}$
= £21. 0*s*. 4*d*. × 324 = £6809. 8*s*.

∴ the widow's share = £6809. 8*s*. $\times\frac{121}{324}$ = £21. 0*s*. 4*d*. × 121
= £2543. 0*s*. 4*d*. *Ans.*

Ex. 6. Gunpowder being composed of nitre 15 parts, charcoal 3 parts, and sulphur 2 parts; find how much of each is required for 18 maunds of powder.

The whole number of parts = (15 + 3 + 2) = 20.

∴ of every 20 parts, $\frac{15}{20}$ or $\frac{3}{4}$ is nitre, $\frac{3}{20}$ is charcoal, $\frac{2}{20}$ or $\frac{1}{10}$ is sulphur.

Hence, required nitre = $\frac{3}{4}$ of 18 mds. = 13 mds. 20 sr.
...............charcoal = $\frac{3}{20}$ of 18 mds. = 2 mds. 28 sr.   *Ans.*
............sulphur = $\frac{1}{10}$ of 18 mds. = 1 md. 32 sr.

Examples LXXXIX.

1. After detaching $\frac{11}{58}$ and $\frac{12}{35}$ of a company of soldiers, the general had 1110 left; required his original force.

2. If a person lay out $\frac{5}{9}$ of his income in board and lodging, $\frac{1}{6}$ in clothes and save *Rs.*600 a year; what is his income?

3. What is the capacity of a vessel, out of which when a third of it is empty, 35 gallons being drawn, there remains $\frac{3}{8}$ of the whole content?

4. In an orchard, $\frac{1}{3}$ are apple trees, $\frac{1}{5}$ peach trees, $\frac{2}{7}$ pear trees, and the remainder which is 38, cherry trees. How many trees are there in the orchard?

5. After taking out of a purse $\frac{2}{5}$ of its contents, $\frac{3}{8}$ of the remainder was found to be *Rs.*6. 11*a*. 8*p*.; what sum did it contain at first?

6. If $\frac{11}{20}$ of an estate be left to the elder and the remainder to the younger of two children, and the difference of their legacies be *Rs.*2250; find the value of the estate.

7. Of a field $\frac{1}{5}$ is meadow, $\frac{5}{8}$ is arable and the remainder is 1 ac. 3 ro. 26 po.; find the quantities of meadow and arable land.

8. *A* had at first £1. 8*s*.; and *B*, when he had paid $2\frac{3\frac{4}{7}}{1\frac{2}{9}}$ of £1. 11*s*. 6*d*. to *A*, found that he had remaining $\frac{1}{45}$ of what *A* then had; what had *B* at first?

9. A man pays away $\frac{1}{4}$ of his money, then $\frac{1}{3}$ of what remains, and then $\frac{1}{5}$ of the second remainder; after which he has 7*s*. 6*d*. left; how much had he at first?

10. A post is divided into 4 parts; the first part is $\frac{3}{7}$ of the whole length, the second part is $\frac{2}{3}$ of the first, the third $\frac{7}{8}$ of the second, and the fourth is 2 yds. 1 ft. 4 in.; find the length of the post.

11. Out of *Rs*.43. 12*a*., $\frac{1}{5}$ is paid to *A* and $\frac{1}{7}$ to *B*, after this $\frac{4}{11}$ of the remainder is paid to *A* and the rest to *B*; find the sums respectively received by *A* and *B*.

12. A gentleman left his eldest son $\frac{5}{8}$ of his money, to the younger $\frac{5}{8}$ of the remainder, and the rest to his wife; upon dividing the money, it was found that the eldest son had *Rs*.7500 more than the younger; how much was left to each?

13. *A* and *B* have *Rs*.18. and *Rs*.12 respectively; and if *A* gives *B* $\frac{2\frac{3}{8}}{4\frac{3}{4}}$ of the difference of $\frac{2\frac{3}{13}}{13\frac{6}{13}}$ of their respective sums, and $\frac{4}{7}$ of $2\frac{1}{2}$ of *A*'s present sum be added to $\frac{4}{15}$ of $\frac{1}{2}$ of *B*'s, *C*'s money will be $1\frac{1}{2}$ of this sum; find it.

14. A person had a legacy left to him, which he thus divided amongst 3 charities. To one he gave $\frac{3}{10}$, to the second $\frac{3}{7}$ of the remainder, and to the third $\frac{3}{4}$ of what now remained; and he then had *Rs*.1500 left. Find the amount of the legacy, and how much was given to each charity.

15. What number is that of which the fourth, fifth and sixth parts together exceed the half of the number by 112?

16. A person making his will, gave to one child $\frac{13}{20}$ of his estate, and the rest to another. When these legacies came to be paid, the one turned out to be £1200 more than the other; what did the testator die worth?

17. *A*, *B* and *C* rent a pasture for *Rs*.400. *A* puts in 8 cattle, *B* 9 and *C* 11; how much should each pay for his share?

18. A person dies worth *Rs*.100000, and leaves $\frac{1}{3}$ of his property to his wife, $\frac{1}{4}$ to his son, and the rest to his daughter. The wife at her death leaves $\frac{3}{5}$ of her legacy to the son, and the rest to the daughter; but the son adds his fortune to his sister's and gives her $\frac{1}{3}$ of the whole. How much will the sister gain by this, and what fraction will her gain be of the whole?

313. Pipes and Cisterns.

If one or more pipes fill or empty a cistern in 8 min., they fill or empty $\frac{1}{8}$th of it per min., and *conversely*, if they fill or empty $\frac{1}{8}$th of it per min., they fill or empty the whole in 8 min. Similarly, if they fill or empty a cistern in $5\frac{1}{2}$ hours, they fill or empty $\frac{1}{5\frac{1}{2}}$ or $\frac{2}{11}$ of it in 1 hr.; and *conversely*, if they fill or empty $\frac{2}{11}$ of it per hour, they will fill or empty the cistern in $(1 \div \frac{2}{11})$ or $5\frac{1}{2}$ hrs.

Ex. 1. Two pipes can separately fill a cistern in 10 and 15 minutes. If both the pipes are opened, how soon will the cistern be filled?

The first pipe fills $\frac{1}{10}$ of the cistern in 1 min.
... second ...... $\frac{1}{15}$ ....

∴ both the pipes fill $(\frac{1}{10}+\frac{1}{15})$ or $\frac{1}{6}$ of the cistern in 1 min.

Hence they fill the cistern in $(1 \div \frac{1}{6})$ min. = 6 min. *Ans.*

Ex. 2. Pipes *A* and *B* can fill a cistern in 3 min. and 5 min. respectively, and *C* can empty it in $7\frac{1}{2}$ min. In what time will the cistern be filled when *A*, *B* and *C* are all turned on?

The first pipe fills in 1 min. $\frac{1}{3}$ of the cistern;
......second.............. ..$\frac{1}{5}$................

The third pipe empties in 1 min. $(1 \div 7\frac{1}{2})$ or $\frac{2}{15}$ af the cistern;

∴ with all open, $(\frac{1}{3}+\frac{1}{5}-\frac{2}{15})$ or $\frac{2}{5}$ of the cistern is filled in 1 min.

Hence the cistern will be filled in $(1 \div \frac{2}{5})$ min. = $2\frac{1}{2}$ min. *Ans.*

Ex. 3. Two taps take 4 hours and 6 hours respectively to fill a cistern. When the waste pipe is left open along with the two taps, the cistern is filled in 24 hours. In what time does the waste pipe empty the cistern?

The first tap fills $\frac{1}{4}$ of the cistern in 1 hour.
... ...second........$\frac{1}{6}$..................................

The three together fill $\frac{1}{24}$..................................

∴ the waste pipe empties $(\frac{1}{4}+\frac{1}{6}-\frac{1}{24})$ or $\frac{3}{8}$ of the cistern in 1 hour.
Hence, the waste pipe will empty the cistern in $(1 \div \frac{3}{8}) = 2\frac{2}{3}$ hrs. *Ans.*

Ex. 4. A cistern which would be filled in 8 hours requires 2 hours more to be filled, owing to a leak in the bottom. If the cistern is full, in what time will the leak empty it?

Had there been no leak, $\frac{1}{8}$ of the cistern would have been filled in 1 hr.; but the leak allows only $\frac{1}{10}$ to be filled in 1 hour.

∴ $(\frac{1}{8}-\frac{1}{10})$ or $\frac{1}{40}$ of the cistern is emptied by the leak in 1 hour.
Therefore the leak requires $(1 \div \frac{1}{40})$ or 40 hours to empty the cistern. *Ans.*

Examples XC.

1. Two taps, *A* and *B*, fill a cistern in 10 and 20 hours respectively. In what time will they fill it together?

2. A cistern is filled by two taps in 10 and 15 hours respectively and is emptied by a tap, *C*, in 8 hours. If all the three taps are open, in what time will the cistern be filled?

3. A cistern is fed by a spout which can fill it in 3 hrs. How long would it take to fill it, if the cistern has a leak which would empty, when full, in 17 hrs.?

4. Two pipes together can fill a cistern in 8 min., and one of them alone in 24 min. How long would the other alone take?

5. A cistern has three pipes connected with it, two to supply and one to draw off. The first alone can fill $\frac{2}{3}$ of the cistern in 3 hours, and the second $\frac{3}{4}$ in 4 hours; the third can empty $\frac{3}{4}$ of the cistern in 5 hours. If all the pipes be opened together at once, when will the cistern be full?

6. A cistern is filled by two spouts in 20 and 24 minutes respectively and emptied by a tap in 30 minutes; what portion of it will be filled in 15 minutes when they are all left open together?

7. A cistern has three pipes *A*, *B* and *C*; *A* and *B* can fill it in 3 and 4 hours respectively, and *C* can empty it in 1 hour; if these pipes be opened in order at 1, 2 and 3 o'clock, when will the cistern be empty?

8. A cistern is provided with three spouts *A*, *B* and *C*. *A* can fill it in 30 minutes, *B* in 40 and *C* can empty it in 2 hours. *A*, *B* and *C* be opened successively for a minute each, in what tim will the cistern be filled; and how much of the content of th cistern will have passed out by *C*?

9. A cistern can be filled by three pipes; by the first in 10 hours, by the second in 9, and by the third in 8 hours. It is supplied by the first pipe till $\frac{1}{4}$ of it is full, then the second is also turned on till it becomes half full, and then all three begin to run. How long would it take to fill the cistern?

10. A tank can be filled by one pipe in 6 min., and by a second in 5 min., there is also a tap by which the tank can be emptied. If the tank be empty at first, and the pipes and tap be all left open, the tank is filled in 3 min. If the pipes are then closed, in what time will the tank be emptied by the tap?

11. A cistern can be filled by two pipes, *A*, *B* in 4 and 5 min. respectively and emptied by *C* in 144 seconds. *B* is opened 2 min. after *A*, *C* is opened 1 min. after *B*. The cistern contains 361 gallons just before *C* is opened. In what time will it be filled or emptied after the opening of *C* and how many gallons will go out by *C*?

12. Three taps, *A*, *B* and *C* can fill a cistern, *A* by itself in

24 min., *B* in 10 min., and *C* in 27 min. They are all turned on at once, but after $4\frac{1}{3}$ min. *B* and *C* are turned off. How much longer will *A* by itself take then to fill the cistern?

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314. Time and Work. The following points (if remembered) will greatly help students in solving problems concerning *Time and Work.*

(1) If **1** man can do a piece of work in a certain time, then in the same time **2** men will do *twice* as much, **3** men *thrice* as much, and so on. *Conversely*, if **3** men can do a piece of work in a certain time, **1** man will do $\frac{1}{3}$ of the work in the same time.

(2) If one or more men can do a piece of work in **6** days, they can do $\frac{1}{6}$ of the work in 1 day; so, if a piece of work can be done in $6\frac{1}{2}$ days, $(1 \div 6\frac{1}{2})$ or $\frac{2}{13}$ths of the work can be done in 1 day; and *conversely*, if $\frac{1}{6}$th of a piece of work is done in 1 day, the whole work can be done in **6** days; so, also if $\frac{2}{13}$ths of a piece of work can be done in 1 day, the whole work will be done in $(1 \div \frac{2}{13})$ or $6\frac{1}{2}$ days.

(3) If **5** men can do a piece of work in **7** days, then they will do $\frac{1}{7}$th of the work in one day; therefore 1 man will do $\frac{1}{5} \times \frac{1}{7} = \frac{1}{35}$ of the work in 1 day, or the whole work in 35 or (5×7) days.

(4) If **1** man can do a piece of work in **5** days, then he will do $\frac{1}{5}$th of the work in 1 day; therefore 3 men will do $\frac{3}{5}$ths of the work in 1 day, and therefore the whole work in $(1 \div \frac{3}{5})$ or $(5 \div 3)$ days.

(5) If 1 man can do $\frac{3}{5}$ of a piece of work in **7** days, he can do $\frac{3}{5} \times \frac{1}{7}$ of it in one day, and therefore the whole work in $1 \div (\frac{3}{5} \times \frac{1}{7})$ or $(7 \div \frac{3}{5})$ days.

(6) If *A* can do $\frac{1}{5}$ of a work in 1 day, and *B* $\frac{1}{4}$ in 1 day, then *A* and *B* together will do $(\frac{1}{5} + \frac{1}{4})$ or $\frac{9}{20}$ of the work in 1 day, and therefore both will finish the work in $(1 \div \frac{9}{20})$ or $2\frac{2}{9}$ days.

Ex. 1. *A* can do a piece of work in 5 days, *B* in 6 and *C* in 7; how much of it can they jointly do in 2 days, and how long will they take to do the whole work?

A can do $\frac{1}{5}$ of the work in 1 day; *B* can do $\frac{1}{6}$ of the work in 1 day; *C* can do $\frac{1}{7}$ of the work in 1 day.

$\therefore$ *A*, *B* and *C* can jointly do $(\frac{1}{5} + \frac{1}{6} + \frac{1}{7})$ or $\frac{107}{210}$ of the work in 1 day.

Hence, in 2 days they will do $\frac{107}{210} \times 2$ or $\underline{1\frac{2}{105}}$ of the work. *Ans*

Also, they can jointly do the whole work in $(1 \div \frac{107}{210})$ or $\underline{1\frac{103}{107}}$ days.

Ex. 2. If *A* and *B* together can perform a piece of work in 10 days, and *A* himself can do it in 18 days, what time will it take *B* to do it alone?

A and *B* can do the work in 10 days ;

∴ they can do $\frac{1}{10}$ of the work in 1 day ;

A can do the work in 18 days ;

∴ he can do $\frac{1}{18}$ of the work in 1 day ;

∴ *B* can do $(\frac{1}{10} - \frac{1}{18})$ or $\frac{2}{45}$ of the work in 1 day.

Hence *B* can do the whole work in $(1 \div \frac{2}{45})$ or $\underline{22\frac{1}{2}}$ days. *Ans.*

Ex. 3. *A* does $\frac{7}{10}$ of a piece of work in 14 days ; he then calls in *B*, and they finish the work in 2 days ; how long would *B* take to do the whole work by himself ?

A does $\frac{7}{10}$ of the work in 14 days ;

∴ he does $\frac{7}{140}$ or $\frac{1}{20}$ of the work in 1 day ;

∴ in 2 days, *A* does $\frac{2}{20}$ or $\frac{1}{10}$ of the work.

But $(1 - \frac{7}{10})$ or $\frac{3}{10}$ of the work remains to be done ;

∴ *B* does $(\frac{3}{10} - \frac{1}{10})$ or $\frac{1}{5}$ of the work in 2 days ;

∴ *B* can do $\frac{1}{10}$ of the work daily, and

∴ *B* can do the whole work in $(1 \div \frac{1}{10})$ or $\underline{10}$ days. *Ans.*

Ex. 4. If *A* and *B* can do a piece of work in 18 days, *A* and *C* in 12 days, and *B* and *C* in 9 days, find the time in which *A*, *B* and *C* can together finish it, and also each working singly.

A and *B* can do $\frac{1}{18}$ of the work in 1 day ;
A and *C*..........$\frac{1}{12}$.............................. ;
B and *C*..........$\frac{1}{9}$.............................. ;

∴ 2 men like $A + 2$ men like $B + 2$ men like *C* can do $(\frac{1}{18} + \frac{1}{12} + \frac{1}{9})$ or $\frac{1}{4}$ of the work in 1 day ;

∴ *A*, *B* and *C* can do $\frac{1}{8}$ of the work in 1 day.

Hence they can jointly do the whole work in $(1 \div \frac{1}{8})$ or 8 days. *Ans.*

Also *A* can do $(\frac{1}{8} - \frac{1}{9})$ or $\frac{1}{72}$ of the work in 1 day, and

∴ the whole work in $\underline{72}$ days.

B...........$(\frac{1}{8} - \frac{1}{12})$ or $\frac{1}{24}$ of the work in 1 day, and

∴ the whole work in $\underline{24}$ days.

C...........$(\frac{1}{8} - \frac{1}{18})$ or $\frac{5}{72}$ of the work in 1 day, and

∴ the whole work in $\underline{14\frac{2}{5}}$ days. *Ans.*

Ex. 5. 5 men or 10 women or 15 boys can do a piece of work in 16 days. In how many days will 2 men, 3 women and 4 boys do it ?

Since 5 men can do the work in 16 days ;

∴ 1 man will do the work in (16×5) days ;

∴ 1 man in one day will do $\frac{1}{16 \times 5}$ of the work ;

$\therefore$ 2 men in one day will do $\frac{2}{16 \times 5}$ or $\frac{1}{40}$ of the work.

Similarly, 3 women in one day will do $\frac{3}{160}$ of the work ;

and 4 boys............................ $\frac{1}{60}$ of the work.

$\therefore$ 2 men + 3 women + 4 boys in one day will do $(\frac{1}{40}+\frac{3}{160}+\frac{1}{60})$ or $\frac{29}{480}$ of the work.

Hence, they will take $(1 \div \frac{29}{480})$ or $\frac{480}{29} = \underline{16\frac{16}{29}}$ days. *Ans.*

Ex. 6. *A* and *B* can do a piece of work in 10 days, *B* and *C* in 15 days, and *A* and *C* in 25 days ; they all work at it together for 4 days ; *A* then leaves, and *B* and *C* go on together for 5 days, and then *B* leaves ; in how many days will *C* complete the work ?

A and *B* can do $\frac{1}{10}$ of the work daily ; *B* and *C* $\frac{1}{15}$ daily, and *A* and *C* $\frac{1}{25}$ daily ; $\therefore$ 2 men like *A* + 2 men like *B* + 2 men like *C* can together do $(\frac{1}{10}+\frac{1}{15}+\frac{1}{25})$ or $\frac{31}{150}$ of the work daily, and $\therefore$ $A+B+C$ can do $\frac{31}{300}$ of the work daily. Hence in 4 days, they do $(\frac{31}{300} \times 4) = \frac{31}{75}$ of the work.

$\therefore$ when *A* leaves, $(1 - \frac{31}{75})$ or $\frac{44}{75}$ of the work remains to be done.

Now, *B* and *C* together in 5 days do $(\frac{1}{15} \times 5)$ or $\frac{1}{3}$ of the work.

$\therefore$ when *B* leaves, $(\frac{44}{75} - \frac{1}{3})$ or $\frac{19}{75}$ of the work remains to be done and this work *C* finishes by himself.

Again, *C* in one day can do $(\frac{31}{300} - \frac{1}{10})$ or $\frac{1}{300}$ of the work.

Hence, *C* finishes the work in $(\frac{19}{75} \div \frac{1}{300})$ or $\frac{19}{75} \times 300 = \underline{76}$ days. *Ans.*

Ex. 7. If 10 excavators can dig 12 loads of earth in 16 hours, whilst 12 others can dig 9 loads in 15 hours ; find the time in which they will jointly dig 100 loads.

The *first* set of men can dig $\frac{12}{16}$ or $\frac{3}{4}$ load in 1 hour ; the *second* set $\frac{9}{15}$ or $\frac{3}{5}$ load in 1 hour.

$\therefore$ they can jointly dig $(\frac{3}{4}+\frac{3}{5})$ or $\frac{27}{20}$ loads in 1 hour ;

$\therefore$ they can dig 1 load in $(1 \div \frac{27}{20})$ or $\frac{20}{27}$ hour.

Hence they can dig 100 loads in $(\frac{20}{27} \times 100)$ or $\underline{74\frac{2}{27}}$ hours. *Ans.*

Ex. 8. *A* can do a piece of work in 10 days, *B* in 9 days and *C* in 12 days. All begin together ; but *A* leaves after 4 days and *B* 2 days before the work is done. How long did the work last ?

A can do $\frac{1}{10}$ of the work in 1 day ; *B* $\frac{1}{9}$ in 1 day ; and *C* $\frac{1}{12}$ in 1 day.

A in 4 days does $\frac{4}{10}$ or $\frac{2}{5}$ of the work. Now, *C* worked 2 days more than *B*, and during that time did $\frac{2}{12}$ or $\frac{1}{6}$ of the work.

Therefore the work done by *B* and *C* together is $(1 - \frac{2}{5} - \frac{1}{6})$ or $\frac{13}{30}$ of the work. Now, *B* and *C* in 1 day can do $(\frac{1}{9}+\frac{1}{12})$ or $\frac{7}{36}$ of the work ; therefore they took $(\frac{13}{30} \div \frac{7}{36})$ or $\frac{78}{35}$ days $= 2\frac{8}{35}$ days.

Hence the whole time occupied $= (4 + 2\frac{8}{35} + 2)$ or $\underline{8\frac{8}{35}}$ days. *Ans.*

Ex. 9. If *A* can do as much work in 5 hours as *B* can do in

6 hours, or as C can do in 9 hours, how long will it take C to complete a piece of work, one-half of which has been done by A working 12 hours and B 24 hours?

Since 5 hrs. work of $A = 9$ hrs. work of C;

$\therefore$ 1 hr. of $A = \frac{9}{5}$ hrs. of C, or 12 hrs. of $A = \frac{9}{5} \times 12$ or $21\frac{3}{5}$ hrs. of C.

Since 6 hrs. work of $B = 9$ hrs. work of C;

$\therefore$ 24 hrs. work of $B = 9 \times 4$ or 36 hrs. of C.

Hence 12 hrs. of A + 24 hrs. of $B = (21\frac{3}{5} + 36)$ or $57\frac{3}{5}$ hrs. of C.

But 12 hrs. work of A + 24 hrs. work of $B = \frac{1}{2}$ of the work;

$\therefore$ C can finish the remaining half in $\underline{57\frac{3}{5}}$ hrs. *Ans.*

Ex. 10. A is thrice as good a workman as B; and together they finish $\frac{3}{5}$ of a work in 9 days. In how many days will it be done by each separately?

Since 3 days' work of $B = 1$ day's work of A;

$\therefore$ 9 days' work of $B = 3$ days' work of A;

$\therefore$ 9 days' work of B + 9 days' work of $A = 12$ days' work of A

But 9 days' work of B + 9 days' work of $A = \frac{3}{5}$ of the work;

$\therefore$ 12 days' work of $A = \frac{3}{5}$ of the work, *i. e.*,

A can do $\frac{3}{5}$ of the work in 12 days.

Hence A does the whole work in $(12 \div \frac{3}{5})$ or $\underline{20}$ days,
and therefore B does the whole work in 3×20 or $\overline{60}$ days. } *Ans.*

Examples XCI.

1. A alone can do a piece of work in 11 days, and B alone can do it in 17 days: find how long they would take to do it together.

2. A, B and C can complete a piece of work in 10, 12 and 15 days respectively. How long would it take them if they work together?

3. A can finish a piece of work in $2\frac{1}{2}$ days and B in $3\frac{1}{3}$ days; if they work together what part of the work will they finish in $1\frac{2}{7}$ days?

4. A and B can do a piece of work in 12 days; when C joins them they can do it in 9 days; in what time can C do it working alone?

5. A man alone can do a piece of work in 10 days which, if his son helps him, he can do in 6 days; in what time would his son working alone do the work?

6. A can reap $\frac{4}{5}$ of a field in $2\frac{2}{5}$ days, and B can reap $\frac{3}{5}$ of it in $4\frac{1}{5}$ days; in what time could A and B working together reap the field?

7. If A and B can do a piece of work in 24 days, A and C in

16 days, and B and C in 12 days; find the time in which A, B and C can together finish it.

8. A and B can do a piece of work in 6 days which B and C can do in 4 days, and A and C in 3 days. Find the time in which each can separately do it.

9. A and B can do a piece of work in 8 days, A and C in $10\frac{2}{3}$ days, and B and C in $9\frac{1}{3}$ days; in how many days can A alone do it?

10. A, B and C can finish a piece of work in 12 hours, also A and B can do it in 16 hours, and A and C in 18 hours; what part of the whole work can B and C do in $9\frac{1}{7}$ hours?

11. A, B and C can do a piece of work together in 20 days, A alone can do it in 40 days, and B alone in 60 days. In what time can C alone do it?

12. A performs $\frac{2}{3}$ of a piece of work in 13 days, and with the help of B finishes it in 6 days. In what time could each of them do the piece of work separately?

13. A can do $\frac{2}{3}$ of a piece of work in 4 hours, B can do $\frac{3}{4}$ of the remainder in 1 hour, and C can then finish it in 20 minutes; in what time can A, B and C together do it?

14. A certain number of men mow 4 acres of grass in 3 hours, and a certain number of others mow 8 acres in 5 hours; how long will they be in mowing 11 acres, if all work together?

15. A can mow $2\frac{1}{2}$ acres in $6\frac{2}{3}$ days, and B $2\frac{1}{3}$ acres in $5\frac{1}{3}$ days; they mow together a field of 10 acres; how long will it take them to do it, and how many acres will each mow?

16. A and B can do a piece of work in 4 days, working 6 hours a day; B and C can do it in 4 days, working 5 hours a day; and A and C can do it in 4 days, working 4 hours a day. In how many days of 8 hours will each do it separately?

17. A can do a piece of work in 27 days, A and B can do it in 15 days; A works alone for 12 days, and A and C together for 5 days, and B finishes it in 7 days; find in what time B and C together could do it.

18. A can do a piece of work in 27 days and B in 15 days; A works at it alone for 12 days, B then works 5 days and afterwards C finishes it in 4 days; in what time could C have done the whole work?

19. A and B can do a piece of work, each, in 24 days; A and B work together for 6 days, when B goes away and C works with A for 3 days, then B rejoins them, and the work is finished in 2 days more. How long would it have taken A, B and C to do the piece of work, if they had all worked together?

20. A can do a piece of work in 6 days and B in 9 days. They begin together. But 2 days before the completion of the work, A leaves off. In how many days is the work finished?

21. *A* is twice as good a workman as *B*; and together they finish a work in 8 days. In how many days can it be done by each separately?

22. 8 men or 12 women or 16 children can do a piece of work in 15 days. In how many days will 3 men, 4 women and 5 children do it?

23. *A* is thrice as good a workman as *B*. If the time taken by *B* to do a piece of work exceed that taken by *A* by 8 days, find in how many days each can do it.

24. *A* is twice as good a workman as *B* and thrice as good as *C*. Working together for 10 days they can finish a work. They all begin together. But after working for 3 days *A* leaves off. After 5 days more *B* also leaves off. In how many days more will *C* finish the work?

25. *A* can do a pie:ce of work in 10 days. *B* in 9 days and *C* in 12 days. All begin together; *A* leaves after $1\frac{3}{8}$ days, *B* leaves $\frac{3}{4}$ day before the work is done. How long did the work last?

26. A man can do as much work in 3 days as a boy can do in 5. How long will a man take to finish a work, $\frac{1}{5}$ of which has been done by a bpy in 8 days?

27. If *A* in 2 days can do as much work as *C* in 3 days, and *B* in 5 days as much as *C* in 4 days; what time will *B* require to execute a piece of work which *A* can accomplish in 6 weeks?

28. If *A* can do as much work in 5 hours as *B* can do in 6 hours, or as *C* can do in 9 hours, how long will it take *A* to complete a piece of work, one-half of which has been done by *B* working 12 hours, and *C* working 24 hours?

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315. Equations. A statement of the equality of two arithmetical expressions is called an **arithmetical equation**.

Thus, $8=5+3$ is an *arithmetical equation*, for it asserts that 8 is equal to the sum of 5 and 3. The numbers 8, 5 and 3 are called **terms** of the equation. If one of the terms be unknown, it can be easily found from the above statement.

316. Although equation is an instrument of great power in all mathematical calculations, yet it is surprising to see that in no text-book of Arithmetic the method of solution by equations has received due favour. The following simple results are very useful in solving equations.

(i) *If eqncls be added to equals the sums are equal.*
Thus, $15-13=12$; $\therefore 15=12+3$, (adding 3 to each side of the equation).

(ii) *If equals be taken from equals the remainders are equal.*
Thus, $15=12+3$, $\therefore 15-3=12$, (taking 3 from each side of the equation).

(iii) *If equals be multiplied by equals the products are equal.*

Thus, $15=12+3$; $\therefore 15\times4=(12+3)\times4$, (multiplying each side by 4).

(iv) *If equals be divided by equals the quotients are equal.*

Thus, $15=12+3$; $\therefore 15\div3=(12+3)\div3$, (dividing each side by 3).

Hence from (i) and (ii) we see that *any term of an equation may be transferred from one side of the equation to the other, if its sign be changed, plus becoming minus and minus becoming plus.*

317. In a problem, the number to be found is called the **unknown quantity** or **unknown term**, and the numbers given are called the **known quantities** or **known terms**. To combine them and thus reduce their number, we **transpose** all the terms into which the unknown quantity enters to one side of the equation and the known terms to the other side, changing the sign of each term so transposed.

Ex. 1. If to the sum of $\frac{1}{3}$ and $\frac{1}{4}$ of a number 5 be added, the sum is 19 ; find the number.

$(\frac{1}{3}+\frac{1}{4})$ of the number $+5=19$; $\therefore \frac{7}{12}$ of the number $+5=19$.

Transposing the terms, we have

$\frac{7}{12}$ of the number $=19-5=14$; $\therefore$ number $=14\times\frac{12}{7}=\underline{24}$. *Ans.*

Ex. 2. What is the number from which if you take away 15, the remainder is $\frac{2}{3}$ of the original number ?

The number $-15=\frac{2}{3}$ of the number.

Transposing the terms, we have

the number $-\frac{2}{3}$ of the number $=15$;

$\therefore \frac{1}{3}$ of the number $=15$; $\therefore$ the number $=3\times15=\underline{45}$. *Ans.*

Ex. 3. A boy loses $\frac{1}{4}$ of his money, and then gains 6*ps.* ; he then loses $\frac{1}{3}$ of what he has, and then gains 4*ps.* ; he afterwards loses $\frac{3}{8}$ of what he has, and then finds that he has 6*a.* 1*ps.* left. How much had he at first ?

$\frac{1}{4}$ of the money is lost ; $\therefore \frac{3}{4}$ of it remains ; 6*ps.* is then gained ; $\therefore$ money now remaining $=\frac{3}{4}$ of original money $+$ 6*ps.* ; of this $\frac{1}{3}$ is lost ;

$\therefore \frac{2}{3}$ of ($\frac{3}{4}$ of original money $+$ 6*ps.*) remains ; 4*ps.* is then gained ;

$\therefore$ money now remaining $=\frac{2}{3}$ of ($\frac{3}{4}$ of original money $+$ 6*ps.*) $+$ 4*ps.* ; of this amount $\frac{3}{8}$ is lost ;

$\therefore \frac{5}{8}$ of [$\frac{2}{3}$ of ($\frac{3}{4}$ of original money $+$ 6*ps.*) $+$ 4*ps.*] remains,

$=\frac{5}{8}$ of ($\frac{1}{2}$ of original money $+$ 4*ps.* $+$ 4*ps.*),

$=\frac{5}{16}$ of original money $+$ 5*ps.* ;

$\therefore \frac{5}{16}$ of original money $+$ 5*ps.* $=$ 6*a.* 1*ps.* ;

$\therefore \frac{5}{16}$ of original money $=$ 25*ps.* $-$ 5*ps.* $=$ 20*ps.*

$\therefore$ original money $=$ 20*ps.* $\times\frac{16}{5}=$ 64*ps.* $=$ *Re.* $\underline{1}$. *Ans.*

Ex. 4. From a tank $\frac{4}{5}$ths full of water 12 gals. are drawn, and the tank is then found to be $10\frac{1}{2}$ gals. more than half full; find how many gals. it will hold.

After drawing 12 gals. the quantity of water remaining $= \frac{4}{5}$ of tank $-$ 12 gals.; and it is then found that the tank is $10\frac{1}{2}$ gals. more than half full;

$\therefore$ $\frac{4}{5}$ of tank $-$ 12 gals. $= \frac{1}{2}$ of tank $+ 10\frac{1}{2}$ gals.
$\therefore$ $\frac{4}{5}$ of tank $- \frac{1}{2}$ of tank $= 10\frac{1}{2}$ gals. $+$ 12 gals. $= 22\frac{1}{2}$ gals.;
$\therefore$ $\frac{3}{10}$ of tank $= 22\frac{1}{2}$ gals.; $\therefore$ tank holds $22\frac{1}{2} \times \frac{10}{3}$ or 75 gals. *Ans.*

Examples XCII.

1. If to $\frac{3}{7}$ of a number 18 be added the sum is 42; find the number.

2. If to the sum of $\frac{3}{14}$ and $\frac{5}{8}$ of a number 34 be added the sum is 128; find the number.

3. If from the sum of $\frac{4}{5}$ and $\frac{7}{10}$ of a number 41 be taken the remainder is 97; find the number.

4. What is the number to which if you add 60 the sum is 5 times the original number?

5. There is a number, to which 3 is added and $\frac{1}{10}$ of the result taken; to this 5 is added and $\frac{1}{15}$ of the result taken: then the result is $1\frac{1}{3}$; what is the number?

6. The sum of two numbers is 5760, and their difference is equal to one-third of the greater. What are the numbers?

7. The sum of four fractions is $2\frac{19}{60}$, and one common result is obtained by adding the fraction $\frac{3}{10}$ to the first, subtracting $\frac{1}{3}$ from the second, multiplying the third by $\frac{7}{8}$ and dividing the fourth by $\frac{10}{11}$. Find the four fractions.

8. A person after paying away one-third of his money together with *Rs.*10, finds that he has remaining *Rs.*15 more than its half; what money had he?

9. *A* spends $\frac{3}{16}$ of his mouey and then earns *Rs.*5; he afterwards spends $\frac{4}{7}$ of what he then has, and has then *Rs.*10. 8*a.* left; find how much he had at first.

10. Out of $\frac{2}{3}$ of my income I pay to one person *Rs.*100 and to another *Rs.*150, and then find that I have *Rs.*50 less than $\frac{7}{12}$ of my income left; find my income.

11. Out of a cask two-thirds full of wine 8 gals. are drawn, and it is then found to be 2 gals. less than half-full; how many gals. is the cask able to hold?

12. An army in a defeat loses $\frac{1}{6}$ of its number and 8000 prisoners; after being reinforced by 6000 men it again loses $\frac{1}{4}$ of its number in retreat; and 36000 are then left, what was the original force?

318. Irregular Distributions.

Again means a second time. *As much again* means as much once and as much a second time, (*i. e.*) twice as much. *Half as much again* means as much once and half as much a second time, *i. e.*, $1\frac{1}{2}$ times as much.

Proceed as in the following Examples.

Ex. 1. Divide *Rs.*11875 among *A*, *B* and *C* so that as often as *A* gets *Rs.*4, *B* shall get *Rs.*3, and as often as *B* gets *Rs.*6, *C* shall get *Rs.*5.

As often as *A* gets *Rs.*4, *B* gets *Rs.*3 ; ∴ *B*'s share $=\frac{3}{4}$ of *A*'s.

As often as *B* gets *Rs.*6, *C* gets *Rs.*5 ; ∴ *C*'s share $=\frac{5}{6}$ of *B*'s.

∴ *C*'s share $=\frac{5}{6}$ of $\frac{3}{4}$ of *A*'s $=\frac{5}{8}$ of *A*'s ;

∴ *A*'s share + *B*'s + *C*'s $=(1+\frac{3}{4}+\frac{5}{8})$ of *A*'s $=2\frac{3}{8}$ times *A*'s share.

Hence $2\frac{3}{8}$ times *A*'s share $=$ *Rs.*11875,

∴ *A*'s share $=$ *Rs.*$11875\div2\frac{3}{8}=$ *Rs.*5000.

∴ *B*'s share $=\frac{3}{4}$ of *Rs.*5000 $=$ *Rs.*3750.

and *C*'s share $=\frac{5}{8}$ of *Rs.*5000 $=$ *Rs.*3125.

Otherwise thus : If *A* gets *Rs.*8, *B* gets *Rs.*6 and *C* gets *Rs.*5.

Now, $8+6+5=19$; and $11875\div19=625$.

∴ *A* gets $\frac{8}{19}$ of *Rs.*11875 $=$ *Rs.*$8\times625=$ *Rs.*5000 ; &c.

Ex. 2. Divide *Rs.*640 among *A*, *B* and *C*, so that *A* may have 3 times as much as *B*, and *C* $\frac{1}{3}$ of what *A* and *B* together have.

A's share $=3$ times *B*'s share ; *C*'s share $=\frac{1}{3}$(*A*'s + *B*'s).

∴ *C*'s share $=\frac{1}{3}$(3 *B*'s + *B*'s) $=\frac{4}{3}$ *B*'s.

∴ *A*'s share + *B*'s + *C*'s $=(3+1+\frac{4}{3})$ of *B*'s $=5\frac{1}{3}$ of *B*'s share.

Hence $5\frac{1}{3}$ of *B*'s share $=$ *Rs.*640 ; ∴ *B*'s share $=$ *Rs.*$640\div5\frac{1}{3}=$ *Rs.*120.

∴ *A*'s share $=$ *Rs.*$120\times3=$ *Rs.*360 and *C*'s $=\frac{4}{3}\times$ *Rs.*120 $=$ *Rs.*160.

Ex. 3. The sum of *Rs.*155 is to be divided amongst 3 men, 5 women and 8 boys, so that for every 3*a.* a man gets, a woman gets 2*a.*, and a boy 1*a.* 6*p.* ; find the share of each.

A woman's share $=\frac{2}{3}$ of a man's ; a boy's share $=\frac{1}{2}$ of a man's ;

∴ a man's share + a woman's + a boy's $=(1+\frac{2}{3}+\frac{1}{2})$ of a man's ;

∴ 3 men's shares + 5 women's + 8 boys' $=(3+\frac{10}{3}+4)$ of a man's $=10\frac{1}{3}$ times a man's share.

Hence $10\frac{1}{3}$ times a man's share $=$ *Rs* 155 ;

∴ a man's share $=$ *Rs.*$155\div10\frac{1}{3}=$ *Rs.*15 ; a woman's share $=\frac{2}{3}$ o *Rs.*15 $=$ *Rs.*10 and a boy's share $=\frac{1}{2}$ of *Rs.*15 $=$ *Rs.*7. 8*a.*

Ex. 4. Divide *Rs.*8424 among *A*, *B* and *C*, so that *A* shall receive $\frac{4}{5}$ as much as *B* and *C* together and *B* $\frac{4}{5}$ of what *A* and *C*; together receive.

A's share $= \frac{4}{9}$ of (B's $+ C$'s), and B's share $= \frac{4}{5}$ of (A's $+ C$'s).

$\therefore$ A's share $= \frac{4}{9}$ B's $+ \frac{4}{9}$ C's $= \frac{4}{9}$ of $\frac{4}{5}$ of (A's $+ C$'s) $+ \frac{4}{9}$ C's $= \frac{16}{45}$ of (A's $+ C$'s) $+ \frac{4}{9}$ C's $= \frac{16}{45}$ A's $+ \frac{16}{45}$ C's $+ \frac{4}{9}$ C's $= \frac{16}{45}$ A's $+ \frac{4}{5}$ C's ;

$\therefore$ A's $- \frac{16}{45}A$'s $= \frac{4}{5}C$'s, or $\frac{29}{45}A$'s $= \frac{4}{5}C$'s ; $\therefore$ A's $= \frac{4}{5} \times \frac{45}{29}C$'s $= \frac{36}{29}C$'s.

$\therefore$ B's $= \frac{4}{5}A$'s $+ \frac{4}{5}C$'s $= \frac{4}{5} \times \frac{36}{29}C$'s $+ \frac{4}{5}C$'s $= \frac{260}{145}C$'s $= \frac{52}{29}C$'s.

$\therefore$ A's share $+ B$'s $+ C$'s $= (\frac{36}{29} + \frac{52}{29} + 1)$ of C's $= \frac{117}{29}$ of C's share ; hence $\frac{117}{29}$ of C's share $=$ *Rs.*8424.

$\therefore$ C's share $=$ *Rs.*8424 $\div \frac{117}{29} =$ *Rs.*2088,
$\therefore$ A's share $= \frac{36}{29}$ of *Rs.*2088 $=$ *Rs.*2592,
and B's share $= \frac{52}{29}$ of *Rs.*2088 $=$ *Rs.*3744. } *Ans.*

Examples XCIII.

1. Divide *Rs.*6488. 7*a.* 10*p.* amongst three persons A, B and C, so that $\frac{1}{17}$ of A's share $= \frac{1}{23}$ of B's $= \frac{1}{18}$ of C's.

2. Divide *Rs.*75. 8*a.* between A, B and C giving B half as much again as A less *Re.*1, and C as much as A and B together.

3. Divide *Rs.*1400 among A, B and C in such a manner that as often as A gets *Rs.*5, B shall get *Rs.*4, and as often as B gets *Rs.*3, C shall get *Rs.*2.

4. Divide *Rs.*352. 9*a.* among A, B and C, so that B may get twice, and C 3 times as much as A.

5. Divide *Rs.*1800 among A, B and C, so that A shall receive 3 times as much as B, and B and C together $\frac{1}{2}$ as much as A.

6. Divide *Rs.*12540 among A, B and C, so that A shall receive $\frac{2}{3}$ as much as B and C together, and B $\frac{2}{3}$ of what A and C together receive.

7. Divide *Rs.*2008 among A, B and C, so that B's share may be $\frac{7}{8}$ of A's share, and C's share $\frac{5}{8}$ of B's.

8. Divide *Rs.*95. 10*a.* 8*p.* among 10 men, 6 women and 4 children, giving a woman 3 times as much as a child and a man twice as much as a woman.

9. Divide £1650 among A, B, C and D, so that A may have half as much as B, B a third as much as C and C a fourth as much as D.

10. If $\frac{3}{4}$ of A's money $= \frac{2}{3}$ of B's $= \frac{5}{6}$ of C's and A, B and C's money together amount to *Rs.*8260 ; how much has each ?

11. If $\frac{5}{8}$ of A's money $= \frac{7}{8}$ of B's $= \frac{4}{5}$ of C's $= \frac{9}{10}$ of D's and A, B, C and D together have *Rs.*23078 ; determine how much money each has.

12. If $\frac{7}{10}$ of A's money $= \frac{9}{11}$ of B's, and C's money $= \frac{2}{3}$ ($\frac{4}{5}$ of

A's + $\frac{3}{8}$ of B's), and C's money − A's money = *Rs.*667 ; find how much A, B and C each has.

319. Travelling round a Circle.

When two or more persons start simultaneously from the same place to travel round a circular course either in the same direction or in opposite directions, (i) they should first be together again at an interval of time which is the L. C. M. of the times during which one of the walkers gains one complete round over each of the others, for each pair will be together after this time ; (ii) they should first be together at the starting post again at an interval of time which is the L. C. M. of the times during which each makes one complete round, for in that interval each shall make an integral number of rounds.

Ex. 1. A can go round a circular course in 18 min., B can go round it in 24 min., and C in 32 min. If they start simultaneously from the same point and travel in the same direction, in what time will they come together again ?

Take 1 for the length of the course ;
then A travels $\frac{1}{18}$, B $\frac{1}{24}$ and C $\frac{1}{32}$ of the course in 1 min.
∴ A gains on B $(\frac{1}{18}-\frac{1}{24})$ or $\frac{1}{72}$ of the course in 1 min.
∴ A gains on B one complete round in $(1 \div \frac{1}{72})$ or 72 min
Hence A and B will be together after 72 min.

Again, A gains on C $(\frac{1}{18}-\frac{1}{32})$ or $\frac{7}{288}$ of the course in 1 min.
∴ A gains on C one complete round in $(1 \div \frac{7}{288})$ or $\frac{288}{7}$ min.
Hence A and C will be together after $\frac{288}{7}$ min.
Therefore A, B and C will be together after a time which is the L. C. M. of 72 and $\frac{288}{7}$, but the L. C. M. of 72 and $\frac{288}{7}$ is 288.
∴ A, B and C are first together after 288 min. *Ans.*

Ex. 2. In the above question, if A and B travel in the same direction but C in the opposite direction, when will they meet again ?
As in the above question,

A and B will be together at the end of 72 min. ;
Again, A and C together pass over $(\frac{1}{18}+\frac{1}{32})$ or $\frac{25}{288}$ of the course in 1 min.

∴ they come together at the end of $(1 \div \frac{25}{288})$ or $\frac{288}{25}$ min.
Now, the L. C. M. of 72 and $\frac{288}{25}$ is 288 ;
∴ A, B and C will be together at the end of 288 min. *Ans.*

Ex. 3. A, B and C start from the same point and travel in the same direction round an island 6 miles in circumference, A at the rate of 3, B at the rate of $2\frac{1}{2}$ and C at the rate of $1\frac{1}{2}$ miles an hour, in how many hours will they come together again ?

A gains on B $(3-2\frac{1}{2})$ or $\frac{1}{2}$ mile in 1 hour, ∴ he gains 6 miles or a complete round in $(6 \div \frac{1}{2})$ or 12 hours.
Hence A and B are together at the end of every 12 hours.

Again, *A* gains on *C* $(3-1\frac{1}{4})$ or $1\frac{3}{4}$ miles in 1 hour, ∴ he gains 6 miles or a complete round in $(6+1\frac{3}{4})$ or $\frac{24}{7}$ hours.

Hence, *A* and *C* are together at the end of every $\frac{24}{7}$ hours. Therefore *A*, *B* and *C* are together at the end of any number of hours which is a common multiple of 12 and $\frac{24}{7}$;

but the L. C. M. of 12 and $\frac{24}{7}$ is 24;

hence *A*, *B* and *C* are first together at the end of 24 hrs. *Ans.*

Ex. 4. In the above question, when will they be togethe again at the starting point?

Here, *A* takes $\frac{6}{3}$ or 2 hrs., *B* $(6+2\frac{1}{2})$ or $\frac{12}{5}$ hrs. and *C* $(6\div1\frac{1}{4})$ or $\frac{24}{5}$ hrs. to make one round.

Now, the L. C. M. of 2, $\frac{12}{5}$ and $\frac{24}{5}$ is 24;

∴ they will be together again at the starting point 24 hrs. after.

Examples XCIV.

1. Two persons *A* and *B* start from the same point to walk round a circular course in the same direction. *A* takes 9 min. and *B* takes 24 min. to complete one round; in what time will they be together again?

2. Three persons, *A*, *B* and *C* can respectively go round a circular path in 8, 18 and 30 min. If they start simultaneously from the same point and travel in the same direction, when will they meet again?

3. *A*, *B* and *C* start from the same point and travel in the same direction round an island 73 miles in circumference. *A* at the rate of 10, *B* at the rate of 14 and *C* at the rate of 26 miles a day; in how many days will they come together again?

4 There is a park $1\frac{7}{8}$ miles in circumference. Five persons start from the same point to travel round it in the same direction at the respective rates of 3, $3\frac{3}{4}$, 4, $4\frac{1}{2}$ and 5 miles per hour. When will they be together again at the starting point?

5 *A*, *B* and *C* start from the same point and travel in the same direction round an island 36 miles in circumference, *A* at the rate of 3 miles, *B* at the rate of $3\frac{1}{2}$ miles and *C* at the rate of 4 miles an hour; when will they be together again?

6. An island is 43 miles in circumference. Three men *A*, *B* and *C* start from the same place to walk round it, at the rates of 4, $4\frac{3}{8}$ and $5\frac{1}{2}$ miles per hour respectively In how many hours will they come together again, supposing them to travel in the same direction?

7. In the above question, if *A* and *B* travel in the same direction and *C* in the opposite direction, when will they come together again for the first time?

8. An island is 120 miles in circumference. Three persons *A*, *B* and *C* start from the same place to walk round at the respective

rates of 10, 12 and 15 miles per hour. When will they next meet supposing (i) that they walk in the same direction, (ii) that A walks in one direction and B and C in opposite directions?

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320. Chain Rule. If we wish to express one quantity A in terms of another quantity R, and have data from which we can form the following series of relations, *viz.*,

$$a\,A = m\,M \ldots\ldots\ldots\ldots\ldots\ldots(1)$$
$$b\,M = n\,N \ldots\ldots\ldots\ldots\ldots\ldots(2)$$
$$c\,N = p\,P \ldots\ldots\ldots\ldots\ldots\ldots(3)$$
$$d\,P = q\,Q \ldots\ldots\ldots\ldots\ldots\ldots(4)$$
$$e\,Q = r\,R \ldots\ldots\ldots\ldots\ldots\ldots(5)$$

which may be as numerous as we choose, then will

$$A = \frac{mnpqr}{abcde} R.$$

Hence we see that the quantity required is found by dividing the product of the numbers on the right-hand side of these equations by the numbers on the left-hand side.

Ex. 1. If 3 lbs. of tea be worth 4 lbs. of coffee, and 6 lbs. of coffee be worth 20 lbs. of sugar, and 15 lbs. of sugar be worth 24 lb of rice; how many lbs of rice are equal to 18 lbs. of tea?

lbs. reqd. rice = 18 lbs. tea,
3 lbs. tea = 4 lbs. coffee,
6 lbs. coffee = 20 lbs. sugar,
15 lbs. sugar = 24 lbs. rice;

$$\therefore \text{ lbs. reqd. rice} = \frac{18 \times 4 \times 20 \times 24}{3 \times 6 \times 15} = 128$$

321. In the preceding equations the quantity on the right-hand side of one equation is of the *same kind* as that on the left-hand side of the next equation, and thus the Chain of quantities from one kind to another is unbroken. And not only must they be of the *same kind* but also of the *same denomination*; for if not, the one or more missing links must be supplied.

Ex. 2. If 3 lbs. of rice be worth 5 oz. of tea, and 4 lbs. of tea be worth 9 lbs. of coffee, how many lbs. of coffee are worth 48 lbs. of rice?

Here, we must either supply the missing link 16 oz. tea = 1 lb. tea, or we must express 5 oz. tea as $\frac{5}{16}$ lb. tea; so that we have

lbs. reqd. coffee = 48 lbs. rice,
3 lbs. rice = 5 oz. tea,
16 oz. tea = 1 lb. tea,
4 lbs. tea = 9 lbs. coffee;

$$\therefore \text{ lbs. reqd. coffee} = \frac{48 \times 5 \times 1 \times 9}{3 \times 16 \times 4} = 11\tfrac{1}{4}.$$

lbs. reqd. coffee = 48 lbs. rice,
3 lbs. rice = $\frac{5}{16}$ lb. tea,
4 lbs. tea = 9 lbs. coffee;

$$\therefore \text{ lbs. reqd. coffee} = \frac{48 \times \frac{5}{16} \times 9}{3 \times 4} = \frac{48 \times 5 \times 9}{16 \times 3 \times 4} = 11\tfrac{1}{4}.$$

322. It is unnecessary to name the quantity on the *left-hand* side of any equation; for it must be the same as the quantity on the right-hand side of the preceding equation.

Ex. 3. If $\frac{1}{5}$ of a sheep be worth £$\frac{2}{3}$, and $\frac{3}{7}$ of a sheep be worth $\frac{1}{14}$ of an ox, what must be given for 100 oxen?

$$\pounds s.\ \text{reqd.} = 100 \text{ oxen},$$
$$\tfrac{1}{14} = \tfrac{3}{7} \text{ sheep},$$
$$\tfrac{1}{5} = \pounds\tfrac{2}{3},$$
$$\pounds s.\ \text{reqd.} = \frac{100 \times \frac{3}{7} \times \frac{2}{3}}{\frac{1}{14} \times \frac{1}{5}} = \frac{100 \times 3 \times 2 \times 14 \times 5}{7 \times 3} = \underline{2000.}$$

Ex. 4. If 1 lb. of standard gold, of which 11 parts out of 12 are fine gold, be worth £46. 14*s* 6*d*., find the value of 595 gold rupees of Bombay, each weighing 7 dwts. 10½ grs. of which 187 parts are fine gold and 13 alloy.

£46. 14*s*. 6*d*. = £$46\frac{29}{40}$ = £$\frac{1869}{40}$; ∴ 40 lbs. standard = £1869;
7 dwts. 10½ grs. = $7\frac{7}{16}$ dwts = $\frac{119}{16}$ dwts.; 16 Bombay rupees = 119 dwts.;
187 + 13 = 200; ∴ 187 parts out of 200 are fine; hence

$$\pounds s.\ \text{reqd.} = 595 \text{ Bombay rupees},$$
$$16 = 119 \text{ dwts. Bombay standard.}$$
$$20 \times 12 = 1 \text{ lb.}$$
$$200 = 187 \text{ lbs. fine},$$
$$11 = 12 \text{ lbs. English standard},$$
$$40 = \pounds 1869;$$
$$\therefore \pounds s.\ \text{reqd.} = \frac{595 \times 119 \times 187 \times 12 \times 1869}{16 \times 20 \times 12 \times 200 \times 11 \times 40} = \pounds 878\tfrac{401453}{512000}$$
$$= \underline{\pounds 878.\ 15s.\ 8\tfrac{1159}{6400}d.}$$

Examples XCV.

1 When 25 yards of muslin are equal to 16 yds. of calico, 21 yds. of calico to 13 yds. of flannel, 40 yds. of flannel to 27 yds. of linen, 58½ yds. of linen to 28 yds. of silk, 47 yds. of silk to 35 yds. of velvet; find how many yards of velvet are equal in value to 60 yds. of muslin.

2. If 16 mangoes be equal in price to 25 apples, and 18 oranges equal to 12 mangoes, and 20 lemons equal to 27 oranges, and lemons cost 9*d*. a dozen, what is the cost of 15 apples?

3. If 12 of *A* count for 13 of *B*, 6 of *B* for 18 of *C*, and 13 of *C* for 2 of *D*; how many of *A* count for 100 of *D*?

4. If £3 = 20 thalers; 25 thalers = 93 francs; 27 francs = 5 scudi; and 62 scudi = 135 gulden; how many gulden = £1?

5. If 16 darics make 17 guineas, 19 guineas make 24 pistoles, 31 pistoles make 38 sequins; how many sequins are there in 1581 darics?

6. If 72 carlini be worth 25 shillings, 4 shillings worth 5 francs and 8 scudi worth 45 francs, how many carlini are equal to 100 scudi?

7. If 35 metres = 39 yards, and 17 metres = 9 toises, and 5 plethera = 124 toises, how many yards are there in 1575 plethera?

8. If 6 horses cost as much as 24 cows, 10 cows as much as 8 buffaloes, 4 buffaloes as much as 15 asses, 8 asses as much as 32 sheep, and if the price of 9 sheep be *Rs.*25, find the cost of 8 horses.

9. If $\frac{1}{7}$ of a sheep be worth £$\frac{2}{3}$, and $\frac{4}{5}$ of a sheep worth $\frac{1}{12}$ of an ox; how much must be given for 300 oxen?

10. If 40 lbs. of standard gold, of which 11 parts out of 12 are fine, be coined into 1869 sovereigns; how many grains of pure gold are there in 1 sovereign?

11. If 1 lb of standard gold, of which 11 parts out of 12 are fine, be worth £46. 14*s.* 6*d.*, find the value of 550 Madras gold rupees, each weighing 7 dwts. 12 grs., of which 916 parts out of 1000 are fine.

12. If 1 lb of standard silver, of which 37 parts out of 40 are fine, be worth 66*s.*, find the value of an Arcot Rupee, weighing 7 dwts. 9 grs, of which 941 parts out of 1000 are fine.

Examples worked out.

Ex. 1. What least number must be added to $8\frac{1}{9}$, that the result being divided by $1\frac{2}{3}$, the quotient shall be an integer?

$$8\tfrac{1}{9} \div 1\tfrac{2}{3} = \tfrac{73}{9} \div \tfrac{5}{3} = \tfrac{73}{9} \times \tfrac{3}{5} = \tfrac{73}{15} = 4\tfrac{13}{15}.$$

Now, the least number that should be added to $4\frac{13}{15}$ to make it an integer is $\frac{2}{15}$, for $\frac{13}{15}+\frac{2}{15}=1$.

Then the question reduces to "What number divided by $1\frac{2}{3}$ will give $\frac{2}{15}$ as quotient?"

Hence the required number $=\frac{2}{15} \times 1\frac{2}{3} = \frac{2}{15} \times \frac{5}{3} = \frac{2}{9}$. *Ans.*

Ex. 2. Find two least integers such that $\frac{5}{6}$ of the first shall be equal to $\frac{7}{8}$ of the second.

If $\frac{5}{6}$ of 1st number be $=1$, then also $\frac{7}{8}$ of 2nd number $=1$.

∴ 1st number $=(1 \div \frac{5}{6}) = \frac{6}{5}$, and 2nd number $=(1 \div \frac{7}{8}) = \frac{8}{7}$.

Now to transform these fractions to least integers, multiply each of them by the L. C. M. of their denominators, and divide the numbers thus found by their G. C. M.

The L. C. M. of 5 and 7 is 35; ∴ from 1st we have $\frac{6}{5} \times 35 = 42$, and from 2nd $\frac{8}{7} \times 35 = 40$ Now the G. C. M. of 42 and 40 is 2.

Hence the numbers are $\frac{42}{2}$ and $\frac{40}{2}$, or 21 and 20. *Ans.*

Ex. 3. By selling an article for £12. 7*s.* 6*d.*, I cleared $\frac{4}{5}$ of what it cost me; what was the original cost?

Taking 1 for the original cost, the gain is $\frac{4}{5}$, and the selling price $(1+\frac{4}{5})$ or $\frac{9}{5}$.

∴ $\frac{9}{5}$ of the original cost = £12. 7*s.* 6*d.*;

∴ the original cost = £12. 7*s.* 6*d.* ÷ $\frac{9}{5}$ = £12. 7*s.* 6*d.* × $\frac{5}{9}$

= £1. 7*s.* 6*d.* × 5 = £6. 17*s.* 6*d.* *Ans.*

Ex. 4. By selling 15 seers of tea at *Rs* 5. 4*a*. per seer, a grocer clears $\frac{1}{8}$ of his outlay. He then raises the price to *Rs*.6. per seer and sells 50 seers more. What does he gain on the whole outlay for 65 seers ?

Taking 1 for the original cost, the selling price is $(1+\frac{1}{8})$ or $\frac{9}{8}$.

∴ the original cost = *Rs*.5. 4*a*. ÷ $\frac{9}{8}$ = *Rs*.4. 10*a*. 8*p*.

∴ in the 1st case gain per seer = *Rs*.5. 4*a*. − *Rs*.4 10*a*. 8*p*. = 9*a*. 4*p*.

in the 2nd case ... = *Rs*.6. − *Rs*.4. 10*a*. 8*p*. = *Re*.1. 5*a*. 4*p*.

Now, gain on 15 seers = 9*a*. 4*p*. × 15 = *Rs*. 8. 12*a*.

and gain on 50 seers = *Re*.1. 5*a*. 4*p*. × 50 = *Rs*.66. 10*a*. 8*p*.

∴ his whole gain = *Rs*.75. 6*a*. 8*p*. *Ans*.

Ex. 5. Find the least number of sovereigns that contains an exact number of 20-franc pieces of 15*s*. 11¼*d*. each.

Here, 15*s*. 11¼*d*. = 191¼*d*. = $\frac{765}{4}$*d*. and a sovereign = 240*d*.

∴ $\frac{765}{4}$*d*. × no. of 20-franc pieces = 240*d*. × no. of sovereigns ;

∴ no. of 20-franc pieces = 240 × $\frac{4}{765}$ × no. of sovereigns ;

= $\frac{64}{51}$ × no. of sovereigns.

Hence the least no. of sovereigns that will make an exact number of 20-franc pieces is 51. *Ans*.

Ex. 6 A man bought 4 sorts of rice at an average price of *Rs*.6 a maund. If the prices increase by a common difference of 5*a*. per maund, find the cost of each sort per maund.

The price of 4 sorts at *Rs*.6 per maund = *Rs*.6 × 4 = *Rs* 24.

Each maund of second sort cost 5*a*. more than a md. of 1st sort,

....third............10*a*..

...................fourth............15*a*........ ....

∴ these 3 maunds cost *Re*.1. 14*a*. more...............................

Now, leaving out this sum, the cost of 4 maunds is *Rs*.24 − *Re*.1. 14*a*. or *Rs*.22. 2*a*. ; ∴ the cost of 1 md. = *Rs*.22. 2*a*. ÷ 4 = *Rs*.5. 8*a*. 6*p*.

Hence the cost of 1 md. of 1st sort = *Rs*.5 8*a*. 6*p*.
........................2nd sort = *Rs*.5. 8*a*. 6*p*. + 5*a*. = *Rs*.5. 13*a*. 6*p*.
... .3rd sort = *Rs*.5. 13*a*. 6*p*. + 5*a*. = *Rs* 6. 2*a*. 6*p*.
..............................4th sort = *Rs*.6. 2*a*. 6*p*. + 5*a*. = *Rs*.6. 7*a*. 6*p*. } *Ans*.

Ex. 7. *A* and *B* undertake to do a piece of work for *Rs*.12. 8*a*., *A* can do the work alone in 20 days and *B* in 15 days. They work together for three days, and then with the assistance of *C* finish it in 5 days more. How should the sum be divided ?

Here, *A* and *B* each worked for (5+3) or 8 days, and *C* for 5 days.

As *A* can do $\frac{1}{20}$ of the work in 1 day, he did $\frac{8}{20}$ or $\frac{2}{5}$ of the work in 8 days.

...*B*..............$\frac{1}{15}$.................................he did $\frac{8}{15}$..

∴ *A* and *B* did in 8 days $(\frac{2}{5}+\frac{8}{15})$ or $\frac{14}{15}$ of the work.

Hence the work done by C in 5 days $=(1-\frac{14}{15})$ or $\frac{1}{15}$ of the work.

$\therefore$ A received $\frac{2}{5}$ of *Rs.*12. 8*a* $=$ *Rs.*5
B............$\frac{8}{15}$ of *Rs.*12. 8*a.* $=$ *Rs* 6. 10*a.* 8*p.* } *Ans.*
and C............$\frac{1}{15}$ of *Rs.*12. 8*a.* $=$ 13*a.* 4*p.*

Ex. 8. A man's income from Government Securities is $\frac{3}{4}$ of what he receives from his landed property. An income-tax of 5*p.* in the rupee is charged on the first and of 4*p.* in the rupee on the second, and he has to pay altogether *Rs* 31 as income-tax. Find his total income.

Suppose his income from landed property to be *Rs.*4,
then.......................Government Securities is *Rs* 3 ;
income-tax on 1st $=(4\times4)$ or 16*p.* and on second $=(3\times5)=15p$,
and 16*p.* + 15*p.* = 31*p.* = *Re.* $\frac{31}{192}$.

$\therefore$ he has to pay *Re.* $\frac{31}{192}$ as tax on every *Rs.*7 of income.
$\therefore$ ....*Re.*1 as tax............... *Rs.*$7\times\frac{192}{31}$..................
$\therefore$ ....*Rs.*31 as tax.............. *Rs.*$7\times\frac{192}{31}\times31$..............
Hence, required income $=$ *Rs.*$7\times\frac{192}{31}\times31$ $=$ *Rs.*1344. *Ans.*

Ex. 9. A can do as much work in one day as B can do in 2 days, or as C can do in 3 days or as D can do in 4 days. They together finish a piece of work in 8 days. How many days would each take to do it singly ?

Suppose A's one day's work to be 1, then B's one day's work $\frac{1}{2}$, C's $\frac{1}{3}$ and D's $\frac{1}{4}$.

$\therefore$ A, B, C and D's one day's work

$=(1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4})$ or $\frac{25}{12}$ times A's work per day ;

$\therefore$ A, B, C and D's 8 days' work $=8\times\frac{25}{12}$ or $\frac{50}{3}$ times A's work per day.

But A, B, C and D's 8 days' work $=$ whole work ;

$\therefore$ $\frac{50}{3}$ times A's work per day $=$ whole work ;

or A's work per day $=\frac{3}{50}$ of the whole work.

Hence A can do the whole work in $(1\div\frac{3}{50})$ or $16\frac{2}{3}$ days.

Therefore B's time $=(2\times16\frac{2}{3})$ or $33\frac{1}{3}$ days ; C's time $=(3\times16\frac{2}{3})$ or 50 days, and D's time $=(4\times16\frac{2}{3})$ or $66\frac{2}{3}$ days.

Examples XCVI.

1. Determine the least number which must be added to $3\frac{1}{8}$ that when the result is divided by $\frac{1}{4}$, the quotient shall be an integer.

2. What least number must be subtracted from $8\frac{2}{3}$, that when the difference is divided by $\frac{8}{7}$, the quotient shall be an integer ?

3. If a pound weight of standard gold is worth £61. 18*s.* 9*d.*, find the least integral number of pounds of gold that can be coined into an integral number of sovereigns.

4. If the rupee is worth 1*s.* 9*d.* and the mohur 30*s*, find the least number of pounds which can be paid exactly in rupees or mohurs.

5. By selling an article for *Rs.*460, I cleared $\frac{3}{20}$ of the prime cost. Find the cost price.

6. By selling a horse for *Rs* 2520, a man lost $\frac{4}{25}$ of what it cost him. What did it cost him?

7. Find the least number of sovereigns that contains an exact number of thalers and of dollars; 48 thalers being worth £7. 3*s.* and 8 dollars £1. 13*s.*

8 *A* has twice as much money as *B*. They play together, and at the end of the first game *B* wins from *A* one-third of *A*'s money; what fraction of the sum which *B* now has must *A* win back in the second game that they may have exactly equal sums?

9. How many maunds of rice at *Rs.*4 per maund must a corn-merchant mix with 1 maund of rice at *Rs.*5 per maund, that by selling the mixed rice at *Rs* 4. 8*a.* per maund, he may gain $\frac{1}{10}$ of his outlay?

10. Find the least number that must be added to 75¼, that the sum being severally divided by $\frac{3}{4}$, $\frac{5}{12}$, $\frac{7}{8}$ or $\frac{9}{16}$, the quotient in each case shall be an integer.

11. *A* and *B* undertake to do a piece of work for *Rs.*7. 8*a.* *A* can do it alone in 8 days and *B* in 6 days. With the assistance of *C* it is finished in 3 days. How should the money be divided?

12 *A* and *B* engage to do a piece of work for *Rs.*40 *A* can do it alone in 16 days and *B* can do it in 12 days. After working together for 4 days, *A* leaves off, when *C*, who can alone finish the work in 8 days, joins. How should the sum be distributed after the work is completed?

13. A man derives his income from three sources. His income from Government Securities is $\frac{1}{4}$ of his income from trade, and his landed property yields an income equal to $\frac{2}{3}$ of the sum of both. The rate of tax on income from trade is 6*p.* per rupee, on Securities 5*p.* per rupee, and on landed property 4*p.* per rupee. If his total income-tax amounts to *Rs.*260, find his gross income.

14. By selling tea at *Rs.*2. 10*a.* 8*p.* per lb., a grocer clears $\frac{1}{8}$ of his outlay; he then raises the price to *Rs.*3. What does he clear on every *Rs.*200 of his outlay by this price?

15. A tradesman buys 5 mds. 24 sr. of goods for *Rs.*150 intending to gain $\frac{3}{25}$ of his outlay by the sale; but *Rs.*10. 8*a.* worth at this calculation being damaged, at what price shall he sell the remainder per maund, to gain as much upon the whole outlay as he intended?

16. *A* can do as much work in 1 day as *B* in 3 days, *C* in

5 and *D* in 7 days. They together complete a piece of work in 8 days. In how many days will each do it singly?

17. A cloth-merchant bought a bale of cloth containing 150 pieces each, of cloths $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4 and $4\frac{1}{2}$ yds. in length for *Rs.*626. 9*a.* If the prices increase by a common difference of 3*a.*, at what price per piece must he sell them that he may gain *Rs.*100 by the transaction?

18. By selling a horse for *Rs.*345, I lost $\frac{2}{25}$ of the prime cost. What would have been my gain had I sold it for *Rs.*380?

19. *A* and *B* can finish a piece of work in $1\frac{1}{2}$ days, *A* and *C* in 2 days and *B* and *C* in 3 days. If *Rs.*6 be paid for the piece of work, what are a day's wages of each workman?

20. An elastic ball after striking the ground rises to $\frac{3}{4}$ of the height from which it fell. After striking the ground the third time it rises $3\frac{3}{8}$ inches: from what height did it fall at first?

Miscellaneous Examples III.

1. Divide the sum of 10 and $\frac{1}{10}$ by their difference, and also the difference by their sum; and find the sum and difference of the two quotients.

2. Add together $1\frac{3}{4}$, $2\frac{2}{3}$, and $3\frac{1}{2}$; multiply this sum by the product of these fractions; subtract from the result the difference of $2\frac{2}{3}$ and $1\frac{1}{2}$; and divide the remainder by the sum of $5\frac{1}{2}$ and $1\frac{1}{3}$ of $3\frac{3}{4}$.

3. Simplify $\frac{3}{4}$ of $\frac{£3.\ 10s.}{£4.\ 4s.} \div \frac{4}{5}\left(\frac{7}{12}+\frac{29}{36}\right)$ of $\frac{2 \text{ tons } 4 \text{ cwt.}}{3 \text{ tons } 6 \text{ cwt.}}$.

4. Divide *Rs.*19000 among *A*, *B*, *C* and *D*, so that *B* may receive $\frac{3}{4}$ of *A*, *C* $\frac{2}{3}$ of *B* and *D* $\frac{5}{6}$ of *C*.

5. If $\frac{5}{8}$ of $\frac{9}{10}$ of *A*'s money $=\frac{7}{9}$ of $1\frac{2}{3}$ of *B*'s and the difference of their moneys be *Rs.*291, find *A*'s and *B*'s money.

6. If 3 men and 2 boys can do a piece of work in 15 days, and 2 men and 3 boys can do the same in 18 days, in what time will a man and a boy jointly do the work?

7. In an orchard, $\frac{1}{3}$ of the trees are apple trees, $\frac{1}{4}$ pear trees, $\frac{1}{5}$ cherry trees, $\frac{1}{6}$ filbert trees, and there are 12 walnut trees; what is the number of each sort?

8. If *A* can do half a piece of work in 3 hours, which is twice as much as *B* can do, and *A*, *B* and *C* can together do the whole in $2\frac{1}{2}$ hours; shew that *C* can do in 5 hours as much as *B* can do in 9 hours.

9. A 38-gallon cask of wine cost a wine-merchant *Rs.*250; but he lost 8 gallons of it by leakage; how must he sell the remainder per gallon to gain $\frac{1}{10}$th of the prime cost?

10. A man owns $\frac{5}{16}$ of an estate. He sells $\frac{1}{5}$ of his share and then finds that his remaining share is worth *Rs.*525. 10*a*. What is the value of the whole property?

11. *A* can do a piece of work in 8 days, *B* in 12 days and *C* in 15 days. They all work together for 3 days at the same piece of work. In what time will *B* finish the remaining work?

12. A grocer mixes 5 maunds of rice at *Rs.*4 per md. with $3\frac{1}{3}$ mds. at *Rs.*4. 8*a.* per md. At how much per maund must he sell the mixed rice that he may gain $\frac{3}{32}$ of his outlay?

13. If *A* takes 8 days to complete a piece of work, *B* takes 9 days to do $\frac{3}{4}$ of the same, and if *B* takes 10 days to complete a piece of work, *C* takes 8 days to do $\frac{2}{3}$ of the same; in what time will *B* and *C* together finish a work which *A* alone can do in 25 days?

14. Reduce $\frac{5}{8}$ of *Rs.*3. 12*a.* + $\frac{1}{9}$ of *Rs.*7. 6*a.* − $\frac{1}{10}$ of *Rs.*8. 4*a.* 6*p.* to the fraction of *Rs.*20. 10*a.*

15. Reduce $1\frac{1}{3}$ of £7. 6*s.* 8*d.* + $\frac{5}{24}$ of £9. 13*s.* 4*d.* − $\frac{1}{10}$ of £10. 3*s.* 4*d.* to the fraction of £25. 10*s.*

16. If 2 men and 1 boy do a piece of work in 5 days, 1 man and 2 women do it in 6 days and 1 woman and 2 boys do the same in 8 days, in what time will a man, a woman and a boy do it jointly?

17. Of the population of a certain town $\frac{1}{3}$ can read, $\frac{1}{4}$ can write, $\frac{1}{5}$ can read and write and the remaining 130 can neither read, nor write. Find the total population of the town.

18. Simplify—

(*a*) $\dfrac{\frac{5}{12}+\frac{3}{10}}{\frac{5}{12}-\frac{3}{10}}$ of $\dfrac{8}{9\frac{1}{3}}+\dfrac{\frac{7}{8}+\frac{1}{6}}{\frac{7}{8}-\frac{1}{6}}$ of $\dfrac{13}{14}\times\dfrac{\frac{15}{16}+\frac{3}{4}}{\frac{15}{16}-\frac{3}{4}}$ of $\dfrac{2}{3}\left(\dfrac{3}{4}+\dfrac{1}{5}\right)$.

(*b*) $\dfrac{9\frac{1}{3}}{11\frac{1}{5}}$ of $\dfrac{Rs.3.\ 12a.}{Rs.4.\ 8a.}\div\dfrac{7\frac{1}{4}}{11\frac{3}{8}}$ of $\dfrac{£2.\ 6s.}{£3.\ 9s.}$.

19 The adult population of a country is 22815210; the adult females are $\frac{5}{16}$ of the whole population, and the adult males are $\frac{18}{17}$ of the adult females; find the whole population.

20. The wages of *A* and *B* together for $22\frac{1}{2}$ days amount to the same sum as the wages of *A* alone for $38\frac{1}{4}$ days. For how many days will this sum pay the wages of *B* alone?

21. A farmer paid a corn-rent of 5 qrs. of wheat and 3 qrs of barely, Winchester measure. What was the value of his rent when wheat was at 60*s.* and barley 54*s.* per quarter, Imperial measure, it being assumed that 32 Imperial gallons are equivalent to 33 Winchester gallons?

22. A man's debts amount to $\frac{3}{14}$ of his property, but before paying them he loses $\frac{7}{8}$ of his property: afterwards he recovers a portion equal to $\frac{1}{5}$ of what he has left, and then loses $\frac{1}{2}$ of what he has got. Can he pay his debts? What part of his property remains over?

23. A man can do 4 times a certain work in 9 hours, a woman 3 times the work in 10 hours, and a child twice the work in 11 hours;

if a man, a woman and a child work together, in what time can they do 7 times the work ?

24. Five brothers join in paying a sum of money ; the eldest pays a third of it, and the others pay the remainder in equal shares, and thereby each of them pays *Rs.*840 less than the eldest brother. What is the sum of money ?

25. If 9 men or 16 women can do a piece of work in 144 days, in what time would 7 men and 9 women do it, working together ?

26. Out of a cistern, which is $\frac{2}{3}$rds full, 20 gallons are drawn, the cistern is then found to be $\frac{3}{8}$ths full. How much will the cistern hold ?

27. The product of three numbers is 340 ; the first is $7\frac{3}{8}$, the second is less than the first by $1\frac{7}{20}$. Find the third number.

28. *A* owned $\frac{5}{8}$ of a mine and sold $\frac{3}{4}$ of his share to *B*, who sold $\frac{1}{2}$ of his share to *C*, who sold $\frac{3}{5}$ of his share to *D* ; *D*'s share was worth *Rs.*20565. What was the worth of *B*'s remaining share, and what the worth of the whole mine ?

29 There are two fractions whose sum is $1\frac{1}{28}$, and whose difference is $\frac{7}{18}$; find the fractions, and the quotient of the greater by the less.

30. If a turkey cost £$\frac{5}{7}$ and a goose £$1\frac{3}{10}$, how many turkeys and geese, an equal number of each, can be bought for £14. 4*s.* ?

31. A boy, in flying his kite, lost $\frac{3}{8}$ of the string ; he then added 65 ft., and then found that it was $\frac{5}{6}$ of the original length. What was the length at first ?

32. If $2\frac{1}{3}$ of $(A+\frac{2}{7}$ of $A)=\frac{1}{3}$ of $(B-\frac{1}{3}B)$, find the value of *A* in terms of *B*.

33. A man bequeathed $\frac{5}{12}$ of his estate to one son, $\frac{7}{12}$ of the remainder to another son, and the balance to his widow. The sons' shares differ by *Rs.*1320 ; find the widow's share.

34. A man gives away in charity $\frac{1}{8}$ of his income, and pays $\frac{1}{16}$ of it in rates and taxes ; with these deductions he has *Rs.*4736. 8*a.* 8*p.* left. What is his gross income ?

35. Find the whole annual cost of a house, of which the rent is *Rs.*360 ; the poor-rate being 2*a.* 8*p.* in the rupee, the gas-rate $\frac{3}{4}$ of the poor-rate, and the paving-rate $\frac{3}{8}$ of the gas-rate.

36. What sum must be added to or subtracted from £12. 7*s.* 6*d.*, so that £5. 3*s.* 4*d.* shall be the same fraction of the sum or difference that £3. 6*s.* 8*d.* is of £8. 6*s.* 8*d.* ?

37. Divide *Rs.*4200 among *A*, *B*, *C* and *D* so that *A* may get twice as much as *B*, *A* and *C* may get thrice as much as *B* and *A*, and *D* may get four times as much as *B* and *C*.

38. The sum of $\frac{3}{4}$ and $\frac{1}{3}$ of a man's debts amounts to *Rs* 198.

7*a*. 4*p*. and his assets are *Rs*.45. 1*a*. 8*p*. ; how much in the rupee will his creditors lose ?

39. One-third of *A*'s money is equal to $\frac{1}{2}$ of *C*'s and $\frac{1}{3}$ of *C*'s is equal to $\frac{3}{8}$ of *B*'s ; *B* gives to *A* $\frac{1}{2}$ of his money and to *C* $\frac{1}{4}$ of the remainder, and has 2*s*. 6*d*. left. What amount had each at first ?

40. Express $\frac{3}{8}$ of $\frac{4}{13}$ of £1. 10*s*.+$\frac{2}{7}$ of $\frac{5}{6}$ of 5*s*. 4*d*.−$8\frac{1}{2}$ of $\frac{1}{4\frac{1}{4}}$ of 5*s*. $3\frac{3}{4}$*d*. as the fraction of 2*s*. $1\frac{1}{2}$*d*.

41. Find the value of

$$\frac{1}{4} \text{ of } \left(\frac{4\frac{7}{8} \text{ of } 6\frac{2}{3}}{7\frac{8}{27}}\right) \times \frac{3\frac{3}{5}-3\frac{1}{6}}{3\frac{3}{5}+2\frac{1}{6}} \text{ of } Rs\ 184.\ 11a.\ 5p.$$

42. What sum must be added to or subtracted from *Rs* 8. 12*a* 6*p*. so that the sum or difference shall be the same fraction of *Rs*.20. 10*a* that *Rs*.7. 6*a*. 6*p*. is of *Rs*.18. 8*a*. 3*p*. ?

43. *A* can do in 6 days as much work as *C* can do in 4 days, and *B* in 10 days as much as *C* in 8 days : what time would *B* require to finish a piece of work which *A* can do in 12 weeks ?

44. When rice is at 12 sr. per rupee, the expenses of a family amount to *Rs*.140 ; but they amount to *Rs* 134 only, when the price falls to 16 sr. per rupee. What will the expenses be, when rice is at 18 sr. per rupee ?

45. *A* can do in 2 days as much work as *B* in 3 days, and *B* in 5 days as much work as *C* in 4 days ; what time will *C* require to finish a piece of work which *A* can do in 9 days?

46. *A* can by himself perform a certain quantity of work in 5 days, *B* twice as much in 7 days, and *C* four times as much in 11 days ; in what time can *A*, *B* and *C* together perform three times the original work ?

47. *A* was owner of $\frac{5}{17}$ of a privateer, and sold $\frac{8}{11}$ of $\frac{3}{8}$ of his share for £$12\frac{4}{33}$; what was the value of $\frac{1\frac{1}{7}}{5\frac{3}{8}}$ of $\frac{7}{12}$ of the vessel at the same rate ?

48. How much ore must be raised, that on losing $\frac{17}{40}$ in roasting, and $\frac{13}{16}$ of the residue in smelting, there may result 506 tons of pure metal ?

49. Simplify—

$$\frac{\frac{1}{2}+\frac{1}{4}-\frac{1}{12}}{\frac{1}{2}-\frac{1}{4}+\frac{1}{12}} \text{ of } \frac{\frac{1}{2}-(\frac{1}{4}+\frac{1}{12})}{\frac{1}{3}-(\frac{1}{4}-\frac{1}{12})}+\frac{\frac{1}{3}\times\frac{1}{4}-\frac{1}{12}}{\frac{1}{3}\times(\frac{1}{4}-\frac{1}{12})} \text{ of } \frac{\frac{1}{3}-\frac{1}{4}\times\frac{1}{12}}{(\frac{1}{3}-\frac{1}{4})\times\frac{1}{12}}.$$

50 *A* is $1\frac{1}{2}$ times as good a working person as *B*, and twice as good as *C*. They all three can do a piece of work together in 6 days. They begin together, but after working for 2 days *A* goes away. After 2 days more *B* goes away, and *C* then completes the work alone. In how many days from the commencement is the whole work finished ?

51. *A* and *B* are engaged to do a piece of work, which can be done by each in 15 and 20 days respectively. If *A* leaves off 3 days before the completion of the work, how should a sum of *Rs.*12. 8*a.* be distributed among them ?

52. *A* and *B* undertake to do a piece of work in 15 days for *Rs.*22. 8*a.* After working for 12 days they call *C* to their help, and finish it in time. *A* could have done the work alone in 25 days. If they give *C* *Rs.*2. 4*a.*, how many days would *B* take to finish the work ?

53. A man is thrice as good a workman as a boy. If the time taken by a boy to do a piece of work exceed that taken by a man by $4\frac{1}{2}$ days, find the time in which a man can do it.

54. *A* and *B* can do a piece of work in 6 days, *B* and *C* in 7 days, and *A*, *B* and *C* can do it in 4 days. How long will *A* and *C* take to do it ?

55. There is a leak in the bottom of a cistern. When the cistern was in thorough repair it could be filled in $\frac{3}{4}$ of an hour. It now takes 10 min. longer. If the cistern were full, how long would it be in leaking itself to become empty ?

56. 10 men can do a piece of work in 30 days. After working for 10 days, a certain number of men are allowed to leave off and then the work is finished in $43\frac{1}{3}$ days from the commencement. How many men are allowed to leave off ?

57. The work which can be done by a certain number of men in 60 days, can be done by 15 men more in 40 days. Find the number of men required to do the work in 60 days.

58. There are two numbers of which the difference is 91. A third number is contained in them 13 and 20 times respectively. Find the numbers.

59. The number 483 divided by another gives 4 for the quotient and 7 for the remainder ; find what number, when multiplied by the remainder, will give that divisor.

60. A boy was told to divide one half of a certain number by 7, and the other half by 9, and then to add the two quotients. To save trouble he divided the number by 8, and his result was 6 wrong. What was the number ?

61. At 3 o'clock I had completed $\frac{3}{8}$ of my journey, and at 5 o'clock $\frac{5}{8}$ of the same ; when did I start and arrive ?

62 40 men can do a piece of work in a certain number of days ; if only 30 men be employed it requires 6 days more. Find the time in which 60 men can do it.

63. 20 men do a piece of work in 24 days. After working for 6 days, an additional number of men is taken for assistance, and the work is finished in 21 days from the beginning. Required the additional number of men.

64. There are 4 casks of different sizes. The 1st is filled with water, the rest are empty. The 2nd cask is filled from the 1st and $\frac{4}{7}$ths of the original water in the 1st remains. The third is then filled from the 2nd, and $\frac{1}{4}$th of the water in the 2nd remains. The water in the 3rd is then poured into the 4th, and fills $\frac{9}{10}$ths of it. Had the 3rd and 4th casks been filled from the contents of the 1st, 15 seers would still have remained in the 1st. Find the size of each cask.

CHAPTER VI.

The Theory of Decimals.

323. In the Notation of Integers, it has been seen that the figures in the units' place alone retain their *absolute* values, whilst the *local* values of figures in other situations increase **tenfold** for every figure we advance towards the left-hand from that place. Therefore, in beginning at the *left*-hand figure of any number and proceeding towards the *right* hand, it follows that the *local* value of every figure will be **tenth** part of that which immediately precedes it; and if we suppose figures to be situated to the right of the units' place, and this kind of tenfold *sub-division* to be extended to them, it is manifest that the local values of such figures in order from the place of units, will be a *tenth*, a *hundredth*, a *thousandth &c.*, parts of their absolute values.

Hence we are enabled to represent integers and fractions by one uniform system of notation, by merely marking the **place of units**; and whilst *Integers* are expressed by figures in the units' place and in places to the *left* of it, *Fractions* will be represented by figures situated in places, on the *right* of the units, called the places of *tenths*, *hundredths*, *thousandths*, *&c.*

324. In this manner originates the system of **Decimals**, being merely an extension of the Notation of Integers; and though there are decimals of all denominations as *Decimal Integers*, yet from the circumstance of the system representing only *tenth*, *hundredth*, *thousandth &c.*, parts of the unit, all *fractions* belonging to it are termed **Decimal Fractions**, in contradistinction to *Vulgar Fractions*, whereof the denominations may be any parts whatever.

Whence, **Decimals** may be *defined* to be *Fractions* whose denominators are 10, 100, 1000, &c., these denominators not being *written* as in Vulgar Fractions, but *expressed* by the position of a *dot* or *point*, called the **decimal point**.

I. NOTATION AND NUMERATION OF DECIMALS.

325. If we suppose the digit 1 to occupy the units' place, the following scheme will point out the denominations of the figures to

the left and right of it, and it may be extended so as to include both integers and fractions of all local values whatever.

&c.	&c.	Ten-thousands	Thousands	Hundreds	Tens	**Units**	Tenths	Hundredths	Thousandths	Ten-thousandths	&c.	&c.
...	...	5	4	3	2	**1**	2	3	4	5	...	...

A mixed quantity, formed of integers and fractions is separated into its *integral* and *fractional* portions by means of the *Decimal Point* placed on the right of the units' place towards the *top*, to distinguish it from the sign of Multiplication), which dispenses with the description of the local denominations, given above.

Thus, in 54321·2345, the figures 54321 on the *left* of the point denote so many integers, and the figures 2345 on the *right* of it, so many fractions, namely, 2 *tenths*, 3 *hundredths*, 4 *thousandths*, 5 *ten-thousandths*, and so on.

326. A number thus expressed, composed of units and *decimal* parts of unity, or of *decimal* parts of unity only, is called a **decimal number**, or simply a **decimal**. The part to the left of the point is called the **integral**, and to the right the **decimal** parts of the given number.

Thus, 452·3678 is a *decimal* ; 452 is the *integral* part and 3678 the *decimal* part.

327. From what has been said above, it appears that the *expressing* and *reading* of *Decimals* will evidently be conducted upon the respective principles of the *Notation* and *Numeration* of integers ; also, inasmuch as *Integers* denote assemblages of two or more *units*, *Decimals* will represent assemblages of two or more tenth, hundredth, &c., *parts* of a unit. Thus, to express,

(1) 45 units 3 tenths 2 hundredths 6 thousandths 8 ten-thousandths we write 45·3268.

(2) 45 units 2 hundredths 8 ten-thousandths we write 45·0208.

(3) 2 hundredths 8 ten-thousandths we write 0·0208 or simply ·0208.

328. In reading a decimal, we read off the decimal part as an integer annexing the denomination of its *last* figure on the right hand. Thus,

(1) 45·3268 is read 45 and 3268 *ten-thousandths*.

(2) 3·141596 is read 3 and 141596 *millionths.*

(3) ·00047 is read 47 *hundred-thousandths.*

329. In practice, however, we do not annex the decimal denomination, but saying (*decimal*) *point* read off the figures of the decimal *separately* in order. Thus,

(1) 45·3268 is read 45, point 3, 2, 6, 8.

(2) 3·141596 is read 3, point 1, 4, 1, 5, 9, 6.

(3) ·00047 is read point 0, 0, 0, 4, 7.

II. RELATION OF DECIMALS TO VULGAR FRACTIONS.

330. From the statements made in the preceding Articles, it is obvious that every magnitude made of one or more decimals is equivalent to, and may be expressed by, one or more vulgar fractions having 10, 100, 1000, &c., for their denominators; and that all *mixed* quantities expressed decimally may be represented by means of *whole* numbers and *vulgar fractions* of similar denominations.

Thus, $24\cdot387 = 24 + \frac{3}{10} + \frac{8}{100} + \frac{7}{1000}$; $\cdot045 = \frac{0}{10} + \frac{4}{100} + \frac{5}{1000}$.

331. *To convert a decimal into an equivalent vulgar fraction.*

Rule. Write down the given number for the numerator (omitting the decimal point), and for the denominator write 1 followed by as *many* ciphers as there are figures in the decimal part.

Ex. 1. $\cdot327 = \frac{327}{1000}$; for $\cdot327 = \frac{3}{10} + \frac{2}{100} + \frac{7}{1000} = \frac{327}{1000}$.

Ex. 2. $\cdot0459 = \frac{459}{10000}$; for $\cdot0459 = \frac{0}{10} + \frac{4}{100} + \frac{5}{1000} + \frac{9}{10000} = \frac{459}{10000}$

Ex. 3. $13\cdot816 = \frac{13816}{1000}$; for $13\cdot816 = 13 + \frac{8}{10} + \frac{1}{100} + \frac{6}{1000}$
$= 13\frac{816}{1000} = \frac{13816}{1000}$.

In these instances, we see that the reduction to a common denominator, so tedious in vulgar fractions, is entirely dispensed with, and the *immediate* comparison of fractional quantities is one of the great advantages of the system.

332. *Conversely*, every vulgar fraction having 10, 100, 1000, &c., for its denominator, may be immediately represented by an equivalent decimal.

Rule. Write down the numerator and by beginning at the figure on the *right* hand, mark off by the decimal point as many figures as there are ciphers in the denominator. If the number of figures in the numerator be less than the number of ciphers in the denominator, prefix in the numerator the necessary number of ciphers.

Ex. 1. $\frac{15243}{1000} = 15\cdot243$; for there are 3 o's in the denominator.

Ex. 2. $\frac{5243}{10000} = \cdot5243$; for there are 4 o's in the denominator.

Ex. 3. $\frac{243}{100000} = \cdot00243$; for there are 5 o's in the denominator

Examples XCVII.

1. Express as decimals :—

(1) Four-tenths ; eighteen-hundredths ; six-thousandths.
(2) Three and seven-tenths ; one and fifty-four hundredths.
(3) Twenty-four and seventy-nine thousandths ; one millionth.
(4) Five-tenths, seven hundredths and nine thousandths.
(5) Eight ten-thousandths ; seventy millionths ; five ten-millionths.
(6) Three hundred and fifteen, eight thousandths and fifty millionths.

2. Express the following decimals in words :—

(1) ·5 ; ·35 ; ·326 ; ·1 ; ·01 ; ·0001 ; 5·37 ; ·0025.
(2) ·43268 ; ·003405 ; ·0000456 ; 98·7654321 ; ·100001.

3. Transform the following decimals into vulgar fractions :—

(1) ·7 ; ·71 ; ·751 ; ·1461 ; ·03 ; ·0037 ; ·00057 ; ·0000457.
(2) 3·7 ; 52·19 ; 475·913 ; 276·00483 ; 1475·2045709.

4. Convert the following decimals into equivalent vulgar fractions in their lowest terms :—

(1) ·5 ; ·25 ; ·75 ; ·625 ; ·1875 ; 2·56 ; ·432 ; ·00625 ; ·222464.
(2) 4·375 ; ·8125 ; 47·256 ; 4·3125 ; 13·00085 ; ·0006875.
(3) ·1015625 ; 1·075 ; 3·01875 ; 7·0046875 ; 13·0005859375.

5. Express as mixed numbers with the fractional parts in their lowest terms :—

(1) 5·2 ; 6·024 ; 41·015 ; 72·75 ; 9·4158 ; 307·251 ; 8·9125.
(2) 376·275 ; 970·00004 ; 7321·0505 ; 23·0671386671875.

6. Express the following as decimals :—

(1) $\frac{3}{10}$; $\frac{7}{10}$; $\frac{13}{10}$; $\frac{47}{10}$; $\frac{13}{100}$; $\frac{407}{1000}$; $\frac{801}{100}$; $\frac{559}{10}$; $\frac{27}{1000}$.
(2) $\frac{17}{10000}$; $\frac{965}{1000000}$; $\frac{308}{10000}$; $\frac{9}{1000000}$; $\frac{57}{10000000}$; $\frac{4876}{100}$.
(3) $\frac{213}{100}$; $\frac{213}{10000}$; $\frac{213}{1000000}$; $\frac{800507}{100000000}$; $\frac{76859}{1000000000}$; $\frac{20403005}{10000}$.
(4) 16 tenths ; 75 hundredths ; 31 thousandths ; 4 ten-thousandths ; 275 ten-thousandths ; 79 millionths ; 1001 ten-millionths ; 79 hundred-millionths ; two hundred and sixty-one hundred-thousandths ; 328 billionths.

---

333. *Ciphers annexed to the right hand of a decimal fraction have no effect upon its value.*

Thus, $·37=\frac{37}{100}$; $·370=\frac{370}{1000}=\frac{37}{100}$; $·3700=\frac{3700}{10000}=\frac{37}{100}$, and so on ; as appears also from the consideration, that there are *no* thousandths, &c., in addition to the tenths and hundredths expressed by ·37.

334. *Hence an integer or a whole number can be expressed as a decimal by writing ciphers in the decimal part.*

Thus, $317{\cdot}000 = 317 + \frac{0}{10} + \frac{0}{100} + \frac{0}{1000} = 317 + 0 + 0 + 0 = 317$;
$\therefore 317 = 317{\cdot}000$.

Again, $31{\cdot}72 = 31 + \frac{7}{10} + \frac{2}{100} = 31 + \frac{72}{100}$; and
$31{\cdot}720 = 31 + \frac{7}{10} + \frac{2}{100} + \frac{0}{1000} = 31 + \frac{72}{100}$; $\therefore 31{\cdot}72 = 31{\cdot}720$.

335. *Every cipher affixed to the left hand of a decimal fraction after the point diminishes its value tenfold.*

Thus, $\cdot 43 = \frac{43}{100}$; $\cdot 043 = \frac{43}{1000}$; $\cdot 0043 = \frac{43}{10000}$; &c.;
where each fraction is a tenth part of that which immediately precedes it; and indeed this is evident from the circumstance of every figure being reduced *one* denomination lower by means of *each* cipher.

336. Hence, *Multiplication* and *Division* of a decimal by 10, 100, 1000, &c., are immediately effected, by shifting the decimal point *one*, *two*, *three*, &c., places towards the *right* and *left* respectively, adding ciphers, if necessary.

Ex. 1. $23{\cdot}45 \times 10 = 234{\cdot}5$; for $23{\cdot}45 \times 10 = \frac{2345}{100} \times 10 = \frac{2345}{10} = 234{\cdot}5$

Ex. 2. $23{\cdot}45 \times 10000 = 234500$; for $23{\cdot}45 \times 10000 = \frac{2345}{100} \times 10000$
$= 2345 \times 100 = 234500$.

Ex. 3. $23{\cdot}45 \div 10 = 2{\cdot}345$; for $23{\cdot}45 \div 10 = \frac{2345}{100} \times \frac{1}{10} = \frac{2345}{1000} = 2{\cdot}345$.

Ex. 4. $23{\cdot}45 \div 10000 = {\cdot}002345$; for $23{\cdot}45 \div 10000 = \frac{2345}{100} \times \frac{1}{10000}$
$= \frac{2345}{1000000} = {\cdot}002345$.

Examples XCVIII.

1. Multiply :—

(1) ·8 separately by 10, 100, 1000, 100000, 10000000.

(2) ·0053 separately by 100, 10000, 1000000, 100000000.

(3) 13·0014 separately by 10, 100, 1000, 10000, 1000000.

(4) 8·003056 separately by 100, 10000, 10000000.

2. Divide :—

(1) ·71 separately by 10, 100, 10000, 1000000.

(2) 73·58 separately by 1000, 10000, 1000000, 10000000.

(3) ·007 separately by 100, 1000, 100000, a million.

(4) ·1 by 100; ·001 by 10000; 5742·6 by 10000000.

---

337. The operation of Addition, Subtraction, Multiplication and Division of decimals are performed in the same way as in the case of whole numbers. Hence it is an advantage to use decimals in preference to vulgar fractions.

III. ADDITION OF DECIMALS.

338. RULE. Place the numbers so that all the decimal points may be in the same vertical line, to ensure the combination of those

of the *same* denominations ; and add them together as in integers, taking care to place t e decimal point in the sum, immediately under those of the numbers proposed.

Ex. Add together 25·61, 4·805, ·0096, 653·27, 23

25·61	For 25·61+4·805+·0096+653·27+23
4·805	$=\frac{2561}{100}+\frac{4805}{1000}+\frac{96}{10000}+\frac{65327}{100}+23$
·0096	
653·27	$=\frac{256100+48050+96+6532700+230000}{10000}$
23·	
706·6946	$=\frac{7066946}{10000}=706{\cdot}6946.$

339. Hence, decimals are said to be reduced to a *common denominator* when ciphers are supplied so that there is the *same* number of decimal places in each.

Examples XCIX.

1. Add together :—

(1) ·295, 3·086, 12·87, ·0051, 729·54, 7·419, 3·0256.
(2) 3608·26, 360·826, 36·0826, 3·60826, ·360826, ·22314.
(3) 36·053, ·0079, ·000952, ·417, 85·5803, ·0000501.
(4) 16, 12·2, 371·057, ·8241, 9·1, 1·235, 23·000358.
(5) 17·215, 3·0567, ·009, 2·07195, 365, 54·75.
(6) 231·8, 45·001, 2·7169, 4567·21, ·00087, 6·05.
(7) 20·02, 576·89174, 1·0008159, ·423564, 29, 7·21685.
(8) 61, 3·16004, ·0478, 21·805, 1·00006, 12·9871.
(9) ·00625, 30·698, 2·7535, 19·84, ·1875, 8·096.
(10) 100, ·1, ·01, ·001, ·0001, ·00001, ·000001, ·0000001.

2. Find the values of :—

(1) 69·563+1307·2345+16·27+18·03+59·327+116·2491+3·0002.
(2) 15·063+·002857+308·62+769·3276+58·739127+·69325.
(3) 77·3+160·6734+26·345+46+31·1+117·154+·0002+2343·008 +1·0000123+213·7+2·913+14·769+·007871.
(4) ₨1159·217+₨387·61+₨71·316+₨91·204+₨74·031.
(5) £573·162+£83·017+£92·159+£30·031+£99·999.
(6) 1596·131 cwt.+702·021 cwt.+·170038 cwt.+319·7 cwt.+5·93 cwt.
(7) 97·316 yds.+1597·308 yds.+316·2917 yds.+·03 yd.+159·1 yds.
(8) 27 tenths+345 hundredths+17 thousandths+4256 millionths.

IV. SUBTRACTION OF DECIMALS.

340. RULE. Place the less number under the greater as in Addition, suppose ciphers to be supplied if necessary, in the upper line ;

and the difference, found as in integers, will have as many decimal places as are contained in each, either expressed or understood.

Ex. 1. Subtract 34·917 from 41·62.

41·62
34·917
6·703

For $41{\cdot}62 - 34{\cdot}917 = \frac{4162}{100} - \frac{34917}{1000} = \frac{41620 - 34917}{1000}$

$= \frac{6703}{1000} = 6{\cdot}703.$

Ex. 2. Is ·90437532 more nearly represented by ·90438 or by ·90437 ?

·90438 − ·90437532 = ·00000468 ; ·90437532 − ·90437 = ·00000532.

∴ ·90438 is nearer to ·90437532 than ·90437

Examples C.

1. Subtract :—

(1) ·3806 from ·57031 ; 7·998 from 19·201 ; 3·4796 from 56·036.

(2) ·013096 from ·13096 ; ·21968 from 1·026103 ; 6·90086 from 7.

(3) ·99999 from 9 ; ·00071961 from ·03107 ; ·5968 from 1·11315.

(4) ·01 from ·1 ; ·0009 from ·001 ; ·672163 from 1·29613.

2. Find the difference between :—

(1) 27·903 and ·054 ; 7295·06 and 254·738 ; 35·08989 and 3·508989.

(2) 2·057 and 1·0097 ; 3·025 and ·003025 ; ·7053 and ·6729

(3) 5·0009 and ·089898 ; 136·159 and 136·0159 ; 13 and 5·90516.

3. Find the values of :—

(1) 1500·5 − 714·286 ; 15·903 − 4·696843 ; ·001 − ·00001.

(2) R45·21 − R38·793 ; R8·264 − R6·03176 ; R5·71021 − R2·369684.

(3) £83·6 − £83·47916 ; £70·151 − £15·8261 ; £70·107 − £69·89706

(4) 6·4 mds. − ·000064 md. ; 23·5 tons − ·9876 ton ; 1·44 ft. − ·00144 ft.

4. What number subtracted from 13·007 leaves 3·594 ?

5. What number added to 13·265 makes up 100·0008 ?

6. Simplify : —

(1) 5 − 3·22 + 2·333 − 1·4444 ; 2·194 + 15·367 − 10·009 − 11·25 + 5·8.

(2) 227·9 − (420·015 + 27·291) + 865·21 − 1·057.

(3) 17·073 + 1·3591 − 10·84 − (11·03796 − 15·8 + 6·9).

(4) 105·09 − 211·748 − 21·1748 − 15·73241 + 670·6 − ·0053.

7. Find the complement of ·7781513 ; ·000456 ; 98·654321 ; 9542·425 ; 998·899 ; and ·00001. (See Art. 58.)

8. Whether is 3·1415926535 more accurately represented by 3·1415926 or by 3·1415927 ?

9. Express in the decimal notation, the value of 8·0625 − 6·04 − ·00375 + 1·09236 − $\frac{85878}{10000}$.

V. MULTIPLICATION OF DECIMALS.

341. RULE. Multiply together the numbers proposed as if they were integers ; and the product will contain as many places of decimals, as there are decimal places in the multiplicand and multiplier together. If there are not figures enough, prefix the necessary number of ciphers.

Ex. 1. Multiply ·627 by 1·59.

```
  ·627
  1·59
  5643
 3135
 627
99693
```

The number of decimal places in the multiplicand and multiplier is 3 and 2 respectively ; therefore the number in the product is 3+2=5.

∴ the required product = ·99693.

For $·627 \times 1·59 = \frac{627}{1000} \times \frac{159}{100} = \frac{99693}{100000} = ·99693$.

Ex. 2. Multiply 7·5 by ·000084.

```
   7·5
·000084
   300
  600
  6300
```

The number of decimal places in the multiplicand and multiplier is 1 and 6 respectively ; therefore the number in the product is 1+6=7. But there are only 4 figures in the product ; therefore prefix 3 ciphers.

∴ the product = ·0006300 = ·00063.

Examples CI.

1. Multiply :—

(1) ·718 by ·57 ; 16·8 by ·0024 ; 144 by ·0525 ; 12·5 by ·062216.

(2) 270·56 by ·37025 ; ·00579 by 3796·8 ; 36·2185 by ·229.

(3) 421·619 by ·547 ; 34·6875 by 119·808 ; ·007853 by ·00476

(4) 384·759375 by ·00032 ; ·00082175 by 2·38645 ; ·002 by ·0004.

(5) ·0000051472 by ·0625 ; 948·7096 by ·007089 ; 170·71 by ·0325.

(6) ·00015625 by 8·192 ; ·00025 by ·0000625 ; ·00711858 by ·00024.

2. Find the values of :—

(1) 3·51 × ·075 ; ·0167 × ·008448 ; ·354178 × ·005 ; 3·12 × 2·0001.

(2) 3·005 × 40·23 ; 1·279 × ·0008787 ; 35·04 × ·0008 × 5·25.

(3) ·275 × 2·75 × 27·5 ; 3·24 × ·0028 × 2·9375 ; 11·01 × 110 × ·1102.

(4) 1·02 × 102 × 10·2 × ·102 ; 5·107 × ·05107 × ·05 × 700.

(5) ·4 × ·05 × ·006 × ·0007 × 800000 ; ·004 × ·04 × ·4 × ·0004 × 40000.

(6) ·01 × ·001 × ·0001 × ·00001 × 100000 ; ·845 × ·0017 × 7·4 × ·09 × 10000.

3. Find the values of :—

(1) 7·94 × 2·638 + 32·56 × ·00457 − ·007853 × ·00476 − ·000076 × 18·9.

(2) 592·9 × 61·6 × ·0064 + 1562·5 × ·0625 × 2·5 − 45·08 × 64·4 × ·092.

(3) (37·1 − 19·08) × ·703 ; 37·1 − 19·08 × ·703 ; $(·05)^2 + (·025)^2$ + ·00025.

(4) $(36·73)^2 - (25·894)^2$; $(·888)^2 - (·8008)^2$; $(3·025)^2$ − 3·025 × ·003025.

4. Multiply 325 tenths by 547 millionths; 128 thousandths by 78125 ten millionths.

VI. DIVISION OF DECIMALS.

342. *When the divisor is an integer.*

RULE. Divide, as if dividend and divisor were whole numbers; and when, in the process of division, the decimal point of the dividend is arrived at, place a decimal point in the quotient. If the division do not terminate with the last digit of the dividend, annex ciphers to the dividend and continue the operation until it terminates or the required number of decimal places in the quotient is obtained.

Ex. Divide 187·5 by 25; 1770·89 by 4735 and 3217 by 625.

```
(1) 25)187·5(7·5      (2) 4735)1770·890(·374      (3) 625)3217·0000(5·1472
       175                     14205                        3125
       ---                     -----                        ----
        125                     35039                         920
        125                     33145                         625
        ---                     -----                         ---
                                 18940                        2950
∴ the quotient = 7·5.            18940                        2500
                                 -----                        ----
                                                               4500
             ∴ the quotient = ·374.                            4375
                                                               ----
                                                                1250
                                                                1250
                                                                ----
                                             ∴ the quotient = 5·1472.
```

(1) For $187·5 \div 25$
$= \frac{1875}{10} \times \frac{1}{25} = \frac{1875}{25} \times \frac{1}{10}$
$= 75 \times \frac{1}{10} = \frac{75}{10} = 7·5.$

343. When the divisor does not exceed 20, or when it can easily be separated into factors none of which exceeds 20, the division should be performed by the method of **short division**.

Ex. Divide 56·787 by 12, and 1·21968 by 693.

```
(1)  12)56·78700                  (2)        ⎧  7)1·21968
        4·73225 Ans.                    693 ⎨  9) ·17424
                                             ⎩ 11) ·01936
                                                  ·00176 Ans.
```

344. *When the divisor is a decimal.*

RULE. Make the divisor a whole number by removing its decimal point altogether, and shift the decimal point of the dividend as many places to the right as there were decimal figures in the divisor, annexing for this purpose ciphers, if necessary, to the dividend. Then, divide as if the terms were integers; in the quotient, count off as many decimal places from the right as there are in the altered dividend, prefixing ciphers, if necessary.

Ex. Divide 10·836 by 5·16 ; ·1875 by 2·5 and 62·5 by ·025.

```
(1) 5·16)10·836          (2) 2·5)·1875           (3) ·025)62·5
    516)1083·6(2·1           25)1875(·075            25)62500(2500
        1032                    175                     50
         516                    125                     125
         516                    125                     125
```

∴ the quotient = 2·1. ∴ the quotient = ·075. ∴ the quotient = 2500.

345. In the course of the division, if there be any remainder after the last figure from the altered dividend has been brought down, add ciphers to the right of the dividend, and proceed as in Art. 342.

Ex. Divide ·01029 by 1·68.

```
1·68)·01029
168)1·029000(6125
    1008
      210
      168
       420
       336
        840
        840
```

Here the altered dividend has 3 decimal figures, and we have added to it 3 ciphers ; therefore in the quotient, we must count off 6 decimal places.

∴ the quotient = ·006125.

For $\cdot01029 \div 1\cdot68 = \frac{1029}{100000} \div \frac{168}{100} = \frac{1029}{168} \times \frac{1}{1000}$
$= 6\frac{1}{8} \times \frac{1}{1000} = (6 + \frac{125}{1000}) \times \frac{1}{1000}$
$= \frac{6\cdot125}{1000} = \cdot006125.$

346. In this case also the method of **short division** can advantageously be employed when the divisor has been made an nteger, as in Art. 343.

Ex. Divide 90·65 by ·049, and 171·99 by 27·3

```
(1)    ·049)90·65              (2)   27·3)171·99
     49 { 7)90650                    273 {  3)1719·9
        { 7)12950                        {  7) 573·3
             1850 Ans.                   { 13)  81·9
                                                 6·3 Ans.
```

347. If the division do not terminate, the quotient may be required to a given number of decimal places, as in the following examples.

Ex. Divide ·02 by 1·7 ; 1 by ·013 and 1 by ·007, each to 5 places of decimals.

```
(1) 1·7)·02        (2) ·013)1·000           (3) ·007)1·000
    17)·20000           13)1000·00000            7)1000·00000
       ·01176...Ans.        76·92307...Ans.         142·85714...Ans.
```

348. An integral divisor ending with ciphers may be deprived of the ciphers, if we remove the decimal point of the dividend one place *to the left* for every cipher withdrawn.

Thus ·78÷60 = ·078÷6 ; ·78÷600 = ·0078÷6, and so on.

Ex. Divide 1·5625 by 25000, and 7 by 796·3 to 5 places of decimals.

(1)
```
       25000)1·5625
   25 { 5)·0015625
        5)·0003125
          ·0000625  Ans.
```

(2)
```
 796·3)7·0
 7963)70·00000(·00879.  Ans.
      63704
       62960
       55741
        72190
        71667
```

349. In the above divisions, it should be very carefully noticed that *for each digit in the decimal part of the dividend there is a digit in the decimal part of the quotient.*

Examples CII.

1 Divide :—

(1) 783·5 separately by 5, 25, 125, 625 and 6250.

(2) 773·682 separately by 6, 13, 78, 169, 507 and 1014.

(3) ·00750116 separately by 677, 1354, 2708 and 10832.

(4) 35·9424 by 7·02 ; ·278831 by ·653 ; 11·444495 by 4·735.

(5) 1·68 by ·024 ; 971·7 by ·123 ; 142·025 by ·0437 ; 84·375 by ·00375.

(6) ·020872522 by ·08635 ; ·0020925 by ·000864 ; ·39538 by 5300.

(7) ·1 by ·01 ; ·01001 by ·001 ; 92·7 by ·06 ; 99 by ·0009 ; ·001 by ·0001.

(8) 9864·1698175 by 35·0645 ; 124·59993 by 3194·87.

(9) 1·365 separately by 1·25, 12·5, ·00125 and 12500.

(10) 7·835 separately by ·5, ·25, 12·5, 6·25, ·625, ·0625 and 625000.

(11) ·0003738028 by ·0476 ; ·0064096 by 2·003 ; 614·50824 by ·0010201.

(12) 2 and 22 hundredths by 74 ten-thousandths.

2. Find the values of (to 5 places of decimals) :—

(1) 3÷·876 ; ·0257÷·0041 ; 325·46÷·0187 ; ·0719÷27·53.

(2) ·5÷76·91342 ; 11·121÷3·4571 ; 16·1÷63572·45 ; 25÷19.

(3) ·046÷·00762089 ; ·32165÷·0035216 ; 314159·26÷·008597.

3 Find the quotient, by *short division*, of :—

(1) 3·6288 separately by ·3, ·7, ·9, 6·3, 12·6, ·189 and ·024.

(2) ·0255 separately by ·03, ·005, 3·4, 60, ·0102 and 2·55.

4. Divide, by *short division*, to 5 places of decimals :—

(1) ·009384 separately by 7, ·07, ·007, 1·8, ·0018 and ·00063.

(2) 57982·6966 by ·00000076 ; 346·72361 by ·00016.

5. Find the values of :—

(1) ·01385×61·37÷2·77 ; ·399×·007÷·000019 ; 24·01×·0039÷133·77.

(2) $(3{\cdot}124+1{\cdot}25)\times(2{\cdot}237-{\cdot}026)\div{\cdot}0003$; $(2{\cdot}05)^2\times2{\cdot}24\div{\cdot}0041$.

(3) $3833336\div(8{\cdot}99\times20{\cdot}8)$; ${\cdot}000785\div{\cdot}0005-{\cdot}000075+15$.

(4) $206{\cdot}59\div1872\times{\cdot}001$ to 5 places ; $15{\cdot}8402+3{\cdot}689\div672{\cdot}4$ to 6 places.

VII. REDUCTION OF FRACTIONS TO DECIMALS.

350. *To reduce a vulgar fraction to a decimal.*

RULE. Reduce the fraction to its lowest terms ; then divide the numerator with as many ciphers annexed to the right of it, as may be deemed necessary, by the denominator ; and the quotient comprising as *many* decimal places (counting from the right) as there are *ciphers* annexed, will be the decimal required.

Ex. 1. Reduce $\frac{3}{8}$, $\frac{1}{6400}$ and $4\frac{7}{125}$ to decimals.

(1) 8)3·000
 ·375
∴ decimal = ·375. *Ans.*

(2) 100)1·0
64 { 8) ·01000
 8) ·00125000
 ·00015625
∴ decimal = ·00015625. *Ans*

(3) 125 { 5)7·0
 5)1·4
 5) ·28
 ·056
∴ decimal = 4·056. *Ans*

For $\frac{3}{8}=\frac{30}{80}=\frac{\frac{30}{8}}{10}=\frac{3\frac{6}{8}}{10}=\frac{3}{10}+\frac{6}{80}=\frac{3}{10}+\frac{3}{40}=\frac{3}{10}+\frac{30}{400}=\frac{3}{10}+\frac{\frac{30}{4}}{100}$

$=\frac{3}{10}+\frac{7\frac{2}{4}}{100}=\frac{3}{10}+\frac{7}{100}+\frac{2}{400}=\frac{3}{10}+\frac{7}{100}+\frac{1}{200}=\frac{3}{10}+\frac{7}{100}+\frac{10}{2000}$

$=\frac{3}{10}+\frac{7}{100}+\frac{\frac{10}{2}}{1000}=\frac{3}{10}+\frac{7}{100}+\frac{5}{1000}=\frac{375}{1000}={\cdot}375.$

Similarly the other two can be explained.

Ex. 2. Convert $5\frac{5}{640}+{\cdot}75$ of $1\frac{1}{5}$ of $7\frac{1}{2}$ into a decimal.

640 { 10)5·0
 8) ·5000
 8) ·0625000
 ·0078125

$\cdot75$ of $1\frac{1}{5}$ of $7\frac{1}{2}={\cdot}75$ of $\frac{6}{5}$ of $\frac{15}{2}$
$={\cdot}75\times9=6{\cdot}75$.

∴ the reqd. decimal $=5{\cdot}0078125+6{\cdot}75$
$=11{\cdot}7578125.$

Ex. 3. Find the value of $\frac{{\cdot}01491\times2{\cdot}204}{{\cdot}0071\times2{\cdot}1\times{\cdot}029}$.

Since $\frac{1491\times2204}{71\times21\times29}=76$,

∴ $\frac{{\cdot}01491\times2{\cdot}204}{{\cdot}0071\times2{\cdot}1\times{\cdot}029}=76.$

There are 8 decimal places in the numerator and 8 in the denominator, ∴ the quotient is an integer.

351. In some cases if the division do not terminate, neither is the corresponding decimal *finite*, and the vulgar fraction is expressed only *approximately* by the decimal fraction thus found ; five or six figures are generally sufficient for all practical purposes, but the approximation will be nearer, the further the division is continued, inasmuch as by *every succeeding* step of the operation a

decimal fraction of an *inferior* denomination is added to the value already obtained.

Ex. Express $\frac{15}{22}$ as a decimal, as far as the sixth place.

$$22\begin{cases}2)15\cdot 0\\ 11)7\cdot 5000000\end{cases}$$
$$\cdot 6818181\ldots\ldots$$

$\therefore$ the decimal reqd. $=\cdot 681818.$

Note. The following results are useful, and should be verified and remembered :—

$\frac{1}{4}=\cdot 25$; $\frac{1}{2}=\cdot 5$; $\frac{3}{4}=\cdot 75$; $\frac{1}{8}=\cdot 125$; $\frac{3}{8}=\cdot 375$;
$\frac{5}{8}=\cdot 625$; $\frac{7}{8}=\cdot 875$; $\frac{1}{16}=\cdot 0625$; $\frac{1}{5}=\cdot 2$; $\frac{1}{32}=\cdot 03125$.

Examples CIII.

1. Reduce to decimals :—

(1) $\frac{1}{2}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{8}$; $\frac{3}{5}$; $\frac{3}{16}$; $\frac{4}{25}$; $\frac{399}{128}$; $\frac{13}{1600}$; $\frac{9}{125}$; $\frac{97}{128}$.

(2) $\frac{17}{2500}$; $\frac{288}{6250}$; $\frac{2604}{625}$; $3\frac{4}{25}$; $4\frac{5}{16}$; $6\frac{8}{5120}$; $7\frac{51}{120}$.

(3) $\frac{7}{8}$; $\frac{17}{320}$; $\frac{34567}{80000}$; $1\frac{71}{1024}$; $7\frac{3}{6250}$; $3\frac{47}{3125}$; $4\frac{88}{78125}$.

2. Convert into decimals :—

(1) $1\frac{3}{16}$ of $11\frac{1}{4}$; $3\frac{1}{8}$ of $4\frac{1}{4}$ of $5\frac{1}{5}$; $3\frac{1}{8}$ of $2\frac{4}{5}$; $\frac{3}{5}+\frac{7}{8}+\frac{9}{10}+\frac{7}{32}$.

(2) $\dfrac{12\frac{3}{4}}{10\frac{1}{5}}$; $\dfrac{1\frac{1}{10}}{62\frac{1}{2}}$; $\dfrac{5\frac{3}{8}}{7\frac{1}{2}}$ of $\dfrac{21\cdot 25}{\cdot 046875}$; $\dfrac{14\frac{1}{10}}{125}$; $\dfrac{6\frac{3}{4}}{11\frac{1}{4}}$; $\dfrac{17\frac{1}{8}}{12\frac{1}{2}}$.

(3) $17\frac{3}{16}+25\frac{5}{8}+6\frac{25}{32}+13\frac{6}{25}+20\frac{73}{10000}$; $3\frac{3}{8}$ of $\frac{1}{512}$.

(4) $1\cdot 26$ of $66\frac{2}{3}+5\frac{1}{3}$ of $1\cdot 0375$; $3\frac{5}{8}$ of $\cdot 003-\cdot 0011$ of $7\frac{1}{4}$.

3. Express as decimals, as far as the sixth decimal place :—

$\frac{1}{8}$; $\frac{3}{7}$; $\frac{3}{11}$; $\frac{5}{18}$; $\frac{7}{26}$; $\frac{13}{48}$; $2\frac{1}{12}$; $8\frac{9}{14}$; $24\frac{6}{17}$; $7\frac{15}{16}$.

4. Arrange in order of magnitude, by reducing to decimals :—

(1) $\frac{2}{3}, \frac{5}{6}, \frac{3}{8}$. (2) $\frac{5}{12}, \frac{7}{15}, \frac{3}{4}$. (3) $\frac{5}{24}, \frac{7}{32}, \frac{5}{8}$. (4) $\frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{7}{8}$.

5. Simplify the following :—

(1) $\dfrac{\cdot 003125\times\cdot 48}{\cdot 0000125}$. (2) $\dfrac{\cdot 0075\times 2\cdot 1}{\cdot 0175}$. (3) $\dfrac{4\cdot 255\times\cdot 0064}{\cdot 00032}$.

(4) $\dfrac{2\cdot 004\times 3\cdot 375}{4\times\cdot 167}$. (5) $\dfrac{\cdot 0295\times\cdot 00152}{3\cdot 04\times 1\cdot 18}$. (6) $\dfrac{1\cdot 18\times 3\cdot 04}{\cdot 152\times 2\cdot 95\times\cdot 00125}$.

(7) $3\cdot 25-2\cdot 765+3\cdot 125\times 8-\cdot 607095+\cdot 027$.

(8) $\dfrac{22\cdot 4}{\cdot 25}+\dfrac{250}{\cdot 8}+\dfrac{1\cdot 2}{\cdot 0075}$; $5\cdot 634\times\cdot 0057+\dfrac{4\cdot 2325}{8465}+\dfrac{\cdot 39424}{5632}$.

VIII. G. C. M. AND L. C. M. OF DECIMALS.

352. *To find the G. C. M. and the L. C. M. of Decimals.*

RULE. Make the same number of decimal places in each of the numbers (Art. 339) ; find their G. C. M. or L. C. M. as if they

were integers, and then mark off the said number of decimal places in the result, prefixing ciphers, if necessary.

Ex. Find the G. C. M. and the L. C. M. of 1·6, ·24 and 14

Here, the numbers are equivalent to 1·60, ·24 and 14·00.

The G. C. M. of 160, 24 and 1400 = 8 ; their L. C. M. = 16800.

∴ the G. C. M. reqd. = ·08 ; and the L. C. M. reqd. = 168·00 = 168.

Examples CIV.

1. Find the G. C. M. of :—

(1) 1353·6 and 231·48. (2) 4·2237 and 755·82. (3) 36·795 and 57·98.
(4) 376·1034 and 1081. (5) ·14, 1·8 and ·024. (6) ·009, 1·8 and ·24.
(7) 2·4, ·48, ·64 and 1·92. (8) ·016, ·0024, 4·8 and 74.

2. Find the L. C. M. of :—

(1) 1·5, 35, ·063 and 7·2. (2) 6·3, ·12, ·084 and ·0014.
(3) 2·4, ·39 and 3·76. (4) ·312, ·0124, 3·41 and 37·2.
(5) 4·2237 and 755·82. (6) 1·36652 and 246·8642.

IX. RECURRING DECIMALS.

353. In the conversion of a vulgar fraction into a decimal, we find that the division performed according to the Rule laid down in Art. 350 terminates in some cases and does not terminate in others. Thus, $\frac{5}{8}$ = ·625, and here the division terminates ; but $\frac{3}{11}$ = ·272727..........., and in this case the division does not terminate and can be extended to an unlimited length. The former is called a **terminating** or **finite** decimal, and the latter a **non-terminating** decimal.

354. It has already been shewn in Art. 331 that to reduce a vulgar fraction in its lowest terms to a decimal is the same as reducing it to an equivalent one having 10 or some power of 10 for its denominator. Thus, it follows that no vulgar fraction can be reduced to a terminating decimal, unless it can be expressed as one having 10 or some power of 10 for its denominator. Now, no number can, by multiplication, be made a power of 10, unless it be composed of prime factors, each of which is 2 or 5. Hence, to find whether a vulgar fraction can be expressed as a terminating decimal or not, we have the following Rule.

RULE. *Reduce the given vulgar fraction to its lowest terms, and resolve its denominator into its prime factors ; if these prime factors be only 2 and 5, it can be expressed as an exact or terminating decimal ; otherwise, it cannot.*

Ex. 1. Can $\frac{3}{50}$ and $\frac{81}{1250}$ be expressed as a terminating decimal?
(1) Yes ; for 50 = 2 × 5 × 5, and involves factors of 2 and 5 only.
(2) Yes ; for 1250 = 2 × 5^4, and involves factors of 2 and 5 only.

Ex. 2. Can $\frac{187}{576}$ be expressed as a finite decimal?

No; for $576 = 2^6 \times 3^2$, and involves other factors than 2 and 5.

Examples CV.

1. Which of the following fractions can be expressed as finite or terminating decimals :—

$\frac{5}{12}$; $\frac{13}{16}$; $\frac{9}{48}$; $\frac{11}{22}$; $\frac{81}{80}$; $\frac{19}{64}$; $\frac{85}{192}$; $\frac{79}{406}$; $\frac{91}{560}$; $\frac{512}{875}$; $\frac{18241}{83886}$.

2. Write down those numbers between 1 and 25, of which if any one be the denominator of a fraction in its lowest terms, that fraction can be reduced to a terminating decimal.

355. *In non-terminating decimals the figures of the quotient must recur over and over again.*

Take the fraction $\frac{5}{7}$. To reduce it to a decimal, we annex ciphers to 5 and divide by 7. Since the division does not terminate, we cannot have the remainder 0, and the only *possible* remainders that can arise are 1, 2, 3, 4, 5, and 6 and consequently after *six steps at most* (after as many divisions at least as there are units in the denominator) we must come to the given numerator or to one of the remainders that has occurred before, and therefore from that point we must have a recurrence of the remainders, and therefore of the quotient figures in the same order over and over again. Thus,

```
7)5·0(·714285
  49
   10    20    40
    7    14    35
   30    60     5
   28    56
```

Here, after 6 figures, we get 5 for remainder, and therefore the whole process will recur again from the beginning.

$\therefore \frac{5}{7} = \cdot 714285714285714 2 \ldots\ldots\ldots\ldots\ldots\ldots$

356. When, beginning from a certain point in the decimal part of a number, the figures repeat themselves indefinitely and in the same order, the number is called a **recurring, circulating, repeating** or **periodic** decimal; and the whole set of figures which recurs constantly in the same order is called the **period** or **repetend.**

357. The *period* is termed a *simple* or a *compound repetend* according as it consists of *one* or *more* figures; and the *extent* of the period is denoted by means of *dots* (·) placed over the *first* and *last* of the figures which compose it.

Thus, $\frac{8}{3} = 2\cdot6666\ldots\ldots = 2\cdot\dot{6}$; and $\frac{5}{37} = \cdot135135\ldots\ldots = \cdot\dot{1}3\dot{5}$.

So, $\cdot\dot{5}\dot{7} = \cdot575757\ldots$; $\cdot\dot{0}2\dot{4} = \cdot024024024\ldots$; $\cdot3\dot{2}\dot{6} = \cdot3262626\ldots$

The several *periods* in the above are 6, 135, 57, 024 and 26.

358. Recurring Decimals are either **Pure** or **Mixed.**

(i) A **pure circulating decimal** is one which recurs from the first figure of the decimal part; as, $\cdot\dot{3}$, $\cdot\dot{6}7\dot{8}$.

(ii) A **mixed circulating decimal** is one which recurs after some figures and thus consists of a *non-recurring* and a *recurring* part ; as, $\cdot 1\dot{7}$, $\cdot 24\dot{6}\dot{8}$.

359. A vulgar fraction in its lowest terms, whose denominator contains neither of the prime factors 2 and 5, produces a *pure circulating decimal*, whereas one, whose denominator contains 2 or 5 and one or more other prime factors, produces a *mixed circulating decimal.*

Thus, $\frac{2}{3} = \cdot\dot{6}$; $\frac{1}{7} = \cdot\dot{1}4285\dot{7}$; $\frac{7}{22} = \frac{7}{2 \times 11} = \cdot 3\dot{1}\dot{8}$.

Ex. 1. Convert $\frac{7}{3}$ and $\frac{4}{27}$ into decimals.

(1)
```
3)7(2·3...
  6
  --
  10
   9
  --
   1
```

Here, a repetition of 1 in the remainder gives a repetition of the figure 3 in the quotient.

$\therefore \frac{7}{3} = 2\cdot\dot{3}$.

(2)
```
27)4·0(·148...
    27
    ---
    130
    108
    ---
     220
     216
     ---
       4
```

The figure 4 occurs again in the remainder after 3 steps, therefore the digits 1, 4 and 8 must recur in the quotient.

$\therefore \frac{4}{27} = \cdot\dot{1}4\dot{8}$.

For $\frac{7}{3} = 2\frac{1}{3} = 2 + \frac{10}{30} = 2 + \frac{\frac{10}{3}}{10} = 2 + \frac{3\frac{1}{3}}{10} = 2 + \frac{3}{10} + \frac{1}{30} = 2 + \frac{3}{10} + \frac{10}{300}$

$= 2 + \frac{3}{10} + \frac{\frac{10}{3}}{100} = 2 + \frac{3}{10} + \frac{3\frac{1}{3}}{100} = 2 + \frac{3}{10} + \frac{3}{100} + \frac{1}{300} = 2\cdot 33\ldots$

Similarly, the second can be explained.

Ex. 2. Reduce $\frac{5}{36}$ to a decimal.

```
36)5·0(·138...
   36
   ---
   140
   108
   ---
    320
    288
    ---
     32
```

Here, the remainder 32 which occurred after the second step occurs again in the third, and therefore the figure 8 will recur in the quotient.

$\therefore \frac{5}{36} = \cdot 13\dot{8}$.

Examples CVI.

Reduce to recurring decimals :—

1. $\frac{2}{9}$; $\frac{8}{11}$; $\frac{18}{99}$; $\frac{5}{37}$; $\frac{129}{55}$; $\frac{5}{3}$; $\frac{44}{9}$; $\frac{21}{11}$; $\frac{5}{7}$; $\frac{97}{13}$; $\frac{8}{63}$; $9\frac{17}{33}$.

2. $\frac{4}{15}$; $\frac{5}{41}$; $\frac{8}{55}$; $3\frac{59}{90}$; $4\frac{45}{56}$; $7\frac{25}{182}$; $5\frac{20}{101}$; $3\frac{988}{2475}$.

3. [illegible] ; $\frac{214}{37}$; $\frac{808}{999}$; [illegible] ; $\frac{425}{[illegible]}$; $\frac{318}{416}$; [illegible] ; $\frac{506}{[illegible]}$.

4. $\frac{1000}{407}$; $\frac{20000}{5291}$; $\frac{7624}{[illegible]}$; $\frac{28172}{[illegible]}$; $\frac{997}{1875}$; $\frac{541}{1054}$; $\frac{661}{728}$.

---

360. (i) In a given recurring decimal, the period may be supposed to begin at any point we please after the first repeating figure.

Thus, $15{\cdot}453873873\ldots = 15{\cdot}45\dot{3}8\dot{7} = 15{\cdot}453\dot{8}7\dot{3} = 15{\cdot}4538\dot{7}3\dot{8}$ = &c.

(ii) Sometimes the period is made to commence in the *integral* part.

Thus, $6\dot{4}{\cdot}2\dot{5} = 64{\cdot}\dot{2}5\dot{4} = 64{\cdot}2\dot{5}4\dot{2}$ = &c.

(iii) The number of digits in the period may be repeated as often as we please without altering the value of the decimal.

Thus, $8{\cdot}5\dot{4}\dot{6} = 8{\cdot}5\dot{4}64\dot{6} = 8{\cdot}5\dot{4}6464\dot{6}$ = &c.

(iv) In the conversion of a fraction to a recurring decimal, we may often shorten the work by expressing the remainder at some step as a fraction. Thus,

$\frac{1}{7} = {\cdot}142\frac{6}{7}$; $\therefore \frac{6}{7} = {\cdot}142\frac{6}{7} \times 6 = {\cdot}857\frac{1}{7}$; and $\therefore \frac{1}{7} = {\cdot}142857\frac{1}{7} = {\cdot}\dot{1}4285\dot{7}$.

361. When recurring decimals have the same number of non-recurring figures and also the same number of recurring figures, they are said to be **similar**.

Thus, ${\cdot}34\dot{2}5\dot{8}$ and $6{\cdot}17\dot{8}6\dot{3}$ are *similar* recurring decimals.

362. *All recurring decimals can be made similar.*

RULE. Extend each decimal as far as the farthest non-recurring figure in any of them ; then find the L. C. M. of the numbers of figures in each period, and extend each period so many places further.

Ex. Make $4{\cdot}23\dot{8}$, ${\cdot}1\dot{2}3\dot{4}$ and $54{\cdot}0\dot{2}\dot{3}$ similar.

$4{\cdot}23\dot{8}$ =	$4{\cdot}23\vert\dot{8}8888\dot{8}$	Here, we see that the first term has the largest number of non-recurring figures, *i. e.*, 2 figures. So extend each decimal 2 places.
${\cdot}1\dot{2}3\dot{4}$ =	${\cdot}12\vert\dot{3}4234\dot{2}$	
$54{\cdot}0\dot{2}\dot{3}$ =	$54{\cdot}02\vert\dot{3}2323\dot{2}$	

The periods which consist of 1, 3, 2 figures respectively, are then extended 6 places, for 6 is the L. C. M. of 1, 2 and 3.

Examples CVII.

1. In the following recurring decimals begin the period at the fifth decimal place :—

$3{\cdot}\dot{2}\dot{5}$; $4{\cdot}\dot{7}$; $290{\cdot}0\dot{2}$; ${\cdot}3\dot{6}$; $21{\cdot}\dot{1}\dot{4}$; ${\cdot}0\dot{3}5\dot{2}$; ${\cdot}706\dot{5}$; ${\cdot}04\dot{6}\dot{3}$; $\dot{3}{\cdot}4\dot{5}$.

2. Extend ${\cdot}5\dot{7}$, $2{\cdot}\dot{3}\dot{4}$ and ${\cdot}0\dot{6}4\dot{5}$ so that they may have the same number of figures in the period.

3. Extend ${\cdot}1\dot{2}\dot{3}$, ${\cdot}1\dot{2}3\dot{4}$ and ${\cdot}\dot{1}23\dot{4}$ so that they may have the same number of recurring figures.

4. Convert the following vulgar fractions into recurring decimals by the method of Art. 360 (iv) :—

$\frac{11}{13}$; $\frac{1}{14}$; $\frac{5}{17}$; $\frac{2}{19}$; $\frac{8}{23}$; $\frac{1}{29}$; $\frac{6}{31}$; $\frac{10}{53}$.

5. Make the following recurring decimals similar :—

(1) $3\cdot07\dot{6}$, $9\cdot\dot{2}4\dot{5}$, $\cdot2\dot{0}\dot{3}$. (2) $\cdot\dot{8}$, $\cdot\dot{8}\dot{7}$, $\cdot\dot{8}7\dot{6}$.

(3) $\cdot4\dot{1}\dot{4}$, $\cdot0\dot{3}5\dot{2}$, $6\cdot\dot{1}01\dot{3}$. (4) $\cdot550\dot{7}$, $\cdot04\dot{6}\dot{3}$, $1\cdot\dot{4}1\dot{3}$, $\cdot\dot{7}06\dot{5}$.

(5) $\cdot7\dot{8}5\dot{4}$, $\cdot\dot{5}\dot{9}$, $14\cdot5\dot{7}$, $\cdot00\dot{4}\dot{5}$. (6) $9\cdot70\dot{1}\dot{2}$, $4\cdot40\dot{3}$, $10\cdot8492\dot{1}3\dot{7}$, $\cdot2\dot{1}86\dot{5}$

363. *To find the vulgar fraction which shall be equivalent to a pure recurring decimal.*

RULE. Make the period the *numerator* of a fraction whose *denominator* shall consist of as many *nines* as there are figures in the said period : and this reduced to its simplest terms will be the vulgar fraction required.

Ex. Convert $\cdot\dot{6}$ and $\cdot\dot{9}\dot{6}$ into equivalent vulgar fractions in their lowest terms.

(1) $\cdot\dot{6}=\frac{6}{9}=\frac{2}{3}$. (2) $\cdot\dot{9}\dot{6}=\frac{96}{99}=\frac{32}{33}$.

Proof. For the sake of conciseness, let x and y represent their values respectively ; then, we shall have

$x = \cdot6666\ldots$ | $y = \cdot9696\ldots$

$\therefore$ 10 times $x = 6\cdot6666\ldots$ | $\therefore$ 100 times $y = 96\cdot9696\ldots$

whence, subtracting in each case, the former from the latter, we obtain

9 times $x = 6$, | 99 times $y = 96$,

and $\therefore$ $x=\frac{6}{9}=\frac{2}{3}$. | and $\therefore$ $y=\frac{96}{99}=\frac{32}{33}$.

364. *To find the vulgar fraction which shall represent the value of a mixed recurring decimal*

RULE. Make the non-recurring and the recurring parts taken *together*, diminished by the non-recurring part *alone*, the numerator of a fraction whose denominator shall consist of as many *nines* as there are recurring figures, followed by as many *ciphers* as there are non-recurring figures ; and this reduced to its lowest terms will be the vulgar fraction required.

Ex. Convert $\cdot2\dot{7}$, $\cdot24\dot{5}\dot{7}$ and $\cdot011\dot{3}\dot{6}$ into equivalent vulgar fractions in their lowest terms.

(1) $\cdot2\dot{7}=\dfrac{27-2}{90}=\dfrac{25}{90}=\dfrac{5}{18}$. (2) $\cdot24\dot{5}\dot{7}=\dfrac{2457-24}{9900}=\dfrac{2433}{9900}=\dfrac{811}{3300}$.

(3) $\cdot011\dot{3}\dot{6}=\dfrac{1136-11}{99000}=\dfrac{1125}{99000}=\dfrac{1}{88}$.

Proof. For the sake of conciseness, suppose x and y to represent the values of (1) and (2) respectively ; then, we shall have

$x = \cdot 27777\ldots\ldots$	$y = \cdot 2457575757\ldots\ldots$
$10x = 2\cdot 7777\ldots\ldots$	$100y = 24\cdot 57575757\ldots\ldots$
$100x = 27\cdot 7777\ldots\ldots$	$10000y = 2457\cdot 575757\ldots\ldots$

whence, subtracting the second line from the third in each case, we find

$$90x = 27 - 2 = 25, \qquad 9900y = 2457 - 24 = 2433,$$

$$\therefore x = \frac{27-2}{90} = \frac{25}{90} = \frac{5}{18}. \qquad \therefore y = \frac{2457-24}{9900} = \frac{2433}{9900} = \frac{811}{3300}.$$

365. The above method is also applicable if there should be some integral figures in the decimal, but the equivalent vulgar fraction is improper. If it is required as mixed number, we may either reduce this to mixed number or apply the method given below and thus obtain it at once in that form.

Ex. Express $2\cdot\dot{2}\dot{7}$ and $4\cdot 5\dot{8}\dot{3}$ as vulgar fractions.

(1) $2\cdot\dot{2}\dot{7} = \frac{227-2}{99} = \frac{225}{99} = \frac{25}{11} = 2\frac{3}{11}$; or $2\cdot\dot{2}\dot{7} = 2 + \cdot\dot{2}\dot{7} = 2 + \frac{27}{99} = 2\frac{3}{11}$.

(2) $4\cdot 5\dot{8}\dot{3} = \frac{4583-45}{990} = \frac{4538}{990} = \frac{2269}{495} = 4\frac{289}{495}$;

or $4\cdot 5\dot{8}\dot{3} = 4 + \cdot 5\dot{8}\dot{3} = 4 + \frac{583-5}{990} = 4 + \frac{578}{990} = 4\frac{289}{495}$.

366. It follows from the Rule that $\cdot\dot{9} = \frac{9}{9} = 1$; $\cdot 0\dot{9} = \frac{9}{90} = \frac{1}{10} = \cdot 1$. Similarly, $\cdot 06\dot{9} = \cdot 07$; $\cdot 025\dot{9} = \cdot 026$. Hence, whenever $\cdot\dot{9}$ occurs at the end of a decimal, it should be omitted, and the preceding figure increased by 1.

367. The following equivalent forms with their converses should be verified and committed to memory :—

$\frac{1}{3} = \cdot\dot{3}$; $\frac{2}{3} = \cdot\dot{6}$; $\frac{1}{6} = \cdot 1\dot{6}$; $\frac{5}{6} = \cdot 8\dot{3}$; $\frac{1}{9} = \cdot\dot{1}$; $\frac{1}{11} = \cdot\dot{0}\dot{9}$.

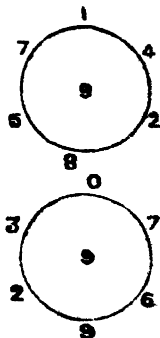
$\frac{1}{7} = \cdot\dot{1}4285\dot{7}$; $\frac{2}{7} = \cdot\dot{2}8571\dot{4}$; $\frac{3}{7} = \cdot\dot{4}2857\dot{1}$; $\frac{4}{7} = \cdot\dot{5}7142\dot{8}$;

$\frac{5}{7} = \cdot\dot{7}1428\dot{5}$; $\frac{6}{7} = \cdot\dot{8}5714\dot{2}$.

Also $\frac{1}{13} = \cdot\dot{0}7692\dot{3}$; $\frac{3}{13} = \cdot\dot{2}3076\dot{9}$;

$\frac{4}{13} = \cdot\dot{3}0769\dot{2}$; $\frac{9}{13} = \cdot\dot{6}9230\dot{7}$;

$\frac{10}{13} = \cdot\dot{7}6923\dot{0}$; $\frac{12}{13} = \cdot\dot{9}2307\dot{6}$.



Students should carefully notice the decimals equivalent to vulgar fractions with denominators 7 and 13. All are pure circulating decimals, and the same digits 142857, 076923 and 153846 occur in all respectively. Now, if these digits be placed round a circle, and read off, beginning with 1, 2, 4, 5, 7, 8 ; 0, 2, 3, 6, 7, 9 and 1, 3, 4, 5, 6, 8, in turn, with the other digits in order as they stand round the circle, decimals equivalent respectively to $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, $\frac{6}{7}$; $\frac{1}{13}$, $\frac{3}{13}$, $\frac{4}{13}$, $\frac{9}{13}$, $\frac{10}{13}$, $\frac{12}{13}$ and $\frac{2}{13}$, $\frac{5}{13}$, $\frac{6}{13}$, $\frac{7}{13}$, $\frac{8}{13}$, $\frac{11}{13}$, will be obtained.

Ex. Express $\cdot 382\dot{1}4285\dot{7}$ as a vulgar fraction.

$$\cdot 382\dot{1}4285\dot{7} = \frac{382\frac{1}{7}}{1000} = \frac{2675}{7000} = \frac{107}{280}.$$

Examples CVIII.

1. Convert the following recurring decimals into vulgar fractions in their lowest terms :—

(1) $\cdot\dot{5}$; $\cdot\dot{0}2\dot{7}$; $\cdot\dot{5}3\dot{4}$; $\cdot\dot{4}26\dot{3}$; $\cdot\dot{5}\dot{6}$; $\cdot\dot{2}5\dot{9}$; $\cdot\dot{7}22\dot{7}$; $\cdot\dot{6}2026\dot{8}$.

(2) $\cdot 3\dot{6}2\dot{1}$; $\cdot 47\dot{5}4\dot{3}$; $\cdot 0\dot{5}$; $\cdot 00\dot{4}9\dot{5}$; $3\cdot\dot{5}4\dot{5}$; $\cdot 1\dot{9}\dot{6}$; $\cdot 16\dot{5}2\dot{7}$; $\cdot 541\dot{6}$.

(3) $0\dot{4}3\dot{2}$; $21\cdot 9\dot{0}4\dot{5}$; $\cdot 6\dot{7}6190\dot{4}$; $\cdot 008\dot{4}9713\dot{3}$; $\cdot 811\dot{3}\dot{6}$; $\cdot 444\dot{1}0\dot{8}$.

(4) $\cdot 24\dot{1}25\dot{4}$; $1\cdot 0\dot{4}2857\dot{1}$; $2\cdot 6\dot{4}2857\dot{1}$; $3\cdot\dot{8}643\dot{0}1\dot{8}$; $13\cdot 94\dot{2}3076\dot{9}$.

(5) $6\cdot\dot{7}6923\dot{0}$; $50\cdot\dot{2}3076\dot{9}$; $4\cdot 15\dot{0}7692\dot{3}$; $\cdot\dot{0}1234567\dot{9}$; $\cdot 27\dot{8}4615\dot{3}$.

2. Express the following as finite decimals :—

$\cdot 0\dot{9}$; $4\cdot 36\dot{9}$; $\cdot 457\dot{9}$; $25\cdot\dot{9}9\dot{9}$; $15\cdot 8\dot{9}$; $3\cdot 78\dot{9}\dot{9}$; $59\cdot\dot{9}9\dot{9}$; $\cdot 00\dot{9}$.

3. Required the least numbers of which $\cdot 476190$ is the recurring quotient ; and find the error in the corresponding fraction when $\cdot 47619$ is taken to represent it.

4. Prove that $\frac{1}{9} = \frac{\cdot\dot{1}}{1} = \frac{\cdot\dot{2}}{2} = \frac{\cdot\dot{3}}{3} = \frac{\cdot\dot{4}}{4} = \frac{\cdot\dot{5}}{5} = \frac{\cdot\dot{6}}{6} = \frac{\cdot\dot{7}}{7} = \frac{\cdot\dot{8}}{8} = \frac{\cdot\dot{9}}{9}$.

5. Prove that $\frac{1}{11} = \frac{\cdot\dot{4}\dot{5}}{5} = \frac{\cdot\dot{5}\dot{4}}{6} = \frac{\cdot\dot{6}\dot{3}}{7} = \frac{\cdot\dot{7}\dot{2}}{8} = \frac{\cdot\dot{8}\dot{1}}{9} = \frac{\cdot\dot{9}\dot{0}}{10}$.

X. ADDITION OF RECURRING DECIMALS.

368. *To find the accurate sum of several recurring decimals.*

RULE. Write down the decimals under one another making them all similar (Art. 362), and afterwards extend two places more to make sure that we are carrying the correct figure to the last place of the second extension. Add in the usual way. Then in the sum the first extension will give the *non-recurring part*, and the second *the recurring part.*

Ex. Add together $32\cdot 0\dot{1}01\dot{1}$, $76\cdot 0\dot{9}1\dot{4}$, $5\cdot 1\dot{3}7\dot{5}$, $98\cdot 86\dot{3}$.

32·01	011101110111	01
76·09	149149149149	14
5·13	753753753753	75
98·86	333333333333	33
212·10	$\dot{2}4733734634\dot{7}$ *Ans.*	

Here, the greatest number of non-recurring figures is 2 ; so extend each decimal 2 places. The periods, consist of 4, 3, 1 figures, of which the L. C. M. is 12 ; so extend each to 12 places, and two places more to ensure accuracy of the last figure retained. In the sum, 10 is the non-recurring part and 247337346347 is the recurring part.

369. *To find the sum of several recurring decimals approximately correct to a given number of decimal places.*

RULE. Set down the decimals under one another repeating the period of each 2 or 3 places more than what is required in the sum. Then add in the usual way, taking care that the last figure retained be increased by 1, if the succeeding figure be 5, or greater than 5.

Ex. Find the sum of $13.\dot{5}$, $2.0\dot{2}\dot{5}$, $111.000\dot{4}$, $3.14\dot{1}5\dot{9}$, and $2.\dot{0}2\dot{4}$ *approximately correct* to 6 decimal places.

```
 13·555555|55
  2·025252|52
111·000444|44
  3·141591|59
  2·024024|02
131·746868|12
```

Here, by carrying out the decimals to 8 places, we ensure the accuracy of the first 6 places. Also in the sum, as we stop at 8 and the succeeding figure is 1, we need not increase 8 by 1.

Examples CIX.

1. Add together accurately :—

(1) $4.\dot{6}+.2\dot{5}\dot{1}+.02\dot{5}1\dot{4}$; $.4187+.\dot{3}0\dot{6}+.1\dot{2}\dot{5}$; $2.001+.\dot{1}81\dot{8}+.\dot{5}$.

(2) $.1\dot{0}\dot{1}+.2431\dot{8}\dot{3}+.12\dot{3}\dot{6}+45.2\dot{9}$; $3.\dot{0}9\dot{0}+.4\dot{0}7\dot{1}+35.13+76.\dot{5}$.

(3) $27.642\dot{3}\dot{5}+9.2642\dot{0}3\dot{7}+5.49\dot{2}\dot{5}+1.49\dot{8}+.\dot{6}0330\dot{6}$.

(4) $4.003\dot{5}\dot{8}+.08\dot{3}889\dot{4}+36.1612+1.00\dot{6}$.

(5) $3.1416+8.25\dot{1}4285\dot{7}+.03\dot{4}+23.257\dot{6}3\dot{5}+5.456\dot{2}\dot{7}$.

2. Find the values (app. correct to 7 places of decimals) of :—

(1) $7.\dot{9}\dot{0}+.3416+3.\dot{2}4\dot{5}+1.\dot{8}$; $6.\dot{1}2\dot{7}+3.\dot{8}0\dot{1}+1.0\dot{3}1\dot{3}+6$.

(2) $45.67\dot{0}\dot{1}+41.209+513.3\dot{1}\dot{7}+6.74\dot{9}\dot{3}+4.45\dot{6}\dot{7}$.

(3) $.7395+71.\dot{3}+16.2\dot{8}\dot{4}+162.7\dot{3}5\dot{4}+18.\dot{2}\dot{9}+1.\dot{6}+3.9\dot{7}$.

(4) $.13\dot{8}+.\dot{1}4285\dot{7}+2.4\dot{1}\dot{8}+2.0\dot{6}+42.\dot{6}\dot{3}+.008\dot{4}9713\dot{3}$.

XI. SUBTRACTION OF RECURRING DECIMALS.

370. The RULES given for Addition are also applicable in the Subtraction of recurring decimals.

Ex. 1. Subtract $5.98\dot{7}6\dot{5}$ from $28.0354\dot{7}\dot{1}$.

```
28·0354|717171|71
 5·9876|576576|57
22·0478|140595   Ans.
```

(Answer: $22.0478\dot{1}4059\dot{5}$)

Here, the periods have 2 and 3 figures ; their L. C. M. is 6 ; therefore the recurring part in the difference contains 6 figures.

Ex. 2. Find (1) the difference of $2.02\dot{3}4\dot{1}$ and $.62\dot{8}$ *approximately correct* to 6 decimal places ; (2) the complement of $.6\dot{1}4285\dot{7}$.

```
(1)  2·023413|413          (2)  1·0000000|00
      ·628888|888                ·6142857|14
     1·394524|525                ·3857142|86
```

∴ difference = 1·394525.   ∴ complement = ·3857143.

Examples CX.

1. Find the accurate difference of :—

(1) $17.21\dot{6}\dot{3} - 12.\dot{4}\dot{6}$. (2) $.306\dot{8}\dot{4} - .23\dot{4}\dot{6}$. (3) $.36\dot{8}0\dot{1} - .24\dot{9}\dot{2}$.

(4) $15.\dot{6}2\dot{3} - 11.\dot{2}\dot{7}$. (5) $365.27\dot{3}2\dot{1} - 148.9\dot{7}$. (6) $25.4\dot{7} - 16.85\dot{7}\dot{8}$.

(7) $6.734\dot{5}\dot{9} - 3.0\dot{7}2\dot{6}$. (8) $.\dot{7}1428\dot{5} - .0011\dot{3}\dot{6}$. (9) $7.321\dot{4} - 1.\dot{2}0\dot{7}$.

2. Find the values (app. correct to 6 places of decimals) of :—

(1) $.\dot{0}\dot{4} - .00\dot{7}6923\dot{8}$. (2) $78.\dot{3}\dot{1} - 19.\dot{6}8\dot{4}$. (3) $142.3\dot{4}\dot{5} - 109.\dot{3}\dot{2}$.

(4) $314.2\dot{9}0\dot{5} - 180.41\dot{6}\dot{2}$. (5) $52.8\dot{6} - 8.3\dot{7}23\dot{5}$. (6) $3.85\dot{6}\dot{4} - 2.038\dot{7}$.

3. Find the complements of $.045\dot{6}\dot{3}$; $.0\dot{7}8\dot{9}$; $25.64\dot{2}037\dot{0}$.

4. Find the values of :—

(1) $5.\dot{7}89\dot{2} - 2.3\dot{6}\dot{8} + 17.5\dot{4} + .2105 - 12.97\dot{6}\dot{1} - 3.21\dot{5}$.

(2) $14.89\dot{7}\dot{6} - 27.\dot{3}15\dot{0} - 49.\dot{8}\dot{1} + 15.763 + 183 + 21.\dot{0}\dot{5}$.

(3) $18.\dot{7}13\dot{0} - 5.\dot{8}\dot{7} + 161.02\dot{3}\dot{5} + 21 - 8.004$.

(4) $7.\dot{5} + 12.\dot{3}\dot{0} - 59.736\dot{5} + 90.02\dot{8} - 6.12\dot{5}\dot{7} - 20.\dot{7}\dot{1}$.

XII. MULTIPLICATION OF RECURRING DECIMALS.

371. *To multiply a recurring decimal by an integer or by a terminating decimal.*

RULE. Proceed in the usual way, extending the decimal 2 or 3 places beyond the end of the period, in order to ensure the correctness of the last digit retained, and in the product point off as many decimal places as there are decimal places in both the multiplicand and multiplier. The product will also be a recurring decimal of the same kind as the multiplicand, *i. e*, with a period containing the same number of digits.

Ex. 1. Multiply $37.83\dot{4}5\dot{9}$ and 7, and $37.82\dot{3}\dot{6}$ by 11.

(1)
$$\begin{array}{r|l} 37.83459 & 459 \\ 7 & \\ \hline 264.84\dot{2}1\dot{6} & \end{array}$$

(2)
$$\begin{array}{r|l} 37.8236 & 36 \\ 11 & \\ \hline 416.05\dot{9}\dot{9} = 416.06. & \end{array}$$
(Art 366)

Ex. 2. Multiply $6.\dot{3}91782\dot{5}$ by 6.924.

$$\begin{array}{r|l} 6.\dot{3}91782\dot{5} & 917 \\ 6.924 & \\ \hline 255\dot{6}71303\dot{3} & \\ 127\dot{8}35651\dot{1} & \\ 575\dot{2}60433\dot{3} & \\ 383\dot{5}06955\dot{5} & \end{array}$$

$$\begin{array}{r|l} 255671303 & 671 \\ 1278356518 & 356 \\ 57526043326 & 043 \\ 383506955506 & 955 \\ \hline 44.2567\dot{0}2665\dot{1} & 025 \end{array}$$
Ans.

372. *To multiply one recurring decimal by another.*

RULE. Convert the given decimals into equivalent vulgar

fractions, and multiply as in Art. 270. Then reduce the resulting fraction to a decimal.

Ex. Multiply $\cdot 08\dot{9}$ by $\cdot 02\dot{8}$.

$$\cdot 08\dot{9} = \frac{89-8}{900} = \frac{81}{900} = \frac{9}{100}; \quad \cdot 02\dot{8} = \frac{28-2}{900} = \frac{26}{900} = \frac{13}{450}.$$

$\therefore$ the product reqd. $= \frac{9}{100} \times \frac{13}{450} = \frac{13}{5000} = \frac{26}{10000} = \underline{\cdot 0026.}$ *Ans.*

Examples CXI.

1. Multiply :—

(1) $\cdot 3764\dot{2}$ by 9 ; $\cdot 376\dot{4}\dot{2}$ by 11 ; $\cdot 37\dot{6}4\dot{2}$ by 37 ; $\cdot 008\dot{3}7\dot{6}$ by 762.

(2) $\cdot 4322443\dot{1}\dot{8}$ by 88 ; $\cdot 785\dot{3}98\dot{1}$ by 3457 ; $6\cdot 34\dot{2}8\dot{7}$ by $5\cdot 01723$.

(3) $3\cdot\dot{5}$ by $\cdot 8\cdot$; $3\cdot\dot{9}\dot{1}$ by $\cdot 012$; $3\cdot 54\dot{2}6\dot{8}$ by $\cdot 144$; $15\cdot\dot{0}7\dot{3}$ by $2\cdot 4$.

(4) $2\cdot 3\dot{8}5714\dot{2}$ by $5\cdot 6$; $27\cdot 38\dot{4}4\dot{3}$ by $26\cdot 7$; $9\cdot 385\dot{0}78\dot{7}$ by $7\cdot 659$.

2. Find the values of :—

(1) $4\cdot\dot{8} \times \cdot\dot{2}\dot{4}$; $7\cdot\dot{6}\dot{3} \times 8\cdot 8\dot{3}$; $19\cdot 7\dot{2} \times 29\cdot\dot{4}\dot{5}$; $7\cdot\dot{5} \times \cdot 015\dot{9}\dot{6}$.

(2) $6\cdot\dot{3}\dot{6} \times \cdot\dot{5}7142\dot{8}$; $1\cdot\dot{1}\dot{8} \times \cdot\dot{5}3846\dot{1}$; $559\dot{8}\cdot 9\dot{2}4\dot{3} \times 8\cdot 24\dot{7}$.

(3) $2\cdot 2\dot{7} \times \cdot 24\dot{9}$; $\cdot 07\dot{3} \times 2\cdot\dot{7}\dot{2}$; $49\cdot\dot{3} \times \cdot 29\dot{9}5\dot{4}$; $\cdot 12\dot{8}3\dot{7} \times 2\cdot 52\dot{2}\dot{7}$.

(4) $\cdot 0\dot{0}2\dot{1} \times 48\cdot 92\dot{6}$; $\dot{4}2857\dot{1} \times \cdot\dot{3}$ of $3\cdot\dot{8}$; $44\cdot 20\dot{6}4\dot{5} \times 1\cdot 582\dot{3}70\dot{7}$.

XIII. DIVISION OF RECURRING DECIMALS.

373. *To divide a recurring decimal by a whole number or by a terminating decimal.*

RULE. Proceed as in ordinary division, bringing down the digits of the period in succession. The quotient will also be a recurring decimal.

Ex. Divide $8\cdot 985\dot{4}$ by 12 and $\cdot 655\dot{9}90\dot{3}$ by $48\cdot 76$.

```
(1) 12)8·98544444...(·7487870̇3̇
       84
        58
        48           84
        105          84
         96           44
          94          36
          84           8
          104
           96

(2) 4876)65 5990399...(134534...
         4876
         16839
         14628
          22110        16839
          19504        14628
           26063        22119
           24380        19504
            16839        2615
```

$\therefore$ the quotient $= \underline{\cdot 7487\dot{8}70\dot{3}}$. $\therefore$ the quotient $= \underline{\cdot 0134534\ldots}$

374. *To divide one recurring decimal by another.*

RULE. Convert the given decimals into vulgar fractions, and divide as in Art. 274. Then reduce the resulting fraction to a decimal.

Ex. Divide $1.1\dot{3}$ by $.00\dot{0}13\dot{2}$.

$$1.1\dot{3}=\frac{113-11}{90}=\frac{102}{90}=\frac{17}{15};\quad .00\dot{0}13\dot{2}=\frac{132}{999900}=\frac{1}{7575}.$$

$$\therefore \text{ the quotient reqd.}=\frac{17}{15}\div\frac{1}{7575}=\frac{17}{15}\times\frac{7575}{1}=\underline{8585}.\ \textit{Ans.}$$

Examples CXII.

1. Divide :—

(1) $.\dot{3}$ by 5, by 7 ; $37.\dot{0}8\dot{7}$ by 5, by 45 ; $.332\dot{5}$ by 125 ; $.\dot{4}6153\dot{8}$ by 30.

(2) $3.4579\dot{5}\dot{4}$ by 8 ; $37.635\dot{8}4\dot{2}$ by 7 ; $539.634\dot{3}\dot{6}$ by 112.

(3) $235.4\dot{7}$ by 24×20 ; $.74\dot{7}\dot{6}$ by $.07$; $9.\dot{4}\dot{0}$ by 1.5 ; $3.\dot{6}$ by 2.4.

(4) $.02834\dot{2}01\dot{2}$ by 14.156 ; $20.13\dot{9}7\dot{2}$ by 42.1 ; $.101\dot{0}\dot{1}$ by $.00036$.

2. Find the values of :—

(1) $3.\dot{8}\div2.\dot{7}\dot{3}$; $1.\dot{9}\dot{0}\div.58\dot{3}$; $60.\dot{4}\dot{5}\div7.3\dot{8}$; $11.8\dot{3}\div.24\dot{9}$; $.\dot{3}\dot{7}\div.\dot{1}4\dot{8}$

(2) $4.0\dot{3}\div.1\dot{4}0\dot{7}$; $.012\dot{3}\dot{6}\div.0\dot{5}\dot{1}$; $9.5\dot{3}\div3.208\dot{3}$; $6.\dot{8}9\dot{1}\div15.\dot{4}\dot{5}$.

(3) $.89\dot{1}\div1.\dot{2}\dot{9}$; $.0\dot{0}5\dot{7}\div.21\dot{3}$; $.12\dot{5}\div.25\dot{1}$; $7.3\dot{9}\div.07\dot{9}$.

(4) $411.3\dot{5}1\dot{9}\div19.588\dot{1}$; $14.\dot{4}7619\dot{0}\div2.15\dot{9}\dot{0}$; $77.670\dot{2}\dot{7}\div9.\dot{4}8\dot{6}$.

XIV. SIMPLIFICATION OF DECIMAL FRACTIONS.

Ex. 1. Simplify $\dfrac{.13\times.14\times.01-.12\times.14\times.02+.12\times.13\times.01}{.01\times.2\times.01}$.

$$\text{The given fraction}=\frac{.000182-.000336+.000156}{.00002}=\frac{.000002}{.00002}=\underline{.1}.\ \textit{Ans.}$$

Ex. 2. Find the value of $\dfrac{2.8\text{ of }2.\dot{2}\dot{7}}{1.1\dot{3}\dot{6}}+\dfrac{4.\dot{4}-2.8\dot{3}}{1.\dot{6}+2.\dot{6}2\dot{9}}$ of $\dfrac{6.8\text{ of }3}{2.25}$.

$$\text{The value}=\frac{2\frac{4}{5}\times2\frac{3}{11}}{1\frac{135}{990}}+\frac{4.4444\ldots-2.83333\ldots}{1.6666\ldots+2.629629\ldots}\text{ of }\frac{20.4}{2.25}$$

$$=\frac{14}{5}\times\frac{25}{11}\times\frac{990}{1125}+\frac{1.6\dot{1}}{4.\dot{2}9\dot{6}}\times\frac{2040}{225}=\frac{28}{5}+\frac{\frac{145}{90}}{\frac{4292}{999}}\times\frac{136}{15}$$

$$=\frac{28}{5}+\frac{145}{90}\times\frac{999}{4292}\times\frac{136}{15}=\frac{28}{5}+\frac{17}{5}=\frac{45}{5}=\underline{9}.\ \textit{Ans.}$$

Examples CXIII.

Simplify :—

1. $1.7\dot{2}$ of $.\dot{2}7\dot{6}$ of 15.

2. $1.8\dot{3}$ of $.\dot{9}5\dot{4}$ of $.\dot{4}2857\dot{1}$ of 2.25.

3. $.\dot{6}\dot{5}$ of $4.\dot{1}\dot{1}$ of $\frac{3\frac{3}{8}}{13}$ of $2.\dot{4}3\dot{2}$.

4. $\frac{2\frac{2}{9}}{3\frac{1}{4}}$ of $.000\dot{6}$ of $\frac{.\dot{4}\dot{5}}{.0024}$.

5. $\frac{.064+12.25}{.9375}$.

6. $\frac{13.5+.078-.003}{.005}$.

7. $\frac{.011\times133.1-.723\times.00723}{1.1377}$.

8. $\frac{5.1183}{.00705}$ of $11.\dot{1}$ of $.2\dot{9}$ of $.1\dot{1}\dot{7}$.

9. $\frac{.12(.02\times.03-.04\times.01)+.16\times.21}{.1\times.023\times.01}$.

10. $\frac{2.5+1.25-2.125}{3.75+2.\dot{3}-4.25}$.

11. $\frac{.0\dot{3}-.\dot{0}\dot{3}}{.12\dot{3}}$.

12. $\frac{1+5.4\times6.4}{1+2.\dot{3}\times3.\dot{3}}$.

13. $\frac{.005}{\frac{2}{3}\text{ of }13\frac{1}{2}}$ of $\frac{26.25}{\frac{3}{4}\text{ of }2.75}$.

14. $\left(37+\frac{3.7\dot{0}3\dot{7}}{100}\right)\times.54$.

15. $\frac{.\dot{4}2857\dot{1}}{.01\dot{7}1428\dot{5}}$.

16. $\frac{7\frac{1}{2}\times3\frac{3}{8}}{.75\times36.6}+\frac{2.\dot{5}\text{ of }15}{2\frac{1}{8}\text{ of }3\frac{3}{4}}+\frac{7.25}{11\frac{3}{8}}$.

17. $\frac{.04275}{3.0\dot{5}}\times\frac{4.21\dot{6}}{.\dot{3}4\dot{2}}\times\frac{2.\dot{7}}{1.5318}$.

18. $\frac{.125}{100}-\frac{.0625}{25}-2.25-\frac{.005\times1.25}{25}+3.1-\frac{.5}{1000}$.

19. $\frac{.\dot{8}5714\dot{2}+.\dot{1}4285\dot{7}}{.\dot{5}7142\dot{8}-.\dot{4}2857\dot{1}}$.

20. $\frac{8.\dot{5}7142\dot{8}\times1.7}{2\frac{2}{9}\text{ of }1.\dot{2}8571\dot{4}}\times\frac{.21\dot{6}\text{ of }.625}{.48}$.

21. $\frac{2.\dot{6}\text{ of }2.8\dot{3}}{6.\dot{2}\text{ of }.\dot{8}5714\dot{2}}+\frac{4\frac{4}{9}\text{ of }4.0\dot{3}\dot{6}}{3.75\text{ of }1.\dot{7}}$.

22. $\frac{2.375}{3.1\dot{6}}$ of $\frac{4.\dot{4}}{.0625}+\frac{8.\dot{8}}{7}$ of $\frac{16}{5.625}$.

23. $\frac{4.\dot{2}-3.1\dot{4}}{1.\dot{3}+2.\dot{1}0\dot{2}}$ of $\frac{1.3\text{ of }4}{.\dot{3}7\text{ of }8.\dot{8}\dot{1}}$.

24. $\frac{.044\times2.1}{.000035}+\frac{3.\dot{0}7692\dot{3}}{2.\dot{3}\times5.\dot{6}}$.

25. $\frac{3.30208\dot{3}}{16.51041\dot{6}}+\frac{6.\dot{6}\times.375}{1\frac{4}{9}\text{ of }.\dot{5}3846\dot{1}\text{ of }\frac{3}{4}}+\frac{2.\dot{7}\dot{2}}{11.\dot{0}\dot{9}}$.

26. $\frac{.1\times.1\times.1+.01\times.01\times.01}{.2\times.2\times.2+.02\times.02\times.02}$.

27. $\frac{.375\times.375-.025\times.025}{.375-.025}$.

28. $\frac{.02\times.9\times.15-.14\times.06\times.03+.13\times.01\times.04}{.05\times.04\times.03}$.

29. $.\dot{6}$ of 3.3 of $\frac{1.75}{2.625}$ of $17+.4$ of $5.75-\frac{1.\dot{7}1428\dot{5}}{2.\dot{0}9523\dot{8}}$.

30. $\frac{2.375}{3.1\dot{6}}$ of $\frac{.4\frac{4}{9}}{.0625}+\frac{8.\dot{8}}{5\frac{5}{9}}$ of $.\dot{5}7142\dot{8}-\left\{\frac{2.8\text{ of }2\frac{8}{11}}{1.1\dot{3}\dot{6}}-4\frac{8}{11}\text{ of }\frac{4.\dot{4}-2.8\dot{3}}{1\frac{1}{3}+2.\dot{6}2\dot{9}}\right\}$.

XV. REDUCTION OF DECIMALS.

375. A general view having now been taken of decimals, we proceed to show how they may be made to change their denominations when they are considered as belonging to a particular unit; and in what ways they may be adapted to the particular computations in which they are most frequently employed.

376. Reduction of Decimals can conveniently be classed under the two following heads :—

(1) To reduce a **decimal** of one denomination to a lower denomination : and conversely,

(2) To reduce a quantity of one denomination to a **decimal** of a higher denomination.

377. Case I. To reduce a decimal of one denomination to a lower denomination. (**Descending Reduction**).

RULE. Multiply the decimal of the given denomination by the number which connects the lower denomination with one (or unit) of the given denomination.

Ex. Reduce *Rs* 7·15 to *pies*, and ·045 of £7 to *farthings*.

(1)
```
   Rs.7·15
       16
  a 114·40
       12
  p.1372·8
```
∴ the reqd. result = 1372·8*p*.

(2)
```
  £7
  ·045
  £·315
    20
  s.6·300
    12
  d.75·6
     4
  q.302·4
```
∴ the reqd. result = 302·4*q*.

378. Case II. To reduce a quantity of one denomination to a decimal of a higher denomination. (**Ascending Reduction**).

RULE. Divide the number of the given denomination by the number which connects that denomination with one (or unit) of the higher denomination.

Ex. Reduce 3333 pies to the decimal of a *rupee*, and 21¾ grs. to the decimal of an *oz. Troy*.

(1)
```
  12)3333p.
  16) 277·75a.
  Rs.  17·359375
```
∴ the reqd. decimal = *Rs*.17·359375.

(2)
```
  24 { 8)21·75 grs.
     { 3) 2·71875
      20) ·90625 dwt.
          ·0453125 oz.
```
∴ the reqd. decimal = ·0453125 oz.

379. Sometimes we employ both the *descending* and the *ascending* process in reducing a decimal of one denomination to a decimal of another denomination.

Ex. Reduce ·78936 of a guinea to the decimal of £1.

```
      ·78936 gui.
          21
20) 16·57656s.
    £·828828
```

∴ the reqd. decimal = £·828828.

Examples CXIV.

Reduce :—

1. £·02375 ; £·00375 ; £3·5478 ; £·00625 ; £·28125 to *pence.*
2. ·03125*s.* ; £ 894791$\dot{6}$; ·00$\dot{1}$ guinea ; £·4708$\dot{3}$; £·38$\dot{3}$ to *farthings.*
3. *Rs* 5·00625 ; *Rs.*2·76543 ; ·775625 of *Rs.*5 ; *Rs.*3·049 to *pies.*
4. *Re.*·972916 ; *Re.*·40972$\dot{2}$; *Re.*·68125 ; *Re*·634375 ; *Re.*·3$\dot{4}$0$\dot{5}$ to *pice.*
5. ·7859 cwt. to *ounces* ; 4·349$\dot{5}\dot{4}$ miles to *yards* ; ·549675 days to *seconds* ; 2·5384375 of a day to *seconds.*
6. ·619791$\dot{6}$ ℔. Troy to *grains* ; ·678$\dot{5}$7142$\dot{8}$ week to *minutes.*
7. 3·687$\dot{4}$ acres to *sq. yds.* ; ·0475 gallon to *pints* ; 2·274025 mds. to *chataks* ; ·825 of a lea. to *yards.*
8. 8·4*d.* ; ·335*s.* ; 6·375*d.* ; ·4068*q.* to the decimal of £1.
9. 37·9872 sec. to the dec. of a *day* ; 420·8138 sq. yds. to the decimal of an *acre* ; 2·25 of 3·5 ac. to *poles.*
10. 47·733 ℔s. to the dec. of a *ton* ; 1 oz. Avoir. to the dec. of 1 *oz. Troy* ; 3·$\dot{6}$ cwt. to the dec. of a *ton.*
11. £·625 to the decimal of a guinea, and of half-a-guinea.
12. 527·3994 yds. to the dec. of a *mile* ; ·54375 ℔s. Troy to *ounces Avoir.* 1 oz. to the dec. of a *cwt.*

380. The preceding two cases of Art. 376 enable us

(i) To reduce a decimal of one denomination to a **compound quantity** of lower denominations ; and

(ii) To reduce a **compound quantity** to a decimal of a higher denomination.

381. Case I. *To reduce a decimal of one denomination to a compound quantity of lower denominations.*

RULE. Multiply the decimal by the numbers which connect the successive denominations in order ; and the integral parts of the products *taken out*, as they occur, will be the value required.

Ex. 1. Find the values of *Rs.*3·46875 and £5·6125.

```
(1) Rs.3·46875            (2) £5·6125
          16                      20
    a.7·50000                s.12·2500
       12                        12
    p.6·0                     d.3·00
```

The reqd. value = *Rs.*3. 7*a.* 6*p.*   The reqd. value = £5. 12*s.* 3*d.*

Ex. 2. Find the values of 4·215 of *Rs*.7 and $31\cdot258\dot{3}$ of £2.

(1) 4·215
 7
Rs.29·505
 16
a.8·080
 12
p.0·96

The reqd. value = *Rs*.29. 8*a*. 0·96*p*.

(2) $31\cdot258\dot{3}$
 2
£62·5166 ..
 20
s.10·3333...
 12
d.3·9999...

The reqd. value = £62. 10*s*. 4*d*.

382. Case II. To reduce a compound quantity to a decimal of a higher denomination.

RULE. Divide the lowest denomination by the number which connects it with the next, and to the left of the quotient affix the number of this denomination; and continue the process till the required denomination is obtained.

Ex. 1. Express *Rs* 5. 1*a*. 6*p*. as the decimal of *Re*.1, and £3. 18*s*. 11¼*d*. as the decimal of £1.

(1) 12)6*p*.
16)1·5*a*.
Rs.5·09375

The reqd. decimal = *Rs*.5·09375.

(2) 4)1*q*.
12)11·25*d*.
20)18·9375*s*.
£3·946875

The reqd. decimal = £3·946875.

Ex. 2. Reduce 7 fur. 25 po. to the decimal of a *mile*, and 14⅖ oz. Avoir. to the decimal of 1 *oz*. *Troy*.

(1) 40)25 po.
8)7·625 fur.
·953125 mi.

To reqd. decimal = ·953125 mi.

(2) 5)2·
16)14·4 oz.
 ·9 lb.
 7000
 6300 grs.

24 { 8)6300 grs.
 3)787·5
20)262·5 dwts.
 13·125 oz. Troy.

The reqd decimal = 13·125 oz. Troy.

Examples CXV.

1. Find the values of :—

(1) *Rs*.5·07125 ; *Rs*.80·075 ; $\cdot01\dot{6}$ of a rupee ; $30\cdot\dot{3}\dot{6}$ of ·75 of *Rs*.10.
(2) ·45 of £1 ; ·16875 of £3 ; 2·36875 of £6 ; £·5675 ; £·0484 ; £$\cdot\dot{7}$.
(3) ·340625 of £1 ; ·615 of 1*s*. ; $\cdot48\dot{3}\dot{5}$ of £1 ; £5·6125 ; ·4375 of £1.
(4) ·375 of a guinea ; 1·025 of a guinea ; $\cdot763541\dot{6}$ of £1 ; $\cdot458\dot{3}$ of 1*s*.
(5) ·375 of a cwt. ; ·6875 of a yard ; 13·3375 acres ; ·655 of a day.
(6) *Rs*.$5\cdot798958\dot{3}$; $\cdot871\dot{6}$ of a ton ; 2·5384375 days ; 22·25 of 17 half-crs.
(7) $\cdot0000355113\dot{6}\dot{3}$ mile ; $\cdot10\dot{7}1428\dot{5}$ of a cwt. ; ·09375 of an acre.
(8) ·00625 of 1 md. ; $\cdot013\dot{8}$ of 3·5 moidores ; $3\cdot2\dot{3}$ of 1¼ acres.

2. Reduce :—

(1) $5\frac{1}{4}d.$; $\frac{3}{4}d.$; $8s.$ $11\frac{1}{4}d.$; $1s.$ $3\frac{1}{4}d.$; £1 $14s.$ $10\frac{1}{2}d.$ to the decimal of $1s$·

(2) $12s.$ $6\frac{3}{4}d.$; $15s.$ $9\frac{3}{4}d.$; $17s.$ $0\frac{3}{4}d.$; $1\frac{3}{4}d.$; £2·$15s.$ $9\frac{3}{4}d.$ to the dec. of £1.

(3) $7a$ $6p.$; $8a$ $3p.$; $13a$ $6\frac{3}{4}p.$; *Rs.*53. $13a.$ $8p.$ to the decimal of *Re.*1·

(4) $18s.$ $11\frac{1}{2}d.$ to the dec. of a guinea ; $4\frac{3}{4}$ guineas to the dec. of £50.

(5) *Rs.*2. $13a.$ $10p.$ to the dec. of *Rs.*5 ; *Rs.*35. $14a.$ $6p.$ to the dec. of *Rs.*25 ; *Rs* 6. $6a.$ $8p.$ to the dec. of *Rs.*10. $8a.$

(6) $12s.$ $6\frac{3}{4}d.$ to the decimal of £1, of £100 and of £·001.

(7) 10 oz 11 dwts. $21\frac{1}{3}$ grs. to the dec. of 1 lb. *Troy* ; and of 1 lb. *Avoir.*

(8) 9 cwt. 13 lbs. 4 oz. 3·84 drs. to the dec. of a *ton* ; 4 cwt. 1qr. $10\frac{1}{4}$ lbs. to the dec. of 1cwt. ; 17 cwt. 3qrs. 17 lbs. 8·7 oz. to the dec. of a *ton*.

(9) 12hrs. 55 min. $23\frac{1}{18}$sec. to the dec. of a *day* ; 5 days 12hrs. 25min· 37·92 sec. to the dec. of a *week* ; 1 cwt. 3qrs. 4 lbs. to the dec· of a *ton*.

(10) 11yds. ; 3fur. 66yds. and 6yds. 2ft. $7\frac{1}{2}$in. each to the dec. of a *mile*.

(11) ·002 of 2·75 pag. to the dec. of *Rs.*3·46 ; 4 mds. 8sr. $1\frac{1}{2}$ch. to the dec. of 14 mds. ; 3 sr. 4 ch. 2 to. 11 m. to the dec. of 1 md.

(12) 6 fur. 100 yds 2 ft. 3 in. to the dec. of a *mile* ; 3 ro. 31 po. $16\frac{1}{2}$yds. to the dec. of an *acre* ; 13 cub. ft. 1323 cub. in. to the dec. of a *cub. yard*.

383. *To multiply or divide a quantity by a decimal, or to find the value of a decimal of a quantity.*

RULE. (1) Express the given quantity, when necessary, as a simple quantity, and perform the required operation ; or (2) reduce the decimal to a fraction in its lowest terms, and proceed as in fractions. (Arts. 302 and 303.)

Note. When the decimal is *recurring* and the value is required to be *exact*, the second method is advantageous.

Ex. 1. Find the value of ·432 of *Rs.*6. $10a.$ $8p.$

(1) *Rs.*6 $10a.$ $8p.$ × ·432 = $1280p.$ × ·432 = $552·96p.$ = <u>*Rs.*2 $14a.$ $0·96p.$</u>

(2) ·432 of *Rs.*6. $10a$ $8p.$ = $\frac{432}{1000}$ of *Rs.*6. $10a.$ $8p.$ = $\frac{54}{125}$ of *Rs.*6. $10a.$ $8p.$

= $\frac{1}{125}$ of *Rs* 360 = *Rs.*$2\frac{22}{25}$ = <u>*Rs.*2. $14a.$ $0·96p.$</u>

Ex. 2. Find the value of $4·23\dot{4}\dot{5}$ of £2. $15s.$

$4·23\dot{4}\dot{5}$ of £2. $15s.$ = $4\frac{2322}{9900}$ of £2. $15s.$ = $4\frac{129}{550}$ of £2. $15s.$

= £2. $15s.$ × 4 + £2 $15s.$ × $\frac{129}{550}$ = £11 + $55s.$ × $\frac{129}{550}$

= £11 + $12·9s.$ = <u>£11. $12s.$ $10·8d.$</u>

Ex. 3. Find the value of $3\cdot\dot{3}$ of $\dfrac{4\cdot\dot{4}}{\cdot\dot{7}3\dot{5}}$ of 1 sq. ft. 3 sq. in.

Value required $= 3\frac{3}{9}$ of $\dfrac{4\frac{4}{9}}{\frac{735}{999}}$ of $1\frac{3}{144}$ sq. ft. $= 3\frac{1}{3} \times 4\frac{4}{9} \times \frac{999}{735} \times \frac{147}{144}$ sq. ft.

$= \frac{10}{3} \times \frac{40}{9} \times \frac{999}{735} \times \frac{49}{48}$ sq. ft. $= \frac{370}{18}$ sq. ft.

$= 20\frac{5}{9}$ sq. ft. $=$ <u>20 sq. ft. 80 sq. in.</u>

Ex. 4. Find the value of $2\cdot8680\dot{5}$ of *Re.*1. 8*a.* + $\cdot8\dot{3}$ of *Rs.*2 − 1·8 of *Rs.*2. 8*a.*

$2\cdot8680\dot{5}$ of *Re.*1. 8*a.* $= 2\frac{78125}{90000}$ of *Re.*1. 8*a.* $= 2\frac{125}{144}$ of *Re.*1. 8*a.*

$=$ *Re.*1. 8*a.* × 2 + *Rs.*$\frac{3}{2} \times \frac{125}{144}$ = *Rs.*3 + *Re.*1. 4*a.* 10*p.*

$=$ *Rs.*4. 4*a.* 10*p.*

$\cdot8\dot{3}$ of *Rs.*2 $= \frac{75}{90}$ of *Rs.*2 $= \frac{5}{6}$ of *Rs.*2 $=$ *Rs.*$\frac{5}{3}$ = *Re.*1. 10*a.* 8*p.*

1·8 of *Rs.*2. 8*a.* $= \frac{18}{10}$ of *Rs.*$2\frac{1}{2} = \frac{9}{5} \times$ *Rs.*$\frac{5}{2}$ = *Rs.*$\frac{9}{2}$ = *Rs.*4. 8*a.*

∴ value required = *Rs.*4. 4*a.* 10*p.* + *Re.*1. 10*a.* 8*p.* − *Rs.*4. 8*a.*

= <u>*Re.*1. 7*a.* 6*p.*</u>

Examples CXVI.

1. Find the values of :—

(1) 1·85 of *Re.*1. 10*a.* 8*p.* ; 2·375 of *Rs.*6. 10*a.* 8*p.* ; ·775625 of *Rs.*50.
(2) ·925 of 6*s.* 8*d.* ; ·7365 of 6*s.* 8*d.* ; ·59375 of 19*s.* 4*d.* ; ·78125 of £6.
(3) ·00390625 of £1. 12*s.* ; ·0474609375 of £10. 13*s.* 4*d.* ; ·07 of £2. 10*s.*
(4) 6·156510416 of *Rs.*40 ; ·001953125 of *Rs* 400 ; 1·46875 of 3 bighas.
(5) ·046875 of 1 md. 8 sr. ; 4·106 of 4 mds. 32sr. 8ch. ; ·045 of 4 miles.
(6) ·7385 of 13*s.* 4*d.* ; 1·625 of 2 tons 4 cwt. ; 27·138 of 2 mi. 450 yds.
(7) ·3792 of £3. 18*s.* $1\frac{1}{2}$*d.* ; ·0013 of £3. 17*s.* $10\frac{1}{2}$*d.* ; ·365 of £1. 0*s.* 10*d.*
(8) £3. 14*s.* $6\frac{3}{4}$*d.* × 2·46875 ; £874. 13*s.* 4*d.* × 1·875.
(9) £1205. 6*s.* 8*d.* ÷ 51·2 ; £503. 12*s.* $6\frac{3}{4}$*d.* ÷ 26·312.
(10) *Rs.*47. 13*a.* × 24·5775 ; *Rs.*149. 5*a.* × 345·67 ; *Rs.*239. 9*a.* 6*p.* ÷ 13·53.
(11) ·2775 of 1 sq. yd. 3ft. 72in. ; ·9765625 of 2 tons 18cwt. 3qrs. 14lbs.
(12) 225 days 14 hrs. 36 min. ÷ 8·71846 ; 27 lbs. 13 oz. 15 drs. × ·4352.

2. Find the values of :—

(1) $\cdot\dot{5}$ of *Rs.*2. 6*a.* 4·8*p.* ; $3\cdot\dot{0}\dot{6}$ of *Rs.*2. 1*a.* ; $\cdot\dot{3}$ of *Rs.*3. 8*a.* 4*p.*
(2) $\cdot\dot{7}1428\dot{5}$ of 10*s.* 6*d.* ; $\cdot4\dot{2}\dot{8}$ of £3. 8*s.* ; $3\cdot958\dot{3}$ of *Rs.*8.
(3) $\cdot3\dot{4}8\dot{1}$ of £4. 18*s.* 8*d.* ; $4\cdot\dot{0}09\dot{9}$ of *Rs.*16. 13*a.* 4*p.* ; $\cdot00015\dot{7}4\dot{0}$ of *Rs.*81.

(4) $\cdot 53\dot{5}7142\dot{8}$ of 2 cwt. 3qrs. $17\frac{1}{2}$lbs. ; $13\cdot 26\dot{3}79\dot{8}$ of 3 mi. 7 fur. $22\frac{1}{2}$po.

(5) $\cdot 208\dot{3}$ of $\cdot 3\dot{4}2857\dot{1}$ of $2\frac{1}{3}$ cwt. ; $1\cdot 91\dot{6}$ of 8*s.* ; $3\cdot 0\dot{7}$ of 11*s.* 3*d.* ; $3\cdot 24\dot{2}$ of $7\frac{1}{2}$ bighas ; $3\cdot\dot{6}$ of 4 qrs. 4 bus.

(6) $\cdot\dot{8}4615\dot{3}$ of $\cdot 0\dot{8}\dot{1}$ of *Rs.*6. 8*a.* ; $\cdot 0\dot{1} \times \cdot 10\dot{1}$ of *Rs.*749. 4*a.* ; $\cdot\dot{1} \times \cdot\dot{4}\dot{7}$ of *Rs.*3601. 2*a.* ; $\cdot 4\dot{6}9\dot{4}$ of *Rs.*5. 3*a.* 2*p.*

3. What is the value of $\cdot\dot{2}3\dot{4}$, when the unit is worth £20, and the worth of $\cdot\dot{3}$ of $\cdot\dot{3}$, when the unit is valued at *Rs.*108 ?

4. What is the value of $\cdot 58\dot{3}$, when the unit is 3 oz. 5 dwts. ?

5. Find the respective values of :—

(1) ·45 of *Rs.*35 + ·75 of *Rs.*2. 5*a.* 4*p.* + 3·245 of *Re.*1. 10*a.* 8*p.*

(2) 8·71875 of 5*a.* 4*p.* + 1·146875 of *Rs.*3. 5*a.* 4*p.* − ·0625 of *Rs.*10. 8*a.*

(3) ·375 of a guinea + ·1875 of a crown + ·3 of 7*s.* 6*d.* − ·875 of 2*d.*

(4) ·5*s.* + ·7 of a crown + £·125 ; £·6 + ·3125*s.* + $\cdot\dot{2}$ of a guinea.

(5) 1·125 of *Rs.*13. 8*a.* + 44·045 of 7*a.* 6*p.* − ·0625 of *Rs.*3. 12*a.* + 1·025 of 2*a.* 6*p.* − 2·56 of *Rs.*5. 7*a.* 6*p.*

(6) ·175 of 28 mds. + ·195 of 1 md. 16 sr. + ·145 of 14 sr. + ·15 of 8 ch.

(7) ·625 of £1. 1*s.* + $\cdot\dot{5}\dot{4}$ of 8*s.* 3*d.* + $\cdot 0\dot{2}\dot{7}$ of £2. 15*s.*

(8) $\cdot\dot{7}$ of 7*s.* 6*d.* − $\cdot\dot{8}\dot{4}$ of 16*s.* 6*d.* + $\cdot 9\dot{2}\dot{7}$ of £2. 10*s.* 5*d.*

(9) $\cdot\dot{2}8571\dot{4}$ of £30 + £$6\cdot\dot{8}5714\dot{2}$ + $\cdot\dot{6}$ of $\cdot\dot{7}1428\dot{5}$ of £·6 + $1\cdot\dot{3}$ of $\cdot\dot{4}2857\dot{1}$*s.*

(10) $\cdot\dot{8}5714\dot{2}$ of 2·0625 tons + $\cdot\dot{5}7142\dot{8}$ of 3·375 cwt. + $\cdot\dot{7}1428\dot{5}$ of 1·25qrs. + $\cdot\dot{2}8571\dot{4}$ of 10·5 lbs.

---

384. *To find what decimal one compound concrete quantity is of any other of the same kind.*

RULE. Express the first quantity as the fraction of the second, as in Art. 306, and then reduce this fraction to a decimal.

Ex. 1. Reduce 3*s.* $11\frac{3}{4}$*d.* to the decimal of £1. 19*s.* $4\frac{1}{2}$*d.*

3*s.* $11\frac{3}{4}$*d.* = $47\frac{3}{4}$*d.*, and £1. 19*s.* $4\frac{1}{2}$*d.* = $472\frac{1}{2}$*d.*

$47\frac{3}{4}d. \div 472\frac{1}{2}d. = \frac{191}{4} \times \frac{2}{945} = \frac{1}{10}$; ∴ the reqd. decimal = ·1. *Ans.*

Ex. 2. Express $\frac{3}{8}$ of *Rs.*3. 12*a.* + ·625 of *Rs.*5 − $\cdot 5\dot{4}\dot{5}$ of *Rs.*4. 9*a.* 4*p.* as the decimal of *Rs.*100.

$\frac{3}{8}$ of *Rs.*3. 12*a.* = $\frac{3}{8} \times 60a.$ = $22\frac{1}{2}$*a.* = *Re.*1. 6*a.* 6*p.*

·625 of *Rs.*5 = *Rs* 3·125 = *Rs.*3 2*a.*

$\cdot 5\dot{4}\dot{5}$ of *Rs.*4. 9*a.* 4*p.* = $\frac{540}{990}$ of $73\frac{1}{3}$*a.* = $\frac{6}{11} \times \frac{220}{3}$*a.* = 40*a.* = *Rs.*2. 8*a.*

∴ the first quantity $=$ *Re.*1. 6*a.* 6*p.* $+$ *Rs.*3 2*a.* $-$ *Rs.*2. 8*a.*
$=$ *Rs.*2. 0*a.* 6*p.* $=$ *Rs.*$2\frac{1}{32}$.

∴ the reqd. decimal $= 2\frac{1}{32} \div 100 =$ ·0203125. *Ans.*

Examples CXVII.

1. In the following Examples, reduce the first of the two given quantities to the decimal of the second :—

(1) *Rs.*11. 2*a.* 2*p.* ; *Rs.*178. 2*a.* 8*p.* (2) *Rs.*12 0*a* 6*p.* ; *Rs.*25. 7*a.*

(3) *Re.*1. 11*a.* ; *Rs* 2. 8*a.* (4) *Rs* 2. 13*a.* 10*p.* ; *Rs.*50.

(5) 5*s.* ; 13*s.* 4*d.* (6) 13*s.* $6\frac{3}{4}$*d.* ; 15*s.* 6*d.*

(7) £3. 6*s.* $8\frac{1}{2}$*d.* ; £7. 10*s.* (8) $3\frac{1}{2}$ guineas ; £2. 15*s.* $5\frac{1}{8}$*d.*

(9) $1\frac{1}{2}$*d.* ; 7*s.* $10\frac{1}{2}$*d.* (10) 7*s.* 8·1942*d.* ; 15*s.* 9*d.*

(11) $\frac{7}{8}$ of 10*s* ; 13*s.* 4*d.* (12) $\frac{3}{4}$ of 2*s.* 6*d.* ; $\frac{5}{8}$ of $1\frac{1}{2}$ guineas.

(13) 3·45 of 10*s.* 6*d.* ; half-a-crown. (14) ·052$\dot{7}$ of £1. 7*s.* 6*d.* ; 13*s.* 4*d.*

(15) 3 hrs. 26 min. 37 sec. ; 13 days 20 hrs. 23 min.

(16) 1 cwt. 2 qrs. $3\frac{1}{4}$ lbs. ; 1 ton 4 cwt. 1 qr. 24 lbs.

(17) 10 lbs. 11 oz. 12 dwts. 7 grs. ; 9 lbs. 8 oz. Avoir.

(18) 5 ac. 3 ro. 15 po. ; 1 ac. 2 ro. 32 po.

(19) $3\frac{1}{8}$ of £4. 15*s.* 4*d.* ; ·$\dot{2}\dot{7}$ of 16*s.* 3*d.* (20) 2 sr. 4 ch. ; 1 md. 8 sr.

(21) 2[illegible] of £2. 6*s.* $5\frac{3}{4}$*d.* ; £18. 17*s.* $10\frac{3}{4}$*d.*

(22) ·101 of 1 lb. 5 oz. ; $\frac{7}{8}$ of 1 qr. 22 lbs. 8 oz.

(23) 1 bi. 11 k. 8 ch. ; 16 k. 14 ch.

(24) 1 md. 3 sr. $8\frac{3}{8}$ ch. ; 1 md. 16 sr.

2. Express 3*s.* $5\frac{11}{100}$*d.* as the decimal of a dollar of 4*s.* $1\frac{1}{2}$*d.*

3. Express £5·456 as the decimal of a rupee of 1*s.* 10*d.*

4. Express ·375 of a guinea $+ \frac{3}{16}$ of a crown $+$ ·3 of 7*s.* 6*d.* $- \frac{3}{8}$ of 2*d.* as the decimal of 16*s.*

5. Find the value of £·0375 $+$ ·625*s.* $+$ ·75*d.* $+$ 3*s.* 3·5*d.* and reduce the result to the decimal of 7*s.* 6*d.*

6. Find the value of ·$\dot{2}4\dot{6}$ of *Rs.*4. 10*a.* $+$ ·25$\dot{9}$ of *Rs.*12. 8*a.* $+$ ·0$\dot{2}$ of *Rs.*33. 12*a.* and reduce the result to the decimal of *Rs.*30.

7. Find the value of $\frac{3}{4}$ of $\frac{1}{9\frac{1}{2}}$ of £1. 18*s.* $+ \frac{2}{3}$ of ·375 of 15*s.* $+ \frac{3}{8}$ of ·4$\dot{2}\dot{9}$ of 8*s.* 3*d.* and express the result as the decimal of £5.

8. Express $\frac{5}{8}$ of 12*s.* 6*d.* $+$ ·625 of 7*s.* 6*d.* $-$ ·5$\dot{8}\dot{5}$ of 16*s.* 6*d.* as the decimal of £1.

9. Express £874. 13*s.* 4*d.* $\times$ 3·75 as the decimal of £1000.

10. What decimal of a crown is the difference between $6\frac{1}{2}$ half-guineas and £3·525 ?

11. Express the difference between $\cdot\dot{3}7\dot{8}$ of 13*s.* $10\frac{1}{2}$*d.* and $\cdot\dot{3}\dot{7}\dot{8}$ of 16*s.* 6*d.* as the decimal of ·426 of £1. 17*s.* 6*d.*

12. Express £9 + 2·7*s.* + 3·6*d.* as the decimal of £(2 − ·2) + (6 − ·6)*s.* + (8 − ·8)*d.*

XVI. APPROXIMATION.

385. It has already been shewn in Art. 351, that in converting a vulgar fraction to a decimal, where the division does not terminate (which is often denoted by dots (...) placed at the end of the quotient), an **approximation** to its true value can always be found to any degree of accuracy. Thus $\frac{5}{17}$ = ·29411764....If we wish to *approximate* to the result by terminating the operation at the 5th place, we write $\frac{5}{17}$ = ·29412, but if at the fourth place, we write $\frac{5}{17}$ = ·2941, and so on. From this it is evident that we increase the last figure retained by 1, if the succeeding figure be 5, or greater than 5.

386. The reason for the above is obvious from the following considerations If we take ·29412 to represent ·29411764 . instead of ·29411, it is clear that ·29412 is greater, and ·29411 less than the true value of the decimal ; but ·29412 is greater than the true value by ·00000236..., and ·29411 is less than the true value by ·00000764...

Now ·00000236...is less than ·00000764...

Therefore ·29412 is nearer the true value than ·29411.

387. Contracted Addition and Subtraction. These methods have already been explained in Art. 369.

388. Contracted Multiplication. In multiplying one long decimal by another, it is generally required to get the product *approximately* correct, *i. e*, as far as a certain decimal place. The following Rule enables us to shorten the work.

RULE. Mark off in the decimal part of the multiplicand as many figures as is one more than the number of decimal places we are required to retain in the product ; under the last of these marked figures place the units' figure of the multiplier, writing the figures in a reverse order. Omit decimal points of both the multiplicand and the multiplier and add 0's (if necessary) in the multiplicand, so that every figure of the multiplier shall have a figure above it. Begin the multiplication with the right hand figure of the multiplier and multiply in succession by each of the others, in each case beginning the multiplication from the figure above the one we are multiplying by, but carrying to it the *nearest ten* from its product with the next figure on the right. Place the units' figure of all these partial products in the same vertical line ; add as usual, and mark off the required number of decimal places in the result, striking out the last figure.

Note. In carrying the *nearest ten*, if the product is a number from 5 to 14 carry **1**; from 15 to 24 carry **2**; from 25 to 34 carry **3**; from 35 to 44 carry **4**; and so on. If the product is 4 or less than 4, reject it. (Art. 385.)

Ex. 1. Multiply 459·63524 by 25·4637, retaining 3 places; ·00040635 by 241·6358, retaining 6 places; and ·453 by ·01694, retaining 4 places of decimals.

(1)	(2)	(3)
4596352,4	4063,50	4530,0
736452	8536142	496100
91927048	812700	453
22981762	162540	272
1838541	4064	41
275781	2438	2
13789	122	00768
3217	20	
11704·0138	3	for ·01694 may be written as 0·01694.
	·0981887	

Ex. 2. Multiply $3\cdot 2567834$ by $4\cdot 2089542$, retaining 7 places, and $1\cdot 8\dot{2}35\dot{7}$ by $\cdot 0\dot{7}8\dot{5}$, retaining 6 places of decimals.

(1)	(2)
325678340	18235723,57
24598024	758758700
1302713360	1276500
65135668	145886
2605426	9118
293109	1276
16284	146
1303	9
65	1
13·70765218	·1432936

389. Contracted Division. In dividing one decimal by another where the quotient is required to be approximately correct only to a certain number of decimal places, we use the following Rule :—

RULE. Make the divisor a whole number; and determine first of all—by inspection or by taking one step in the ordinary way—the highest number of *integral* figures in the quotient, and then the whole number of figures in the quotient; from the left of the divisor cut off this number of figures, and one more for *approximation*; and strike out the rest. Proceed one step with this new divisor, but in multiplying its first figure by the quotient figure, carry the *nearest ten* from its product with the next figure on the right. Instead of bringing down a figure to the remainder, strike off another figure from the divisor, and proceed as before, until no figure is left in the divisor.

If the number of figures in the divisor be less than the number of figures to be cut off, proceed in the ordinary way until the number

of figures still to be found in the quotient is one less than the number of figures in the divisor, and then apply the Rule.

Ex. 1. Divide 2508·928065051 by 92·410357, approximately correct to 4 places of decimals.

```
9,2,4,1,0,3,5)2508928065·051(27·1498
              1848207
               660721
               646872
                13849
                 9241
                 4608
                 3696
                  912
                  832
                   80
                   74
```

Making the divisor a whole number, we find by inspection that there will be 2 figures in the *integral* part of the quotient; and 4 places of decimals are to be retained. Hence, 6 figures are retained in the divisor and 1 more for *approximation*, so that the divisor is 924103,5. In the next stage the divisor is 92410,3'; 3 being retained for *approximation*, and so on.

Ex. 2. Divide ·257917 by 2·03458 approximately correct to 7 places of decimals.

```
2,0,3,4,5,8)25791·70(·1267667
            203458
             544590
             406916
             137674
             122075
              15599
              14242
               1357
               1220
                137
                122
                 15
                 14
```

Here, by inspection, we find that the quotient will contain no integral part; and as 7 places of decimals are to be retained, the divisor must consist of 8 figures, with 1 for approximation. But as there are only 6 figures in the divisor, proceed in the usual way of division for 2 figures in the quotient, when the number of figures still to be obtained will be one less than the number of figures in the divisor. Then apply the Rule.

Ex 3. Divide ·549532676 by 931·2167, retaining 7 places of decimals.

```
9,3,1,2,1)5495·32676(·0005901
          46561
           8392
           8381
             11
              9
```

By inspection, we determine that there will be 3 ciphers after the decimal point in the quotient; hence only (7 − 3) or 4 figures are required in the quotient. Therefore we retain 5 figures in the divisor, one for approximation.

390. Series. The value of a *Series* is frequently required to be obtained correct to a certain number of decimal places. In such cases proceed as in the following Examples.

Ex. 1 Find the value, correct to 7 places of decimals, of

$$1+\frac{1}{1.2}+\frac{1}{1.2.3}+\frac{1}{1.2.3.4}+\&c.$$

1	$=1$	$=1\cdot$	
$\frac{1}{1.2}$	$=\frac{1}{2}$	$=\cdot5$	
$\frac{1}{1.2.3}=\frac{1}{3}\times\frac{1}{2}$	$=\frac{1}{3}\times\cdot5$	$=\cdot1666666$	67
$\frac{1}{1.2.3.4}=\frac{1}{4}\times\frac{1}{1.2.3}$	$=\frac{1}{4}\times\cdot166666667$	$=\cdot0416666$	67
$\frac{1}{1.2.3.4.5}=\frac{1}{5}\times\frac{1}{1.2.3.4}$	$=\frac{1}{5}\times\cdot041666667$	$=\cdot0083333$	33
$\frac{1}{1.2.3.4.5.6}=\frac{1}{6}\times\frac{1}{1.2.3.4.5}$	$=\frac{1}{6}\times\cdot008333333$	$=\cdot0013888$	89
$\frac{1}{1.2.3.4.5.6.7}$	$=\frac{1}{7}\times\cdot001388889$	$=\cdot0001984$	12
$\frac{1}{1.2.3.4.5.6.7.8}$	$=\frac{1}{8}\times\cdot000198412$	$=\cdot0000248$	01
$\frac{1}{1.2.3.4.5.6.7.8.9}$	$=\frac{1}{9}\times\cdot000024801$	$=\cdot0000027$	56
$\frac{1}{1.2.3.4.5.6.7.8.9.10}$	$=\frac{1}{10}\times\cdot000002756$	$=\cdot0000002$	76
$\frac{1}{1.2.3.4.5.6.7.8.9.10.11}$	$=\frac{1}{11}\times\cdot000000276$	$=\cdot0000000$	25
		$1\cdot7182818$	26

The next and the following terms need not be considered, as they will all give 0's only up to the 7th decimal place.

Ex. 2. Find the value, correct to 5 places of decimals, of

$$\tfrac{3}{16}+(\tfrac{3}{16})^2+(\tfrac{3}{16})^3+(\tfrac{3}{16})^4+\ldots\text{to infinity.}$$

Let s denote the sum of the given series.

Then $s=\tfrac{3}{16}+(\tfrac{3}{16})^2+(\tfrac{3}{16})^3+(\tfrac{3}{16})^4+\ldots\ldots$

$\therefore\ \tfrac{16}{3}s=1+\tfrac{3}{16}+(\tfrac{3}{16})^2+(\tfrac{3}{16})^3+(\tfrac{3}{16})^4+\ldots\ldots$

Hence by subtraction, we get

$(\tfrac{16}{3}-1)s=1$; or $\tfrac{13}{3}s=1$; $\quad\therefore\ s=\tfrac{3}{13}=\cdot\underline{23076}.$

Ex. 3 Find the value, correct to 7 decimal places, of

$$\frac{1}{3\cdot5}+\frac{2}{3^2.5^3}+\frac{3}{3^3.5^5}+\frac{4}{3^4.5^7}+\&c.$$

Let s denote the sum of the series,

$$\text{then } s = \frac{1}{3.5} + \frac{2}{3^2.5^3} + \frac{3}{3^3.5^5} + \frac{4}{3^4.5^7} + \&c.$$

$$\therefore \frac{1}{3.5^2}s = \qquad \frac{1}{3^2.5^3} + \frac{2}{3^3.5^5} + \frac{3}{3^4.5^7} + \&c.$$

By subtraction, we have

$$\left(1 - \frac{1}{3.5^2}\right)s \text{ or } \frac{74}{75}s = \frac{1}{3.5} + \frac{1}{3^2.5^3} + \frac{1}{3^3.5^5} + \frac{1}{3^4.5^7} + \&c.$$

$$\therefore \frac{74}{75}s \times \frac{1}{3.5^2} = \qquad \frac{1}{3^2.5^3} + \frac{1}{3^3.5^5} + \frac{1}{3^4.5^7} + \&c.$$

Again, by subtraction, we get

$$\frac{74}{75}s \times \left(1 - \frac{1}{3.5^2}\right) = \frac{1}{3.5}, \text{ or } \frac{74}{75}s \times \frac{74}{75} = \frac{1}{15}.$$

$$\therefore s = \frac{75 \times 75}{74 \times 74 \times 15} = \frac{375}{5476} = \cdot 0684806\ldots$$

391. Abbreviated Method of dividing a number by 9, 99, 999, &c.

RULE. Point off in the dividend as many decimal places (counting from the right) as there are *nines* in the divisor; then again twice as many decimal places, next three times as many, and so on. Then add these several numbers as in Addition of Decimals. The integral part will give the quotient and the recurring part the remainder.

Ex. Divide 578921 by 99 by the abbreviated method.

5789·21	$578921 \div 99 = 578921 \times \frac{1}{99} = 578921 \times \cdot\dot{0}\dot{1}$
57·8921	$= 578921 \times \left\{\frac{1}{100} + \frac{1}{(100)^2} + \frac{1}{(100)^3} + \&c.\right\}$
·578921	$= \frac{578921}{100} + \frac{578921}{10000} + \frac{578921}{1000000} + \&c.$
·00578921	$= 5789{\cdot}21 + 57{\cdot}8921 + {\cdot}578921 + {\cdot}00578921$
·0000578921, &c.	$+ \&c.$
5847·6868681021	$= 5847{\cdot}6868\ldots$

Hence the quotient is 5847 and remainder 68.

Examples CXVIII.

1. Multiply (by the *contracted method*) :—

(1) ·43429448 by ·6931472 retaining 7 places of decimals.
(2) 459·63524 by 25·4637 .... 6 ....
(3) 583·26784 by ·00985 .... 2 ....
(4) ·0008127 by 483·2716 .... 6 ....
(5) 3670·257 by 12·61158 .... 3 ....
(6) ·86858896 by 1·0986123 .... 5 ....

(7) 52·687640812 by 18·703216231 retaining 6 places of decimals.
(8) 1·050625 by itself .... 4 ....
(9) $27\cdot54\dot{3}\dot{6}$ by $8\cdot34\dot{7}$ .... 5 ....
(10) $\cdot012\dot{3}4\dot{5}$ by $49\cdot\dot{3}\dot{6}$ .... 5 ....

2. Divide (by the *contracted method*) :—

(1) 3789·436 by 265·5984 retaining 2 places of decimals.
(2) 742·876315 by 4967·358 .... 4 ....
(3) 185·37612 by ·08764032 .... 4 ....
(4) 154·362904 by ·000541398 .... 7 ....
(5) 10·926954 by ·3547808034 .... 3 ....
(6) 2 by 15·314865 .... 5 ....
(7) 1 by 3·1415926535 .... 6 ....
(8) $2\cdot347\dot{2}\dot{1}$ by $3\cdot279\dot{2}\dot{4}$ .... 7 ....
(9) $176\cdot804\dot{3}\dot{2}$ by 25·123456 .... 3 ....
(10) $66\cdot02\dot{0}3\dot{7}$ by $248\cdot7\dot{2}\dot{2}$ .... 5 ....

3. Find the respective values of :—

(1) $\frac{1}{5}+\frac{1}{5^2}+\frac{1}{5^3}+\frac{1}{5^4}+$&c. .... to infinity.

(2) $\frac{1}{7}+\frac{1}{7^2}+\frac{1}{7^3}+\frac{1}{7^4}+$&c. .... to ....

(3) $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+$&c. .... to ....

(4) $1+\frac{1}{1.3}+\frac{1}{1.3.5}+\frac{1}{1.3.5.7}+$&c....to infinity to 7 places of decimals.

(5) $\frac{1}{5}+\frac{1}{3}\times\frac{1}{5^3}+\frac{1}{5}\times\frac{1}{5^5}+\frac{1}{7}\times\frac{1}{5^7}+$&c .... to 6 ....

(6) $\frac{1}{10^2}\times\left\{1-\frac{3}{10^2}+\frac{3.4}{1.2}\times\frac{1}{10^4}-\frac{3.4.5}{1.2.3}\times\frac{1}{10^6}+\text{\&c.}\right\}$ to 6 ....

(7) $16\times\left\{\frac{1}{5}-\frac{1}{3}\times\frac{1}{5^3}+\frac{1}{5}\times\frac{1}{5^5}-\frac{1}{7}\times\frac{1}{5^7}+\text{\&c.}\right\}-\frac{4}{239}$ to 6 ....

(8) $\frac{1}{4}+\frac{3}{4^2}+\frac{1}{4^3}+\frac{3}{4^4}+\frac{1}{4^5}+\frac{3}{4^6}+$&c .... to infinity.

(9) $5\times\left\{1-\frac{1}{50}-\frac{1}{2\times(50)^2}-\frac{1\times3}{6\times(50)^3}-\frac{1\times3\times5}{24\times(50)^4}-\frac{1\times3\times5\times7}{120\times(50)^5}-\text{\&c. to inf.}\right\}$ to 5 places of decimals.

(10) $\frac{2}{5}+\frac{4}{5^2}+\frac{2}{5^3}+\frac{4}{5^4}+\frac{2}{5^5}+\frac{4}{5^6}+$&c .... to infinity.

4. Divide (by the *abbreviated* method) :—

(1) 2916433 and 75061382 separately by 9.

(2) 51647901 and 7204561 separately by 99.

(3) 7204561 and 580844 separately by 999.

(4) 591608 and 7391684 separately by 9999.

(5) 236916 by 9999 and 720532876 by 99999.

Examples worked out.

Ex. 1. A man owns $\frac{3}{16}$ of a house, and sells $\cdot 1\dot{3}5\dot{1}$ of his share ; what fraction of the house does he still own ?

He sells $\cdot 1\dot{3}5\dot{1}$ of $\frac{3}{16} = \frac{1350}{9990}$ of $\frac{3}{16} = \frac{15}{111}$ of $\frac{3}{16}$.

$\therefore$ he has left $(1 - \frac{15}{111})$ of $\frac{3}{16} = \frac{96}{111}$ of $\frac{3}{16} = \frac{6}{37}$. *Ans.*

Ex. 2. A vessel's cargo, $\frac{2}{3}$ of which is worth £$6666\cdot\dot{6}$, gets damaged, and the owner in consequence sells $\frac{\cdot 8\dot{3} + \cdot 041\dot{6}}{1\cdot 05}$ of it for half the original value of the whole cargo. What is the value of the remainder at the same rate and what the loss on the whole cargo ?

The whole cargo is worth $\frac{3}{2}$ of £$6666\cdot\dot{6}$ = £$9999\cdot\dot{9}$ = £10000.

He sells $\frac{\cdot 83333\ldots + \cdot 04166\ldots}{1\cdot 05} = \frac{\cdot 874\dot{9}}{1\cdot 05} = \frac{\cdot 875}{1\cdot 05} = \frac{875}{1050} = \frac{5}{6}$.

$\therefore$ he has remaining $(1 - \frac{5}{6})$ or $\frac{1}{6}$.

Now since $\frac{5}{6}$ of the cargo sells for $\frac{1}{2}$ of £10000 = £5000 ;

$\therefore$ $\frac{1}{6}$ of the cargo must sell for $\frac{1}{5}$ of £5000 = £1000.

Hence loss = £(10000 − 5000 − 1000) = £4000.

Ex. 3. A woman had a certain number of eggs ; she sold ·25 of the number and 3 more to one person, ·375 of the remainder to a second, and $\cdot\dot{6}$ of what still remained to a third, when she had only 15 left. How many had she at first ?

After selling $\cdot\dot{6}$ or $\frac{2}{3}$ of the second remainder, she had $(1 - \frac{2}{3})$ or $\frac{1}{3}$ of the eggs left. Therefore $\frac{1}{3}$ of the second remainder = 15 ; $\therefore$ the second remainder = $15 \times 3 = 45$.

Again, ·375 or $\frac{3}{8}$ of the first remainder being sold, $\frac{5}{8}$ remained ;

$\therefore$ $\frac{5}{8}$ of the first remainder = 45 ; $\therefore$ the first remainder = $45 \times \frac{8}{5} = 72$.

Next, after selling ·25 or $\frac{1}{4}$ of what she now had and 3 more, she had 72 left ; $\therefore$ $\frac{3}{4}$ of the number = $72 + 3 = 75$.

$\therefore$ the whole number of eggs = $75 \times \frac{4}{3} = 100$.

Ex. 4. *A* owns $\cdot 58\dot{3}$ of an estate and *B* the rest. If $\frac{4}{5}$ of *B*'s share is *Rs.* 5000 less than *A*'s, what is the worth of the whole estate ?

Since *A*'s share $=\frac{525}{900}$ or $\frac{7}{12}$ of the estate ;

∴ *B*'s share $=(1-\frac{7}{12})$ or $\frac{5}{12}$ of the estate.

∴ $\frac{3}{5}$ of *B*'s share $=(\frac{3}{5}\times\frac{5}{12})$ or $\frac{1}{4}$ of the estate ; and the difference of their shares $=(\frac{7}{12}-\frac{1}{4})$ or $\frac{1}{3}$ of the estate.

Therefore $\frac{1}{3}$ of the estate $=$ *Rs.*5000 ;

∴ the whole estate $=$ *Rs.*5000 × 3 $=$ *Rs.*15000.

Miscellaneous Examples IV.

1. Find the sum, difference, product and the quotients of 30·33 and ·0337 ; and find the sum of all the results.

2. Reduce ($\frac{8}{25}$ of 2·45 $-\frac{3}{100}$ of ·02) + 1000 to a decimal.

3. Find the sum of 3·102 + ·00071 + 5·876 + 1·2 + ·31907 + ·027 + 310·68 + ·0000743 + 38·691 + ·1041457.

4. Which is the greater, ·39 of a guinea, or $\cdot 4\dot{0}\dot{9}9$ of £1 ?

5. Divide the sum of 8·25 and 4·125 by their difference.

6. Divide the product of 1·075 and ·0101 by ·43.

7. Divide the difference between 3·1047 and ·0731 by the sum of 1·27 and 11·384.

8. If ·3 of an estate is sold for *Rs.*4504, find the value of ·48 of it at the same rate.

9. A man, who possesses $\cdot\dot{2}\dot{7}$ of a ship, sells $\cdot 41\dot{6}$ of his share for *Rs* 32400 ; what is the ship worth ?

10. In a school of 200 children there are 4 classes, of which the first contains ·24, the second ·36, and the third ·18 of the whole ; of how many does the fourth class consist ?

11. If $\cdot\dot{6}$ of the number of apples in a basket exceeds ·6 of the number by 57·4 ; find the number of apples.

12. Divide 8·064 by $84\dot{6}+\frac{20}{7}$ of $\cdot 291\dot{6}$.

13. Divide $\frac{\cdot 052}{1\cdot 3}$ of 1·56 by $\frac{\cdot 0624}{14\cdot 4}$ of 25·92.

14. A butcher bought an equal number of calves and sheep for £265 ; for the calves he gave £3·75 a head, and for the sheep £2·875 a head ; how many did he buy of each kind ?

15. A gentleman having given $\cdot\dot{5}$ of the money in his purse for a horse, and ·375 of the remainder for a sheep, had £1·6875 still left ; what sum had he at first ?

16. Divide *Rs.*870 between *A*, *B* and *C*, so that ·75 of *C*'s share shall $=$ ·5 of *A*'s $=$ $\cdot\dot{6}$ of *B*'s.

17. A coal-dealer bought 198 mds. of coal for *Rs.*32·5875, of which he sold 100 mds. for *Re.*·2375 a maund. At what price per seer must he sell the remainder so as to gain *Rs.*2·1875 by his bargain ?

18. *A* had *Rs.*2568. 11*a.* 4*p.*, which was *Rs.*$431{\cdot}88541\dot{6}$ less than ·6 of $\cdot\dot{7}$ of 2·5 times *B*'s money. How much money had *B* ?

19. How many oranges at £·084375 a dozen ought to be given for 378 eggs at ·0625*s.* each ?

20. What number must be subtracted from the product of 9·27 and 8·0003 to give the sum of 19, 27·9652, ·003, 5·0267 and 17·09 ?

21. *A* has shares in an estate to the amount of ·25 of it and of $\cdot\dot{3}\dot{6}$ of it. *B* has shares in the same estate to the amount of $\cdot 25\dot{7}\dot{2}$ of it ; find the difference in value between the properties of *A* and *B*, when $\cdot\dot{3}\dot{6}$ of the estate is worth *Rs.*50000.

22. Divide 9·614 by ·0000019 and $\dfrac{2\frac{1}{8}}{5\frac{1}{3}}$ by $\cdot 000\dot{3}$ and multiply the sum of the quotients by ·0005.

23. Express the value of $\dfrac{133\frac{1}{7}}{83\frac{8}{9}} \div \left(1+\dfrac{2}{3+\dfrac{4}{5+\frac{6}{7}}}\right)$ of a rupee in decimals of £1, when the value of the rupee is 1*s.* 5½*d.*

24. Simplify $\cdot 0576 \times 1{\cdot}97 + \cdot\dot{1}4285\dot{7} \div 2\tfrac{1}{7} + \cdot 0454864$.

25. Divide 1001 by 390625 ; ·1001 by ·000390625 and 10·01 by 390·625. Multiply $1{\cdot}\dot{1}\dot{8}$ by $\cdot\dot{5}3846\dot{1}$.

26. Find the value (to three places of decimals) of

$$1+\frac{1}{4}+\frac{1\times3}{1\times2}\left(\frac{1}{4}\right)^2+\frac{1\times3\times5}{1\times2\times3}\left(\frac{1}{4}\right)^3+\frac{1\times3\times5\times7}{1\times2\times3\times4}\left(\frac{1}{4}\right)^4+\text{\&c. to infinity.}$$

27. Simplify :—

$$\frac{3.\dot{3}}{6.0625} \text{ of } \frac{9.7}{2.4\dot{2}} \div \frac{2.5}{1.09}\,(7.25+2.75)\times\frac{£3.\ 6s.\ 8d.}{£10.\ 13s.\ 4d.}.$$

28. Subtract $\cdot 0\dot{3}$ from $\cdot\dot{0}\dot{3}$ and divide the result by $\cdot 1\dot{0}\dot{2}$.

29. Find the value of $\cdot 01\dot{6}$ of *Rs.*260. 2*a.* 6*p.* + $\cdot\dot{3}5\dot{1}$ of *Rs.*13. 14*a.* + $1{\cdot}0\dot{0}03\dot{3}$ of *Rs.*7. 14*a.* 3*p.*

30. Find how much more than $\dfrac{\cdot 0338184}{\cdot 03416}$ of $1{\cdot}1\dot{6}$ of ·6 of $\cdot 5\dot{8}\dot{7}$ of *Rs.*52. 1*a.* 4*p.* I need to pay a bill of *Rs.*21. 4*a.*

31. A person owns $\frac{7}{8}$ of an estate, and sells $\cdot 3\dot{5}7142\dot{8}$ of his share ; what part of the whole estate has he still left ?

32. *A* and *B* can do a piece of work in 15·75 days, *B* and *C* can do it in $18{\cdot}\dot{6}$ days and *A* and *C* in $16{\cdot}\dot{3}$ days. In what time would *A*, *B* and *C* singly perform the whole work ?

33. There is a number which, when multiplied by 4·255 and divided by ·0016, gives 851 ; find the number.

34. Shew that, whether the value of $3\frac{1}{4}+4\frac{3}{8}-5\frac{1}{2}+16\frac{5}{8}-\frac{1}{2}\frac{1}{4}+10-14\frac{5}{8}$ be found by vulgar fractions or by decimals, the results coincide.

35. The owner of ·375 of a mine sold $\cdot\dot{6}$ of his share for *Rs.*25200 ; find the value of ·875 of the mine.

36. A cistern of water lost ·12 of its contents by leakage, then 26 gals. were drawn off, and it was then ·75 full ; how many gals. did it contain at first ?

37. In a cricket match, one side of 11 men made a certain number of runs, one player obtained ·25 of the number, each of three others ·1, each of two others ·0625, and the rest 39 amongst them ; find the whole number of runs.

38. Reduce to their simplest forms :—

(1) $\dfrac{\cdot005}{\frac{1}{13}\text{ of }11\frac{1}{7}}$ of $\dfrac{49\frac{1}{91}}{\frac{7}{4}\text{ of }2\cdot25} \div \left(\dfrac{1}{21}+\dfrac{1}{27}\right)$. (2) $\cdot70\frac{4}{5}$. (3) $\dfrac{4}{\frac{9}{10}}-\dfrac{\frac{4}{5}}{10}$.

39. Five bells which toll at intervals of 1·2, 1·5, 1·75, 1·8, 2·1 seconds respectively, begin tolling simultaneously ; how long after will they all toll simultaneously again ?

40. Reduce £24. 16*s.* $4\frac{1}{2}$*d.* and £167. 10*s.* $6\frac{1}{4}$*d.* $\frac{1}{2}$*q.* to decimals of the same denomination, so as to find how often the former is contained in the latter.

41. Find the value of $\dfrac{\cdot093\dot{1}\dot{8}}{\cdot56\dot{8}\dot{1}}$ of $2\frac{1}{12}$ of 2·5 days.

42. A woman has a certain number of eggs ; she sells $\cdot\dot{3}$ of the number and one more to one person, $\cdot\dot{3}$ of the remainder to a second person, and ·5 of the remainder to a third person ; after these sales she has 15 eggs left. How many had she at first ?

43. A clerk copied ·55 of *Rs.*50 instead of 5·5 of *Rs.*50 ; what was the amount of the error ?

44. From a rod 2·078 miles long, portions are cut off each equal to ·0037 of an inch, how many such portions can be cut off and what will be the remainder ?

45. Express the sum of $\cdot\dot{5}7142\dot{8}$ of a vis, $\frac{3}{8}$ of $\dfrac{1}{3\frac{3}{8}}$ of $\frac{8\frac{1}{3}}{8\frac{1}{4}}$ of a maund and $\frac{8801}{10188}$ of a cwt. as the decimal of 1 ton. (a vis = 3 lbs. 2 oz. ; one maund = $82\frac{2}{7}$ lbs. Avoir.)

46. The difference in the values of the two shares into which a certain property is divided is *Rs.*48·575, and one share is ·51 of the whole. Find the value of the property and of each share.

47. *A* has an income = ($\cdot6$ of $\cdot8\dot{3}+3\cdot5$) of *B*'s income. If *A* after spending *Rs.*645 per annum, find that he has exceeded his income by ·075 of it, find *B*'s income.

48. *A* can reap $\cdot\dot{4}$ of a field in $2\cdot\dot{6}$ days and *B* can reap ·6 of it in [illegible] days ; *A* and *B* work together till they have reaped ·75 of

the field. *A* then leaves, and *B* completes the work. If *A* earn *Rs* 2. 8*a*. a day, what ought the reaping of the field to cost ?

49. Out of a bag of silver, I take *Rs*.25 more than ·5 of the whole sum which it contained ; then *Rs*.15 more than ·2 of what then remained ; and then *Rs*.10 more than ·25 of what then remained ; after this *Rs* 5 remained. What did the bag contain at first ?

50. *A* has shares in an estate to the amount of $\cdot 15 \div \cdot 3\bar{6}$ of it, *B* has shares in the same estate to the amount of $\cdot\dot{4}7\dot{2}$ of it ; find the difference in value between the properties of *A* and *B*, when $\cdot 0\dot{5}\dot{6}$ of the estate is worth £373·3.

CHAPTER VII.

Rules of Practice and Invoices.

392. We shall here shew how the primitive fractions as defined in Art. 228, may be applied to the *practical* calculation of prices, when the price of a unit of any denomination is supposed to be given ; and the tediousness of the *enunciations* of the rules at length, will be a sufficient excuse for the mere *indications* of the processes to be employed, by means of examples.

393. An **aliquot part** of a number is such that we may make up the number by taking the part a certain *integral* number of times. Its relation with the whole can therefore be expressed by a fraction which has unity for its numerator and an integer for its denominator.

Thus, 5*a*. 4*p*., being $\frac{1}{3}$ of *Re*.1, is an *aliquot* part of a rupee ; 10*s*, being $\frac{1}{2}$ of £1, is an *aliquot* part of a pound.

Table of Aliquot Parts.

Of a **Rupee**.	
8*a*.	= $\frac{1}{2}$*Re*.
5*a*. 4*p*.	= $\frac{1}{3}$*Re*.
4*a*.	= $\frac{1}{4}$*Re*.
2*a*. 8*p*.	= $\frac{1}{6}$*Re*.
2*a*.	= $\frac{1}{8}$*Re*.
1*a*. 4*p*.	= $\frac{1}{12}$*Re*.
1*a*.	= $\frac{1}{16}$*Re*.

Of a £.	
10*s*.	= $\frac{1}{2}$£.
6*s*. 8*d*.	= $\frac{1}{3}$£.
5*s*.	= $\frac{1}{4}$£.
4*s*.	= $\frac{1}{5}$£.
3*s*. 4*d*.	= $\frac{1}{6}$£.
2*s*. 6*d*.	= $\frac{1}{8}$£.
2*s*	= $\frac{1}{10}$£.
1*s*. 8*d*.	= $\frac{1}{12}$£.
1*s*. 4*d*.	= $\frac{1}{15}$£.
1*s*. 3*d*.	= $\frac{1}{16}$£.
1*s*.	= $\frac{1}{20}$£.

Of a **Maund**.	
20 sr.	= $\frac{1}{2}$ md.
10 sr.	= $\frac{1}{4}$ md.
8 sr.	= $\frac{1}{5}$ md.
5 sr.	= $\frac{1}{8}$ md.
4 sr.	= $\frac{1}{10}$ md.
2 sr. 8 ch.	= $\frac{1}{16}$ md.
2 sr.	= $\frac{1}{20}$ md.
1 sr. 4 ch.	= $\frac{1}{32}$ md.
1 sr.	= $\frac{1}{40}$ md

OF AN Anna.

6*p*.	$= \frac{1}{2}a.$
4*p*.	$= \frac{1}{3}a.$
3*p*.	$= \frac{1}{4}a.$
2*p*.	$= \frac{1}{6}a.$
1*p*.	$= \frac{1}{12}a.$
2*ps*.	$= \frac{1}{2}a.$
1*ps*.	$= \frac{1}{4}a$

OF A Ton.

10 cwt.	$= \frac{1}{2}$ ton.
5 cwt.	$= \frac{1}{4}$ ton.
4 cwt.	$= \frac{1}{5}$ ton.
2 cwt. 2 qr.	$= \frac{1}{8}$ ton.
2 cwt.	$= \frac{1}{10}$ ton.
1 cwt. 1 qr.	$= \frac{1}{16}$ ton.
1 cwt.	$= \frac{1}{20}$ ton.

OF A Bigha.

10 kâthas	$= \frac{1}{2}$ big.
5 k.	$= \frac{1}{4}$ big.
4 k.	$= \frac{1}{5}$ big.
2 k. 8 ch.	$= \frac{1}{8}$ big.
2 k.	$= \frac{1}{10}$ big.
1 k. 4 ch.	$= \frac{1}{16}$ big.
1 k.	$= \frac{1}{20}$ big.

OF AN Acre.

2 ro.	$= \frac{1}{2}$ ac.
1 ro.	$= \frac{1}{4}$ ac.
20 po.	$= \frac{1}{8}$ ac.
16 po.	$= \frac{1}{10}$ ac.

OF A Week.

$3\frac{1}{2}$ da.	$= \frac{1}{2}$ wk.
$1\frac{3}{4}$ da.	$= \frac{1}{4}$ wk.

OF A Shilling.

6*d*.	$= \frac{1}{2}s.$
4*d*.	$= \frac{1}{3}s.$
3*d*.	$= \frac{1}{4}s.$
2*d*.	$= \frac{1}{6}s.$
$1\frac{1}{2}d.$	$= \frac{1}{8}s.$
$1\frac{1}{3}d.$	$= \frac{1}{9}s.$
1*d*.	$= \frac{1}{12}s.$

OF A Cwt.

2 qrs.	$= \frac{1}{2}$ cwt.
1 qr.	$= \frac{1}{4}$ cwt.
16 lbs.	$= \frac{1}{7}$ cwt.
14 lbs.	$= \frac{1}{8}$ cwt.

OF A Katha.

8 ch.	$= \frac{1}{2}$ k.
4 ch.	$= \frac{1}{4}$ k.
2 ch.	$= \frac{1}{8}$ k.
1 ch.	$= \frac{1}{16}$ k.

OF A Rood.

20 po.	$= \frac{1}{2}$ ro.
10 po.	$= \frac{1}{4}$ ro.
8 po.	$= \frac{1}{5}$ ro.
5 po.	$= \frac{1}{8}$ ro.
4 po.	$= \frac{1}{10}$ ro.
2 po.	$= \frac{1}{20}$ ro.
1 po.	$= \frac{1}{40}$ ro.

OF A Month.

1 wk.	$= \frac{1}{4}$ mo.
2 wk.	$= \frac{1}{2}$ mo.
15 da.	$= \frac{1}{2}$ mo.
10 da.	$= \frac{1}{3}$ mo.

OF A Seer.

8 ch.	$= \frac{1}{2}$ sr.
4 ch.	$= \frac{1}{4}$ sr.
2 ch.	$= \frac{1}{8}$ sr.
1 ch.	$= \frac{1}{16}$ sr.

OF A Quarter.

14 lbs.	$= \frac{1}{2}$ qr.
7 lbs.	$= \frac{1}{4}$ qr.
4 lbs.	$= \frac{1}{7}$ qr.
3 lbs. 8 oz.	$= \frac{1}{8}$ qr.
2 lbs.	$= \frac{1}{14}$ qr.
1 lb. 12 oz.	$= \frac{1}{16}$ qr.
1 lb.	$= \frac{1}{28}$ qr.

OF A lb. AVOIR.

8 oz.	$= \frac{1}{2}$ lb.
4 oz.	$= \frac{1}{4}$ lb.
2 oz.	$= \frac{1}{8}$ lb.
1 oz.	$= \frac{1}{16}$ lb.

OF AN Oz. AVOIR.

8 dr.	$= \frac{1}{2}$ oz.
4 dr.	$= \frac{1}{4}$ oz.
2 dr.	$= \frac{1}{8}$ oz.
1 dr.	$= \frac{1}{16}$ oz.

OF A Mile.

4 fur.	$= \frac{1}{2}$ mi.
2 fur.	$= \frac{1}{4}$ mi.
1 fur.	$= \frac{1}{8}$ mi.

OF A Furlong.

110 yd.	$= \frac{1}{2}$ fur.
55 yd.	$= \frac{1}{4}$ fur.

394. **Practice** is a short method of finding the value of any quantity by means of *aliquot parts*, when the value of a unit of any denomination is given It is therefore another method of solving questions in *Compound Multiplication.*

395. Practice may be either **Simple** or **Compound**.

It is *Simple Practice*, when the value of one unit of a certain denomination is given, and the value of a number of these units is

required ; but in *Compound Practice*, the given quantity is not wholly expressed in the same denomination as the unit whose value is given.

Thus, to find the value of 350 articles at 15*a*. 8*p*. each is *Simple Practice* ; and to find the value of 14 mds. 15 sr. 7 ch. at *Rs*.2. 5*a*. 8*p*. per maund is *Compound Practice*.

I. SIMPLE PRACTICE.

396. The RULE for Simple Practice will be best understood by the following Examples.

Ex 1. Find the value of 1298 things at *Rs*.8. 14*a*. 6*p*. each.

If the cost of a thing be *Re*.1 ; then the total cost is *Rs*.1298.

	Rs.	*a*.	*p*.	
8*a*. = ½ of *Re*. 1.	1298	0	0	= price @ *Re*.1 each.
			8	
	10384	0	0	= price @ *Rs* 8 ......
4*a*. = ½ of 8*a*.	649	0	0	= price @ 8*a*. ....
2*a*. = ½ of 4*a*.	324	8	0	= price @ 4*a*. ......
6*p*. = ¼ of 2*a*.	162	4	0	= price @ 2*a* ......
	40	9	0	= price @ 6*p*. ......
	Rs.11560	5	0	= price @ *Rs*.8. 14*a*. 6*p* each.

Note 1. It is generally most convenient, when possible, to use the *aliquot part* of the denomination next superior to the highest denomination of the price proposed.

Here, *Rs* 8. 14*a*. 6*p*. is less than *Rs* 9 by 1*a* 6*p*. Hence the calculation may be shortened thus :—

1*a*. = $\frac{1}{16}$ of *Re*.1	*Rs*.1298. 0*a* 0*p*.			= price at *Re* 1 each.
	9			
	Rs.11682	0	0	= price at *Rs*.9 each.
6*p*. = ½ of 1*a*.			*Rs*.81. 2*a*.	= price at 1*a*. ......
			Rs.40. 9*a*.	= price at 6*p*. ......
	121	11	0	= price at 1*a*. 6*p*. ......
	Rs.11560	5	0	= price at *Rs*.8. 14*a* 6*p*....

Ex. 2. Find the cost of 345 things at £3. 17*s*. 10½*d*. each.

	£.	*s*.	*d*.	
10*s*. = ½ of £ 1.	345	0	0	= cost @ £1 each
			3	
	1035	0	0	= cost @ £3 each.
5*s*. = ½ of 10*s*.	172	10	0	= cost @ 10*s*. ......
2*s*. 6*d*. = ½ of 5*s*.	86	5	0	= cost @ 5*s*. ......
3*d*. = $\frac{1}{10}$ of 2*s*. 6*d*.	43	2	6	= cost @ 2*s*. 6*d*. ......
1½*d*. = ½ of 3*d*.	4	6	3	= cost @ 3*d*. ......
	2	3	1½	= cost @ 1½*d*. .. ...
	£1343	6	10½	= cost @ £3. 17*s*. 10½*d*......

Otherwise thus :—As £3. 17*s*. 10½*d*. is the difference between £4 and 2*s*. 1½*d*., we can simplify the process thus :—

	£.	s.	d.	£.	s.	d.	
2*s*. = $\frac{1}{10}$ of £1.	345	0	0	345	0	0	= cost @ £1 each.
1½*d*. = $\frac{1}{16}$ of 2*s*.	34	10	0			4	
	2	3	1½	1380	0	0	= cost @ £4 each
	£36	13	1½	36	13	1½	= cost @ 2*s*. 1½*d*.....
				£1343	6	10½	= cost @ £3. 17*s*. 10½*d*...

Note 2. Sometimes by introducing a *subsidiary* aliquot part we can easily find the required aliquot part ; thus, taking the preceding example, we have

	£.	s.	d.	
2*s*. = $\frac{1}{10}$ of £1.	345	0	0	= cost at £1 each.
~~6~~*d*. = ¼ of ~~2~~*s*.	34	10	0	= cost at 2*s*. ......
1½*d*. = ¼ of 6*d*.	~~8~~	~~12~~	~~6~~	
	2	3	1½	= cost at 1½*d*. ......
	36	13	1½	= cost at 2*s*. 1½*d*......

Ex. 3. Find the value of 456⅝ mds. at *Rs*.8. 5*a*. 10*p*. per maund.

Since *Re*.⅝ = 10*a*., the cost of 456⅝ mds. at *Re*.1 is *Rs*.456. 10*a*. ; we therefore proceed as before, thus :—

	Rs.	a.	p.	
4*a*. = ¼ of *Re*.1	456	10	0	= value @ *Re*.1 each.
			8	
	3653	0	0	= value @ *Rs*.8 ......
1*a*. = ¼ of 4*a*.	114	2	6	= value @ 4*a*. ......
6*p*. = ½ of 1*a*.	28	8	7½	= value @ 1*a*. .. ...
3*p*. = ½ of 6*p*.	14	4	3¾	= value @ 6*p*. ......
1*p*. = ⅓ of 3*p*.	7	2	1⅞	= value @ 3*p*. ......
	2	6	0⅝	= value @ 1*p*. ......
	Rs 3819	7	7¾	= value @ *Rs*.8. 5*a*. 10*p*. each.

Ex. 4. Find the cost of 2864$\frac{3}{7}$ cwt. at 9*s*. 10¾*d*. per cwt.

Since £$\frac{3}{7}$ would introduce a fraction of a farthing, it will be better to find separately the cost of 2864 cwt. and $\frac{3}{7}$ of a cwt. and then add.

	£.	s.	d.	
5*s*. = ¼ of £1.	2864	0	0	= cost @ £1 each.
4*s*. = ⅕ of £1.	716	0	0	= cost @ 5*s*. ...
10*d*. = ⅙ of 5*s*.	572	16	0	= cost @ 4*s*. ......
~~6~~*d*. = $\frac{1}{10}$ of ~~5~~*s*.	119	6	8	= cost @ 10*d*. ...
	~~71~~	~~12~~	~~0~~	
¾*d*. = ⅛ of 6*d*.	8	19	0	= cost @ ¾*d*.......
	£1417	1	8	= cost @ 9*s*. 10¾*d*. each.
		4	2$\frac{25}{28}$	= cost of $\frac{3}{7}$ of a cwt.
	£1417	5	10$\frac{25}{28}$	= cost @ 9*s*. 10¾*d*. per cwt.

s.	d.
9	10¾
	3
7)29	8¼
4	2$\frac{25}{28}$.

Ex. 5. Find the price of 2108 cwt. of sugar at £1. 6*s*. $2\frac{3}{8}$*d*. each.

	£	*s*.	*d*.	
4*s* = $\frac{1}{5}$ of £1.	2108	0	0	= price at £1 each.
2*s*. = $\frac{1}{2}$ of 4*s*.	421	12	0	= price at 4*s*. ....
2*d*. = $\frac{1}{12}$ of 2*s*.	210	16	0	= price at 2*s*. ....,
$\frac{1}{4}$*d*. = $\frac{1}{8}$ of 2*d*.	17	11	4	= price at 2*d*. ....
$\frac{1}{8}$*d*. = $\frac{1}{2}$ of $\frac{1}{4}$*d*.	2	3	11	= price at $\frac{1}{4}$*d*. ....
	1	1	$11\frac{1}{2}$	= price at $\frac{1}{8}$*d* ....
	£2761	5	$2\frac{1}{2}$	= price at £1. 6*s*. $2\frac{3}{8}$*d*. each.

Examples CXIX.

Find by Practice the values of the following articles :—

1. 3467 at 2*a*. 6*p*.
2. 659 at 13*a*. 2*p*.
3. 1448 at 10*a*. 8*p*.
4. 1281 at 5*a*. 4*p*.
5. 2370 at 13*a*. 4*p*.
6. 659 at 1*s*. $7\frac{3}{4}$*d*.
7. 1250 at 2*s*. $3\frac{3}{4}$*d*.
8. 328 at 8*s*. $5\frac{1}{2}$*d*.
9. 7351 at 14*s*. $9\frac{1}{4}$*d*.
10. 2345 at *Rs*.2. 14*a*. 8*p*.
11. 1600 at *Rs* 2. 5*a*. 6*p*.
12. 140321 at 13*a*. $11\frac{1}{2}$*p*.
13. 632 at *Rs*.14. 5*a*. 6*p*.
14. 7777 at 17*s*. $8\frac{3}{4}$*d*.
15. 1298 at 17*s*. $9\frac{3}{4}$*d*.
16. 537 at £1. 7*s*. $2\frac{1}{2}$*d*.
17. 2937 at £2. 11*s*. $10\frac{3}{4}$*d*.
18. 1684 at £8. 5*s*. $1\frac{1}{2}$*d*.
19. 412 at £5. 14*s*. $5\frac{1}{2}$*d*
20. 6439 at *Rs*.16. 15*a*. $7\frac{3}{4}$*p*.
21. 295 at *Rs*.5. 11*a*. $7\frac{3}{7}$*p*.
22. 3655 at £9. 16*s*. $10\frac{1}{2}$*d*.
23. 3546 at £5. 15*s*. $7\frac{3}{4}$*d*.
24. 65437 at *Rs*.4. 13*a*. 2*p*.
25. $1449\frac{2}{7}$ at *Rs*.11. 6*a*. 6*p*.
26. $237\frac{1}{2}$ at 13*a*. 8*p*.
27. $1128\frac{5}{8}$ at *Rs*.2. 15*a*. 11*p*.
28. $7432\frac{1}{2}$ at *Rs*.6. 12*a*. 4*p*.
29. $6147\frac{3}{4}$ at 17*s*. $6\frac{1}{2}$*d*.
30. $2763\frac{3}{8}$ at 13*s*. $6\frac{3}{4}$*d*.
31. $217\frac{1}{4}$ at £2. 17*s*. $7\frac{1}{2}$*d*.
32. $769\frac{5}{8}$ at *Rs*.16. 4*a*.
33. $674\frac{5}{8}$ at £3. 19*s*. $6\frac{1}{2}$*d*.
34. 22·6 at 5*a*. 1*p*.
35. 169·875 at £2. 17*s*. $10\frac{1}{4}$*d*.
36. 359·3125 at £1. 6*s*. 2*d*.
37. 3764·6 at *Rs*.27. 4*a*. 10*p*.
38. $178\cdot\dot{6}$ at *Rs*.3. 5*a*. 2*p*.
39. 821·5 at *Rs*.6. 15*a*. 2*p*.
40. 861 at *Rs*.5. 7*a*. $5\frac{3}{4}$*p*.
41. 45656 at 6*a*. $2\frac{3}{8}$*p*.
42. 2841 at 5*s*. $10\frac{3}{14}$*d*.
43. 2731 at £4. 8*s*. $9\frac{3}{8}$*d*.
44. 567384 at 5*a*. $10\frac{1}{2}$*p*.
45. 30000 at *Rs*.4. 2*a*. $4\frac{3}{8}$*p*.
46. $5109\frac{7}{18}$ at £4. 16*s*. $4\frac{3}{4}$*d*.

II. COMPOUND PRACTICE.

397. The RULE for Compound Practice will be easily shewn by the following Examples.

Ex. 1. Find the value of 8mds. 6sr. 12ch. at *Rs*.5. 6*a*. 8*p*. per md.

	Rs.	*a.*	*p.*	
5 sr. = $\frac{1}{8}$ of 1 md.	5	6	8	= value of 1 md.
			8	
	43	5	4	= value of 8 mds.
1 sr. = $\frac{1}{5}$ of 5 sr.		10	10	= value of 5 sr.
8 ch. = $\frac{1}{2}$ of 1 sr.		2	2	= value of 1 sr.
4 ch. = $\frac{1}{2}$ of 8 ch.		1	1	= value of 8 ch.
			$6\frac{1}{2}$	= value of 4 ch.
	Rs.44	3	$11\frac{1}{2}$	= value of 8 mds. 6 sr. 12 ch.

Ex. 2. What is the price of 3 cwt. 2 qrs. 16 lbs. at £3. 7*s*. 8*d* per cwt.?

	£.	*s.*	*d.*	
2 qrs. = $\frac{1}{2}$ of 1 cwt.	3	7	8	= price of 1 cwt.
			3	
	10	3	0	= price of 3 cwt.
14 lbs. = $\frac{1}{4}$ of 2 qrs.	1	13	10	= price of 2 qrs.
2 lbs. = $\frac{1}{7}$ of 14 qrs.		8	$5\frac{1}{2}$	= price of 14 lbs.
		1	$2\frac{1}{2}$	= price of 2 lbs.
	£12	6	6	= price of 3 cwt. 2 qrs. 16 lbs.

Ex. 3. Find the value of 11 mds. 4 sr. 8 ch at *Re*.1. 14*a*. 4*p*. per seer.

	Rs.	*a.*	*p.*	
8 ch. = $\frac{1}{2}$ of 1 sr.	1	14	4	= value of 1 seer.
4 sr. = 1 sr. × 4 }			4	
1 md. = 4 sr. × 10 }	7	9	4	= value of 4 sr.
			10	
	75	13	4	= value of 1 md.
			11	
	834	2	8	= value of 11 mds.
	7	9	4	= value of 4 sr.
		15	2	= value of 8 ch.
	Rs.842	11	2	= value of 11 mds. 4 sr. 8 ch.

Ex. 4. Find the value of 365 mds. 37 sr. 8 ch. at *Rs*.126. 6*a*. 8*p*. per maund.

	Rs.	a.	p.	
20 sr. = $\frac{1}{2}$ of 1 md.	126	6	8	= value of 1 md.
			10	
	1264	2	8	= value of 10 mds.
			10	
	12641	10	8	= value of 100 mds.
			3	
	37925	0	0	= value of 300 mds.
	7585	0	0	= value of 60 mds.
	632	1	4	= value of 5 mds.
10 sr. = $\frac{1}{2}$ of 20 sr.	63	3	4	= value of 20 sr.
5 sr. = $\frac{1}{2}$ of 10 sr.	31	9	8	= value of 10 sr.
2 sr. 8 ch. = $\frac{1}{2}$ of 5 sr.	15	12	10	= value of 5 sr.
	7	14	5	= value of 2 sr. 8 ch.
	*Rs.*46260	9	7	= value of 365mds. 37sr.8ch.

Ex. 5. Find the rent of 71 bighas 6 kat. 14 ch. at *Rs.*8. 12*a.* per bigha.

	Rs.	a.	p.	
4 kat. = $\frac{1}{5}$ of 1 big.	8	12	0	= rent of 1 bigha.
			10	
	87	8	0	= rent of 10 bighas.
			7	
	612	8	0	= rent of 70 bighas.
	8	12	0	= rent of 1 bigha.
2 kat. = $\frac{1}{2}$ of 4 kat.	1	12	0	= rent of 4 kat.
8 ch. = $\frac{1}{4}$ of 2 kat.		14	0	= rent of 2 kat.
4 ch. = $\frac{1}{2}$ of 8 ch.		3	6	= rent of 8 ch.
2 ch. = $\frac{1}{2}$ of 4 ch.		1	9	= rent of 4 ch.
			$10\frac{1}{2}$	= rent of 2 ch.
	Rs 624	4	$1\frac{1}{2}$	= rent of 71 big. 6 kat. 14 ch.

Examples CXX.

Find by Practice the value, rent, &c. (as the case may be) of :—

1. 15 mds. 25 sr. 11 ch. at *Rs.*12. 10*a.* 8*p.* per maund.
2. 8 mds. 11 sr. 7 ch. at *Rs.*6. 10*a.* 8*p.* per maund.
3. 18 mds. 5 sr. 6 ch. at *Rs.*27. 14*a.* 8*p.* per maund.
4. 777 mds. 20 sr. 12 ch. at *Rs* 40. 10*a.* 8*p.* per maund.
5. 373 mds. 39 sr. 7 ch. at *Rs.*25. 2*a.* 4*p.* per maund.
6. 3 cwt. 2 qrs. 17 lbs. at £1. 5*s.* 8*d.* per quarter.
7. 57 cwt. 3 qrs. 14 lbs. at £5. 9*s.* 6*d.* per cwt.
8. 45 oz. 6 dwts. 7 grs. at 5*s.* 10*d.* per oz.
9. 37 cwt. 3 qrs. 2 lbs. at £3. 14*s.* $7\frac{1}{2}$*d.* per cwt.

6. How much will the carriage of 5 packages, each containing 4 cwt. 3 qrs. 21 lbs., come to, at *Rs.*6. 4*a.* per ton ?

7. What is the dividend on *Rs.*57348. 5*a.* 4*p.* at 7*a.* 6*p.* in the rupee ?

8. What is the dividend on £1710. 14*s.* 6*d.* at 13*s.* 4½*d.* per £ ?

9. Find the price of 111 things at £11. 11*s.* 11*d.* per every 11.

10. Find the weight of 2697 packages, each weighing 19 lbs. 10 oz. 18 dwts. 22 grs.

11. What distance will a train travel in 3 hours 39 min. 22 sec. at a speed of 49 miles 7 fur. 52 yds. per hour ?

12. Find the rent for 11 mo. 2 wks. 6 days from March 1, 1889 at *Rs.*38. 4*a.* 6*p.* per month.

13. Find the produce of 14 bighas 18 k. 2 ch. at 12 mds. 8 sr. per bigha.

14. Find the rent of 375·3675 bighas at *Rs.*29. 15*a.* per bigha

15. Find the value of 143·7526 gallons of spirit at *Rs* 11. 14*a.* per gallon.

16. A bankrupt owns *Rs.*7953·75 and pays 12*a.* 3*p.* in the rupee ; what is the value of his assets ?

17. A bankrupt's debts amount to *Rs.*35483. 5*a.* 4*p.* ; find what his creditors will lose, if he pay 10*a.* 3½*p.* in the rupee.

18. When exchange is at 2*s.* 1½*d.* per rupee, what is the value of *Rs.*4032. 8*a.* 8*p.* in English money ?

19. Find the rent for 7 mo. 3 wks 4 days from Feb. 1, at *Rs.*60 per month.

20. If 1 lb. Avoir. is 1 lb. 2 oz 11 dwts. 16 grs. Troy, what is the weight (Troy) of 1 cwt. 2 qrs. 25 lbs. 10 oz. 6 drs. ?

III. INVOICES.

399. Every tradesman sells his goods at two prices, **cash** and **credit.** When payment is made at the time of purchase, it is called *cash*, but otherwise *credit.* Both these sales are **entered** in a book called the **Day-Book**, in the order in which they occur in the course of the day.

The *entries* in the *Day-Book* are **posted** at short intervals in the Ledger, the index of which contains a list of customers' names in alphabetical order. For facility of reference, opposite each name is the page of the *Ledger* in which is collected all the dealings which have taken place with that particular customer.

400. When a buyer has completed his purchases he is presented with a **Bill** containing in detail a written list of the goods bought with a statement of the cost of them attached. An **Invoice** is a copy of the *Bill* which is sent home with the goods or forwarded to a customer living at a distance. Each separate entry in an *Invoice* or a *Bill* is called an **Item.**

401. An **Account** is a statement sent by the seller to the buyer at the end of the term of a credit shewing the totals and dates of each *Invoice* and the sum total of the whole. In such a case the *account* is said to be **rendered** (*i.e.*) sent to the buyer. If the details of the goods are also given, it is called a **Detailed Account** or **Bill of Parcels**.

(i) SPECIMEN OF AN INVOICE.

INVOICE, Calcutta, *4th April*, 1897.
From S. C. AUDDY, ESQ.
58, Wellington Street.

		Rs.	*a.*	*p.*
16 copies of Hall and Steven's Euclid...	at *Rs.*3. 1*a.* 6*p.*	49	8	0
14 copies of Todhunter's Euclid	at *Rs.*2. 6*a.* 6*p.*	33	11	0
25 copies of Lock's Arithmetic	at *Rs* 3. 1*a.* 6*p.*	77	5	6
10 copies of Dicken's Novels	at *Re.*1. 4*a.*	12	8	0
		173	0	6

(ii) SPECIMEN OF AN ACCOUNT.

K. C. SETT & CO. Calcutta, *May 4th*, 1897.
Bought of KHETTER MOHAN DEY & CO.,
45, Radha Bazar Street, Calcutta.

1897		*Rs.*	*a.*	*p.*
January 5	To goods as per invoice................	48	10	6
February 12	To goods as per invoice....................	59	7	3
March 18	To..................ditto....................	85	12	0
April 4	To..................ditto....................	72	6	6
		266	4	3

(iii) SPECIMEN OF A DETAILED ACCOUNT.

H. BALFOUR, ESQ. Calcutta, *July 24th*, 1897.
Bought of MOORE & CO.,
Dhurmtollah Street.

1897			*Rs.*	*a.*	*p.*	*Rs.*	*a.*	*p.*
April 21	40yds.	Irish linen.........at *Re.*1. 4*a.* 8*p.*	51	10	8			
	1doz.	Dusters........... „ 4*a.* 8*p.*	3	8	0			
	25yds.	Towelling .... „ 7*a.* 4*p.*	11	7	4	66	10	0
May 4	23yds.	Flannel............ „ *Re.*1. 3*a.* 4*p.*	27	12	8			
	15yds.	Brown Holland.. „ 7*a.* 8*p.*	7	3	0	34	15	8
June 26	14yds.	Calico .... „ 3*a.* 8*p.*	3	[illegible]	4			
	22yds.	Brussels Carpet „ *Rs.*2. 4*a.* 8*p.*	50	6	8			
	2 Rugs, *Rs.*10. 8*a.*, *Rs.*18. 8*a.*		29	0	0	82	10	0
						184	3	8

Examples CXXII.

Make out invoices for the following :—

1. 10 sr. of sugar at 3*a*. 9*p*. per sr.; 6 sr. of tea at 15*a*. 3*p*. per sr.; 8 sr. of coffee at 14*a*. 3*p*. per sr.; 12 sr. of wheat at 1*a*. 2*p*. per sr.; 10 sr. of rice at 1*a*. 3*p*. per sr.; and 9 sr. of cream at 11*a*. per sr.

2. $4\frac{1}{2}$ yds. of long cloth at 2*a*. 9*p*. per yd.; $7\frac{1}{4}$ yds. of cambric at 4*a*. 6*p*. per yd.; 6 pairs of socks at 1*a*. 3*p*. per pair; 3 pairs of hose at 4*a*. 6*p*. per pair; 1 doz. pairs of socks at 2*a*. $9\frac{1}{2}$*p*. per pair; and $5\frac{1}{2}$ yds. of flannel at 8*a*. 11*p*. per yd.

3. 5 lbs. of black tea at *Re*.1. 5*a*. 4*p*. per lb.; $2\frac{1}{4}$ lbs. of green tea at *Rs*.2. 4*a*. per lb.; $15\frac{1}{2}$ lbs. of lump sugar at 3*a*. 4*p*. per lb.; 17 lbs. of moist sugar at 2*a*. 8*p*. per lb.; $7\frac{1}{2}$ lbs. of raisins at 7*a*. 4*p*. per lb.; and 4 lbs. of currants at 4*a*. 4*p*. per lb.

4. $15\frac{1}{2}$ yds. of flannel at 2*s*. 3*d*. per yd.; 29 yds. of calico at $8\frac{1}{2}$*d*. per yd.; 25 yds. of Irish linen at 2*s*. 4*d*. per yd.; 17 yds. of towelling at 1*s*. 2*d*. per yd.; $12\frac{1}{2}$ yds. of brown holland at $11\frac{1}{2}$*d*. per yd.; and $3\frac{1}{2}$ doz. handkerchiefs at 9*s*. 10*d*. a doz.

5. $39\frac{1}{2}$ yds. of Brussels carpet at *Rs*.2. 10*a*. 8*p*. per yd.; $62\frac{3}{4}$ yds. of Kidderminster carpet at *Re*.1. 12*a*. per yd.; 27 yds. of cocoa-nut matting at 9*a*. 4*p*. per yd.; $34\frac{1}{2}$ yds. of drugget at *Re*.1. 2*a*. per yd.; and $43\frac{1}{2}$ yds. of India matting at 8*a*. 8*p*. per yd.

6. $17\frac{5}{8}$ yds. of calico at 6*a*. 6*p*. per yd.; $35\frac{9}{16}$ yds. of flannel at 14*a*. 2*p*. per yd.; $96\frac{8}{11}$ yds. of sheeting at *Re*.1. 0*a*. 4*p*. per yd. $104\frac{5}{6}$ yds. of holland at 8*a* 6*p*. per yd.; and $12\frac{3}{8}$ yds. of ribbon at 5*a*. 7*p*. per yd.

7. $17\frac{1}{2}$ mds. of coal at *Rs*.8. 14*a*. per md.; carriage of ditto at *Re*.1. 2*a*. per md.; 2 mds. of coke at *Rs*.14. 9*a*. 4*p*. per md.; 62 mds. of gram at *Rs*.2. 2*a*. per md.; 23 sr. of seed at 9*a*. per seer; and 136 mds. of grain at *Rs*.3. 11*a*. per md.

8. 24·25 yds. of cloth at *Rs* 5. 4*a*. per yd.; 12 yds. of flannel at 15*a*. 4*p*. per yd.; 43·75 yds. of calico at 6*a*. per yd.; 12·75 yds. of drugget at *Re*.1. 6*a*. per yd.; 37 yds. of Brussels carpet at *Re*.1. 13*a*. 4*p*. per yd.; and 25·5 yds. of Kidderminster do. at *Re*.1. 4*a*. 8*p*. per yd.

9. $3\frac{1}{2}$ pharas of lime at *Rs*.2. 3*a*. 4*p*. per phara; 15 sr. of ghee at *Rs*.20. 8*a*. per md.; $2\frac{3}{4}$ sr. of tea at *Re*.1. 0*a*. 8*p*. per seer; 20 sr. of flour at *Rs*.2. 3*a*. per md.; $3\frac{1}{2}$ yds. of flannel at *Re*.1. 2*a*. per yd.; and 29 yds. of calico at 9*a*. 7*p*. per yd.

10. Calcutta, June 16th, 1885.—W. Godfrey, Esq. bought of Ghose and Co., 500 envelopes at 14*a*. 8*p*. per 100; 3 boxes of elastic bands at 11*a*. per box; $\frac{1}{2}$ a gross of penholders at 6*a*. 4*p*. per doz.; $2\frac{1}{2}$ reams of Foolscap at 7*a*. per quire; 4 dozen quill pens at 3*a*. per doz.; 13 note-books at 9*a*. each: and 250 official envelopes at *Re*.1 per 100. Make out a copy of the bill and find its amount.

CHAPTER VIII.

Involution and Evolution.

402. A **power** of a number is the number which arises from successive multiplications by itself; the operation by which it is obtained is termed **involution**; and the **degree** or **order** of the **power** is denoted by the *number* of factors employed.

Thus, taking the number 2, we shall have the *powers* of it as follows :—

$2=2$, the first power of 2 ; $2\times2=4$, the second power of 2 ;
$2\times2\times2=8$, the third power of 2 ;
$2\times2\times2\times2=16$, the fourth power of 2 ;
$2\times2\times2\times2\times2=32$, the fifth power of 2 ;
$2\times2\times2\times2\times2\times2=64$, the sixth power of 2 ;
and so on, as far as we please ;

but instead of expressing these multiplications at *length*, which would soon become inconvenient, we denote the same operation by means of **indices** or small figures placed a little above the line to the right of the quantities whose powers are intended to be exhibited ; thus, what is put down above may be denoted by

$2^1=2$; $2^2=4$; $2^3=8$; $2^4=16$; $2^5=32$; $2^6=64$; &c. ;
where the **index** sometimes called the **exponent** is equal to the number of *factors* and is greater by *one* than the number of *operations*.

403. The *second* powers of the *nine* digits are expressed thus :—

$1^2=1$; $2^2=4$; $3^2=9$; $4^2=16$; $5^2=25$; $6^2=36$;
$7^2=49$; $8^2=64$; $9^2=81$;

and their *third* powers as follows :—

$1^3=1$; $2^3=8$; $3^3=27$; $4^3=64$; $5^3=125$; $6^3=216$;
$7^3=343$; $8^3=512$; $9^3=729$.

The *second* and *third* powers of numbers are styled their **squares** and **cubes** ; and the operations by which *all* powers are obtained are merely those of multiplication.

404. A power of a fraction is equal to the fraction formed by **raising** both its numerator and denominator to the power, and the power of a quantity formed by factors is found by raising each factor to the power.

Thus, $(\frac{2}{3})^2=\frac{2}{3}\times\frac{2}{3}=\frac{4}{9}$; $(\frac{2}{3})^3=\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}=\frac{8}{27}$; and so on.

Also $(2\times7)^2=2^2\times7^2=4\times49=196$; $(2{\cdot}5)^3=2{\cdot}5\times2{\cdot}5\times2{\cdot}5=15{\cdot}625$.

Note. A mixed quantity must be represented as a simple fraction or as a decimal, before the process can be applied.

405. A **root** of a number is such a number as being multiplied into itself one or more times produces it; and the operation by which this root is obtained is called **evolution.**

Thus, the second or **square root** of 16 is 4, because the *square* of 4 is 16, or $4^2=4\times4=16$. The third or **cube root** of 512 is 8, since the *cube* of 8 is 512, or $8^3=8\times8\times8=512$; and similarly of vulgar fractions and decimals.

406. This operation is expressed by the sign $\surd$ which is called the **radical sign,** with a small figure placed on its left to *particularize* the root intended: thus,

$$\sqrt[2]{16}=4\ ;\quad \sqrt[3]{512}=8\ ;\ \text{and}\ \sqrt[5]{32}=2\ ;$$

but the *square root* is denoted by the sign $\surd$ *only,* without the small figure, as being of most frequent occurrence.

These operations are also indicated by means of the primitive fractions $\frac{1}{2}$, $\frac{1}{3}$, &c., used as **indices,** so that the *indices* $\frac{1}{2}$, $\frac{1}{3}$, &c., denote operations exactly the reverse of those expressed by the *indices* 2, 3, &c., respectively; thus,

$$4^2=16\ ;\ 16^{\frac{1}{2}}=4\ \text{and}\ 8^3=512\ ;\ 512^{\frac{1}{3}}=8.$$

I. EXTRACTION OF THE SQUARE ROOT.

407. A **perfect square** is a number whose square root can be expressed exactly either by an integer or by a fraction.

Thus, 16 is a *perfect square,* for its square root is 4.

408. In squaring a number we see that its square has the same units' figure as the square of its units' figure, and if a number ends with 0, its square also ends with 0. Now as the squares of the simple numbers 1, 2, 3, 4, 5, 6, 7, 8, 9, and also 10 are respectively 1, 4, 9, 16, 25, 36, 49, 64, 81, and 100, it follows that the square of every number (integral or decimal) must end with either 1, 4, 5, 6, 9, or an even number of 0's. Hence *no number ending with* (i) *the digits 2, 3, 7, 8,* (ii) *an odd number of 0's, can be a perfect square.*

409. By the help of the Multiplication Tables we can immediately obtain the square root of a number not exceeding 400.

Thus, $9\times9=81$, and $15\times15=225$; $\therefore\ \surd(81)=9$, and $\surd(225)=15$.

410. When a number can be easily resolved into its prime factors, its square root can be determined by inspection. In a perfect square, every prime factor that occurs must occur an *even* number of times. Thus,

Ex. 1. Find the square root of 1764.

$$1764=2\times2\times3\times3\times7\times7=2^2\times3^2\times7^2=(2\times3\times7)^2.$$

$$\therefore\ \sqrt{1764}=2\times3\times7=42.\quad \textit{Ans.}$$

Ex. 2. Obtain the square root of 705600.

$$705600 = 10 \times 10 \times 2 \times 2 \times 6 \times 6 \times 7 \times 7 = (10^2 \times 2^2 \times 6^2 \times 7^2)$$
$$= (10 \times 2 \times 6 \times 7)^2.$$

$$\therefore \sqrt{705600} = 10 \times 2 \times 6 \times 7 = \underline{840}. \quad \textit{Ans.}$$

Ex. 3. What is the *least number* which, when multiplied into 51425, will make the product a perfect square?

$$51425 = 11 \times 11 \times 425 = 11 \times 11 \times 25 \times 17 = 11^2 \times 5^2 \times 17.$$

$\therefore$ the required number $= \underline{17}$. *Ans.*

Examples CXXIII.

1. Find the respective values of :—

(1) 31^2. (2) $39^2 \times 48^2$. (3) 925^2. (4) $(31\cdot5)^2$.

(5) $(806^2 + 31^2) \times 59$. (6) $(2\frac{1}{7})^4$. (7) $506^2 + 506^2 - 307^2$.

(8) $(15^2 - 1\cdot31^2) \div 15$. (9) $502^2 + 18^2 - 1376^2$.

(10) $(7\cdot03^2 \times \cdot19)^2 \div (3\cdot14 \times \cdot02)^2$. (11) $1\cdot03(4\cdot07 + 3\cdot16)^2$.

2. Find (*by inspection*) which of the following are square numbers :—

(1) 27 ; 96 ; 524 ; 9450 ; 7805 ; 9604 ; 12321 ; 494208.

(2) 4000 ; 75720 ; 388129 ; 582168 ; 12343225 ; 38812900.

3. Find the square roots of (*using factors*) :—

(1) 49 ; 196 ; 289 ; 361 ; 324 ; 256 ; 121 ; 400 ; 144.

(2) 625 ; 529 ; 900 ; 1296 ; 17424 ; 63504 ; 99225.

(3) 680625 ; 48024900 ; 12446784 ; 2480625 ; 57153600.

4. Find the *least numbers* which, when multiplied into the following numbers, will make the products perfect squares :—

175 ; 693 ; 1456 ; 3465 ; 3456 ; 4536 ; 28413 ; 750750.

5. What must be the least number of soldiers in a regiment, to admit of its being drawn up 2, 3, 4, 5, or 6 deep, and also of its being formed into a solid square?

---

411. *From the number of figures in any proposed quantity, to find the number of figures in its square root.*

Since, the square root of 1 is 1 ;
the square root of 100 is 10 ;
the square root of 10000 is 100 ;
the square root of 1000000 is 1000 : &c. ;

we see immediately that the square root of a number of fewer than 3 figures must consist of only 1 figure ; that of a number of more than 2 figures and fewer th 5, of 2 figures ; that of a number of

more than 4 figures and fewer than 7, of 3 figures, and so on ; whence it follows, that if a dot or full point be placed over every alternate figure, beginning at the *units'* place, the number of such points will be the same as the number of figures in the square root. This is called the **Rule of Pointing.**

Thus, the square root of $1\dot{9}\dot{8}$ consists of 2 figures in its integral part ; the square root of $3\dot{1}4\dot{2}5\dot{6}$ consists of 3 figures in its integral part ; and so on.

412. The extraction of the square root of a number depends on the principle illustrated by the following example :—

Since $28^2=(20+8)^2=20^2+2\times20\times8+8^2$;
$\therefore$ $28^2-20^2=2\times20\times8+8^2$;
$\therefore$ $28^2-20^2\div(2\times20)=8+$a proper fraction.

413. *To extract the square root of a whole number.*

RULE. Point the alternate figures of the number proposed, beginning at the place of units, so as to form as many periods of two figures each as possible, and remember that each period consists of the figure over which the dot is placed and the figure to its left. (The first period may consist of one figure only).

Find the greatest square number contained in the first period on the left hand, put down its root on the right as in division, and subtract it from that period. To the remainder bring down the next period for a dividend, double the root just found for a divisor (called the *trial divisor*), and find how often it is contained in this dividend exclusive of the figure on its right hand ; annex this quotient to the figures in both the quotient and divisor. Multiply the divisor thus *completed* by the last figure of the quotient, and if the product be not greater than the dividend, subtract it from the dividend, but if the product be greater, use a *lower* number for the root figure until it becomes less ; subtract the product as before. To this remainder bring down the period which comes next in order; take twice the number in the root, and see how often it is contained in this dividend with its last figure omitted ; and proceed precisely as before. Repeat the process till every period in succession is disposed of, and the root will thus be obtained.

Note. If at any step the quotient figure is 0, set down 0 in the root, annex it to the trial divisor, bring down the next period and proceed as before.

Ex. 1. Find the square root of 8649.

```
    8649 ( 93
    81
183 | 549
    | 549
```

Place dots over 9 and 6, so that the number is divided into two periods, 86 and 49.

The number whose square is immediately below 86 is 9 (for $9^2=81$ which is next below 86. Hence 9 is put in the root and 81 subtracted from 86.

To the remainder 5 is brought down the next period 49 ; thus the new dividend is 549. Now $2 \times 9 = 18$, is the *trial divisor*, which goes into 54 (549 with 9 omitted) 3 times. Hence **3** is put after 9 in the root and also annexed to 18. Multiply 183 by 3 and the product is 549, which subtracted from the dividend leaves nothing. Therefore 93 is the root obtained.

Ex. 2. Extract the square roots of 804609; 12809241 ; and 21224449.

```
(1) 804609 (897.          (2) 12809241 (            (3) 21224449 (4607.
    64                          9                        16
 169|1646                     65|380                     86|522
    |1521                       |325                       |516
1787|12509                   707|5592                  9207|64449
    |12509                      |4949                      |64449
                            7149|64341
                                |64341
```

414. When an integer (which is a perfect square) ends with an even number of ciphers, it would be sufficient to extract the square root of the significant figures and then to annex to the root *one* cipher for every *two* ciphers in the proposed number.

Ex. Extract the square root of 841000000.

```
841000000 (29000.
 4
49|441
  |441
```

Here are 6 ciphers in the given number ; therefore we add 3 ciphers to 29, the square root of 841.

415. Again, since the square root of ·01 is ·1 ;
the square root of ·0001 is ·01 ;
the square root of ·000001 is ·001 &c. ;

we infer that the quantity proposed must first be made to have an *even* number of decimal places, and then the pointing must proceed from the place of *units* towards the right-hand over every alternate figure as before ; and the number of such points will be the same as the number of decimal places in the square root.

416. If there be no whole number or integral part in the proposed number, we must, in pointing, begin with the *second* figure from that which would be the *units'* place, if there were a whole number, and place dots successively over every alternate figure to the right. If there be a whole number as well as a decimal fraction, it would be the safest method to begin at the *units'* place and point over every alternate figure to the *right and left* of it. The number of dots over the whole numbers and decimals will shew the number of figures in the integral and decimal parts of the root respectively.

Ex. Extract the square roots of 93·7024, ·02819041 and ·00822649.

(1) 93·7024(9·68.
81
186 | 1270
      | 1116
1928 | 15424
        | 15424

(2) ·02819041(·1679.
1
26 | 181
    | 156
327 | 2590
      | 2289
3349 | 30141
        | 30141

(3) ·00822649(·0907
81
1807 | 12649
        | 12649

Examples CXXIV.

1. Find the square roots of :—

(1) 676 ; 1444 ; 16129 ; 21025 ; 288369 ; 998001 ; 71289.

(2) 2025 ; 692224 ; 54756 ; 822649 ; 97574884 ; 10004569.

(3) 33016516 ; 45859984 ; 5774409 ; 62805625 ; 4020025.

(4) 6512490000 ; 5777216064 ; 95481000000 ; 3915380329.

(5) 8260628544 ; 9387030699156 1 ; 78702684186368 0889.

2. Extract the square roots of :—

(1) 22·09 ; 33·64 ; 1082·41 ; 22·8484 ; 187·4161 ; ·128881.

(2) ·0064 ; ·005329 ; ·00053361 ; ·00038025 ; 3659·0401.

(3) 1164·1744 ; 136966·6081 ; 240168·6049 ; 236·144689.

(4) 41605·800625 ; ·00501361708761 ; ·000000491120 64.

3. A certain number of boys spent *Rs.*90. 4*a.*, each spending as many four-anna pieces as there were boys ; what was the number of boys ?

4. A square pavement contains 20736 square stones, all of the same size ; what number composes one of its sides ?

5. A society collected among themselves for certain purposes a fund of *Rs.*459. 6*a.* ; each person paid twice as many pies as there were members in the whole society. Find the number of members.

6. A general, trying to mass his army of 15410 men into a square, found he had 34 men over ; required the number of men in the front.

---

417. If the number is not a perfect square, we can find an approximation to its square root to any required number of decimal places by affixing ciphers to the right-hand of the proposed number and bringing down periods of 2 ciphers each.

Ex. Find the square roots of 11 and ·4, each to 4 places of decimals.

```
(1)  11·00000000(3·3166
     9
  63|200
    |189
  661|1100
     | 661
  6626|43900
      |39756
  66326|414400
       |397956
         16444

(2)  ·4000000000(·6324...
     36
  123|400
     |369
  1262|3100
      |2524
  12644|57600
       |50576
         7024
```

418. When the proposed number is a recurring decimal, extend the recurring part by a repetition of its period and then proceed as in decimals.

Thus, to extract the square root of 4·3157 (recurring 157) to four places of decimals, first extend the recurring part 157 and put 4·3157157157... for 4·3157, and then proceed as usual.

419. When the number of figures to be found in the decimal part of the root is *large*, we may obtain in the usual way one more than half the required number of figures in the root, and then the remaining figures by dividing the last remainder by the last divisor, as in Art. 389.

Ex. Extract the square root of 10 to 8 places of decimals.

```
   10·    (3·16227766...
    9
 61|100
   | 61
 626|3900
    |3756
 6322|14400
     |12644
 63242|175600
      |126484
        49116

6,3,2,4,4)49116(7766
          44271
           4845
           4427
            418
            379
             39
             38
```

The first 5 figures are obtained in the usual way, and the last 4 by Contracted Division.

Examples CXXV.

1. Find the square roots of (each to 4 places of decimals) :—

(1) 2 ; 3 ; 5 ; 6 ; 7 ; 8 ; 12 ; 13 ; 18 ; 20 ; 32 ; 38.

(2) 44 ; 51 ; 72 ; 80 ; 95 ; 638 ; 796 ; 801 ; 1000.

(3) 5713 ; 363 ; 35120 ; 8837 ; 822646 ; 72471438 ; 7432.

2. Extract the square roots of (each to 4 places of decimals) :—

(1) ·1 ; ·2 ; ·3 ; ·4 ; ·5 ; ·6 ; ·7 ; ·8 ; ·9 ; 1·2 ; 1·6.

(2) ·05 ; 5·1 ; 4·9 ; ·16 ; ·016 ; ·01 ; ·51 ; ·051 ; 4·03.

(3) ·002 ; ·225 ; ·021 ; $\cdot\dot{3}$; $4\cdot\dot{5}$; 34·85 ; 321·73025 ; $18\cdot\dot{7}$.

(4) 3·14159 ; 175·250564 ; 12·56636 ; 29·41275 ; $7894\cdot61\dot{9}\dot{3}$.

(5) $2\cdot3\dot{6}\dot{1}$; $5\cdot0\dot{1}3\dot{2}$; $4\cdot02\dot{9}8\dot{1}$; $49\cdot005\dot{2}\dot{1}$; $\cdot\dot{0}\dot{7}$; $57\cdot\dot{5}\dot{7}$; $\cdot019\dot{8}$.

3. Find to 10 decimal places the square roots of :—

(1) ·001728 ; 9·79 ; ·0683 ; ·3467 ; 44·284 ; 1·57 ; 75·347.

(2) ·85 ; ·07 ; ·3 ; 97·9 ; ·0003532 ; 27·773 ; ·0365.

420. The square root of a fraction may be obtained by finding the square roots of its numerator and denominator separately.

(1) If the denominator of the given fraction, or of the fractional part of the mixed number, be a *perfect square*, we apply the Rule directly, whether the numerator be a perfect square or not.

Thus, $\sqrt{\frac{144}{169}} = \frac{\sqrt{144}}{\sqrt{169}} = \frac{12}{13}$; $\sqrt{8\frac{17}{64}} = \sqrt{\frac{529}{64}} = \frac{\sqrt{529}}{\sqrt{64}} = \frac{23}{8} = 2\frac{7}{8}$.

$$\sqrt{\frac{29}{64}} = \frac{\sqrt{29}}{\sqrt{64}} = \frac{5\cdot385164\ldots}{8} = \cdot673145\ldots$$

$$\sqrt{1278\tfrac{7}{25}} = \sqrt{\frac{31957}{25}} = \frac{\sqrt{31957}}{\sqrt{25}} = \frac{178\cdot7652\ldots}{5} = 35\cdot7530\ldots$$

(2) But if the denominator of the given fraction or of the fractional part of the mixed number be *not a perfect square*, we reduce the fraction or the mixed number either (i) to an equivalent fraction whose denominator is a perfect square and extract the square root of both numerator and denominator, or (ii) to a decimal, and proceed in the usual way.

Thus, $\sqrt{\tfrac{5}{7}} = \sqrt{\frac{5\times7}{7\times7}} = \sqrt{\frac{35}{49}} = \frac{\sqrt{35}}{\sqrt{49}} = \frac{5\cdot91607}{7} = \cdot84515\ldots$

or $= \sqrt{(\cdot\dot{7}1428\dot{5})} = \cdot84515\ldots$

$\sqrt{25\tfrac{8}{11}} = \sqrt{\frac{283}{11}} = \sqrt{\frac{283\times11}{11\times11}} = \sqrt{\frac{3113}{121}} = \frac{\sqrt{3113}}{11} = \frac{55\cdot794265\ldots}{11}$

or $= \sqrt{(25\cdot\dot{7}\dot{2})} = 5\cdot072205\ldots$ $\qquad = 5\cdot072205\ldots$

421. If a recurring decimal is a *perfect square*, it would be convenient to reduce it to a vulgar fraction and proceed as in Art. 420 (1), above.

Thus, $\sqrt{(1\cdot\dot{7})} = \sqrt{\frac{16}{9}} = \frac{\sqrt{16}}{\sqrt{9}} = \frac{4}{3} = 1\cdot\dot{3}$.

Examples CXXVI.

1. Find the square roots of :—

(1) $\frac{4}{25}$; $\frac{169}{256}$; $\frac{361}{1369}$; $\frac{841}{1024}$; $\frac{2304}{3481}$; $\frac{7056}{9216}$; $\frac{38416}{81796}$.

(2) $4\frac{25}{36}$; $10\frac{29}{25}$; $345\frac{24}{25}$; $3\frac{22}{49}$; $32\frac{1}{9}$; $41\frac{1288}{1572}$; $564\frac{220}{289}$.

(3) $3083\frac{214}{225}$; $72002\frac{7}{9}$; $30789\frac{268}{289}$; $15061\frac{119}{121}$; $154751\frac{77}{169}$.

2. Find the square roots of (each to 4 places of decimals where the root does not come out exactly) :—

(1) $\frac{5}{8}$; $9\frac{1}{2}$; $6\frac{2}{5}$; $\frac{1·28}{12·5}$; $\frac{4·41}{·64}$; $76\frac{14}{17}$; $\frac{5·04}{·012}$; $\frac{·00841}{1000}$.

(2) $\frac{3}{8}$; $21\frac{8}{11}$; $4\frac{5}{21}$; $\frac{12}{13}$; $\frac{16}{17}$; $21\frac{185}{117}$; $\frac{17}{128}$; $27\frac{27}{128}$.

(3) $\frac{18}{29}$; $\frac{129}{400}$; $\frac{7}{72}$; $\frac{14}{243}$; $287\frac{5}{8}$; $\frac{1}{308}$; $367\frac{2}{7}$; $756·28\frac{18993}{20736}$.

3. Find the square roots of :—

(1) $·\dot{1}$; $·02\dot{7}$; $18·\dot{7}$; $3·36\dot{1}$; $28·\dot{4}$; $·0002\dot{7}$; $4738·02\dot{7}$.

(2) $·0013\dot{4}$; $·071\dot{1}$; $53·\dot{7}$; $·00\dot{4}$; $5·\dot{4}$; $·01\dot{7}$; $·\dot{0}4938271\dot{6}$.

II. EXTRACTION OF THE CUBE ROOT.

422. A **perfect cube** is a number whose cube root can be expressed exactly either by an integer or by a fraction.

The cubes of the digits 1, 2, 3, 4, 5, 6, 7, 8, 9, are respectively 1, 8, 27, 64, 125, 216, 343, 512, 729, and it is important that these last numbers and the corresponding roots should be committed to memory.

423. *Given the number of figures in a number, to find the number of figures in its cube root.*

Since, the cube root of 1 is 1 ;
the cube root of 1000 is 10 ;
the cube root of 1000000 is 100 &c. ;

it follows that the cube root of any number between 1 and 1000 must lie between 1 and 10, *i. e.*, will have one figure in the integral part ; of any number between 1000 and 1000000 must lie between 10 and 100, *i.e.*, will have 2 figures in the integral part ; of any number between 1000000 and 1000000000 must lie between 100 and 1000, *i.e.*, will have 3 figures in the integral part, and so on. Hence, if a point be placed over the *units'* figure of the number, and thence over every third figure to the left of that place, the number of points will shew the number of figures in the integral part of the root. This rule may manifestly be extended to Decimals.

424. *To extract the cube root of a given number.*

RULE. Place a point over the *units'* figure of the given number and thence over every *third* figure to its left, and also to its right if the number be a decimal, adding ciphers, if necessary, to get periods of three ; and remember that each period consists of the figure over

which the dot is placed and the two figures to its left, if there are so many (for the first period may contain 1, 2, 3 figures).

Find the number whose cube is either equal to, or next less than the *first* period on the left hand and place it as the *first* figure of the root. Subtract its cube from the first period, and to the remainder bring down the *next* period.

Multiply the square of the root already obtained by 300 for a *trial* or *partial* divisor, and then find how often this divisor is obtained in the dividend ; this quotient gives the *next* figure of the root. Then, multiply this quotient figure by the product of the previous figure of the root by 30, and place the result below the partial divisor. Below these place the square of this last quotient figure and add the three together for a *Complete* divisor. Multiply this complete divisor by the last figure of the root and subtract. To the remainder bring down the next period to form the next dividend.

Multiply the square of the root already obtained by 300, and find how often this trial divisor is contained in the dividend. Put this quotient as the third figure of the root. Then, multiply the figures of the root already obtained by 30 and the product by the last quotient figure, and place the product below the partial divisor. Then place the square of the last quotient figure, and add the three together, for a complete divisor. Multiply this divisor by the last figure and subtract, and bring down the next period to form the next dividend. Proceed in this way till all the periods have been brought down.

Note. If at any step, the dividend is less than the divisor, put a cipher to the root, two ciphers to the trial divisor, and bring down the next period.

Ex. 1. Find the cube root of 21952.

	$2\dot{1}95\dot{2}$(28
	8
$2^2 \times 300 = 1200$	13952
$2 \times 30 \times 8 = 480$	
$8^2 = 64$	
1744	13952

Here, first divide into periods beginning with 2 ; the first period on the left contains only 2 figures. The trial divisor 1200 goes into the dividend 13952, 8 times.

Ex. 2. Extract the cube root of 12812·904.

	$1\dot{2}81\dot{2}$·$90\dot{4}$(23·4
	8
$2^2 \times 300 = 1200$	4812
$2 \times 30 \times 3 = 180$	
$3^2 = 9$	
1389	4167
$23^2 \times 300 = 158700$	645904
$23 \times 30 \times 4 = 2760$	
$4^2 = 16$	
161476	645904

First divide into periods of three beginning with 2, both left and right. The first period is 12 and the greatest cube root in 12 is 2.

The trial divisor 1200 goes into the dividend 4812, 3 times.

The trial divisor 158700 goes into 645904, 4 times.

We may shorten the process a little as below :—

```
                      12812·904(23·4
 63       12        |  8
  6        189 \    |  4812
 694      1389  }   |
             9 /    |  4167
          1587      |    645904
            2776    |
          161476    |    645904
```

In column II instead of writing ciphers, put 9 the units' figure of 189 two places further to the right; in the same manner, write 2776 in the second step.

To find the trial divisor in the second step, take the sum of $189 + 1389 + 3^2$, which is equal to the product of 69×23.

425. If the number is not a perfect cube, we may obtain its cube root to any required number of decimal places by annexing ciphers and bringing down periods of three ciphers each.

Ex. Extract the cube root of ·3 to 3 decimal places.

```
                      ·300000000(·669
 186      108       | 216
  12       1116 \   |  84000
 1989     11916  }  |
             36 /   |  71496
          13068     |  12504000
           17901    |
          1324701   |  11922309
                    |    581691
```

Since the root is to be extracted to 3 places of decimals, there must be 3 periods of 3 figures in the decimal part; therefore we must affix 8 ciphers to 3.

426. When one more than a half of the figures required in the root have been obtained by the ordinary method, the rest can be found by Contracted Division, as in Art. 389.

427. In extracting the cube roots of vulgar fractions, if the denominator of the fraction be *a perfect cube*, find the cube roots of both the numerator and the denominator separately; but if the denominator of the fraction be *not a perfect cube*, either reduce the fraction to an equivalent fraction whose denominator is a perfect cube and then extract the cube root of numerator and denominator, or reduce the fraction to a decimal and proceed in the ordinary way.

Thus, $\sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}$; $\sqrt[3]{\frac{29}{64}} = \frac{\sqrt[3]{29}}{\sqrt[3]{64}} = \frac{3·072317\ldots}{4} = ·768079\ldots\ldots$

$\sqrt[3]{8\frac{5}{7}} = \sqrt[3]{\frac{61}{7}} = \sqrt[3]{\frac{2989}{343}} = \frac{\sqrt[3]{2989}}{\sqrt[3]{343}} = \frac{14·4048\ldots}{7} = 2·0578\ldots$

or $= \sqrt[3]{(8·\dot{7}1428\dot{5})} = 2·0578\ldots$

Examples CXXVII.

1. Find the cube roots of :—

(1) 1331 ; 15625 ; 46656 ; 2197 ; 185193 ; 117649.

(2) 704969 ; 912673 ; 33076161 ; 15069223 ; 105823817.

(3) 873722816 ; 198767717056 ; 702121283072.

(4) 17·576 ; 132·651 ; 493·039 ; 64481·201 ; 18·609625.

(5) ·007645373 ; ·876467493 ; ·001030301 ; ·000026730899.

(6) $\frac{64}{343}$; $\frac{729}{140608}$; $49\frac{8}{27}$; $7558\frac{197}{512}$; $465\frac{31}{64}$; $57\frac{132}{343}$.

(7) 1034 ; 5·912 ; 5 ; ·078759 ; 3·467 (each to 4 decimal places).

(8) $\frac{8}{7}$; $\frac{8}{13}$; $\frac{5}{7}$; $7\frac{7}{9}$; $18\frac{7}{12}$; $18\frac{500}{729}$ (each to 4 decimal places).

(9) ·002 ; ·003 ; ·013 ; ·024 ; 2·187 (each to 8 places of decimals).

2. Find the cube roots of :—

(1) $\frac{5·12}{33·75}$; $\frac{5030·912}{65536}$; $\frac{5·12}{·03375}$; $\frac{1257·728}{16384}$.

(2) $3845·\dot{2}9\dot{6}$; $·\dot{0}3\dot{7}$; $1587·\dot{9}6\dot{2}$; ·8 ; ·27 ; ·325142 ; 81·812703 (the last four to 4 decimal places).

3. A cubical block of stone contains 50653 solid feet ; find the length of its side.

4. Extract the cube root of 233·744896, and derive the cube root of this number multiplied by ·008.

III. EXTRACTION OF SOME OTHER ROOTS.

428. The directions already employed may by a little management be rendered available for the discovery of some other roots as will be evinced in the following notes.

(1) The **Fourth** root of a number is found by extracting its square root, and then the square root of its square root.

(2) The **Sixth** root of a number is found by extracting its cube root, and then the square root of its cube root ; or by extracting its square root, and then the cube root of its square root.

(3) The **Eighth** root of a number is found by extracting its square root, then the square root of its square root, and lastly the square root of that square root.

(4) The **Ninth** root of a number is found by extracting its cube root, and then the cube root of its cube root.

Ex. Find the *fourth* root of 1679616 and the *sixth* root of 308·915776.

(1) Here the *square* root is found to be 1296 ; and the *square* root of 1296 is 36. Therefore the *fourth* root of 1679616 is 36.

(2) Here the *square* root is found to be 17·576 ; and the *cube* root of 17·576 is 2·6. Therefore the *sixth* root required is 2·6.

Examples CXXVIII.

1. Find the fourth roots of 104976 ; 1500625 ; 4323738·0096.

2. Find the sixth roots of 2985984 ; 24·137569 ; 17596·287801.

3. Find the eighth roots of 214358881 ; 21035·8 ; ·003532 ; $57\frac{57}{86}$ (the last three to 5 decimal places .

4. Find the ninth roots of 262144 ; ·134217728 ; 5159780352.

5. Find the fourth root of $21\frac{204}{2401}$; and the sixth roots of 85·766121 and 260184053769592O1.

Miscellaneous Examples V.

1. What are the prime factors in 45090045, and what is the smallest whole number by which it must be multiplied in order tc make it a perfect square ?

2. What is the difference between the values of

$$\frac{2\frac{4}{5} \text{ of } 5\frac{1}{8}}{27\frac{1}{2}} \text{ of } Rs.11.\ 4a. \text{ and } \frac{5\frac{1}{12}}{2\frac{1}{2}+3\frac{3}{4}} \text{ of } Rs.6.\ 4a. \ ?$$

3. A chain, 11 yds. long, is divided into 50 equal parts, called inks ; find how many square links there are in an acre.

4. Bring $\left\{\left(\frac{5\frac{1}{4}-\frac{1}{3} \text{ of } 2\frac{5}{8}}{\frac{2}{5}\times 4\frac{1}{6}+\frac{1}{12}}+\frac{2\frac{2}{7}}{4\frac{2}{7}}\right)+21\frac{28}{29}\times 3\frac{19}{206}\right\}$cwt. to the fraction of $4\frac{1}{7}$ ton.

5. A merchant bought 264 gallons of spirit at *Rs*.12. 8*a*. $4\frac{1}{2}$*p*. per gal. ; 378 gallons at *Rs*.9. 10*a*. 7*p*. per gal. ; and 420 gallons at *Rs*.12. 15*a*. $6\frac{1}{2}$*p*. per gal. If he sell the whole quantity at *Rs*.12 4*a*. per gal., what profit will he make by the transaction ?

6. Two numbers have for their G. C. M. 179 and for their L. C. M. 56385. What must the greater number be, if the less

$$= 105 \text{ times } \frac{2\frac{2}{7}}{4\frac{5}{6}} \text{ of } \frac{363{\cdot}37}{8{\cdot}4} \ ?$$

7. Which is the greater

$$\frac{3}{5} \text{ of } \frac{13}{16}-\frac{1\frac{2}{3}}{6\frac{2}{3}} \text{ of } \frac{19}{20}+\frac{3}{7} \text{ of } \frac{6\frac{5}{12}}{3\frac{2}{3}} \text{ or } \frac{5}{3} \text{ of } \frac{13}{16}+\frac{6\frac{2}{3}}{1\frac{2}{3}} \text{ of } \frac{19}{20}-\frac{7}{3} \text{ of } \frac{6\frac{5}{12}}{3\frac{2}{3}} \ ?$$

and express the difference as a decimal.

8. Express *Rs*.6. 5*a*. $10\frac{2}{3}$*p*. as the fraction of *Rs*.9. 8*a*. 10*p*.

9. Find by Practice the value of 29764 articles at *Re*.1. 11*a*. $9\frac{3}{4}$*p*. each.

10. Simplify $\frac{2}{5}(6\frac{2}{3}+2\frac{1}{2})£+\frac{2\frac{1}{4}-\frac{2}{3} \text{ of } 1\frac{5}{8}}{\frac{1}{5} \text{ of } 3\frac{1}{8}+1\frac{3}{86}}\times{\cdot}95 \text{ of } 5s.+\frac{16{\cdot}8}{{\cdot}024}\,d.$

11. Simplify $(25{\cdot}4)^2+(24{\cdot}6)^2-12{\cdot}7\times 98{\cdot}4+({\cdot}6)^2$.

12. Prove that

$$\cdot\dot{0}7692\dot{3} = \frac{\cdot 07}{1-\cdot 09} = \frac{\cdot 076}{1-\cdot 012} = \frac{\cdot 0769}{1-\cdot 0003} = \frac{\cdot 07692}{1-\cdot 00004}.$$

13. A man bequeathes his property amounting to *Rs.*49166 in such a way that $\frac{1}{3}$ of his wife's share, $\frac{2}{3}$ of his eldest son's, $\frac{3}{8}$ of his younger son's and $\frac{1}{3}$ of his daughter's shares, are all equal. Find the shares of each.

14. In a subscription list one-half of the subscriptions are a guinea each, one-third a half-guinea each, and the 5 shilling subscriptions which complete the list amount to £12 ; find the whole sum subscribed.

15. *A* can mow 2 ro 32 po. of grass in 1 day ; *B*, 2 ac. 2 ro. in $2\frac{1}{2}$ days ; *C*, 2 ac. 2 ro. 32 po. in 3 days ; what is the size of the smallest field which would employ each of them, working alone, an exact number of days ? In what time would each mow the field ?

16. Express as decimals :—

$$(1)\ \frac{35}{57} + \left(\frac{\cdot 00\frac{2}{3}}{7} + \frac{\cdot 003\frac{1}{3}}{21}\right). \qquad (2)\ \frac{\cdot 0759}{2\frac{5}{7} - \frac{22}{24}} \div \frac{3\frac{4}{15} - \frac{19}{20}}{14\cdot 75}.$$

17. If 1 rupee = 1*s*. $10\frac{1}{2}$*d*., 1 sovereign = 4·84 dollars, and 1 dollar = $5\frac{1}{5}$ francs, find the value in francs of 10 lacs of rupees.

18. A certain number of men agree to subscribe as many pies each as there are subscribers ; the whole subscriptions being *Rs.*5797. 0*a*. 1*p*. How many subscribers are there ?

19 A person after paying an income-tax of 1 anna in the rupee, devotes $\frac{1}{16}$ of the remainder of his income to purposes of charity, and finds that he has left *Rs.*5175 ; what is his income ?

20. Find how much rice a family requires monthly, when a reduction in the price from 7 to 10 measures for the rupee reduces the total monthly expenses from *Rs.*$31\frac{1}{2}$ to *Rs.*30.

21. Simplify :—

$$(1)\ \left(\frac{2\cdot 375}{6\cdot 3} \text{ of } \frac{8\cdot\dot{8}}{\cdot 0625}\right) + \left(\frac{17\cdot\dot{7}}{11\cdot 35} \text{ of } \frac{4}{7}\right). \quad (2)\ \frac{10\frac{5}{8}}{100} - \cdot 10\frac{5}{8}. \quad (3)\ \frac{7}{\frac{8}{9}} - \frac{\frac{7}{8}}{9}.$$

22. Two pieces of cloth of the same length cost *Rs.*111. 12*a*. and *Rs.*144 respectively ; the price of the first was *Rs.*3 1*a*. 8*p*. per yard, what was the price of the other ?

23. Find the value of ·03125 of *Rs.*2 + $\cdot\dot{7}2\dot{9}$ of *Rs.*3. 1*a*. 4*p*. + $\cdot\dot{7}2\dot{9}$ of *Rs.*4. 10*a*.

24. Find the square roots of $39\frac{1}{16}$ and ·00255025.

25. Find the least and the greatest number of six digits such that if they be divided by 240, 275, 320 and 400, the remainders will be 210, 245, 290 and 370 respectively.

26. If one cubic foot of water weighs 62·37 lbs. Avoir.; reduce the weight of $\frac{3\cdot4\dot{4}\dot{5} \times 3\cdot4\dot{4}\dot{5} - 1\cdot5\dot{5}\dot{4} \times 1\cdot5\dot{5}\dot{4}}{\cdot4\dot{1} \times \cdot\dot{4}0\dot{5}}$ cub.ft. to the decimal of a ton

27. A contractor bought 2250 pharas of unslaked lime at Calcutta at the rate of *Rs.*45 for 100 pharas. On slaking it every phara gave 3 cub. ft. of lime, but of this $\frac{1}{18}$th was unserviceable; the carriage of the remainder to the place where it was required (distant 18 miles) cost 4*a.* per 100 cub. ft. per mile. At what rate per cub. ft. must he sell it there, in order to gain *Rs.*90 on his outlay?

28. How many sheep must a person buy at £7 each, that after paying one shilling a score for folding them at night he may gain £79. 16*s.* by selling them at £8 each?

29. Divide 1028·5 by ·00000 , and $\frac{2\frac{3}{8}}{3\frac{1}{4}}$ by $\cdot000\dot{6}$; and multiply the difference of the quotients by ·00025.

30. What decimal multiplied by the sum of 3·5, 6·5, $\frac{7}{18}$ and $\cdot8\dot{3}$ will produce 29?

31. If ·625 of an article cost £13. 16*s.* $1\frac{3}{4}$*d.*, what will ·125 of it cost at the same rate?

32. Which is the greatest, and which the least of the expressions

(1) $\frac{2}{3}+\frac{3}{4}$, (2) 1·41421, (3) $\frac{1}{2}+\frac{1}{5}+\frac{5}{7}$?

33. The year consists of 365·24224 days. In what time would the error arising from taking it as $365\frac{1}{4}$ days amount to 1 day?

34. A person received on the death of his aunt $\frac{1}{10}$ of her property, and spent $\cdot\dot{5}\dot{4}$ of it in paying off his debts, what fraction of his aunt's property did he then possess?

35. What least numbers must be severally subtracted from 9321, 7381 and 5628 that the three remainders shall have 723 for their common measure?

36. Simplify

$$\frac{5\cdot25}{4\cdot2 \text{ of } 5\cdot1\dot{6}} \div \frac{7\cdot25}{3\cdot1} \text{ of } \frac{3\frac{1}{4}}{2\cdot9}\left(3\tfrac{1}{3}+6\cdot\dot{6}\right) \times \frac{Rs.3.\ 6a.}{Rs.10.\ 2a.}.$$

37. What weight must be added to or subtracted from 8 cwt. 3 qrs. 12 lbs. so that 5 cwt. 2 qrs. 18 lbs. shall be the same fraction of the sum or difference that 3 cwt. 2 qrs. $17\frac{1}{2}$ lbs. is of 5 cwt. 1 qr. $26\frac{1}{4}$ lbs.?

38. Find the two greatest numbers of 4 digits whose difference is 50 and G. C. M. 25; and the two least numbers of 5 digits whose difference is 15 and G. C. M. 5.

39. Four concentric circles have the same centre *O*. A straight line through *O* cuts the circles in *A*, *B*, *C* and *D* on the same side of *O*. Four points start simultaneously from *A*, *B*, *C*, *D* and moving

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in the same direction describe the circles in 9 hrs., 13 hrs., 21 hrs., and 37 hrs. respectively. Find after how many hours they will be again in a straight line through O and on the same side of it.

40. A gave ·5 of an orange to B ; $\cdot\dot{3}$ of what remained to C ; how much of the orange had A left for himself?

41. What is the cost of 12 mds. 30 sr. 8 ch. 2 tolas at *Rs.*3. 5*a.* 9*p.* per maund?

42. Reduce 5 bi. 9 kat. 8 ch. to the fraction of an acre ; and $7\frac{3}{8}$ of 3 acres 2 ro. 5 po. to the fraction of $11\frac{1}{3}$ bighas.

43. 7 rix-dollars are worth 2 ducats, and 9 ducats worth 4 moidores, and 20 moidores worth £27 ; how many rix-dollars will there be in £72?

44. Find the value of $3\frac{2}{5}+4\frac{1}{8}+1\frac{1}{40}+3\frac{13}{625}$ both by vulgar fractions and by decimals ; and shew that the two results coincide.

45. Find the values of $3\cdot5+2\cdot8\dot{3}+\cdot\dot{6}+1\cdot175$; $11\cdot7\dot{3}-10\cdot9\dot{1}\dot{6}$; $3\cdot375\times1\cdot\dot{6}\times4\cdot8$; $3\cdot375\div4\cdot5$ and find the product of the results.

46. A man travelled $\frac{3}{4}$ of a journey by sea, $\frac{1}{3}$ of the remainder by rail ; he then drove $\frac{4}{5}$ of the rest, and finished it by walking 10 miles. How long was the journey?

47. Multiply *Rs.*3. 0*a.* 6*p.* by 85·3125 and divide *Rs.*38340. 3*a.* 6*p.* by 441·75.

48. If a pound of silver cost £3. 6*s.*, what is the price of a cup which weighs 10 lbs. 6 oz. 10 dwts., subject to a duty of 1*s.* 6*d.* per oz., and also to a charge of 1*s.* 9*d.* per oz. for workmanship?

49. Reduce $\frac{1}{4}$ of $\frac{1}{2\frac{4}{5}}$ of $\frac{4}{5}$ of 5 cwt. 2 qrs. 14 lbs. 7 oz. to the decimal of $\cdot\dot{4}2857\dot{1}$ of 15 tons 8 cwt. 1 qr. 14 lbs.

50. Reduce $7\frac{8}{9}$ of $(3\frac{3}{4}\div3\frac{17}{19})$ of 5 cwt. 2 qrs. $3\frac{1}{8}$ lbs. to the fraction of $(5\frac{8}{9}-3\frac{1}{4})$ of 3 tons 16 cwt. 3 qrs. $22\frac{2}{3}$ lbs.

51. What fraction of 2 lbs. 10 oz. Avoir. must be added to 1 lb. 8 oz. Troy to give 3 lbs. 7 oz. 10 dwts.?

52. Gold is sold at £3. 17*s.* $10\frac{1}{2}$*d.* per oz and bought at £3. 17*s.* 9*d.* per oz., what is the largest unit of value in which both prices can be expressed as integers? and what is the smallest integral number of ounces, the value of which can be exactly expressed in £ at both prices?

53. Extract the square roots of $\frac{1000\cdot10001}{1000}$ and $\frac{1\cdot\dot{7}\times29\frac{4}{25}}{\cdot000729}$.

54. What length of wire will go round the edges of a cube, the surface of which contains 187 sq. yds. 54 sq. in.? What is the least number of such cubes which will contain an exact number of cubes whose edges are 1 ft. 3 in.?

CHAPTER IX.

Measurements and Duodecimals.

I. SQUARE MEASURE.

429. In Arithmetic, we deal with the areas of **rectangles** only.

(*a*) **A rectangle** is a four-sided figure whose opposite sides are equal and parallel, and whose angles are right angles.

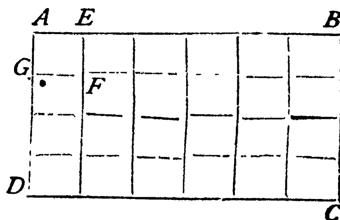
(*b*) **A square** is a rectangle which has all its sides equal.

(*c*) The *length* and *breadth* of a rectangle are called its **dimensions**.

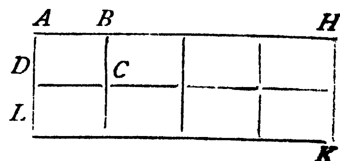
(*d*) The **area** of a figure is the quantity of surface contained in it; and is numerically measured by the multiple it is of a certain fixed *area*, which is assumed for its measuring unit.

430. *To find the area of a rectangle.*

Suppose *ABCD* to represent a rectangular surface as that of a table, of which the length *AB* is 6 feet, and the breadth *AD* 4 feet. Divide *AB* into 6 equal parts, each equal to 1 foot and *AD* into 4 equal parts, each equal to 1 foot, as in the figure, and through the points of division draw straight lines parallel to *AB* and *AD*. Then by this means we shall have divided the whole surface into 6×4 small figures, such as *AEFG*, all equal to one another; and since each of these figures measures a foot every way – 1 foot in length and 1 foot in breadth – it is a *square foot.* Therefore the area of 6×4 small figures, is 6×4 or 24 square feet.



If the side *AB*, instead of containing an exact number of feet, contain some feet and a fraction, as $3\frac{1}{4}$ feet, and *AD* in like manner contain $4\frac{1}{2}$ feet: produce *AB* to *H*, making *AH* equal to four times *AB*; also produce *AD* to *L* making *AL* equal to twice *AD*. Complete the rectangle *AHKL*. Then *AH* is four times *AB*, or $4 \times 3\frac{1}{4}$ feet $= 13$ feet: and *AL* is twice *AD* or $2 \times 4\frac{1}{2}$ feet $= 9$ feet. Therefore the rectangle *AHKL* contains 9×13 or 117 square feet. But the rectangle *AHKL* contains 8 such rectangles as *ABCD*, and therefore *ABCD* contains $\frac{1}{8}$ of 117 square feet, or $\frac{117}{8} = 14\frac{5}{8}$ sq. ft., a result obtained by multiplying $3\frac{1}{4}$ by $4\frac{1}{2}$: for $\frac{13}{4} \times \frac{9}{2} = \frac{117}{8}$. Hence, the number of **square feet** in the **surface** is found by multiplying the ***number*** of feet in its length by the ***number*** of feet in its breadth.



431. As the same method of proof would apply in every case (whether integral or fractional) we have the following Rule :—

RULE. *Express the length and breadth in units of the same denomination, and then multiply the number of the units denoting the length by the number of the units denoting the breadth. The product will give the area in square units of that denomination.*

The above *Rule* is frequently expressed briefly thus :—

Area = Length × Breadth.

We must remember here that it is the *number* of the units in length and the *number* of the units in breadth that we should multiply together ; for length being a concrete quantity cannot be multiplied by another concrete quantity such as breadth. Also, if the numbers giving the length and breadth be given in inches, or in feet, or in yards, or in miles, their product will give the area in square inches, or in square feet, or in square yards, or in square miles.

432. Since area = length × breadth ;

∴ Length = area ÷ breadth, and Breadth = area ÷ length.

Hence, if we divide the number of square units in the area by the number of units *either* the length or breadth, we shall find the number of units in the *other*.

433. The length and breadth of a square being equal, its side will be found by extracting the square root of its area.

To find the breadth of a rectangle, whose length is twice or thrice its breadth, divide the given area by 2 or 3, and then extract the square root of the result.

Note. Students should notice here the difference between **3 square feet** and **3 feet square**. By *3 square feet* is meant an area which is equal to 3 squares, each of which is a foot square. By *3 feet square* is meant the area of a square whose side is 3 feet, and therefore whose area is 3 × 3 or 9 square feet.

Ex. 1. Find the area of a floor 8 ft. 9 in. long by 3 ft. 8 in. broad.

Length = 8 ft. 9 in. = $8\frac{3}{4}$ or $\frac{35}{4}$ ft. ; breadth = 3 ft. 8 in. = $3\frac{2}{3}$ or $\frac{11}{3}$ ft.
∴ area of the floor = $(\frac{35}{4} \times \frac{11}{3})$ sq. ft. = $\frac{385}{12}$ sq. ft. = $32\frac{1}{12}$ sq. ft.
= <u>3 sq. yds. 5 sq. ft. 12 sq. in.</u>

Ex. 2. Find the width of a room 50 ft. 8 in. long having an area of 128 sq. yds. 2 sq. ft. 4 sq. in.

Area = $(128 \times 9 + 2\frac{1}{36})$ sq. ft. = $1154\frac{1}{36}$ sq. ft. = $\frac{41545}{36}$ sq. ft.
Length = 50 ft. 8 in. = $50\frac{2}{3}$ ft. = $\frac{152}{3}$ ft.
∴ width = $(\frac{41545}{36} \div \frac{152}{3})$ ft. = $(\frac{41545}{36} \times \frac{3}{152})$ ft. = $\frac{41545}{1824}$ ft.
= $22\frac{1417}{1824}$ ft. = <u>22 ft. $9\frac{49}{152}$ in.</u>

Note. Since linear feet multiplied by linear feet gives sq. ft., it follows that sq. ft. divided by linear feet gives linear feet, and so on.

Ex. 3. Find the area of a square court whose side is 17 ft. 11 in.

Here, length = breadth = 17 ft. 11 in. = $17\frac{11}{12}$ ft. = $\frac{215}{12}$ ft.

$\therefore$ area = $(\frac{215}{12} \times \frac{215}{12})$ sq. ft. = $\frac{46225}{144}$ sq. ft. = $321\frac{1}{144}$ sq. ft.
= 35 sq. yds. 6 sq. ft. 1 sq. in.

Ex. 4. How many bricks $11\frac{3}{4}$ in. long by $7\frac{1}{2}$ in. wide will be required to pave the floor of a room 18 ft. 9 in. long by 15 ft. 8 in. wide ?

Area of a brick = $(11\frac{3}{4} \times 7\frac{1}{2})$ sq. in. = $\frac{47}{4} \times \frac{15}{2}$ sq. in.
Area of the floor = (225×188) sq. in.

$\therefore$ no. of bricks reqd. = $(225 \times 188) \div (\frac{47}{4} \times \frac{15}{2})$
= $225 \times 188 \times \frac{4}{47} \times \frac{2}{15}$ = 480. *Ans.*

Ex. 5. How many yards of fencing are required to enclose a square garden containing 1 ac. 1 po. 29 yds. $6\frac{3}{4}$ ft. ?

Area = 1 ac. 1 po. 29 yds. $6\frac{3}{4}$ ft. = 44100 sq. ft.
$\therefore$ a side = $\sqrt{(44100)}$ ft. = 210 ft.
$\therefore$ length of fencing reqd. = 4×210 ft. = 840 ft. = 280 yds. *Ans.*

Ex. 6. The area of the floor of a room is 450 sq. ft. ; its length is twice its breadth ; find its length.

Breadth = $\sqrt{(\frac{450}{2})}$ ft. = $\sqrt{(225)}$ ft. = 15 ft. ;
$\therefore$ length = 2×15 ft. = 30 ft. *Ans.*

Ex. 7. A garden roller is 3 ft. 3 in. wide, and its circumference is 6 ft. 9 in. ; how many sq. ft. of ground does it pass over in 8 complete revolutions ?

In each revolution, the roller passes over a space of ground whose area is equal to the product of the width and the circumference.

$\therefore$ area rolled over in one revolution = $(3\frac{1}{4} \times 6\frac{3}{4})$ sq. ft. = $\frac{351}{16}$ sq. ft.
$\therefore$ area passed over in 8 revolutions = $\frac{351}{16} \times 8$ sq. ft.
= $\frac{351}{2}$ sq. ft. = $175\frac{1}{2}$ sq. ft.

434. Carpeting and **matting** always refer to the *floor* only.

The amount of carpet or mat required to cover a floor will be equal to the **area of the floor**. Carpet or mats are sold in strips of different widths and when we know the **width** of a strip, we can find its **length** by dividing the *area of the floor* by the *width* of the carpet or mat.

Ex. 1. Find the cost of carpeting a room 22 ft. 8 in. long and 17 ft. 4 in. broad at *Re.*1. 2*a.* per square yard.

Area of carpet = $(22\frac{2}{3} \times 17\frac{1}{3})$ sq. ft. = $\frac{68}{3} \times \frac{52}{3} \times \frac{1}{9}$ sq. yds.
$\therefore$ required cost = $\frac{68}{3} \times \frac{52}{3} \times \frac{1}{9} \times$ *Rs.*$1\frac{1}{8}$ = *Rs.*$\frac{442}{9}$ = *Rs.*49. 1*a.* $9\frac{1}{3}$*p.* *Ans.*

Ex. 2. What length of mats, $\frac{3}{4}$ of a yard wide, will be required

to cover a floor 15 ft. 8 in. long by 11 ft. 3 in. broad? also find the cost of the mats at 2*a.* 3*p.* per yard.

Area of matting $=(15\frac{2}{3}\times 11\frac{1}{4})$ sq. ft. $=\frac{47}{3}\times\frac{45}{4}\times\frac{1}{9}$ sq. yds. $=\frac{235}{12}$ sq. yds.

$\therefore$ length of matting $=(\frac{235}{12}\div\frac{3}{4})$ yds. $=\frac{235}{9}$ yds. $=26\frac{1}{9}$ yds.

$=$ 26 yds. 0 ft. 4 in. *Ans.*

Also, cost of mats $=2a.\ 3p.\times\frac{235}{9}=\frac{9}{4}\times\frac{235}{9}a.=\frac{235}{4}a.=$ *Rs.*3. 10*a.* 9*p.*

Ex. 3. The cost of painting a surface 8 ft. 9 in. wide at *Rs.*3. 13*a.* 4*p.* per sq. yd. is *Rs.*50. 5*a.*; find the length of the surface.

*Rs.*3. 13*a.* 4*p.* $=$ *Rs.*$3\frac{5}{6}=$ *Rs.*$\frac{23}{6}$; *Rs.*50. 5*a.* $=$ *Rs.*$50\frac{5}{16}=$ *Rs.*$\frac{805}{16}$.

$\therefore$ area of surface $=(\frac{805}{16}\div\frac{23}{6})$ sq. yds. $=\frac{105}{8}$ sq. yds. $=\frac{945}{8}$ sq. ft.

$\therefore$ length of surface $=\frac{945}{8}$ sq. ft. $\div 8\frac{3}{4}$ ft. $=(\frac{945}{8}\times\frac{4}{35})$ ft.

$=\frac{27}{2}$ ft. $=$ 13 ft. 6 in. *Ans.*

Ex. 4. The expense of carpeting a room thrice as long as it is broad at *Rs.*3. 12*a.* per sq. yd. is *Rs* 551. 4*a.*; find the length and breadth of the room.

*Rs.*3. 12*a.* $=$ *Rs.*$3\frac{3}{4}=$ *Rs.*$\frac{15}{4}$; *Rs.*551. 4*a.* $=$ *Rs.*$551\frac{1}{4}=$ *Rs.*$\frac{2205}{4}$.

$\therefore$ area of the room $=(\frac{2205}{4}\div\frac{15}{4})$ sq. yds. $=147$ sq. yds.

$\therefore$ breadth $=\sqrt{(\frac{147}{3})}$ yds. $=\sqrt{(49)}$ yds. $=7$ yds. $=21$ ft.
and length $=21$ ft. $\times 3$ $=63$ ft. } *Ans.*

Examples CXXIX.

1. Find the area of each of the following rectangles:—

(1) 25 ft. by 17 ft. (2) 36 ft. by 13 ft.
(3) 17 ft. 6 in. by 13 ft. 4 in. (4) 19 ft. 4 in. by 16 ft. 8 in.
(5) 12 yds. 1 ft. 5 in. by 2 ft. 9 in. (6) 5 yds. 11 in. by 23 ft. 10 in.
(7) 15 ft. $4\frac{1}{2}$ in. by 14 ft. 4 in. (8) 19 ft. 1 in. by 17 ft. $1\frac{1}{2}$ in.

2. Find the quantity of matting that will be required for each of the following rooms:—

(1) 52 yds. 10 in. by 13 yds. $2\frac{1}{2}$ in. (2) 2 yds. 1 ft. 6 in. by 4 yds. 9 in.
(3) 22 ft. 8 in. by 17 ft. 4 in. (4) 6 yds. $2\frac{1}{4}$ ft. by 2 yds. 1 ft. 5 in.

3. Find the length or breadth of each of the following rooms:—

(1) Area $=$ 150 sq. ft. 27 sq. in. and length $=$ 6 yds. $2\frac{1}{4}$ ft.
(2) ... $=$ 20 sq. yds. 5 sq. ft. 101 sq. in. and length $=$ 15 ft. 7 in.
(3) ... $=$ 11 sq. yds. 3 sq. ft. 30 sq. in. and ... $=$ 37 ft. 2 in.
(4) ... $=$ 17 sq. yds. 2 sq. ft. 131 sq. in. and breadth $=$ 11 ft. 11 in.
(5) ... $=$ 402 sq. ft. 72 sq. in. and breadth $=$ 5 yds. $2\frac{1}{2}$ ft.

4. Find the area of a square whose side is 7 yds. 1 ft. 5 in.

5. How many sq. ft. and inches remain out of 313 sq. ft. of matting, after covering a floor 16 ft. 9 in. long by 12 ft. 11 in. broad?

6. Find the sides of the following squares :—
6sq. yds. 7sq. ft. 52sq. in. ; 241sq. yds. 8sq. ft. 112sq. in. ; 34225sq. in.

7. A piece of canvas of uniform width is 7 ft. $3\frac{3}{8}$ in. long and it covers 2 sq. yards $103\frac{3}{4}$ sq. in.; what is its width ?

8. Find the number of acres in a square field whose side is 4 chains 50 links.

9. How many sods each $10\frac{1}{2}$ in. by $7\frac{1}{2}$ in will be required to cover a piece of ground 25 yards by 14 yards ?

10. How many planks, each 10 ft. 6 in. by 10 in., must be used to cover a floor 42 ft. by 17 ft. 6 in. ?

11. How many postage stamps, each $1\frac{7}{8}$ in. by $\frac{7}{8}$ in. would be required to cover a wall 14 ft. by 10 ft. 6 in. ?

12. How many boards 18 ft. 6 in. long and 7 in. wide will be required to floor a room 10 yds. 1 ft. 9 in. long and 8 yds. 6 in. wide?

13. A postage stamp is one inch long and $\frac{7}{8}$ in. broad ; how many will an album of 32 leaves, 8 in. long and 5 in. broad, contain ?

14. A square space containing 140 sq. yds. 36 sq. in. is to be lengthened by 4 ft. 3 in. in one of its dimensions and shortened by 3 ft. 4 in. in the other ; what will then be its area ?

15. How many tiles 7 in. square will be required for the floor of a room 19 ft. 3 in. long by 13 ft. 5 in. wide ?

16. A garden roller is 3 ft. $7\frac{1}{2}$ in. wide and 5 ft. $10\frac{3}{8}$ in. in circumference ; how many sq feet and inches of ground does it pass over in making 4 revolutions ?

17. A rectangular field is 7 chains 35 links long and 5 chains broad ; 1 ac. 3 ro. is to be cut off from it by a line parallel to its breadth ; where must this line be drawn ?

18. The length of a room is double its width, and the area of the floor is 136 sq. yds. 1 sq. ft. 18 sq. in.; find its length.

19. The length of a room is three times its breadth, and its area is 635 sq. yds. 5 sq. ft. 48 sq. in. ; find its length in feet.

20. How many acres are there in a rectangular tract of country $3\frac{1}{2}$ miles long and $2\frac{3}{4}$ miles broad ?

21. Find the cost of matting each of the following rooms :—

(1) 119 ft. 6 in. long and 83 ft. 4 in. broad at 12*a.* per sq. ft.

(2) 146 ft. 9 in. long and 88 ft. 9 in. broad at 7*a.* 6*p.* per sq. yd.

(3) 25 ft. 8 in. long and 16 ft. 9 in. wide at *Re.*1. 7*a.* per sq. yd.

(4) 98 ft. 4 in. long and 24 ft. 6 in. broad at 3*s.* 9*d.* per sq. foot.

22. What length of carpet will be required to cover each of the following floors :—

(1) 22 ft. 6 in. long and 15 ft. 9 in. wide. Carpet 2 ft. 3 in. wide ?

(2) 35 ft. 4 in. long and 27 ft. 3 in. broad. Carpet 27 in. wide?

(3) 15 ft. 8 in. long and 11 ft. 3 in wide. Carpet $\frac{3}{4}$ yd. wide?

23. Find the expense of carpeting each of the following rooms:—

(1) 34 ft. 8 in. long and 13 ft. 3 in. wide with carpet $\frac{3}{4}$ yd. wide at *Re.*1. 11*a.* per yd.

(2) 31 ft. 6 in. by 23 ft. 9 in. with carpet 18 in. wide at *Re.*1. 6*a.* per yd.

(3) 20 ft. 3 in. long and 17 ft. 4 in. broad with carpet $\frac{5}{8}$ yd. wide at *Rs.*2. 1*a.* 4*p.* per yd.

(4) 26 ft. 8 in. by 20 ft. 3 in. with carpet $\frac{3}{8}$ yd. wide at 4*s.* 8*d.* per yd.

24. The cost of carpeting the floor of a room 12 ft. 6 in. long at 5*a.* per sq. foot is *Rs.*187. 8*a.*; find its width.

25. The breadth of a room is 11 ft. 11 in. and the cost of matting the floor at 10*a.* 8*p.* per sq. ft. is *Rs.*113. 3*a.* 4*p.*; find the length of the room.

26. The length of a room is 23 ft. 6 in. and the cost of carpeting the floor at 4*s.* 6*d.* per sq. yd. is £11. 15*s.*; find the breadth of the room.

27. It costs *Rs.*226. 14*a.* to cover the floor of a room 8 yds. 9 in. long and 6 yds. 24 in. wide, with carpet 2 ft. wide. Find the price of the carpet per yard.

28. If the cost of carpeting a room 11 yds. long and 8 yds. wide with carpet at 3*s.* a yard, be £19. 16*s.*; find the width of the carpet.

29. Find the expense of turfing a plot of ground, which is 40 yds. long and 100 ft. wide, with turfs each a yard in length and 1 ft. in breadth; the turfs, when laid, costing *Rs.*3. 6*a.* per hundred.

30. The expense of carpeting a room, whose breadth is 12 ft. 9 in. with carpet 24 in. wide at *Rs.*3. 13*a.* 4*p.* per yard is *Rs.*126. 4*a.* 2*p.*; find the length of the room.

31. On laying down a plot of ground with sods 2 ft. 6 in. long and 9 in. wide, it is found that it requires 75 sods to form one strip extending its whole length, and that a man can lay down $1\frac{1}{4}$ strips each day; find the surface covered in 8 days.

32. The roller used for a bowling green being 6 ft. 6 in. in circumference and 2 ft, 3 in. in width, is observed to make 12 revolutions from one extremity of the green to the other; find the area rolled, when the roller has passed 10 times the length of it.

33. Find the side of a square court-yard the expense of paving which at *Re.*1. 14*a.* per sq. yard is *Rs.*385. 3*a.* 4*p.*

34. If 1572 tiles each 10 in. square would pave a court, how many would be required if the tiles were each 8 in. wide and 15 in. long?

35. A lawn which is twice as long as it is broad, costs £38. 10*s.* $5\frac{1}{4}$*d.* to turf at 11*d.* per sq. yard. Find its length and breadth.

36. A lawn is half as long again as it is wide; the cost of levelling it at 6*a.* per sq. yd. is *Rs.*1764; find the cost of enclosing it with a fence at *Rs.*3. 12*a.* per yard.

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435. Papering and **painting**, always refer to the *walls* only; **plastering** and **whitewashing** both to the *walls* and the *ceiling*; and **glazing** to the *windows* only.

In finding the area of a *wall* we multiply the length of the wall by its height. In a room there are 4 walls, and the two that are opposite are equal.

The area of a wall running length-wise = length (of the room) × height; and that of a wall running breadth-wise = breadth (of the room) × height.

∴ **area** of the 4 walls = 2 × length × height + 2 × breadth × height
= 2(**length + breadth**) × **height**.

But 2 × (length + breadth) is the **perimeter** or **circuit** of the room, *i. e.*, the sum of the lengths of the 4 walls.

∴ **area** of the walls = **perimeter** (or **circuit**) × **height**.

This expression gives the area of the walls, including that occupied by windows, doors, fire-places, &c. Deductions must be made for these in practice in all questions relating to papering, painting, plastering, &c. unless otherwise mentioned. Paper for covering walls is done up in long rolls, and is of a certain width, and to find the **length** of paper required to cover the walls of a room, we must first find the *area of the walls* and then divide this area by the *width* of the paper, as in Art. 434.

Note. In finding the quantity of paper required to cover the walls of a room, the student must not multiply together the length, the breadth and the height of the room; for this would give the volume necessary to fill the room instead of the quantity of paper required merely to cover the walls. See Art. 441.

436. The area of the **ceiling** of a room is the *same* as that of the floor. Hence to find the area of a ceiling multiply the length of the room by its breadth.

Ex. 1. Find the area of the 4 walls of a room 30 ft. 8 in. long 26 ft. 5 in. broad and 10 ft. 6 in. high.

Area of the 4 walls = 2(30 ft. 8 in. + 26 ft. 5 in.) × 10 ft. 6 in.
$= 2 \times 57$ ft. 1 in. × 10ft. 6in. $= 2 \times 57\frac{1}{12} \times 10\frac{1}{2}$ sq. ft.
$= 2 \times \frac{685}{12} \times \frac{21}{2}$ sq. ft. $= \frac{4795}{4}$ sq. ft. $= \underline{1198\frac{3}{4}}$ sq. ft.

Ex. 2. Find the length of paper 2 ft. wide required to cover the walls of the above room.

Area of the 4 walls $= \frac{4795}{4}$ sq. ft.; width of paper = 2 ft.

∴ length of paper reqd. $= (\frac{4795}{4} \div 2)$ ft. $= \frac{4795}{24}$ yds. $= \underline{199\frac{19}{24}}$ yds.

Ex. 3. Find the cost of papering the above room at 5*a.* 4*p.* per yard

Length of paper $=\frac{4795}{24}$ yds. ; 5*a.* 4*p.* $= 5\frac{1}{3}$*a.* $=\frac{16}{3}$*a.* $=$ *Re.*$\frac{1}{3}$.

$\therefore$ cost required $=$ *Re.*$\frac{1}{3}\times\frac{4795}{24}=$ *Rs.*$\frac{4795}{72}=$ *Rs.*66 9*a.* $6\frac{2}{3}$*p.*

Ex. 4. A room 25 ft. 7 in. long, 18 ft. 11 in. broad and 10 ft. 6 in. high has two doors, each 7 ft. by 3 ft 4 in. and 3 windows, each 6 ft. by 4 ft. 3 in. ; determine the expense of plastering the walls and ceiling at 12*a.* per sq. yard.

Area of the 4 walls $=$ 2 25 ft. 7 in. + 18 ft. 11 in.) × 10 ft. 6 in.

$= 2\times 44$ ft. 6 in. × 10ft. 6in. $= 2\times 44\frac{1}{2}\times 10\frac{1}{2}$ sq ft

$= 2\times\frac{89}{2}\times\frac{21}{2}$ sq. ft. $=\frac{1869}{2}$ sq. ft.

Area of the ceiling $= 25\frac{7}{12}\times 18\frac{11}{12}$ sq ft. $=\frac{307}{12}\times\frac{227}{12}$ sq. ft. $=\frac{69689}{144}$ sq. ft.

Area of 2 doors $= 2\times 7\times 3\frac{1}{3}$ sq ft. $=\frac{140}{3}$ sq. ft., and that of the 3 windows $= 3\times 6\times 4\frac{1}{2}$ sq. ft. $=\frac{153}{2}$ sq. ft.

$\therefore$ whole area to be plastered $=\left(\frac{1869}{2}+\frac{69689}{144}-\frac{140}{3}-\frac{153}{2}\right)$ sq. ft.

$=\frac{186521}{144}$ sq. ft. $=\frac{186521}{1296}$ sq. yds.

$\therefore$ cost of plastering $=\left(\frac{186521}{1296}\times 12\right)$*a.* $= 1727\frac{5}{108}$*a.* $=$ *Rs.*107. 15*a.* $0\frac{5}{9}$*p.*

437. *Having given the length and breadth of a room, and the area of the four walls, to find the height.*

RULE. Height $=$ (area of the 4 walls) ÷ 2 (length + breadth).

Ex. 5. The cost of papering a room 15 ft. long at *Re.*1. 4*a.* per sq foot is *Rs* 710 ; the cost of carpeting the floor of the same room at *Re.*1. 14*a.* per sq. foot is *Rs.*243. 12*a.* Find the height and the breadth of the room.

Area of the floor in sq. ft. $=$ *Rs.*243. 12*a.* ÷ *Re.*1. 14*a.* $=$ 130.

$\therefore$ breadth of the room $=$ 130 sq. ft. ÷ 15 ft. $= 8\frac{2}{3}$ ft.

Area of the 4 walls in sq. ft. $=$ *Rs.*710 ÷ *Re.*1. 4*a* $=$ 568.

Also 2(length + breadth) $= 2(15$ ft. $+ 8\frac{2}{3}$ ft.$) = 2\times 23\frac{2}{3}$ ft. $= 47\frac{1}{3}$ ft.

$\therefore$ height of the room $=(568\div 47\frac{1}{3})$ ft. $= 568\times\frac{3}{142}$ ft. $=$ 12 ft.

Ex. 2. If the cost of papering a room $8\frac{1}{4}$ yds. long and 4 yds. high with paper 2 ft. wide at 2*a.* 8*p.* per yard, be *Rs.*29. 13*a.* 4*p.* ; find the breadth of the room.

*Rs.*29. 13*a.* 4*p.* $=$ *Rs.*$29\frac{5}{6}=$ *Rs.*$\frac{179}{6}$; and 2*a.* 8*p.* $=$ *Re.*$\frac{1}{6}$.

$\therefore$ length of paper required $=\left(\frac{179}{6}\div\frac{1}{6}\right)$ yds. $=$ 179 yds.

$\therefore$ area of the four walls $=(179\times\frac{2}{3})$ sq. yds. $=\frac{358}{3}$ sq. yds.

$\therefore$ 2 (length + breadth) $=\left(\frac{358}{3}\div 4\right)$ yds. $=\frac{179}{6}$ yds.

$\therefore$ the reqd. breadth $=\left(\frac{179}{12}-8\frac{1}{4}\right)$ yds. $=\frac{20}{3}$ yds. $=$ 20 ft.

Ex. 3. The expense of carpeting a room 20 ft. long was *Rs.*75 ; but if the breadth had been 3 ft. less than it was, the expense would have been *Rs.*60. What was the breadth of the room ?

The difference between the two costs $= Rs.(75-60) = Rs.15$.

But this arises from the fact of the breadth being 3 ft. less, and is therefore the cost of 20×3 or 60 sq. ft. of carpet.

Now, since *Rs.*15 is the cost of 60 sq. ft. of carpet

$\therefore$ *Rs.*75 is the cost of 60×5 or 300 sq. ft. of carpet.

Hence the area of the room is 300 sq. ft. and $\therefore$ breadth reqd. of the room $= (300 \div 20$ ft.$) =$ 15 ft.

Examples CXXX.

1. Find the area of the 4 walls of each of the following rooms :—

(1) 25 ft. 7 in. long, 19 ft. 4 in. wide, and 9 ft. 9 in. high.

(2) 15 ft. 6 in. long, 13 ft. 4 in. wide, and 10 ft. 6 in. high.

(3) 23 ft. 5 in. long, 18 ft. 7 in. wide, and 9 ft. 6 in. high.

(4) 20 ft. 10 in. long, 16 ft. broad, and 10 ft. 9 in. high.

2. Find the length of wall paper required for each of the following rooms :—

(1) 18 ft. 9 in. square and 13 ft. 4 in. high, with paper 1 ft. 4 in. wide.

(2) 21 ft. 4 in. long, 17 ft. 5 in. wide and $13\frac{1}{2}$ ft. high, with paper $16\frac{1}{2}$ in. wide.

(3) 28 ft. 6 in. long, 18 ft. 9 in. wide and 12 ft. high, with paper 1 ft. 9 in. wide.

(4) 22 ft. 5 in. long, 12 ft. 1 in. broad and 11 ft. 3 in. high, with paper $\frac{3}{4}$ yd. wide.

3. Find the expense of papering each of the following rooms :—

(1) 20 ft. 6 in. long, 17 ft. 4 in. broad and 9 ft. high, with paper 20 in. wide at 3*a.* 4*p.* per yard.

(2) 29 ft. 7 in. long, 21 ft. 4 in. wide and 12 ft. 11 in. high, with paper $11\frac{3}{4}$ in. wide at $7\frac{1}{2}d.$ per yard.

(3) 12 ft. 6 in. long, 8 ft. 4 in. wide and 9 ft. high, with paper 3 ft. wide, in pieces 20 yds. long, at 12*a.* per piece.

(4) 5 yds. 1 ft. $2\frac{1}{2}$ in. long, 5 yds. $3\frac{1}{2}$ in. broad, and 4 yds. high with paper 9 in. wide at $2\frac{1}{2}d.$ per yard.

4. Find the cost of painting the walls and ceiling of each of the following rooms :—

(1) 12 ft. 6 in. long, 27 ft. 4 in. wide and 20 ft. high at 5*a.* 8*p.* per sq. yd.

(2) 15 ft. long, 10 ft. broad and 9 ft. 9 in. high at 14*a.* per sq. yd.

(3) 35 ft. 4 in. long, 17 ft. 6 in. wide and 20 ft. high at $7\frac{1}{2}d.$ per sq. yd.

5. A room is 20 ft. 6 in. long, 15 ft. 6 in. wide and 16 ft. high; it has two doors, each 8 ft. high and 3 ft. 9 in. wide and 3 windows, one 5 ft. by 7 ft., the other two 5 ft. by 4 ft. each. What will it cost to paper the room with paper one yard wide at 6*a*. 8*p*. per yard?

6. A room whose height is 18 ft. 5 in., breadth 20 ft. 10 in. and length 22 ft. 9 in., has a door 7 ft. 6 in. by 3 ft. 4 in. and two equal windows each 3 ft. 6 in. by 5 ft. 2 in.; find the cost of papering it at *Re*.1. 11*a*. 6*p*. per square yard.

7. A room is to be papered whose length is 23 ft. 8 in., breadth 15 ft. 10 in. and height 11 ft. 9 in.; in it there are two windows each 9 ft. 6 in. high by 5 ft. wide, a fire-place 4 ft. 6 in. high by 6 ft wide, and a door 7 ft. 6 in. high by 3 ft. 6 in. wide. Find the cost of the paper required at 17*s*. 6*d*. per piece of 12 yards, the width of the paper being 26 inches.

8. What will be the expense of papering a room 24 ft. 4 in. long, 19 ft. 8 in. wide and 13½ft. high, with a paper which is 2¾ ft. wide and costs 11*s*. per piece of 12 yds.: the windows and doors not requiring to be papered, making up a sixth part of the whole surface?

9. How many postage stamps, 1 in. long and ⅞ in. broad, will be required to cover the walls of a room 18 ft. long, 12 ft. broad and 10 ft. 6 in. high?

10. Find the expense of whitewashing the ceiling and walls of a square room at 1*a*. 8*p*. per sq. yd., whose floor measures 32 sq. yds. 1 sq. ft. and height 11 ft. 6 in.

11. A room whose height is 11 feet and length twice its breadth, takes 143 yds. of paper 2 ft. wide for its four walls. How much carpet will it require?

12. A room is 20 ft. long and 16 ft. wide; what must be its height in order that the area of the floor and ceiling together may be equal to the area of the walls?

13. A room is 19 ft. 5 in. long and 16 ft. 7 in. broad; and the cost of painting the walls at 7*a*. 6*p*. per sq. yard is *Rs*.43. 3*a*. Find the height of the room.

14. The cost of carpeting a room twice as long as it was broad at *Rs*.2. 8*a*. per sq yd. amounted to *Rs*.61. 4*a*.; and the painting of the walls at 6*a*. a sq. yd. amounted to *Rs*.26. 4*a*. Find the height of the room.

15. The expense of carpeting a room 18 ft. long was £7. 4*s*.; but if the breadth had been 4 ft. less than it was, the expense would have been £5. 8*s*.; what was the breadth of the room?

16. The length of a room is 14 ft.; the cost of painting the walls at *Re*.1. 5*a*. a sq. yd. is *Rs*.75. 13*a*. 4*p*.; and the cost of carpeting the room at *Rs*.2 a sq. yd. is *Rs*.37. 5*a*. 4*p*. Find the height and breadth of the room.

17. The cost of painting the walls of a room 18 ft. long and 9 ft. high at *Re.*1. 8*a.* a sq. foot is *Rs.*810. Find the cost of carpeting the room at *Re.*1. 9*a.* a sq. yard.

18. A room is 14 ft. 9 in. long, 9 ft. 3 in. wide and 10 ft. 6 in. high ; it contains two windows each 5 ft. 6 in. by 4 ft. ; three doors each 6 ft. by 3 ft. and a fire-place $6\frac{1}{2}$ ft. by 4 ft. ; find how many postage stamps it would take to cover the walls, a stamp being $1\frac{5}{8}$ in. by $\frac{3}{4}$ in., supposing that the stamps may be cut if necessary.

19. Two rectangular rooms of the same height were papered ; the first was 16 ft. long and 14 ft wide, and the second 14 ft. long and 12 ft. wide ; the paper was 1 ft. 9 in. wide and *Re.*1. 6*a.* per piece of 12 yds., and the hanging cost 6*a.* per piece ; the whole expense was *Rs.*42 ; find the height of the rooms.

20. The length of a room is 20 ft. ; the cost of papering the walls with paper $2\frac{1}{2}$ ft. wide at 4*a.* per yard is *Rs.*30. 6*a.* 8*p.* ; and that of carpeting the room at *Rs.*3. 5*a.* 4*p.* per sq. yd. is *Rs.*122. 3*a.* $6\frac{2}{3}$*p.* Find the height of the room.

21. A room 10 ft. high and 20 ft. long costs £190 to paint its walls at 5*s.* per sq. foot. What is the cost of the carpet which will cover the floor at *Rs.*3. 2*a.* per sq. yard ?

22. It takes 5904 tiles, each measuring 6 in. by 10 in., to cover the floor of a room 120 ft. long ; what is the width of the room ? What will be the cost of laying down the tiles at 2*a.* 6*p.* per sq. yard ?

23. The cost of carpeting a room is *Rs.*72, and of papering the same room with paper at 1*a.* 8*p.* per sq. foot is *Rs.*106. 4*a.* The length of the room is 18 ft, and if the width had been 4 ft. less the cost of the carpet would have been *Rs.*18 less. Find the height of the room.

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438. Paths and Moats (1) When a path or a moat of a **uniform** width goes all round **outside** a field, its area may be obtained either (i) by **increasing** each dimension by **twice** the *uniform* width of the path or moat, and then subtracting the area of the given field from the increased area ; or (ii) by **increasing** each dimension by the *uniform* width of the path, &c. and then using the following RULE :—

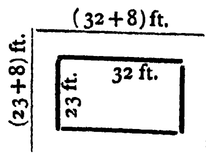
Area = 2(*increased* length + *increased* breadth) × *uniform* width of the path.

(2) But if the path or the moat go round **inside** the field, then either (i) **diminish** each dimension of the field by **twice** the *uniform* width and subtract the diminished area from the area of the field ; or (ii) **diminish** each dimension by the *uniform* width of the path, and use the following RULE :—

Area = 2(*diminished* length + *diminished* breadth) × *uniform* width of the path.

Ex. 1. A rectangular piece of ground is 32 ft. long and 23 ft. broad. Find the cost of enclosing it with a path 4 ft. broad, at the price of *Rs.*3 per sq. yard; (1) when the path is outside the piece of ground, (2) when the path is part of the ground.

(1) *By the first Method.*



$32+2\times4=40$; $23+2\times4=31$

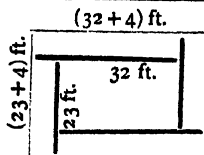
∴ area of the path

$=(40\times31)$ sq. ft. $-(32\times23)$ sq. ft.

$=504$ sq. ft. $=56$ sq. yds.

∴ cost $=Rs.56\times3=Rs.168.$

By the second Method.



$32+4=36$; $23+4=27$.

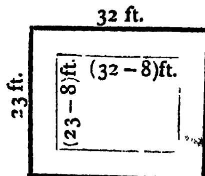
∴ the area of the path

$=2(36+27)\times4$ sq. ft.

$=504$ sq. ft. $=56$ sq. yds.

∴ cost $=Rs.3\times56=Rs.168.$

(2) *By the first Method.*



$32-2\times4=24$; $23-2\times4=15$.

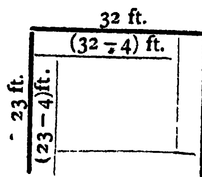
∴ area of the path

$=(32\times23)$ sq. ft. $-(24\times15)$ sq. ft.

$=376$ sq. ft. $=41\frac{7}{9}$ sq. yds.

∴ cost $=Rs.3\times41\frac{7}{9}=Rs.125.\ 5a.\ 4p.$

By the second Method.



$32-4=28$; $23-4=19$.

∴ the area of the path

$=2(28+19)\times4$ sq. ft.

$=376$ sq. ft. $=\frac{376}{9}$ sq. yds

∴ cost $=Rs.3\times\frac{376}{9}=Rs.125.\ 5a.\ 4p.$

Ex. 2. A room 35 ft. long by 18 ft. broad is enclosed by walls 18 in. thick and all round the outside there is a verandah 9 ft. deep. Find the cost of paving the verandah at 8*a.* per sq. yard.

Since the walls are 18 in. or $1\frac{1}{2}$ ft. thick, the external length of the room is $(35+2\times1\frac{1}{2})$ or 38 ft.; and external breadth is $(18+2\times1\frac{1}{2})$ or 21 ft.

Now $38+9=47$; $21+9=30$.

∴ area of the verandah $=2(47+30)\times9$ sq. ft. $=2\times77\times9$ sq. ft.

$=154$ sq. yds.

∴ cost $=Re.\frac{1}{2}\times154=Rs.77.$

Ex. 3. A rectangular court-yard 40 yds. 2 ft. 7 in. long and 75 ft. 5 in. broad, has 2 foot-paths, each $7\frac{1}{2}$ ft. wide, the one running the whole length of it and the other the whole breadth of it, crossing each other at right angles. Find the total expense of paving the court-yard with pebbles at *Rs.*2. 4*a.* per sq. yd., and the foot-paths with flag-stones at *Re.*1. 8*a.* per sq. yd.

Area of court-yard,

(including the paths) $= 122\frac{7}{12} \times 75\frac{5}{12}$ or $\frac{1331255}{144}$ sq. ft.

............(excluding the paths) $= 115\frac{1}{12} \times 67\frac{11}{12}$ or $\frac{1125515}{144}$ sq. ft

∴ area of the paths $= (\frac{1331255}{144} - \frac{1125515}{144})$ sq. ft.

$= \frac{205740}{144}$ sq. ft. $= \frac{635}{4}$ sq. yds.

∴ cost of paving the court-yd. $= \frac{1125515}{1296} \times$ *Rs.*$2\frac{1}{4}$ = *Rs.*$\frac{1125515}{576}$

= *Rs.*1954. 0*a.* $3\frac{2}{3}$*p.*

and cost of paving the paths = *Rs.*$\frac{3}{2} \times \frac{635}{4}$ = *Rs.*238. 2*a.*

∴ the total cost reqd. = *Rs.*2192. 2*a.* $3\frac{2}{3}$*p.*

Examples CXXXI.

1. A field is 300 yds. long and 200 yds. broad; if a belt of tree 30 yds. wide be planted round it inside, find the area of the interior space.

2. A rectangular piece of ground is 60 yds. long and contains half an acre. It consists of a walk 8 ft. wide surrounding a grass plot. Find the area of the plot.

3. A grass plot is 23 ft. 8 in. long and 16 ft. 7 in. broad; round it a walk 10 ft. wide is made and paved at 15*a.* per sq. yd. What is the cost of the paving?

4. How many paving stones, each of them one foot long and $\frac{4}{5}$ of a foot wide, will be required for paving a street 45 ft. wide, surrounding the outside of a square, the side of which is 225 ft.?

5. A hall 70 ft. long and 36 ft. broad is enclosed by walls 18 in. thick, and all round the outside there is a verandah $13\frac{1}{2}$ ft. deep. What will be the cost of paving this verandah at the rate of 12*a.* per sq. yard?

6. A rectangular court is 80 yds. long and 50 yds. broad. It has paths, joining the middle points of the opposite sides 6 ft. wide, and it has also paths of the same breadth running all round it on the inside. The remainder is covered with grass. If the paths cost 13*a.* 4*p.* per sq. ft. and the grass *Re.*1. 8*a.* per sq. yd., find the whole cost of laying out the court.

7. How many flag-stones each 5·76 ft. long and 4·15 ft. wide are required for paving a cloister which encloses a rectangular court 45·77 yds. long and 41·93 yds. wide; the cloister being 12·45 ft. wide?

8. A rectangular court has a path of the uniform width of 3 yds. 1 ft. running round it; the length of the court (including the path)

is 40 yds. and the breadth 30 yds.; find the cost of paving the path at 4*s*. 6*d*. per sq. yd. and of covering the remainder of the court with turf at 13*s*. per 200 sq. ft.

9. The area of a square cricket field is 9 ac. 3 ro. 8·16 po.; a path of the uniform width of 3·9 yds. is made close to the boundary (on the inner side) of the field at a cost of 4*d*. per sq. yd., and the remainder of the field is laid down in turf at a cost of 5*s*. 6*d*. per 100 sq. yds.; find the total cost of preparing the field.

10. A room is 60 ft. long by 29 ft. wide; how many people can be seated in it on chairs $1\frac{1}{2}$ ft. wide, and placed 2 ft. apart from back to back, allowing a clear passage 3 ft. wide down the middle of the room, and a space 15 ft. deep at one end?

11. A box with a lid is to be made of $1\frac{1}{2}$ in. plank; the external dimensions to be 3 ft. 6 in., 2 ft. 6 in. and 1 ft. 9 in. How many square feet of plank will be used in the construction?

12. It costs *Rs*.6435. 6*a*. to level and turf a square cricket ground at 6*a* per sq. yd.; find the cost of enclosing it with an iron railing at *Rs*.3. 12*a*. per yard.

13. A cistern, without a lid, whose floor and sides are $1\frac{1}{2}$ in. thick, is 5 ft. 3 in. long, 3 ft. 7 in wide and 2 ft. $5\frac{1}{2}$ in. high in its external dimensions. Find the area of the inside surface, and the cost of painting the inside at the rate of 2*a*. 8*p*. per sq. foot.

14. A rectangular grass plot measures 320 yds. by 160 yds.; all round it (on the outer side of the boundary) is a gravel path 6 ft. broad; the price for making the grass plot is 4*a*. per sq. yd.; what must be the price of the gravel path per sq. yard, that the path may cost *Rs*.11832 less than the grass plot?

15. A court-yard 15 yds. by 12 yds. is to be paved with pebbles at *Re*.1. 8*a*. per sq. yd. except two foot-paths at right angles to the sides, each 4 ft. wide which meet in the centre, forming a cross; these are to be laid in paving at *Re* 1. 10*a*. per sq. yd.; find the cost of the whole.

16. A cistern $12\frac{3}{4}$ cubits long, $8\frac{1}{4}$ cubits broad and $6\frac{1}{2}$ cubits deep is to be lined with lead. How many sq. cubits of lead will be required? What will be the weight of the lead, if each sq. cubit weigh 8 seers? What will it cost at 2*a*. per seer?

II. CUBIC MEASURE.

439. In Arithmetic, we deal with the volumes of **rectangular solids** only.

(*a*) A **solid** is that which has length, breadth and thickness.

(*b*) A **rectangular parallelopiped** or **solid** is a solid figure bounded by six rectangular surfaces of which every opposite two are equal and parallel; as a *brick*.

(*c*) A **cube** is a rectangular solid bounded by six squares ; as a *die.*

(*d*) The length, breadth and thickness (or height or depth) of a rectangular solid are called its **dimensions**.

(*e*) The **capacity** or **volume** of a solid, is the quantity of space, comprehending length, breadth and thickness, which it contains.

(*f*) The word **content** is also frequently used to denote length, area and capacity or volume.

440. A rectangular solid measuring a yard each way is called a **cubic yard** ; a rectangular solid measuring a foot each way is called a **cubic foot** ; and so on.

441. *To find the volume of a rectangular solid.*

Let $ABCD$ represent a rectangle whose length AB is 4 inches and breadth AD, 3 inches.

Then $ABCD$ will contain 12 square inches.

If on the figure $ABCD$ there be placed a solid in the form of a dice, each edge being one inch in length, on every square as $AEFG$, we shall have a layer of solids containing as many cubic inches as there are square inches in $ABCD$, or 12 cubic inches ; If on this layer we place a second layer of exactly the same form and size, we shall have the whole solid two inches high and containing twice 12 cubic inches ; and so on. Hence the whole number of cubic inches in any such solid will be expressed by the product of the number of square inches in the base or rectangle $ABCD$, multiplied by the number of inches in the height. Therefore the process for measuring the **volume** or **solidity** or **capacity** of any rectangular solid is expressed by the following RULE :—

A E B
G F
D C

RULE. ***Express the length, breadth and height or thickness in units of the same denomination ; their product will give the volume in cubic units of that denomination.***

442. It should be noticed here, as in Art. 431, that if the number of inches in the length, and breadth, and height be mixed numbers or fractions, still their product will give the number of cubic inches in the volume. Also, if the numbers giving the three dimensions be given in feet, or in yards, or in miles, their product will give the volume in cubic feet, or in cubic yards, or in cubic miles.

443. The above *Rule* is more briefly stated thus :—

Volume = Length × Breadth × Height.

$\therefore$ Length = volume ÷ (breadth × height) ; Breadth = volume ÷ (length × height) ; and Height = volume ÷ (length × breadth).

Ex. 1. Find the cubic content of a rectangular solid whose dimensions are $10\frac{1}{2}$ ft. long, $8\frac{1}{4}$ ft. broad and $6\frac{1}{2}$ ft. high.

The cubic content $= 10\frac{1}{2} \times 8\frac{1}{4} \times 6\frac{1}{3}$ cub. ft. $= \frac{21}{2} \times \frac{33}{4} \times \frac{19}{3}$ cub. ft.
$= \frac{4389}{8}$ cub. ft. $= 548\frac{5}{8}$ cub. ft. $=$ 548 cub. ft. 1080 cub. in.

Ex. 2. A square block of stone 2 ft. in thickness, is in cubic content 5 cub. ft. 24 cub. in. ; what is the length of its edge ?

The area of square stone $= 5\frac{1}{72}$ cub. ft. $\div 2$ ft. $= \frac{361}{144}$ sq. ft.

$\therefore$ each edge reqd. $= \sqrt{(\frac{361}{144})}$ ft. $= \frac{19}{12}$ ft. $=$ 1 ft. 7 in.

Ex. 3. How many bricks will be required to build a wall 75 ft. long, 6 ft high and 18 in. thick ; each brick being 9 in. long, $4\frac{1}{2}$ in. wide and 3 in. deep ?

The volume of a brick $= (\frac{9}{12} \times \frac{9}{24} \times \frac{3}{12})$ cub. ft. $= \frac{9}{128}$ cub. ft.

The volume of wall $= 75 \times 6 \times \frac{18}{12}$ cub. ft. $= 75 \times 9$ cub. ft.

$\therefore$ the number of bricks $= 75 \times 9$ cub. ft. $\div \frac{9}{128}$ cub. ft. $= 75 \times 9 \times \frac{128}{9}$
$= 75 \times 128 = 9600.$

Ex. 4. What is the length of a room, whose width is 10 ft. 4 in. and height 10 ft. 6 in. ; and which contains 1519 cub. ft. of air ?

The length $= \dfrac{1519 \text{ cub. ft.}}{10\frac{1}{3} \times 10\frac{1}{2} \text{ sq. ft.}} = 1519 \times \frac{3}{31} \times \frac{2}{21}$ ft. $= 14$ ft.

Ex. 5. A reservoir is 24 ft. 8 in. long by 12 ft. 9 in. wide ; how many cubic feet of water must be drawn off to make the surface sink $3\frac{1}{2}$ ft ?

As the surface sinks $3\frac{1}{2}$ ft., the depth of the quantity of water drawn off is also $3\frac{1}{2}$ ft.

$\therefore$ quantity of water drawn off $= 24\frac{2}{3} \times 12\frac{3}{4} \times 3\frac{1}{2}$ cub. ft.
$= \frac{74}{3} \times \frac{51}{4} \times \frac{7}{2}$ cub. ft. $= 1100\frac{3}{4}$ cub. ft.

Ex. 6. Water flowing at the rate of 60 ft. every 4 min. through a cylindrical pipe, whose sectional area is 16 sq. in. fills a square tank 32 ft. deep in 8 hours. Find the length and breadth of the tank.

The quantity of water flowing through the pipe in 4 min. $= 16$ sq in. $\times 60$ ft. $= \frac{1}{9} \times 60$ cub. ft $= \frac{20}{3}$ cub. ft.

$\therefore$ quantity of water flowing in 8 hours $= (\frac{20}{3} \times 15 \times 8)$ cub. ft.
$= 800$ cub. ft.

$\therefore$ the area of the tank $= 800$ cub. ft. $\div 32$ ft. $= 25$ sq. ft.

As the tank is a square, $\therefore$ each side $= \sqrt{(25)}$ ft. $=$ 5 ft.

Ex. 7. A rectangular pond 60 ft. long, 49 ft. 6 in. broad and 6 ft. 8 in. deep is full of water ; 100 water carts are employed to take away water from it, the water box in each cart being 5 ft. 6 in. long, 4 ft. 6 in. broad and 16 in. high. How much will the water sink down, when each of the carts has been used 3 times ?

The cubic content of each cart $= (5\frac{1}{2} \times 4\frac{1}{2} \times \frac{16}{12})$ cub. ft. $= 33$ cub. ft.

$\therefore$ quantity of water drawn off $= 33$ cub. ft. $\times 100 \times 3 = 9900$ cub. ft.

The area of the surface of water in the tank $=(60 \times 49\frac{1}{2})$ sq. ft. $= 2970$ sq. ft.

$\therefore$ the water will sink down 9900 cub. ft. $\div$ 2970 sq. ft. $=\frac{10}{3}$ or $3\frac{1}{3}$ ft.

Ex. 8. The exterior dimensions of a box made of wood of half an inch in thickness are 5 ft. 4 in. long, 4 ft. 3 in. broad and 3 ft. 9 in. high respectively; find the expense of painting it inside and outside at 6*a.* 9*p.* per square yard.

The wood being $\frac{1}{2}$ in. thick, the inner dimensions are 5 ft. 3 in. 4 ft. 2 in. and 3 ft. 8 in. respectively.

The exterior surface to be painted $= 2(5$ ft. 4 in. $+ 4$ ft. 3 in.$) \times 3$ ft. 9 in. $+ 5$ ft. 4 in. $\times 4$ ft. 3 in. $\times 2 = 2 \times 9$ ft. 7 in $\times 3$ ft. 9 in. $+ 5$ ft. 4 in. $\times 4$ ft. 3 in. $\times 2$

$= 2 \times 9\frac{7}{12} \times 3\frac{3}{4}$ sq. ft. $+ 5\frac{1}{3} \times 4\frac{1}{4} \times 2$ sq. ft.

$= (\frac{1725}{24} + \frac{136}{3})$ sq. ft. $= \frac{2813}{24}$ sq. ft.

The interior surface to be painted $= 2(5$ ft. 3 in. $+ 4$ ft. 2 in.$) \times 3$ ft. 8 in. $+ 5$ ft. 3 in. $\times 4$ ft. 2 in. $\times 2 = 2 \times 9$ ft. 5 in. $\times 3\frac{2}{3}$ ft. $+ 5\frac{1}{4} \times 4\frac{1}{6} \times 2$ sq. ft.

$= (2 \times 9\frac{5}{12} \times 3\frac{2}{3})$ sq. ft. $+ (5\frac{1}{4} \times 4\frac{1}{6} \times 2)$ sq. ft.

$= (\frac{1243}{18} + \frac{175}{4})$ sq. ft. $= \frac{4061}{36}$ sq. ft.

$\therefore$ the whole area to be painted $= (\frac{2813}{24} + \frac{4061}{36})$ sq. ft. $= \frac{16561}{72}$ sq. ft.

$= \frac{16561}{648}$ sq. yds.

$\therefore$ cost reqd. $= \frac{16561}{648} \times 6\frac{3}{4}a. = 172\frac{49}{96}a. =$ *Rs* 10 12*a* $6\frac{1}{8}$*p.*

Examples CXXXII.

1. Find the cubic content of each of the rectangular solids whose dimensions are the following :—

(1) Length 6 ft. 4 in., breadth 5 ft. 3 in. and height 3 ft. 6 in.
(2) ....12 ft. 8 in............. 9 ft. 10 in................ 8 ft. 5 in.
(3) ....15 ft. 7 in.............12 ft. 5 in................ 8 ft. 4 in.
(4) ....10 yds. 7 in............. 8 yds. 2 ft. 3 in.......... 2 yds. 2 ft. 9 in.
(5) .... 6 yds. 2 ft. 3 in....... 4 yds. 1 ft. 7 in.......... 6 ft. 10 in.

2. Find the volume of a cube whose edge is 13 ft. 8 in.

3. Find the cost of a piece of timber, whose length, breadth and thickness are respectively $54\frac{1}{2}$ ft., 5 ft. and 2 ft. 5 in. at 6*a.* per solid foot.

4. What must be the length of a trench 6 ft. 8 in. deep and 9 ft. 2 in. wide, that it may contain 13 cub. yds. 15 cub. ft. 1152 cub. in. ?

5. If 56 cub. ft. 1044 cub. in. of timber are required to floor a room 29 ft. 3 in. broad by 35 ft. 4 in. long; find the thickness of the boards.

6. Find the height of a room 26 ft. 4 in. long, 13 ft. 9 in. broad, the content of which is 44898 cub. ft. 4 cub. in.

7. If 473088 bricks, each 9 in. long, $4\frac{1}{2}$ in. broad and $2\frac{1}{2}$ in. thick be required for a wall $\frac{1}{2}$ a mile long, 7 ft. high and of a certain thickness; find the thickness.

8. A wall is to be built 15 yds. long, 7 ft. high and 13 in. thick, with a door-way 6 ft. high and 4 ft. wide ; how many bricks will it require, if each brick including mortar occupy 108 cub. in. ?

9. Find the edge of a cube which contains 15 cub. ft. 1080 cub. in.

10. The bottom of a cistern contains 16 sq. ft. 128 sq. in. ; how deep must it be to contain 1216 gallons ? 1 gallon contains $277\frac{1}{4}$ cub. in. nearly.

11. A cubic foot of water weighs $31\frac{1}{4}$ seers. Find the length of the side of a cubic vessel whose contents (water) weigh 101 mds. 39 sr. $3\frac{1}{3}$ ch.

12. A space 8 ft. square and 10 ft. high is enclosed all round with earth of the uniform thickness of 2 feet, what is the quantity of earth ?

13. A cubic foot of water weighs $62\frac{1}{2}$ lbs., and a room 18 ft. 9 in. by 13 ft. 4 in. is flooded to the depth of 2 inches ; what is the weight of water in the room ?

14. How many bricks of which the length, breadth and thickness are 12, 9 and 6 in. respectively will be required to build a wall, whereof the length, height and thickness are 64, 9 and $1\frac{1}{2}$ ft. respectively ?

15. If 56 cub. ft. 912 cub. in. be the content of an open cistern, 6 ft. 2 in. long, and 3 ft. 4 in. wide, what will be the cost of lining the inside of it with lead at 10*s.* $1\frac{1}{2}$*d.* per sq. yd ?

16. If a cubic foot of iron weigh 7·8 times as much as a cub. foot of water, find the weight of a block of iron 20·28 ft. long, 2·58 ft. broad and 2·5 ft. thick, supposing a cub. foot of water to weigh 1000 oz. Avoir.

17. The depth of water in a cistern whose base contains 1344 sq. in. is 2 ft. 10 in. Find the depth of the same quantity of water in another cistern whose base contains 1088 sq. in.

18. The weight of a cubic foot of water being $31\frac{1}{4}$ seers, find the weight of a rectangular block of gold 8 in. in length, 2 in. in thickness and 3 in. in breadth ; the weight of a mass of gold being 19·25 times the weight of an equal bulk of water.

19 A cistern whose length, depth and breadth are 6 ft. 3 in., 5 ft. and 4 ft. 2 in. respectively, is filled with water and leaks till the water sinks 7 in. ; find the volume of water left.

20. How many bricks, each 12 in. long, 4 in. wide and 3 in. thick will be required to build a wall 18 ft. 8 in. long, 12 ft. 6 in. high and 9 in. thick, leaving in it a door-way 6 ft. 3 in. high and 2 ft. 8 in. wide ?

21. A cubic foot of water weighs 1000 oz. Avoir. Find in tons the weight of a rain-fall of one inch over an acre of ground.

22. The breadth of a room is twice its height, and half its length, and the volume of air in the room is 4096 cub. ft. Find its length.

23. If in a box 3 ft. 6 in. long, 1 ft. 7 in. deep and 1 ft. 10 in. wide, I pack 160 books each $9\frac{1}{2}$ in. long, $5\frac{1}{4}$ in. wide and $1\frac{3}{4}$ in. thick; find how many more, of a size 7 in. long, $4\frac{1}{2}$ in. wide and 1 in. thick, are required to fill it.

24. A gentleman wishes to raise his lawn (which is 634 yds. long and 340 yds. broad) 2 ft. and for that purpose digs a moat round it 17 yds. broad in every part; supposing the depth of the moat to be uniform, how deep must it be in order that he may have soil sufficient for his purpose?

25. A room 21 ft. long by $13\frac{1}{2}$ ft. wide is surrounded by walls $1\frac{1}{2}$ ft. thick and 14 ft. high. There are two doors each $4\frac{1}{2}$ ft. by 6 ft., and one window 3 ft. by $4\frac{1}{2}$ ft. Find (1) the cost of building the walls at the rate of *Rs.*5. 1*a.* per cub. yard and (2) the number of bricks each measuring 9 in. × 4 in. × $2\frac{1}{4}$ in. required for the work.

26. In the middle of a rectangular field, 350 yds. long by 250 yds. broad, a tank 50 yds. square is dug and the earth is thrown evenly on the field to a height of one yard. Find the depth of the tank and the cost of digging it at 9*p.* per cub. yard.

27. The content of a box, whose length is twice its breadth, and whose breadth is twice its depth, is 1 cub. yd.; find its dimensions, and the cost of gilding it at 14*a.* per sq. ft.

28. Find the cost of painting the surface of a cube, the edge of which is 2 ft. 5 in. at 10*a.* per sq. yd.

29. A closed vessel of metal 1 in. thick, the external dimensions of which are 8 ft. 3 in., 7 ft. 5 in. and 4 ft. 3 in. weighs 3 cwt. 1 qr 8 lbs.; what would be the weight of a solid mass of the metal of the same dimensions?

30. Find the cost of making a road 110 yds. in length and 18 ft. wide; the soil being first excavated to the depth of 1 foot at a cost of 8*a.* per cub. yd.; rubble being then laid 8 inches deep at 8*a.* per cub. yd. and gravel placed on top, 9 in. thick at *Re.*1. 4*a.* per cub. yd.

31. The content of a cistern is the sum of two cubes whose edges are 10 in. and 2 in. and the area of its base is the difference between two squares whose sides are $1\frac{1}{9}$ ft. and $1\frac{2}{9}$ ft. Find its depth.

32. A cubic foot of gold is extended by hammering so as to cover an area of 6 acres. Find the thickness of the gold as a decimal of an inch to 7 places of decimals.

33. A cistern 32 ft. long, 25 ft. wide and 11 ft. deep is emptied in 20 minutes by a pipe whose sectional area is 14 sq. in. How fast does the water flow in the pipe?

34. A cubic foot of water weighs 1000 oz. Avoir.; a pipe whose bore is 5 sq. in., discharges $312\frac{1}{2}$ lbs. of water per minute; find the rate per hour at which the water issues.

35. The contents of a reservoir of square bottom are let out through a rectangular hole 3 in by 4 in. If an edge of the bottom be 6 ft. and the height 4 ft. 8 in., and water flow at the rate of $1\frac{1}{3}$ ft. per second, what time will be spent in discharging the contents?

36. The cubic content of a room 20 ft. long and $12\frac{1}{2}$ ft. high is 4875 cubic feet; find the cost of painting its walls at 6*a.* per sq. yd.

37. An iron chest 5 ft. long, 3 ft. broad and 2 ft. 5 in. high is made with sheet of iron 1 in. thick. Find the inside content of the chest and the weight of the chest, if 1 cub. foot of iron weighs 6 mds. 8 sr.

38. A tank is 100 ft. long, 60 ft. broad and 18 ft. deep; 20 water carts, each carrying 30 mds. of water at a time, are employed to empty the tank. If a cub. foot of water weighs 30 seers, in how many times will the tank be emptied?

39. A hollow rectangular iron pillar 10 ft. in length is made with iron sheets 1 in. thick and the breadth of the hollow part is 6 in. Find the weight of the pillar, if 1 cub. foot of iron weighs 6 mds., and its price at *Rs.*5. 8*a.* per maund.

40. A cistern, measuring 13 ft. in length, 5 ft. in breadth and 4 ft. in depth has a tap which, not being properly opened, discharges 54 gals. an hour less than it would otherwise do and empties the cistern in $7\frac{1}{2}$ instead of 6 hours. How many cub. inches are there in the gallon?

41. Water flows into a rectangular cistern whose dimensions are 12 ft. 1 in. long, 11 ft. 8 in. wide and 5 ft. 4 in. deep through a pipe of 10 sq. in. aperture at the rate of $2\frac{1}{2}$ ft. per second, and flows out through an orifice at the rate of 2 ft. 5·8 in. per second; if the cistern is filled in 2 hours, find the size of the orifice.

42. A stream of water 8 yds. broad at the surface and 6 yds. at the bottom and 2 yds. deep flows at the rate of $1\frac{1}{8}$ miles an hour into a tank 220 yds. long and 56 yds. broad, which holds 74250 tons of water. Find the depth of the tank and the time in which it will be filled, a cubic foot of water weighing 1000 oz. Avoir.

III. DUODECIMALS.

444. Artificers take the dimensions of their work in *yards*, *feet*, *inches*, *parts*, *&c.*; and it is usual to reduce the yards to feet so that the different denominations are *all* connected by the same number 12, or decrease in a *twelve-fold* proportion, from the place of feet towards the right hand. For the sake of uniformity, the denominations after feet are termed *primes*, *seconds*, *thirds*, *&c*, distinguished respectively by accents, ′, ″, ‴, &c., placed a little to the right, contiguous to the figures to which they belong.

Thus, 20 feet, 8 inches, 5 parts, 7 thirds, *&c.* is written 20 ft. 8′. 5″. 7‴. &c. and is read 20 *ft.*, 8 *primes*, 5 *seconds*, 7 *thirds*, &c.

From the circumstance above mentioned, the process is frequently called **Duodecimal** Multiplication or **Duodecimals** ; and it is also sometimes termed **Cross Multiplication** ; but the former of these names are evidently misapplied, because the *different* digits of the various denominations are not connected with each other by the number 12 though the *denominations* themselves are.

445. In Duodecimals, the sub-multiples of the foot whether linear, square or cubic—follow the scale of 12, so that,

LINEAR MEASURE.

1 foot = 12 primes
1 prime = 12 seconds
1 second = 12 thirds
1 third = 12 fourths ; &c.

SQUARE MEASURE.	CUBIC MEASURE.
1 sq. ft. = 12 (superficial) primes	1 cub. ft. = 12 (solid) primes
1 supl. prime = 12...seconds	1 solid prime = 12...seconds
1 ... second = 12...thirds	1 ... second = 12...thirds
1 ... third = 12...fourths ; &c.	1 ... third = 12...fourths ; &c.

446. The whole of the above statements can be briefly stated thus :—

1 linear foot, 1 square foot, 1 cubic foot $\Big\} = 12' = 144'' = 1728''' = 20736^{iv} =$ &c. ;

therefore in linear measure the *inch* is the same as the *prime*, in square measure as the *second*, and in a cubic measure as the *third*. Hence, we can easily pass from quantities expressed in duodecimals to those expressed in foot and inches, and *conversely*.

Ex. 1. Convert 37 ft. 5′ 3″ into *ft.* and *in.*, and 47 ft. $5\frac{3}{4}$ in. into *Duodecimals*.

(*a*) 37 ft. 5′ 3″ = 37 ft. $5\frac{3}{12}$ in. = 37 ft. $5\frac{1}{4}$ in.

(*b*) 47 ft. $5\frac{3}{4}$ in. = 47 ft. $5\frac{9}{12}$ in. = 47 ft. 5′ 9″.

Ex. 2. Express 27 sq. ft. $118\frac{2}{3}$ sq. in. in *Duodecimals*, and 46 sq. ft. 10′ 7″ 8‴ in *sq. ft.* and *sq. in.*

(*a*) 27 sq. ft. $118\frac{2}{3}$ sq. in. = 27 sq. ft. $118\frac{8}{12}''$ = 27 sq. ft. 9′ 10″ 8‴.

(*b*) 46 sq. ft. 10′ 7″ 8‴ = 46 sq. ft. $127\frac{8}{12}''$ = 46 sq. ft. $127\frac{2}{3}$ sq. in.

Ex. 3. Convert 25 cub. ft. $1267\frac{5}{9}$ cub. in. into *Duodecimals*, and 48 cub. ft. 6′ 7″ 8‴ 9^{iv} 4^{v} into *cub ft.* and *cub. in.*

(*a*) 25 cub. ft. $1267\frac{5}{9}$ cub. in. = 25 cub. ft. 105″ 7‴ $6\frac{2}{3}^{iv}$
= 25 cub. ft. 8′. 9″. 7‴. $6^{iv}8^{v}$.

(*b*) 48 cub. ft. 6′ 7″ 8‴ $9^{iv}4^{v}$ = 48 cub. ft. 79″ 8‴ $9\frac{1}{3}^{iv}$
= 48 cub. ft. $956\frac{28}{36}$ cub. in.
= 48 cub. ft. $956\frac{7}{9}$ cub. in.

Examples CXXXIII.

1. Express in yards, feet and inches :—

(1) 7 ft. 3′ 6″ ; 25 ft. 11′ 10″ 8‴ ; 146 ft. 6′ 9″ 9‴.

(2) 18 sq. ft. 11′ 6″ 8‴; 216 sq. ft. 6′ 0″ 10‴ 6iv; 274 sq. ft. 4′ 10″ 0‴ 9iv.

(3) 20 cub. ft. 9′ 6′ 9‴ ; 11 cub. ft. 4′ 8″ 3‴ ; 83 cub. ft. 6′ 5″ 10‴ 8iv 8v.

(4) 25 ft. 8′ 3″ 9‴ ; 24 sq. ft. 11′ 3″ ; 2341 cub. ft. 5′ 6″ 8‴ 8iv.

2. Express in *Duodecimals* :—

(1) 13 ft. $5\frac{1}{4}$ in. ; 19 yds. 2 ft. $6\frac{5}{8}$ in. ; 7 yds. 1 ft. $6\frac{3}{8}$ in. ; 9 yds. 2 ft. $4\frac{3}{4}$ in.

(2) 50 sq. ft. 68 sq. in. ; 47sq. ft. 63sq. in. ; 33sq. yds. 3sq. ft. $28\frac{1}{4}$sq. in.

(3) 4 cub. ft. 1088 cub. in. ; 42 cub. ft. 334 cub. in. ; 39 cub. ft. $1120\frac{1}{3}$ cub. in. ; 18 cub. ft. 1664 cub. in.

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447. Before reckoning areas and volumes by Duodecimals, we must notice the relation which exists between the following products.

SQUARE MEASURE.			CUBIC MEASURE.		
feet × primes =	(superficial)	primes	supl. ft. × primes =	(solid)	primes.
feet × seconds =	„	seconds	„ ft. × seconds =	„	seconds.
feet × thirds =	„	thirds	„ ft. × thirds =	„	thirds
and so on.			and so on.		
primes × primes =	„	seconds	„ primes × primes =	„	seconds.
primes × seconds =	„	thirds	„ primes × seconds =	„	thirds
and so on.			and so on.		
seconds × seconds =	„	fourths	„ seconds × seconds =	„	fourths.
seconds × thirds =	„	fifths, &c.	„ seconds × thirds =	„	fifths, &c.

for 1 ft. × 1′ = 1 × $\frac{1}{12}$ sq. ft. = $\frac{1}{12}$ sq. ft. = 1′ ; 1 ft. × 1″ = 1 × $\frac{1}{144}$ sq. ft. = $\frac{1}{144}$ sq ft. = 1″ ; 1 ft. × 1‴ = 1 × $\frac{1}{1728}$ cub. ft. = $\frac{1}{1728}$ cub. ft. = 1‴ ; &c.

448. The operation employed to compute superficial and solid contents is that of Multiplication, conducted by means of a mixed *Decimal* and *Duodecimal* scale of Notation ; the figures of the feet being expressed and multiplied in the ordinary way, whilst in the other places the number 12 is always made use of instead of 10. The denomination on the left hand of the multiplier is used first, those of the multiplicand being taken as in other cases ; then the next in order, and so on ; and for the reason that we put the first figure of a *partial* product one place to the *left* of that of the preceding one when we begin with the least denomination of the multiplier, the terms of the product here must each be put one place to the *right* of those of the preceding, in order to possess their proper relative values ; and the addition is effected by beginning with the lowest denomination, as in compound quantities. The practical applications of the Rule will be best understood by the following Examples.

Ex. 1. Find the area of a rectangle whose adjacent sides are 5 ft. 3 in. and 4 ft. 9 in.

5 ft.	3′		
4	9		
21 sq. ft.	0′		4 ft. × 3′ = 12′ = 1 ft. 0′ ; *carry* 1 ft.
3	11	3″	4 ft. × 5 ft. + 1 ft. = 21 ft. ;
24 sq. ft.	11′	3″	9′ × 3′ = 27″ = 2′ 3″ ; 9′ × 5 ft. + 2′ = 47′ = 3 ft. 11′.

Thus the area = 24 sq. ft. 135 sq. in.

Ex. 2. Required the area of a square whose side is 7 ft. 8′ 9″.

7 ft.	8′	9″			7 ft. × 9″ = 63″ = 5′ 3″ ; *carry* 5′
7	8	9			7 ft. × 8′ + 5′ = 61′ = 5 ft. 1′ ; &c.
54 sq. ft.	1′	3″			8′ × 9″ = 72‴ = 6′ 0‴ ;
5	1	10	0‴		8′ × 8′ + 6″ = 70″ = 5′ 10″ ; &c.
	5	9	6	9⁗	9″ × 9″ = 81⁗ = 6‴ 9⁗ ; &c.
59 sq. ft.	8′	10″	6‴	9⁗	Thus the area = 59 sq. ft. $106\frac{9}{16}$ sq. in.

Ex. 3. Find the content of a rectangular solid whose lineal dimensions are 5 ft. 6 in., 4 ft. 5 in. and 3 ft. 4 in.

5 ft.	6′		
4	5		
22 sq. ft.	0		
2	3	6″	
24	3	6	
3	4		
72 c. ft.	10	6	
8	1	2	0‴
80 c. ft.	11	8	0‴

Thus the required volume
= 80 cub. ft. 11′ 8″
= 80 cub. ft. 1680 cub. in.

Ex. 4. Required the capacity of a cube the length of whose edge is 2 ft. 9 in.

The capacity = 2 ft. 9′ × 2 ft. 9′ × 2 ft. 9′ = 7 sq. ft. 6′ 9″ × 2 ft. 9′
= 20 cub. ft. 9′ 6″ 9‴ = 20 cub. ft. 1377 cub. in.

Examples CXXXIV.

1. Find by *Duodecimals* the areas of the following rectangles :—

(1) 14 ft. 6 in. by 12 ft. 7 in. (2) 25 ft. 7 in. by 7 ft. 10 in.

(3) 16 ft. 5′ by 12 ft. 11′. (4) 11 ft. 11′ by 2 ft. 3′ 4″.

(5) 9 ft. 4′ 7″ by 5 ft. 6′ 4″. (6) 17 ft. 3′ 4″ by 19 ft. 5′ 11″.

(7) 15 yds. 2 ft. $4\frac{1}{2}$ in. by 9 yds. 2 ft. $4\frac{3}{4}$ in.

(8) 207 ft. $4\frac{1}{4}$ in. by 95 ft. $7\frac{7}{12}$ in. (9) 17 ft. $3\frac{1}{2}$ in. by 12 ft. $6\frac{1}{4}$ in.

(10) 19 yds. 2 ft. $6\frac{5}{8}$ in. by 7 yds. 1 ft. $3\frac{7}{8}$ in.

(11) 10′ 3″ 4‴ by 5′ 0″ 6‴. (12) 13 ft. 2′ 6″ by 1′ 9″ 10‴.

2. Find by *Cross Multiplication* the volumes of each of the following solids :—

(1) 8 ft. 6 in. by 3 ft. 9 in. by 5 ft. 4 in.

(2) 3 ft. 7 in. × 5 ft. 8 in. × 2 ft. 11 in.

(3) 4 ft. 6′ by 5 ft. 7′ by 6 ft. 8′. (4) 8 ft. 9′ × 5 ft. 10′ × 3 ft 6′ 4″.

(5) 18 ft. 7′ 4″ by 17 ft. 3′ 9″ by 11 ft. 11″.

3. Find by *Duodecimals* the area of a square whose side is

(1) 2 yds. 1 ft. $3\frac{5}{8}$ in. (2) 123 ft. $6\frac{7}{8}$ in.

4. Find by *Duodecimals* the volume of a cube whose edge is

(1) 12 ft. 3 in. (2) 11 ft. 6′ 5″. (3) 3 yds. 1 ft. $7\frac{3}{8}$ in.

5. Divide 1532 ft. 9′ 9″ superficial measure by 18 ft. 9′ lineal measure.

IV. SQUARE & CUBIC MEASURES OF BENGAL.

449. In Bengal, the areas of rectangular fields, &c , are found by a method similar to the above, and is called **Suvankar's Method.**

Suvankar gives the following RULE for finding areas :—

Bigha × bigha = bigha	Bigha × chatak = chatak
Bigha × katha = katha	Katha × chatak = ganda
Katha × katha = dhûl	Chatak × chatak = kág.

20 dhûls = 1 katha ; 16 gandas = 1 dhûl ; 16 kags = 1 ganda.

Ex. Find the area of a rectangular field 5 bi. 14 kat. long and 4 bi. 13 kat. broad.

5 bi.	14 kat.	
4	13	
22	16	
3	14	2 dhûl.
26 bi.	10	2 dhûl.

4 bi. × 14 kat. = 56 kat. = 2 bi. 16 kat. ; *carry* 2 bi.
5 bi. × 4 bi. + 2 bi. = 22 bi. ; 13 kat. × 14 kat. = 182 dhûl = 9 kat. 2 dhûl ; *carry* 9 kat. ;
5 bi. × 13 kat. + 9 kat. = 74 kat. = 3 bi. 14 kat.
2 dhûl = 2 × 16 or 32 ga. = 1 ch. 12 ga.

∴ area = 26 bi. 10 kat. 2 dhûl = 26 bi. 10 kat. 1 ch. 12 ga.

450. The volume of a rectangular solid is found in the same way as in Art. 441.

Ex. Find the volume of a wall 48 cubits long, 12 cubits high and $1\frac{1}{2}$ cubits thick.

Volume of the wall = 48 × 12 × $1\frac{1}{2}$ cub. cubits = 864 cub. cubits.

Examples CXXXV

1. Find by *Suvankar's* method the area of :—

(1) 3 bi. by 2 bi. (2) 5 bi. by 16 kat.

(3) 1 bi. 12 kat. by 16 kat. (4) 8 bi. 13 kat. by 3 bi. 16 kat.

( ) 6 bi. 17 kat. by 5 bi. 6 kat. (6) 12 bi. 18 kat. by 10 bi. 12 kat.

(7) 10 bi. 18 kat. by 6 bi. 12 kat. (8) 8 bi. $12\frac{1}{4}$ kat. by 6 bi. 8 kat.
(9) 12 bi. $12\frac{1}{2}$ kat. by 10 bi. 16 kat. 5 ch.
(10) 12 bi. 16 kat. 10 ch. by 8 bi. 10 kat. 6 ch.
(11) 380 cubits by 260 cubits. (12) 724 cubits by 248 cubits.

2. Find the volume from the following dimensions :—

(1) 72, 14, 8 cubits. (2) 312, 16, $1\frac{3}{4}$ cubits.
(3) 480, 62, $5\frac{1}{2}$ cubits. (4) 248, 15, $7\frac{1}{4}$ cubits.
(5) 24 yds., 18 yds., 6 yds. (6) 58 yds., 25 yds., 6 ft.

CHAPTER X.

Miscellaneous Propositions.

451. The Unitary Method. (*Complex Cases.*)

In problems in the preceding Sections we have had to find the change in *one* quantity corresponding to the change in *one* other. In the Examples which follow *three* quantities are given and we have to find the change in *one* of them corresponding to given changes in the other *two.*

Ex. 1. If 40 acres of grass be mowed by 8 men in 7 days, how many acres will be mowed by 24 men in 28 days ?

In 7 days 8 men mow 40 acres,

∴ in 1 day 8 men mow $\frac{40}{7}$ acres,

∴ in 1 day 1 man mows $\frac{40}{8\times 7}$ acres,

∴ in 28 days 1 man mows $\frac{40\times 28}{8\times 7}$ acres,

∴ in 28 days 24 men mow $\frac{40\times 28\times 24}{8\times 7}$ acres or 480 ac. *Ans.*

Ex. 2. If the wages of 29 men for 54 days amount to *Rs.*74. 5*a.*; how many men must work 12 days to earn *Rs.*410 ?

Rs.$74\frac{5}{16}$ can be earned in 54 days by 29 men.

∴ *Re.*1....................in 54 days by $\frac{29\times 16}{1189}$ men,

∴ *Re.*1....................in 1 day by $\frac{29\times 16\times 54}{1189}$ men,

∴ *Rs* 410....................in 1 day by $\frac{29\times 16\times 54\times 410}{1189}$ men

∴ *Rs.*410. ....in 12 days by $\frac{29\times 16\times 54\times 410}{1189\times 12}$ men

or 720 men. *Ans.*

Ex. 3. If 6 men can do a piece of work in 30 days of 9 hours each, how many men will take to do 10 times the amount, if they work 25 days of 8 hours each ?

The work can be done in 30 days of 9 hrs. each by 6 men,

$\therefore$ ... ... 1 day of 9 hrs. each by (6×30) men,

$\therefore$ ... ... 1 day of 1 hr. each by $(6 \times 30 \times 9)$ men,

$\therefore$ ... ... 25 days of 1 hr. each by $\frac{6 \times 30 \times 9}{25}$ men,

$\therefore$ ... ... 25 days of 8 hrs. ... by $\frac{6 \times 30 \times 9}{25 \times 8}$ men.

$\therefore$ 10 times the work... ... 25 days of 8 hrs....by $\frac{6 \times 30 \times 9 \times 10}{25 \times 8}$ men

or 81 men. *Ans.*

Ex. 4. If 252 men in 5 days of 11 hours each, can dig a trench 210 yds. long, 3 yds. wide and 2 yds. deep ; in how many days of 10 hours each, can 24 men dig a trench 420 yds. long, 5 yds. wide and 3 yds. deep ?

The solid content of the first trench $= 210 \times 3 \times 2$ or 1260 cub. yds.

.......................second... $= 420 \times 5 \times 3$ or 6300 cub. yds.

Now, 252 men can dig 1260 cub. yds. in 5 days of 11 hrs. each.

$\therefore$ 1 man.........1260 cub. yds. in (5×252) days of 11 hrs....... .....

$\therefore$ 1 man..... ...1 cub. yd. in $\frac{5 \times 252}{1260}$ days of 11 hrs.. ....

$\therefore$ 24 men ....1 cub. yd. in $\frac{5 \times 252}{1260 \times 24}$ days of 11 hrs.............

$\therefore$ 24 men........6300 cub. yds. in $\frac{5 \times 252 \times 6300}{1260 \times 24}$ days of 11 hrs..........

$\therefore$ 24 men.......6300 cub. yds. in $\frac{5 \times 252 \times 6300 \times 11}{1260 \times 24}$ days of 1 ......

$\therefore$ 24 men.......6300 cub. yds. in $\frac{5 \times 252 \times 6300 \times 11}{1260 \times 24 \times 10}$ days of 10....... ..

or $288\frac{3}{4}$ days. *Ans.*

Ex. 5. If I get 8 chataks of bread for 4*a*. when wheat is *Rs*.7. 8*a*. a maund, what ought a maund of wheat to be when I get 12 ch. of bread for 2*a*. 8*p*. ?

The 4*a*. bread weighs 8 ch. when wheat is at *Rs*.$7\frac{1}{2}$ per md.

$\therefore$ ...1*a*. ... ... 8 ch. ... ... *Rs* $(\frac{15}{2} \times \frac{1}{4})$ per md.

$\therefore$ ...1*a*. ... ... 1 ch. ... ... *Rs*.$(\frac{15}{2} \times \frac{1}{4} \times 8)$ per md.

$\therefore$ ...$2\frac{2}{3}$*a*. ... ... 1 ch. ... ... *Rs*.$(15 \times \frac{8}{3})$ per md.

$\therefore$ ...$2\frac{2}{3}$*a*. ... ... 12 ch. ... ... *Rs*.$(\frac{15}{12} \times \frac{8}{3})$ per md.

or *Rs*.3. 5*a*. 4*p*. *Ans.*

Ex. 6. If 10 cannon, which fire 3 rounds in 5 minutes, kill 270 men in $1\frac{1}{2}$ hours, how many cannon, which fire 5 rounds in 6 minutes will kill 500 men in one hour?

The first set in $1\frac{1}{2}$ hours firing 3 rounds in 5 min. make $(3 \times 1\frac{1}{2} \times 60 \div 5)$ or 54 rounds; and the second set in one hour firing 5 rounds in 6 min. make $(5 \times 60 \div 6)$ or 50 rounds.

Now, in 54 rounds 270 men are killed by 10 cannon,

∴ ...... 1 round 270 men .... 10×54......

∴ ...... 1 round 1 man is .... $\frac{10 \times 54}{270}$ ......

∴ ...... 50 rounds 1 man .... $\frac{10 \times 54}{270 \times 50}$......

∴ ...... 50 rounds 500 men are .... $\frac{10 \times 54 \times 500}{270 \times 50}$......

or 20 cannon. *Ans.*

Examples CXXXVI.

1. If the wages of 4 men for 12 days be *Rs.*6, what would be the wages of 6 men for 10 days?

2. If 9 men earn *Rs* 40. 8*a.* in 24 days, how many must work 16 days to earn *Rs.*450?

3. If a regiment of 939 soldiers consume 351 mds. of wheat in 168 days, how many soldiers will consume 1404 mds. in 56 days?

4. If I pay *Rs.*5 for the carriage of 2 tons for 6 miles, what must I pay for the carriage of 12 tons 17 cwt. for 34 miles?

5. If the wages of 29 men for 54 days amount to £80. 9*s.* 6*d.*, how many men must work 12 days to receive £407?

6. If the gas for 5 burners, lighted 5 hours every day, for 10 days, cost *Rs.*2. 2*a.*, how many burners may be lighted 4 hours every evening for 15 days at a cost of *Rs.*38. 4*a.*?

7. If the carriage of 60 cwt. for 20 miles cost *Rs.*145, what weight can I have carried 30 miles for *Rs.*54. 6*a.*?

8. If 5 men can reap a field of $12\frac{1}{2}$ bighas in $3\frac{1}{2}$ days, working 16 hrs. a day, in what time can 7 men reap a field of 15 bighas working 12 hrs. a day?

9. If 200 men in 12 days of 8 hours each can dig a trench 160 yds. long, 6 wide and 4 deep, in how many days of 10 hours each will 90 men dig a trench 450 yds. long, 4 wide and 3 deep?

10. If the carriage of goods weighing 5 cwt. 2 qrs. 12 lbs. for 150 miles come to *Rs.*32. 11*a.* 4*p.*, what will be the charge for carrying 4 waggon-loads of the same, each weighing 7 cwt. 2 lbs. a distance of 450 miles?

11. If 15 pumps, working 8 hours a day, can raise 1260 tons of water in 7 days ; how many pumps, working 12 hours a day, will be required to raise 7560 tons of water in 14 days ?

12. If with a capital of *Rs.*10000 a person gains by trade *Rs.*500 in 16 months, in how many months will he gain *Rs.*1250 with a capital of *Rs.*4000 ?

13. If when wheat is at *Rs* 3 per maund, the 4*a* loaf weighs 8 chataks, what should be the price of wheat per maund, when 3 sr. 2 ch. of bread cost 12*a.* 6*p.* ?

14. If the 4*d.* loaf weigh 1 lb. 11 oz. 12 drs., when wheat is at 7*s.* 6*d.* per bushel, what ought the 6*d.* loaf to weigh when wheat is at 5*s.* 3*d.* per bushel ?

15. If 15 horses and 148 sheep can be kept for 9 days for *Rs.*757. 8*a.*, what sum will keep 10 horses and 132 sheep for 8 days, supposing 5 horses eat as much as 84 sheep ?

16. If *Rs.*240 be paid for bread for 49 persons for 20 months, when wheat is at *Rs.*3 per maund ; how long will *Rs.*234 find bread for 91 persons, when wheat is at *Rs.*3. 8*a.* per maund ?

17. If 5 men and 7 boys can reap a field of 125 acres in 15 days, in how many days will 10 men and 3 boys reap a field of 75 acres, each boy's work being one-third of a man's ?

18. If 44 cannon, firing 30 rounds an hour for 3 hours a day, consume 300 barrels of powder in 5 days, how long will 40 barrels last 66 cannon, firing 40 rounds an hour for 5 hours a day ?

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452. Problems in simultaneous Equations.

Proceed as in the following Examples.

Ex. 1. If 9 horses and 7 cows cost *Rs.*850, and 5 horses and 8 cows cost *Rs.*575 ; find the cost of a horse and of a cow.

The cost of 9 horses and 7 cows = *Rs.*850,

∴ the cost of 45 horses and 35 cows = *Rs.*850 × 5 or *Rs.*4250.

Again, the cost of 5 horses and 8 cows = *Rs.*575,

∴ the cost of 45 horses and 72 cows = *Rs.*575 × 9 or *Rs.*5175.

Hence, by subtraction, we get

the cost of 37 cows = *Rs.*925 ;

∴ the cost of a cow = *Rs.*925 ÷ 37 = *Rs.*25. *Ans.*

Again, the cost of 9 horses and *Rs.*175 = *Rs.*850, from (1)

∴ the cost of 9 horses = *Rs.*675 ;

∴ the cost of a horse = *Rs.*675 ÷ 9 = *Rs.*75. *Ans.*

Ex. 2. If 2 men and 3 boys can do $\frac{1}{4}$ of a piece of work in 2 days, and 3 men and 5 boys can do $\frac{19}{16}$ of it in 6 days ; in what time can a man do the work ?

In 2 days, 2 men and 3 boys can do $\frac{1}{4}$ of the work,
∴ ... 1 day, 2 men and 3 boys ... $\frac{1}{8}$ ... ...
∴ ... 1 day, 10 men and 15 boys ... $\frac{5}{8}$ ... ...
Again, in 6 days, 3 men and 5 boys ... $\frac{19}{16}$ ... ...
∴ ... 1 day, 3 men and 5 boys ... $\frac{19}{96}$ ... ...
∴ ... 1 day, 9 men and 15 boys .. $\frac{19}{96}\times 3$ or $\frac{19}{32}$ of the work.

Hence, by subtraction, we get
in 1 day, 1 man can do $(\frac{5}{8}-\frac{19}{32})$ or $\frac{1}{32}$ of the work.

∴ a man can do the whole in 32 days. *Ans.*

Ex. 3. If 3 men with 4 boys earn *Rs.*58 in 8 days, and 2 men with 3 boys earn *Rs.*40 in the same time; in what time will 6 men and 7 boys earn *Rs.*210?

Since 3 men and 4 boys earn in 8 days, *Rs.*58,
∴ 3 men and 4 boys earn in 1 day *Rs.*58 ÷ 8 = *Rs.*7. 4*a.*
Also 2 men and 3 boys earn in 1 day *Rs* 40 ÷ 8 = *Rs.*5.

Hence, by subtraction, we get
1 man and 1 boy earn in 1 day *Rs.*2. 4*a.*
∴ 2 men and 2 boys ... ... *Rs.*2. 4*a.* × 2 = *Rs.*4. 8*a.*
But 2 men and 3 boys ... ... *Rs.*5.
∴ by subtraction, 1 boy earns in 1 day 8*a.*

Again, since 1 man and 1 boy earn in 1 day *Rs.*2. 4*a.*
∴ 6 men and 6 boys earn in 1 day *Rs.*2. 4*a.* × 6 = *Rs.*13. 8*a.*
and since 1 boy earns in 1 day 8*a.*
∴ 6 men and 7 boys earn in 1 day *Rs.*14.

∴ the no. of days required $= \frac{Rs.210}{Rs.14} = 15$. *Ans.*

Examples CXXXVII.

1. 6 horses and 7 cows can be bought for *Rs.*2500, and 13 cows and 11 horses can be bought for *Rs.*4610. What is the value of each animal?

2. If 15 lbs. of tea and 17 lbs. of coffee together cost *Rs.*32. 12*a.*, and 25 lbs. of tea and 13 lbs. of coffee together cost *Rs.*43. 1*a.* 4*p.*; find the price of each per pound.

3. The price of 2 turkeys and 9 fowls is £3, and the price of 5 turkeys and 3 fowls is £4. 5*s.*; find the price of a turkey and of a fowl.

4. If 3 men and 5 women do a piece of work in 8 days, which 2 men and 7 children can do in 12 days; find how long 13 men, 14 children and 15 women working together will take to do it.

5. If 5 men with 7 boys can earn *Rs.*76. 8*a.* in 6 days, and 2 men with 3 boys can earn *Rs.*21 in 4 days, in what time will 6 men with 12 boys earn *Rs.*600?

6. If 8 men and 5 boys can reap 29 acres in 3 days, and 6 men and 7 boys can reap 50 acres in 6 days, how long will it take 3 men and 6 boys to reap 15 acres ?

7. If 5 men and 3 boys can reap 23 acres in 4 days, and if 3 men and 2 boys can reap 7 acres in 2 days, how many boys must assist 7 men, in order that they may reap 45 acres in 6 days ?

8. If 2 boys and 1 man can do a piece of work in 4 hours and 2 men and 1 boy can do the same in 3 hours ; find in what time a man, a boy, and a man and a boy together, respectively, can do the same.

9. If 2 men and 5 boys can do $\frac{16}{21}$ of a work in 3 days, and 3 men and 7 boys can do $\frac{11}{15}$ of it in 2 days ; in what time will a boy be able to do the whole work ?

10. A farmer parting with his stock sells to one person 9 horses and 7 cows for *Rs.*300 ; and to another at the same prices, 6 horses and 13 cows for the same sum. What is the price of each ?

11. In a certain employment, 9 men and 7 women received together *Rs* 4. 7*a.* 2*p.* for their wages, and it is found that 7 men receive *Re.*1. 3*a.* 8*p.* more than 5 women ; required the wages of each.

12. If 17 ducks and 20 chickens are worth *Rs.*29. 14*a.* and at the same average prices 15 chickens and 31 ducks are worth *Rs.*38. 6*a.* ; how many ducks are worth 14 chickens ?

453. Least Common Multiple.

Ex. 1. Find the *least* number which, being divided by 2, 3, 4, 5, 6, 7, shall give the remainders 1, 2, 3, 4, 5, 6, respectively.

Since $2-1=1$; $3-2=1$; $4-3=1$; and so on,

∴ the remainder in each case is less than the divisor by 1.

Now, the L. C. M. of 2, 3, 4, 5, 6 and 7 is 420.

∴ the reqd. least number $=420-1=\underline{419}$. *Ans.*

Ex. 2. Find the *least* number which, being divided by 2, 4, 6, 8, 10 and 12, leaves in each case a remainder 1, but when divided by 13, leaves no remainder.

The L. C. M. of 2, 4, 6, 8, 10 and 12 is 120.

∴ the reqd. number $=120k+1$, where k is a positive integer.

Making $k=1$, 2, 3, 4, 5, &c., the reqd. number is one of the numbers 121, 241, 361, 481, 601, &c.

Again, since the number is divisible by 13, therefore of the above numbers that which is divisible by 13 is the reqd. number.

Now, the first of the above numbers which is divisible by 13 (by trial) is 481.

Hence the reqd. least number $=\underline{481}$. *Ans.*

Examples CXXXVIII.

1. Find the *least* number which, when separately divided by 6, 9, 12, 15 and 21, leaves in each case a remainder which is the G. C. M. of the above divisors.

2. Find the *least* number which, being divided by 2, 3, 4, 5, 6, 7 and 8, shall give the remainders 1, 2, 3, 4, 5, 6 and 7 respectively.

3. Find the *least* number which, when divided by 6, 9, 12, 15 and 18, leaves in each case a remainder 3, but when divided by 21 leaves no remainder.

4. Find the *least* number which, when divided by 8, 10, 12, 14 and 16, leaves the remainders 6, 8, 10, 12 and 14 respectively, but when divided by 17 leaves no remainder.

5. Find the *least* number that is divisible by 22, but being divided by 6, 9, 12, 15 and 18 gives 4, 7, 10, 13 and 16 as remainders respectively.

6. Find the *least* number which, being divided by 2, 3, 4, 5, 6 and 7, gives the remainders 1, 2, 3, 4, 5 and 6 respectively.

7. What *greatest* number and what *least* number can be subtracted from 902510 that the remainders may be divisible by 28, 32, 40 and 45?

8. What *least* number must be added to 17346 that the sum may be divisible by 48, 64, 72, 96 and 108?

9. What *least* number must be added to 17287 that the sum being divided by 26, 35, 38, 91 and 95 respectively shall leave in each case a remainder 2?

10. What *greatest* number and what *least* number must be subtracted from 1100 that the remainders being severally divided by 4, 12, 20, 24, 36 and 45 shall give a remainder 3 in each case?

11. What *greatest* number of 7 digits is that, which, being divided by 7, 9, 11, 13 and 15, leaves the remainders 5, 7, 9, 11 and 13 respectively?

12. What *least* number of 8 digits is that, which, being divided by 24, 30, 36, 42 and 50, leaves the remainders 14, 20, 26, 32 and 40 respectively?

454. Races and Games of Skill. If *A* and *B* start from the same place at the same time to run a distance of 100 yds., and if while *A* arrives at the goal, *B* has run only 80 yds.; then *A* **gives** *B* (100 − 80) or 20 yds. in 100, and *B* **gets** or **takes** 20 yds. in 100. Also, if *A* and *B* start at the same time, but *B* 20 yds. in advance of *A* and arrive at the goal at the same instant, then *A* is

said to give B 20 yds. **start**, and that they run a **dead heat**. So in a game of skill, if B can make only 90 **points** while A makes 100, A **gives** B (100 − 90) or 10 points out of 100, and B **gets** 10 out of 100.

Ex. 1. In a race of 500 yds., A can beat B by 50 yds., and B can beat C by 50 yds.; by how much will A beat C in a race of 200 yds.?

A can run 500 yds. while B runs (500 − 50) or 450 yds.

$\therefore A$ ... ... $\frac{500}{450}$ yds. ... ... 1 yd.

$\therefore A$ ... ... $\frac{500}{450} \times 500$ or $\frac{5000}{9}$ yds., while B runs 500 yds

But when B runs 500 yds., C can run (500 − 50) or 450 yds.

$\therefore A$ can run $\frac{5000}{9}$ yds., while C runs 450 yds.

$\therefore A$ ... ... 5000 yds. ... ... 450×9 yds.

$\therefore A$ ... ... 100 yds. ... ... $\frac{450 \times 9}{50}$ yds.

$\therefore A$ ... ... 200 yds. ... ... $\frac{450 \times 9 \times 2}{50}$ or 162 yds.

Hence A can beat C by (200 − 162) or 38 yds. in 200 yds. *Ans.*

Ex. 2. At a game of billiards, A can give B 15 points in 50 and he can give C 20 in 50; how many can B give C in a game of 70?

While A makes 50 points, B makes (50 − 15) or 35;

and ... ... C makes (50 − 20) or 30;

$\therefore$ while B makes 35 points, C makes 30;

$\therefore$ while B makes 70 points, C makes 30×2 or 60.

Hence B can give C (70 − 60) or 10 points in a game of 70. *Ans.*

Examples CXXXIX.

1. A can beat B by 5 yds. in a 100 yds. race, and B can beat C by 10 yds. in a 200 yds. race. By how much can A beat C in a 400 yds. race?

2. In a race of 200 yds., P can beat Q by 31 yds. and R by 18 yds.; by how many yds. could R beat Q in 350 yds.?

3. In a mile race, A can beat B by 20 yds., and B can beat C by 20 yds. How many yds. start can A give C that there may be a dead heat?

4. In a game at rackets A can give C 10 points out of 15, and B can give C 8. How many points can A give B?

5. In a mile race, *A* beats *B* by 60 yds., and *B* beats *C* by 80 yds. By how much will *A* beat *C* in a race of 400 yds.?

6. In a mile race *A* wins, *B* being 11 yds. behind, and *C* 64 yds. behind *A*. How much would *C* be behind *B* in a 2 miles race?

7. In a game of skill, *A* can give *B* 8 points out of 40 and *B* can give *C* 10 points out of 50; how many can *A* give *C* out of a game of 25?

8. In a mile race *A* gives *B* 100 yds. start, and beats him by 20 yds. If B can run the mile in 5 min. 8 sec., how long did *A* take?

9. In a 440 yds. race, *A* beats *B* by 20 yds., and *C* by 41 yds.; *B* can also give *C* a start of 12 sec in a mile race. In what time can each run a mile?

10. In a race *P* gained 25 yds. upon *Q* in every 125 yds., and finally won by 70 yds.; find the length of the course.

11. At billiards *A* can give *B* 15 points and to *C* 20 points out of 50; how many can *B* give *C* in 70 for an even match?

12. *A* runs 20 yds. while *B* runs 21 yds.; *B* runs 31 yds. while *C* runs 30 yds.; if *A* can run a mile in 5 min. 15 sec., what time will *C* take to do it?

13. In a half-mile race *A* gives *B* 10 yds. start and beats him by 20 yds.; *B* gives *C* 30 yds. start in half-a-mile, and is beaten by 60 yds Which runs the faster, *A* or *C*?

14. At a game at fives, out of 15 points *A* can give *B* 3; also *A* can give *C* 7 points; how many points can *B* give *C* so as to make an even match?

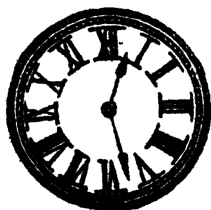
15. *A*, *B*, *C* and *D* run a race over 1 mile. First *A* and *B* race, when *A* wins by 20 yds.; then *C* and *D* race, when *C* wins by 60 yds.; next *B* and *D* race, when *B* wins by 40 yds. If *A* and *C* race, which will win and by how much?

16. In a mile race *A* beats *B* by 80 yds. and *C* by 20 sec., also *B* beats *C* by 5 seconds in the same race. How long will *A* take to run the whole distance?

17. At a game of skill *A* can give *B* 8 points out of 38, and to *C* 10 points out of 95. Of *B* and *C*, which is the better player, and how many points can he give the other in 340?

18. *A* can give *B* 400 yds. and *C* 500 yds. in a mile race; if *C* can run the mile in 10 minutes, in what times can *A* and *B* run the same?

455. Hands of a Clock. On the right is the diagram of a clock with its two hands that move about a common centre. The longer of these hands is called the *minute-hand* and indicates the minutes, and the shorter the *hour-hand* and indicates the hours. The minute-hand takes one hour or 60 minutes to travel round the dial plate once, while the hour-hand moves only 5 minutes round. Therefore the minute-hand travels 12 minutes for every minute of the hour-hand or in other words the minute-hand *gains* $(12-1)$ or 11 minutes for every 12 minutes it advances, or 1 minute for every $\frac{12}{11}$ minutes it advances. Hence, to find the time in which the minute-hand is to gain a certain number of minutes over the hour-hand, multiply the given number of minutes by $\frac{12}{11}$.



It should also be noticed here that if the minute-hand is 15 min. either *before* or *behind* the hour-hand, the hands are *at right angles* ; and if the minute-hand is 30 min. either *before* or *behind* the hour-hand, the hands are *opposite to each other.*

Ex. 1. At what times between 7 and 8 o'clock will the hands of a clock be (i) together, (ii) at right angles, and (iii) opposite to each other?

(i) At 7 o'clock the minute-hand points to 12 and the hour-hand to 7, so that the minute-hand is 5×7 or 35 min. behind the hour-hand. Now, in order that the hands may be together, the minute-hand shall have to gain these 35 min. over the hour-hand. But the minute-hand takes $\frac{12}{11}$ min. to gain 1 min. ; therefore the required time $=(\frac{12}{11} \times 35)$ min. or $38\frac{2}{11}$ min. past 7.

(ii) At 7 o'clock the minute-hand is 35 min. behind the hour-hand. Now to be at right angles, the minute-hand shall be 15 min. either *behind* or *before* the hour-hand ; therefore the minute-hand shall have to gain either $(35-15)$ min. or $(35+15)$ min., *i e*, either 20 min. or 50 min. Hence the required time is either $(\frac{12}{11} \times 20)$ min. or $(\frac{12}{11} \times 50)$ min. past 7, *i. e*, either $21\frac{9}{11}$ min. or $54\frac{6}{11}$ min. past 7.

(iii) At 7 o'clock the minute-hand is 35 min. behind the hour-hand. Now to be opposite to each other, the minute-hand shall be 30 min. either *behind* or *before* the hour-hand, and this can happen only once *behind*, when the minute-hand has gained $(35-30)$ min. or 5 min. over the hour-hand. Therefore the required time is $(\frac{12}{11} \times 5)$ min. or $5\frac{5}{11}$ min. past 7.

Ex. 2 When will the hands of a clock be (i) at right angles, (ii) 10 min. apart, between 10 and 11 o'clock?

(i) At 10 o'clock the minute-hand is 10 min. before the other ; so that the two hands will be at right angles when the minute-hand

has gained either (15 – 10) or 5 min. more, or (45 – 10) or 35 min. more. Hence the required times are either $(\frac{12}{11}\times 5)$ min. or $(\frac{12}{11}\times 35)$ min., *i.e.*, either $5\frac{5}{11}$ min. or $38\frac{2}{11}$ min. past 10.

(ii) At 10 o'clock the minute-hand is 10 min. before the other. Again, the hands will be 10 min. apart, when the minute-hand has gained (10 + 30) or 40 min. over the hour-hand. Hence the required time is $(\frac{12}{11}\times 40)$ min. or $43\frac{7}{11}$ min. past 10.

Ex. 3. I left home between 12 and 1 o'clock P. M., and on my return between 5 and 6 o'clock P.M., the hands have exactly changed places. Find the time of my departure.

When the *H* hand and the *M* hand change places, the *H* hand moves through a space equal to the original interval between *H* and *M* and the *M* hand moves through a space equal to a certain number of rounds of the dial, *minus* the original interval between H and M.

Now the M hand moves 12 times as fast as the H hand and in this case the number of rounds of the dial is evidently 5 ; ∴ the M hand moves through 5 rounds of the dial *minus* the original interval between H and M and this space must be equal to 12 times the original interval between H and M. Hence 5 rounds of the dial = 13 times the original interval between H and M ; ∴ the original interval between H and M $=\frac{5}{13}\times 60$ min. spaces $=23\frac{1}{13}$ min. spaces. (See Fig. Art. 455.)

Again, at 12 o'clock the H and M hands were coincident, ∴ the M hand must have to gain $23\frac{1}{13}$ min. spaces over the H hand, and ∵ it gains 11 min. spaces in every 12 min. spaces, ∴ the time in which it gains $23\frac{1}{13}$ min. spaces $=\frac{12}{11}\times\frac{300}{13}$ min. $=25\frac{25}{143}$ min. Hence the time of my departure is 12. $25\frac{25}{143}$ min. P. M.

Examples CXL.

1. At what times between the hours stated below are the hour and minute-hands of a clock (i) together ? (ii) at right angles ? and (iii) directly opposite to each other ?

(1) 1 and 2.	(2) 2 and 3.	(3) 3 and 4.	(4) 4 and 5.
(5) 5 and 6.	(6) 6 and 7.	(7) 7 and 8.	(8) 8 and 9.
(9) 9 and 10.	(10) 10 and 11.	(11) 11 and 12.	(12) 12 and 1.

2. When will the hour and minute-hands of a clock be (i) 10 min. apart ? (ii) 25 min. apart ? between the hours of :—

(1) 3 and 4.	(2) 4 and 5.	(3) 6 and 7.	(4) 7 and 8.

3. At 3 o'clock the hour and minute-hands of a clock are at right angles. How often will they be at right angles to each other before striking 5 ?

4. At 12 o'clock the two hands of a clock are coincident. How often will they be coincident before striking 4?

5. At 6 o'clock the two hands are in the same straight line. How often will they be in the same straight line before striking 9?

6. In the astronomical clock, the hours are marked upon the dial from 1 up to 24; find the time between 8 and 9 o'clock when the hands are together.

7. I left home between 4 and 5, and on my return between 7 and 8, the hands have exactly changed places. Calculate the hour of departure.

8. It is between 6 and 7, and the number of minute spaces between the two hands of a clock is $\frac{1}{2}$ of what it was $8\frac{8}{11}$ min. ago. What is the time?

9. I left home between 3 and 4, and on my return between 8 and 9, I found that the hands of the watch had exactly changed places. When did I return?

10. If the hands of a clock coincide every 65 minutes, how much does the clock gain or lose in a day?

11. A clock in which the hour-hand has been displaced shews the time to be 16 min. past 3, and the two hands are together; the time is between 3 and 4 o'clock. Find by how many minute divisions the hour-hand has been displaced.

12. A clock is 10 min. too fast at noon; it loses 2 min in an hour; find the true time when the hands are (i) at right angles, (ii) directly opposite, and (iii) coincident, between 4 and 5 o'clock?

13. A clock is 10 min. too slow at noon, and gains $2\frac{1}{2}$ min. in an hour; what will be the true time when the hands are (i) coincident, (ii) directly opposite and (iii) at right angles, for the fourth time afternoon?

14. A clock which loses 10 sec. per hour is set to the correct time at 9-15 A. M. on Monday. What will be the correct time by the clock when its hour and minute hands point in exactly opposite directions between 9 and 10 o'clock P. M. in the evening? How must the hands be altered, so that the clock may show correct time at noon on Tuesday?

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456. Clocks. When a clock indicates 10 minutes *before* the true time, it is said to be 10 min. **too fast**; and when it indicates 10 min. *behind* the true time, it is said to be 10 min. **too slow**.

Thus, if a clock indicate 9-10 when the correct time is 9, it said to be 10 min. *too fast*; but if, on the other hand, it indicates 8-5 at the same hour, it is called 10 min. *too slow*.

Ex. 1. A clock was 10 min. too slow at noon on Monday ; on Friday at the same hour it is 10 min. too fast. When will it again shew correct time ?

From Monday noon to Friday noon, there are 4 days, and in these 4 days the clock has gained (10+10) or 20 min. Therefore in one day the clock gains (20÷4) or 5 min. Now at noon on Friday the clock indicates 12 hrs. 10 min. ; therefore it shall have to gain (12 hrs.−10 min.) or 710 min. in order to shew correct time. But it gains 5 min. per day ; ∴ it will gain these 710 min. in (710÷5) or <u>142</u> days.

Ex. 2. Two clocks are set right at noon on Monday ; one gains 4 min. and the other loses 2 min. in 24 hrs. (i) When will there be a difference of one hour between the times indicated by them ? (ii) What time will the first indicate when the second indicates noon, 4 days after ? (iii) What time will the second indicate, when the first indicates 6 P. M. on the following Friday ? (iv) When will be the true time when the first indicates 3 P. M. on the following Wednesday ? (v) What will be the true time when the second indicates 8 A. M. on the following Saturday ?

(i) Since the first gains 4 min and the second loses 2 min. in 24 hours, ∴ they will differ by (4+2) or 6 min. in 24 hrs. or 1 day. Therefore they will differ by 1 hour or 60 min. in 10 days. Hence the required time is Thursday noon, 10 days later.

(ii) When the second indicates 1 hr. 58 min. or $\frac{719}{30}$ hrs., the first indicates 24 hrs. 4 min. or 6′ *more*.

∴ when the second indicates 1 hr., the first indicates $6 \times \frac{30}{719}$ min. more
∴ ... ... (4 × 24) hrs.........$\frac{180}{719} \times 4 \times 24$ min. „
or $24\frac{24}{719}$ min. „

Hence the first will indicate 12 hrs. $24\frac{24}{719}$ min. P. M., when the second indicates noon, 4 days after.

(iii) From Monday noon to Friday 6 P. M. there are 102 hours. When the first indicates $24\frac{1}{15}$ hrs., the second indicates $23\frac{29}{30}$ or 6′ less.
∴ ... ... 1 hr. ... ... $6 \times \frac{15}{361}$ min. „
∴ ... ... 102 hrs. ... ... $\frac{90}{361} \times 102$ min. „
or $25\frac{155}{361}$ min. „

Hence the second will indicate $25\frac{155}{361}$ min. less, or 5 hrs. $34\frac{206}{361}$ min. P. M. when the first indicates 6 P. M. on the following Friday.

(iv) From Monday noon to Wednesday 3 P. M. there are 51 hours. When the first indicates $24\frac{1}{15}$ hrs., the true time is 4′ less ;
∴ ... ... 1 hr. ... ... $4 \times \frac{15}{361}$ min. less ;
∴ ... ... 51 hrs. ... ... $\frac{60}{361} \times 51$ min. „
or $8\frac{172}{361}$ min. „

Hence the true time is $8\frac{172}{361}$ min. behind or 2 hrs. $51\frac{189}{361}$ min. P. M. when the first indicates 3 P. M. on the following Wednesday.

(v) From Monday noon to Saturday 8 A. M. there are 116 hours.

When the second indicates $23\frac{59}{60}$ hrs., the true time is 2 min. more.

$\therefore$ ... ... 1 hr. ... ... $2 \times \frac{30}{719}$ min. more.

$\therefore$ ... ... 116 hrs. ... ... $\frac{60}{719} \times 116$ min.

or $9\frac{489}{719}$ min. ...

Hence the true time is $9\frac{489}{719}$ min. before, or 8 hrs. $9\frac{489}{719}$ min. A. M. when the second indicates 8 A. M. on the following Saturday.

Ex. 3. Two clocks commence to strike 10 together. The 5th stroke of the second is coincident with the third stroke of the first. If the first strike all its strokes in 18 seconds, find the interval between the 8th strokes of the two clocks.

Since the clocks begin striking together, and the 5th stroke of the second is coincident with the 3rd stroke of the first, it is clear that the time taken by the second for 4 strokes is the same as that taken by the first for 2 strokes, or time for 2 strokes of the second = time for 1 stroke of the first. Again, since the first strikes all its strokes, *i.e.*, 9 strokes after it begins striking, in 18 seconds, we see that each of the strokes of the first comes after an interval of 2 seconds. Therefore each stroke of the second clock comes after an interval of 1 second. The 8th stroke of the first would thus come 14 seconds, and the 8th stroke of the second would come 7 seconds after the striking begins. Therefore the interval between the 8th strokes would be $(14-7)$ or 7 seconds. *Ans.*

Examples CXLI.

1. A clock was 10 min. slow 25 days ago, and to-day at the same hour is 10 min. fast ; when will it again shew true time ?

2. A watch which is 10 minutes too fast at 12 o'clock on Monday, gains 3 min. 10 sec. per day ; what will be the time by the watch at a quarter-past 10 A. M. on the following Saturday ?

3. Of two clocks, one gains 10 min. and the other loses $7\frac{1}{2}$ min. in 24 hrs. ; what will be the difference of the times indicated by them at 6 o'clock A. M. on Friday, if they are together at noon on the preceding Tuesday ?

4. Two clocks point out 12 at the same instant ; one of them gains 7 sec. and the other loses 8 sec in 12 hours ; after what interval will one have gained half an hour on the other and what o'clock will each then shew ?

5. A church clock is set at 12 o'clock on Saturday night ; at noon on Tuesday it is 3 min. too fast ; supposing the rate regular, find the true time when the clock strikes four on Thursday afternoon.

6. A watch set accurately at 12 o'clock indicates 10 min. to 5 at 5 o'clock P. M. What is the time when the watch indicates 5 o'clock ?

7. If one watch loses and another gains at the rate of 1 min. a day, and they are both set right at noon on Monday; what time will be indicated by the latter, when the former points to 10 hrs. $49\frac{1}{11}$ min. P. M on the following Saturday?

8. A clock, which is 4 min. $8\frac{9}{88}$ sec. too fast at half past 9 A. M. on Tuesday, loses 2 min. 45 sec. daily; what will be the time indicated by the clock at a quarter-past 5 P. M., on the following Friday?

9. Two clocks, one gaining 3 min. and the other losing 2 min. per day, are set right at noon. What is the time by the first clock when the second indicates noon a week afterwards?

10. A clock which loses 4 min. in 12 hours is 10 min. too fast at midnight on Sunday. What o'clock will it indicate at 6 o'clock on Wednesday evening?

11. A watch, which was 5 min. 40 sec. fast on Monday at noon, is 2 min. 51 sec. fast at midnight on the following Sunday; what did it lose in a day?

12. A clock which gains $7\frac{1}{2}$ min. in 24 hours is 12 min. fast at midnight on Sunday. What o'clock will it indicate at 4 o'clock on Wednesday afternoon?

13. A clock gains $3\frac{1}{2}$ min. a day; how must the hands be placed at noon so as to point to the true time at 7 hrs. 30 min. P. M.?

14. One clock gains 4 min. in 12 hours, and another loses 4 min. in 24 hours. They are set right at noon on Monday. Determine the time indicated by each clock, when the one appears to have gained $16\frac{1}{2}$ min. on the other.

15. Two clocks begin to strike twelve together; one strikes in 35 seconds, the other in 25; what fraction of a minute is there between their seventh strokes?

16. Two clocks strike 9 together on Tuesday morning. On Wednesday morning one wants 10 min. to 11 when the other strikes 11. How much must the faster be put back, that they may strike 9 together on Wednesday evening?

17. One clock strikes four times while another strikes three. It is observed that they both begin striking a certain hour together, and that the last stroke of one is simultaneous with the last stroke but two of the other. What o'clock is it?

18. A clock, which was 12 min. fast at a quarter to 11 P. M. on Nov. 28, was exactly right at 11-30 P. M. on the following day. How many minutes was it slow at a quarter to 2. P. M. on Dec. 7?

19. A clock, which was 1·4 min. fast at a quarter to 11 P. M. on Dec. 2, was 8 min. slow at 9 A. M. on Dec. 7. When was it exactly right?

20. A watch gains 3 min. every day. How should it be set at 9 P. M. on Friday, so that it should denote right time at 12 noon the next day?

21. Two clocks, of which one gains 3 min. and the other 2 min. a day, are set right at 10 A. M. What time will the first clock denote when the second denotes 40 min. past 8 P. M.?

22. On Monday it was 8 A. M. by a watch, and 55 min. past 7 A. M. by a clock; 3 days afterwards, it was 8 A M by the watch and 58 min. past 7 A. M. by the clock. If the clock gains 30 seconds per day, how much does the watch gain or lose daily?

23. Two clocks begin to strike 12 together. The second stroke of one is coincident with the third stroke of the other. If the first strike all the strokes in 18 seconds, find the time between their 11th strokes.

24. Two clocks commence striking a certain hour at the same instant. The third stroke of one is simultaneous with the fourth stroke of the other, and the first strikes thrice after the second has stopped. What is the hour?

25. At 10 minutes to 2 in the afternoon a clock is 55 seconds slow, and at 6 in the evening it is 30 seconds slow; at what hour will it shew true time?

26. A clock loses 5 min. a day. It shews correct time at noon on a Monday; after how many days will it again shew correct time on a Monday?

27. A clock which was 16 min. too slow 24 days ago, is 16 min. too fast to-day at the same hour. When did it last shew correct time, and when will it shew correct time again?

28 I set my watch at noon on Saturday; at 10-30 A. M. on Monday it had gained $3\frac{1}{2}$ min.; what will be the real time on Tuesday when my watch is at 3-30 P. M.?

29. Two clocks are together at noon on March 1st; if one gains uniformly 3 min. a day, and the other 36 sec. a day, when will the difference between them be $3\frac{1}{4}$ hours?

30. One clock gains 3 min. in 24 hours, and a second gains 3 min. in 15 hours; the first is put right on January 1st, the second on January 2nd; when will they indicate the same time?

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457. Temperature. *Temperature* in bodies is measured by an instrument called a **Thermometer**, which consists of an air-tight glass tube having a reservoir of mercury or alcohol at one end of it. The tube is *graduated* or marked off into small equal lengths, and the divisions are numbered in regular order. It has been ascertained by experiment that the atmospheric pressure remaining the same,

water always boils and freezes at the *same* temperatures. The two marks on the stem of the thermometer which correspond to these are called the **boiling** and **freezing points** respectively.

458. The following **Thermometers** are in common use :—

(1) **Fahrenheit's** Thermometer is commonly used in England. In it the *freezing point* is denoted by 32° and the *boiling point* by 212°. The space between these two points is divided into 180 equal parts, each called a *degree* (1°F).

(2) The **Centigrade** Thermometer is in use on the Continent. The *freezing point* is denoted by 0°, and the *boiling point* by 100°, the intervening space being divided into 100 equal parts each called a *degree* (1°C).

(3) **Reaumur's** Thermometer is used in Russia. The *freezing point* is marked 0° R, and the *boiling point* 80°R.

459. In converting a temperature expressed in the Fahrenheit scale into the Centigrade or the Reaumur scale, and *vice-versâ*, we have the following facts to remember :—

(i) $32°F = 0°C = 0°R$; (ii) $180°F = 100°C = 80°R$.

$\therefore\ 1°F = \frac{5}{9}°C = \frac{4}{9}°R$; $1°C = \frac{9}{5}°F = \frac{4}{5}°R$; $1°R = \frac{9}{4}°F = \frac{5}{4}°C$.

Ex. 1. $85°F = 32°F + 53°F = 0°C + \frac{5}{9} \times 53°C = \frac{265}{9}°C = 29\frac{4}{9}°C$
$= 0°R + \frac{4}{9} \times 53°R = \frac{212}{9}°R = 23\frac{5}{9}°R.$

Ex. 2. $-36°C = 0°C - 36°C = 32°F - \frac{9}{5} \times 36°F$
$= 32°F - \frac{324}{5}°F = -\frac{164}{5}°F = -32\frac{4}{5}°F$
$= -\frac{4}{5} \times 36°R = -\frac{144}{5}°R = -28\frac{4}{5}°R.$

The following relation which connects the three scales will be very useful in these reductions :—

$$\frac{F-32}{180} = \frac{C}{100} = \frac{R}{40}\ ;\ \text{or}\ \frac{F-32}{9} = \frac{C}{5} = \frac{R}{4}.$$

Examples CXLII.

1. What degrees (i) *C.*, (ii) *R.*, correspond to—

(1) 40°*F.* (2) 68°*F.* (3) 118°*F.* (4) 160°*F.* (5) −28°*F*?

2. What temperatures (i) *F.*, (ii) *R.*, are represented by—

(1) 20°*C.* (2) 45°*C.* (3) 58°*C.* (4) 92°*C.* (5) −10°*C*?

3. What temperature (i) *F.*, (ii) *C.*, are the same as—

(1) 35°*R.* (2) 60°*R.* (3) 76°*R.* (4) −10°*R.* (5) −25°*R*?

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460. Time and Distance. If a man walks at the rate of 4 miles an hour, he will walk in 5 hours 4 × 5 or 20 miles ; *conversely*,

if a man walks at the rate of $3\frac{1}{2}$ miles an hour, he takes $(15 \div 3\frac{1}{2}$ or $4\frac{2}{7}$ hours to walk 15 miles, and if he takes $4\frac{1}{2}$ hours to walk 12 miles' he walks at the rate of $(12 \div 4\frac{1}{2})$ or $2\frac{2}{3}$ miles per hour.

461. The following remarks will be found useful ln solving problems of this class.

(i) When two bodies are moving in a straight line in *opposite* directions, the speed of their approach or their *relative* speed is equal to the **sum** of their *absolute* speeds.

(ii) When two bodies are moving in a straight line in the *same* direction, the speed of their approach or their *relative* speed is equal to the **difference** of their *absolute* speeds.

(iii) When two bodies are approaching each other from *opposite* directions, they **meet** when they have travelled the whole distance between them.

(iv) When a boat goes down-stream, the rower is *helped* by the current but the current *opposes* his progress when the journey is up-stream. Hence in the first case the total work done on the boat is the **sum** of the works done by the rower and current separately ; in the second case it is their **difference**.

(v) A railway train in motion passes a particular point, when the train has gained its own **length** over the p in.. ; and of two trains running on parallel rails either in opposite directions or in the same direction, the one passes the other only when it has gained over the other a distance equal to the **sum** of the lengths of the trains.

Ex. 1. If two persons A and B start at the same time from two towns C and D distant 300 miles from each other, when and where will they meet, if they travel respectively at the rate of $6\frac{3}{4}$ and $8\frac{1}{4}$ miles an hour ?

Here, the relative speed of A and B is $(6\frac{3}{4}+8\frac{1}{4})$ or 15 miles per hour ; and since they meet when they have together travelled 300 miles,

therefore the required time is $(300 \div 15)$ or 20 hours.
Also the distance travelled by $A = (6\frac{3}{4} \times 20)$ or 135 miles,
and.................................. $B = (8\frac{1}{4} \times 20)$ or 165 miles. } *Ans.*

Ex. 2. A travelling at the rate of 12 miles an hour starts 15 miles behind B who travels only 10 miles an hour ; find when A will overtake B and the distance travelled by each.

Here the relative speed of A and B is $(12-10)$ or 2 miles per hour ; and since A shall have to gain 15 miles on B, in order to overtake him,

therefore the required time $=(15 \div 2)$ or $7\frac{1}{2}$ hours.
Also the distance A has travelled $=(12 \times 7\frac{1}{2})$ or 90 miles,
and...................B.....................$(10 \times 7\frac{1}{2})$ or 75 miles. } *Ans.*

Ex. 3. A starts from P to walk to Q a distance of 84 miles at 3 miles an hour; two hours after, B starts from Q for P at $3\frac{1}{2}$ miles an hour. When will A and B meet?

When B starts, A has already travelled (3×2) or 6 miles; therefore the distance between them is then only $(84-6)$ or 78 miles.

Now, the relative speed of A and B is $(3+3\frac{1}{2})$ or $6\frac{1}{2}$ miles per hour; therefore to travel 78 miles, they will take $(78 \div 6\frac{1}{2})$ or 12 hours.

Hence, A and B will meet $(12+2)$ or 14 hours after A started. *Ans.*

Ex. 4. A starts from P for Q at the rate of 4 miles an hour. Three hours later B also starts from P for Q and reaches the destination at the same time as A. If B's rate exceeds A's by $\frac{1}{2}$ mile an hour, find the distance between P and Q.

In 3 hours A goes (4×3) or 12 miles ahead of B. Now, since B goes $\frac{1}{2}$ mile faster per hour than A, therefore he overtakes A at the end of $(12 \div \frac{1}{2})$ or 24 hours. But B's rate of walking is $4\frac{1}{2}$ miles an hour, therefore in 24 hours he goes $(24 \times 4\frac{1}{2})$ or 108 miles.

Hence the distance between P and Q is 108 miles. *Ans.*

Ex. 5. A train leaves Calcutta at 6-45 A.M. and travels 30 miles an hour; another train leaves Calcutta at 3 P.M. and travels 40 miles an hour; when and where will the second train overtake the first?

From 6-45 A.M. to 3 P.M. there are 8 hrs. 15 min. or $8\frac{1}{4}$ hours.

The first train starts 8 hrs. 15 min. before the other, and is therefore $(30 \times 8\frac{1}{4})$ or $247\frac{1}{2}$ miles ahead of it. Now, the second train shall have to gain these $247\frac{1}{2}$ miles to overtake the first. But the relative speed of the two trains is $(40-30)$ or 10 miles per hour; therefore the required time is $(247\frac{1}{2} \div 10)$ or $24\frac{3}{4}$ hours.

Hence the second overtakes the first at 3-45 P.M. next day and the distance travelled is $(24\frac{3}{4} \times 40)$ or 990 miles from Calcutta. *Ans.*

Ex. 6. Two trains starting from the same station and travelling in opposite directions, are 315 miles apart in 5 hrs. 15 min.; had they been travelling in the same direction, they would have been 74 miles apart in 7 hrs. 24 min. Find the speed of each train.

Since 5 hrs. 15 min. $= 5\frac{1}{4}$ hours, and 7 hrs. 24 min. $= 7\frac{2}{5}$ hrs.

$\therefore$ the sum of their rates per hour $= (315 \div 5\frac{1}{4})$ or 60 miles,
and the difference.................... $= (74 \div 7\frac{2}{5})$ or 10 miles.

Hence, the rate of the faster $= \frac{1}{2}(60+10)$ or 35 miles per hour;
and ....slower $= \frac{1}{2}(60-10)$ or 26 miles per hour. *Ans.*

Ex. 7. A starts from Calcutta for Mankar, a distance of 91 miles, at 6 A. M., walking $3\frac{1}{2}$ miles an hour; B starts from Mankar 12 hrs. later and reaches Calcutta at the same time as A. What was B's speed per hour?

Here *B* takes 12 hours less than *A* to travel 91 miles.

Now, *A* takes ($91 \div 3\frac{1}{2}$) or 26 hrs. to travel;

∴ *B* takes ($26-12$) or 14 hrs.........

∴ *B*'s speed per hour is ($91 \div 14$) or $6\frac{1}{2}$ miles. *Ans.*

Ex. 8. The distance from *A* to *B* is 16 miles, 4 miles of which is up-hill and 8 down-hill; find the difference between the times in which a person would walk from *A* to *B* and back again respectively, supposing his pace up-hill to be 4 miles, down-hill 8 miles and on level 6 miles per hour.

From *A* to *B* is 4 miles up-hill, 4 mi. level and 8 mi. down-hill;

∴ the whole time taken $=(\frac{4}{4}+\frac{4}{6}+\frac{8}{8})$ hrs. $=2\frac{2}{3}$ hrs. $=$ 2 hrs. 40 min.

From *B* to *A* is 8 miles up-hill, 4 mi. level and 4 mi. down-hill;

∴ the whole time taken $=(\frac{8}{4}+\frac{4}{6}+\frac{4}{8})$ hrs. $=3\frac{1}{6}$ hrs. $=$ 3 hrs. 10 min.

Hence the diff. reqd. $=$ 3 hrs. 10 min. $-$ 2 hrs. 40 min. $=$ 30 min. *Ans.*

Ex. 9. If a man rows 40 miles in 10 hrs. against a stream, the rate of which is 3 miles an hour, how long will he be in rowing 40 miles with the stream?

Since he rows in 10 hrs. a distance of 40 miles against the stream,

∴ ....1 hr..............$\frac{40}{10}$ or 4 miles.

And since the rate of the stream is 3 miles per hour, ∴ he can row ($4+3$) or 7 miles per hour. Therefore, with the aid of the stream, he can row ($7+3$) or 10 miles per hour. Hence to row 40 miles with the stream, he will take $\frac{40}{10}$ or 4 hours. *Ans.*

Ex. 10. A hare is 210 of her own leaps before a greyhound; she takes 7 leaps for every 6 that the greyhound takes, but 3 of the greyhound's leaps are equal to 4 of the hare's; how many leaps will the greyhound take before she is caught?

3 leaps of the greyhound $=$ 4 leaps of the hare;

∴ 1 leap.........................$=\frac{4}{3}$..........................

∴ 6 leaps.......................$=\frac{4}{3}\times 6$ or 8..............

But when the greyhound takes 6 leaps, the hare takes 7 leaps; ∴ the greyhound gains ($8-7$) or 1 leap of the hare on 6 of his own leaps. Hence he will gain 210 of the hare's leaps on (210×6) or 1260 leaps of his own. *Ans.*

Ex. 11. I have to be at a certain place in a certain time, and I find that, if I walk at the rate of 4 miles per hour, I shall be 5 min. too late; if at the rate of 5 miles per hour, I shall be 10 min. too soon. How far have I to go?

Since 4 miles per hour $=$ 1 mi. in 15 min.; 5 miles per hour $=$ 1 mi. in 12 min. Therefore, in each mile ($15-12$) or 3 min. is

gained, *i.e.*, 1 min. in each $\frac{1}{3}$ mile by increasing the speed from 4 to 5 miles per hour.

Now the whole time gained in the reqd. distance is 15 min.

$\therefore$ the required distance $=(15 \times \frac{1}{3})$ or 5 miles. ***Ans.***

Ex. 12. If a snail, on the average, creep 1 ft. $3\frac{1}{2}$ in. up a pole during 12 hours in the night, and slip down 8 in. during the 12 hours in the day; how many hours will he be in getting to the top of a pole 70 ft. high?

Here, 1 ft. $3\frac{1}{2}$ in. $= 15\frac{1}{2}$ in.; and 70 ft. $=840$ in.

In 24 hours, the snail gains $(15\frac{1}{2}-8)$ in. or $7\frac{1}{2}$ in.

Now, by alternate creeping and slipping when the snail shall arrive for the **first time** at such a position of the post whose distance from the top is either $15\frac{1}{2}$ in., or less than $15\frac{1}{2}$ in., it will climb up within the next 12 hours to the top. Hence we should have first to find what *least* number, not less than $(840-15\frac{1}{2})$ or $824\frac{1}{2}$ is a multiple of $7\frac{1}{2}$. Now this number is 825, for $110 \times 7\frac{1}{2}=825$, and then $(840-825)$ or 15 in. remains.

Since the snail, on the average, creeps $7\frac{1}{2}$ in. in 24 hours,

$\therefore$ the snail, ... the first 825 in. in (24×110) hrs.

Again, since the snail creeps $15\frac{1}{2}$ in. in 12 hours.

$\therefore$ ... ... 1 in. in $(12 \div 15\frac{1}{2})$ or $\frac{24}{31}$ hour.

$\therefore$ ... ... 15 in in $\frac{24}{31} \times 15$ or $11\frac{19}{31}$ hours.

Hence the whole time reqd. $=(24 \times 110 + 11\frac{19}{31})$ or $\underline{2651\frac{19}{31}}$ hours. ***Ans.***

Ex. 13. Two guns are fired from the same place after an interval of 23 min.; but a person approaching the place hears the reports after an interval of 22 min. 30 sec. Find his rate of progress, sound travelling at the rate of 1142 ft. per second.

The distance which the man travels in 22 min. 30 sec. or $22\frac{1}{2}$ min. is the same as that travelled by the report in (23 min. $-$ 22 min. 30 sec.) or 30 sec.

But in 30 sec., the sound travels 1142×30 ft.;

$\therefore$ the man travels in $22\frac{1}{2}$ min., a distance of 1142×30 ft.

$\therefore$ ... .. in 1 min., ... $(1142 \times 30 \div 22\frac{1}{2})$ or $\frac{4568}{3}$ ft.

$\therefore$ ... ... in 1 hour $\frac{4568}{3} \times 60$ ft. or $\frac{4568}{3} \times 60 \times \frac{1}{3} \times \frac{1}{1760}$ mi.

or $\underline{17\frac{10}{33}}$ miles. ***Ans.***

Ex. 14. Two trains 110 yds. and 88 yds. long respectively, run at the rates of 30 and 15 miles an hour on parallel rails; find how long a person sitting in the first train would take to pass the other train, and how long the two trains would take to pass each other; supposing the trains were running (i) in opposite directions, (ii) in the same direction.

When running in opposite directions, the relative speed of the two trains is (30+15) or 45 miles per hour; but when running in the same direction, their relative speed is only (30−15) or 15 mi. per hour.

A person sitting in the first train would pass the second train when the distance which he gains over the train passed is equal to the length of the train, which is 88 yds. or $\frac{1}{20}$ mile.

∴ for opposite directions, the time $=(\frac{1}{20}\div45)$ hr. = 4 sec.
and ... same direction, $=(\frac{1}{20}\div15)$ hr. = 12 sec. } *Ans.*

Again, one train passes another when it has gained over the other a distance equal to the sum of the lengths of the two trains, which is here (110+88) or 198 yds. or $\frac{9}{80}$ mile.

∴ for opposite directions, the time $=(\frac{9}{80}\div45)$ hr. = 9 sec.
and ... same direction, $=(\frac{9}{80}\div15)$ hr. = 27 sec. } *Ans.*

Ex. 15. *P* and *Q* are two Railway stations. A train 396 ft. long starts from *P* for *Q* 15 min. after a man started from the same place *P* to travel in the same direction. The train overtakes the man and passes him in 11 seconds. On reaching *Q* it rests for 10 min. and on its way back towards *P* again overtakes and passes the man in 9 seconds. If the interval between the two meetings be 1 hr. 20 min., find the distance of *Q* from *P*, and the speed per hour of both the man and the train.

Since 396÷11=36, and 396÷9=44;

∴ the sum of the rates per second of train and man = 44 ft.
and the difference ... ... = 36 ft.
∴ the train's speed per second = ½ (44+36) = 40 ft.
and the man's ... = ½ (44−36) = 4 ft.

Hence the train travels $\frac{40\times60\times60}{3\times1760}$ or $27\frac{3}{11}$ miles per hour.

and the man ... $\frac{4\times60\times60}{3\times1760}$ or $2\frac{8}{11}$ miles per hour.

The train starts 15 min. after the man; therefore the man is (4×15×60) or 3600 ft. in advance of the train at its starting. Now the train gains (40−4) or 36 ft. per second over the man; therefore to gain these 3600 ft., it will take 3600÷36) or 100 sec. and the distance of *E* (the first place of meeting) from *P* = (40×100) or 4000 ft.

P *E* *F* *Q*

Again, since, 1 hr. 20 min. = (80×60) sec. = 4800 sec.; therefore the distance of *F* (the second place of meeting) from *E* = (4×4800) or 19200 ft.

Also the train takes (1 hr. 20 min. − 10 min.) or 70 min. to travel the distance *EF*+2*QF*; but in 70 min. the train can travel (70×60×40) or 168000 ft.; therefore the distance *EF*+2*QF* = 168000 ft. Therefore 2*QF* = (168000 − 19200) or 148800 ft., and *QF* = 74400 ft.

Hence $PQ=(4000+19200+74400)$ or 97600 ft.

$=$ 18 mi. 853 yds. 1 ft. *Ans.*

Ex. 16. A, B, C and D are four Railway stations. The roads from A to B and from C to D are inclined planes of different inclinations, and that from B to C is level. A train starts from A and stopping at B and C for 5 min. and 10 min. respectively arrives at D. At D it stops for 15 min. and then returns directly to A without stopping at B and C. Then down and up speed of the train along AB are 12·5 miles and $10\frac{5}{7}$ miles per hour respectively, the down and up speeds along CD are 15 miles and 9·375 miles per hour respectively, and along the level BC $11\frac{7}{13}$ miles per hour. If the whole time occupied by the train in going and returning be $4\cdot8\dot{3}$ hours, find the length of $AB+BC+CD$ in miles.

Along AB, to run down 1 mile, the train takes $(1\div12\cdot5)$ or $\frac{2}{25}$ hr.

........................up..$(1\div10\frac{5}{7})$ or $\frac{7}{75}$ hr.

$\therefore$ ....down and up 2 miles............$(\frac{2}{25}+\frac{7}{75})$ or $\frac{13}{75}$ hr.

Along BC, to run 1 mile, the train takes $(1\div11\frac{7}{13})$ or $\frac{13}{150}$ hr.

$\therefore$ ....2 miles, ...$2\times\frac{13}{150}$ or $\frac{13}{75}$ hr.

Along CD, to run down 1 mile, the train takes $(1\div15)$ or $\frac{1}{15}$ hr.

.................................up 1 mile.............. $(1\div9\cdot375)$ or $\frac{8}{75}$ hr.

$\therefore$ ....down and up 2 miles.......................$(\frac{1}{15}+\frac{8}{75})$ or $\frac{13}{75}$ hr.

Here, we see that the average speed in the three planes is the same and the time of going and returning is $(4\cdot8\dot{3}$ hrs. $-$ 30 min.) or $4\frac{1}{3}$ hrs.

Since in $\frac{13}{75}$ hr. the train runs 2 miles,

$\therefore$ ....1 hr.................$2\times\frac{75}{13}$ miles.

$\therefore$ ....$4\frac{1}{3}$ hrs. . ...$(2\times\frac{75}{13}\times4\frac{1}{3})$ or 50 miles.

Hence the required distance is $(\frac{1}{2}\times50)$ or 25 miles. *Ans.*

Examples CXLIII.

1. A and B are 6 miles apart, and walk at the rate of $4\frac{3}{4}$ and $3\frac{1}{4}$ miles an hour respectively. How long will elapse before they meet, (i) if they walk towards each other, (ii) if they walk in the same direction ?

2. A walking $5\frac{1}{4}$ miles an hour gives B walking $3\frac{3}{4}$ miles an hour an hour's start. How long will A take to catch B and how far will he have to walk ?

3. Two men start to meet each other at 9 P. M. from places 31 miles apart ; if one of them walks $4\frac{1}{4}$ miles an hour and the other $3\frac{1}{2}$ miles an hour, when will they meet, and how far will each have travelled ?

4. A and B walk respectively $5\frac{1}{4}$ and $3\frac{1}{2}$ miles an hour. They

are 25 miles apart and walk to meet one another, but *B* starts 2 hours before *A*. How far will *A* have to walk?

5. A mail train leaves Calcutta for Buxar (400 miles) at 8-20 A. M. and travels at the rate of 45 miles an hour; another train leaves Buxar for Calcutta at 9 A. M., and travels at the rate of $47\frac{1}{2}$ miles an hour. Find when they will meet, and at what distance from Calcutta.

6. A policeman sets off after a thief at 7 A. M., the thief having had $2\frac{1}{2}$ hours start. If the thief goes 4 miles an hour, and the policeman $5\frac{1}{4}$ miles an hour, when will the policeman catch the thief?

7. A train starts from a terminus at 9 A. M., travelling 25 miles an hour. An express starts at 10-30 A. M. and travels 43 miles an hour. At what time and how far from the terminus will the express overtake the slow train?

8. *A* starts from Calcutta to walk to Burdwan, a distance of 68 miles, at 3 miles an hour; two hours later *B* starts from Burdwan for Calcutta at $3\frac{1}{2}$ miles an hour. When will *A* and *B* meet?

9. *A* starts from a place *X*, for a place *Y*, a distance of 80 miles at 6 A. M., walking $3\frac{1}{2}$ miles an hour; *B* starts from *X* 2 hours later and reaches *Y* at the same time as *A*. What was *B*'s speed?

10. A Hackney coach starts from Calcutta at 6 o'clock A. M. and runs towards Burranagur which is about $4\frac{3}{4}$ miles distant, at the rate of 9 miles an hour; 10 min after, *A* starts for Burranagur and rides the same road at the rate of $13\frac{1}{2}$ miles an hour. When will the coach be overtaken by *A* and at what distance from Burranagur?

11. Two men *A* and *B* leave Howrah for Karmatar, a distance of 168 miles, just at the same time that another man *C* leaves Karmatar for Howrah. *C*, walking at the rate of 5 miles an hour, meets *A* 14 hours and *B* 15 hours after starting on the road. What are *A*'s and *B*'s speed per hour?

12. Two couriers pass through a place at an interval of 4 hours travelling as the rates of $11\frac{1}{2}$ and $17\frac{1}{2}$ miles an hour; how long and how far must the first travel before he is overtaken by the second?

13. *A* and *B* walk in opposite directions from the same place, and are at the end of 5 hours, 38 miles distant. If *A* walk at the rate of 4 miles per hour, find the rate at which *B* walks.

14. The distance from *P* to *Q* is 143 miles. How many hours before *A*, who walks at $3\frac{1}{4}$ miles an hour, must *B* start to reach *Q* in the same time as *A*, *B*'s rate of walking being $2\frac{3}{4}$ miles an hour?

15 Two men *A* and *B* leave Mogul Serai for Jaunpur (46 miles) 15 min. before *C* who leaves Jaunpur for Mogul Serai. After *A* has walked for 5 hrs. 15 min. he meets *C*, who again overtakes *B* 45 min. later on. If *A* walks at the rate of 4 miles per hour, find at what rate were *B* and *C* walking per hour.

16. *A* sets out to walk from *P* to *Q* 5 min. after *B* left *Q* for *P*. After walking for 45 min. at 30 yds. per minute, he overtakes *B*, and on arriving at *Q* stays there for a time. On his return he again overtakes *B* just at *P* after an interval of 2 hrs. 30 min. from the first meeting. Find the distance between *P* and *Q*, *B*'s rate per minute and the time *A* stays at *Q*.

17. The train which leaves Calcutta at 3·10 P.M. arrives at Ranaghat at 5 o'clock; and the train which leaves Ranaghat at 3-30 P. M. arrives in Calcutta at 5-42 P. M.; when do they pass each other?

18. *M* starts from *C* and travels towards *D* at the rate of 6 miles per hour; two hours afterwards *N* starts from *C* and going 10 miles per hour reaches *D* 4 hours before *M*. Find the distance from *C* to *D*.

19. A train starts to go from *A* to *B*, a distance of 72 miles. Its proper rate of travelling is 20 miles an hour, but after having gone 24 miles it meets with an accident which delays it 10 min. and diminishes its speed to 15 miles an hour; how much will the train be behind time?

20. Two trains starting from the same station and travelling in opposite directions, are 405 miles apart in 6 hrs. 45 min.; had they been travelling in the same direction, they would have been 144 miles apart in 14 hrs. 24 min. Find the speed of each per hour.

21. A man walks a certain distance, and rides back in 3 hrs. 45 min.; he could ride both ways in $2\frac{1}{2}$ hours. How long would it take him to walk both ways?

22. A hare starts 40 yds. before a greyhound, and is not perceived by him till she has been up 40 seconds; she runs away at the rate of 10 miles an hour, and the dog makes after her at the rate of 18 miles an hour. How long will the course hold, and what ground will the greyhound run over?

23. The distance from *A* to *B* is 12 miles, 2 miles of which are up-hill and 3 down-hill; find the difference between the times in which a person would ride from *A* to *B* and back again respectively, supposing his pace up hill to be 4 miles, down-hill 5 miles, and on level ground 10 miles per hour.

24. A person sets out to walk from *A* to *B* at the rate of 4 miles an hour. After he has walked $1\frac{3}{8}$ miles he is overtaken by the coach which started a quarter of an hour after him. At a distance of 13 miles from *A* he meets the coach returning from *B* where it has stayed for half an hour. Find the distance from *A* to *B*.

25. Two places *A* and *B* are distant from each other 324 miles by railway. A train leaves *A* for *B* at the same time that a train leaves *B* for *A*; the trains meet at the end of 6 hours, the train from *A* to *B* having travelled 16 miles an hour more than the other. How many miles did each travel an hour?

26. The road between two towns *A* and *B*, distant 15 miles, goes over a hill whose summit is 3 miles from *A*. Two pedestrians set out at the same time from *A* and *B*, the former going 4 miles an hour up-hill, and $5\frac{1}{4}$ down, the latter $3\frac{1}{2}$ up-hill, and $4\frac{1}{2}$ down; how far will the slower one have to walk when the first arrives at his journey's end?

27. Two trains start at the same time from Calcutta and Buxar, and proceed towards each other at the rate of 30 miles and 50 miles per hour respectively. When they meet, it is found that one train has run 100 miles further than the other. Find the distance between Calcutta and Buxar.

28. Sound travels at the rate of 1140 ft. per second. If a shot be fired from a ship moving at the rate of 10 miles an hour, how far will the ship have moved before the report is heard $14\frac{1}{4}$ miles off?

29. A train, having to perform a journey of 250 miles, is obliged after 103 miles to reduce its speed by $\frac{1}{8}$. The result is that the train arrives at its destination 1 hr. 10 min. behind time; what is its ordinary rate?

30. A stream flows at the rate of $2\frac{1}{4}$ miles an hour; a man rows 18 miles against the stream in 6 hours. How long will he be in returning?

31. If a man rows 10 miles in $2\frac{1}{2}$ hours against a stream, the rate of which is 3 miles an hour; how long would he be in rowing 25 miles with the stream?

32. A man rows down a river 18 miles in 4 hours with the stream and returns in 12 hours; find the rate at which he rows and the rate at which the stream flows.

33. *P* can row from *A* to *B* (a distance of 24 miles) and back in still water in 12 hours; how long will it take him to do the same when there is a stream flowing from *A* to *B* at the rate of 2 miles an hour?

34. A hare is 50 leaps before a greyhound, and takes 4 leaps to the greyhound's 3 leaps; but 2 of the greyhound's = 3 of the hare's; how many leaps must the greyhound take to catch the hare?

35. A hare is 40 of her own leaps before a greyhound and takes 5 leaps for the greyhound's 4; 3 of the greyhound's leaps are equal to 4 of the hare's; how many leaps must the hare take before she is caught?

36. The whole time occupied by a train 120 yds. long, travelling at the rate of 20 miles an hour, in crossing a bridge is 18 seconds; find the length of the bridge.

37. Two guns are fired at the same place after an interval of 21 minutes, but a person approaching the place observes that 20 min. 15 sec elapse between the reports; what was his rate of progress, sound travelling 1125 ft. per second?

38. A person saw the flash of a gun fired from a frigate at sea, distant 1 mile 480 yds., and 2 seconds afterwards saw the flash of another gun fired from a vessel in a line between the frigate and himself, and 4 seconds later still heard the two reports simultaneously; what was the distance between the vessels?

39. A man near the sea-shore sees the flash of a gun fired from a vessel steaming directly towards him and hears the report in 15 sec. He then walks towards the ship at the rate of 3 miles an hour, and sees a second flash 5 min. after the first, and immediately stops; the report follows in 10·5 sec. Find the rate of the ship, the velocity of sound being 1200 feet per second.

40. How many seconds will a train 184 ft. in length, travelling at the rate of 21 miles an hour, take in passing another train 223 ft. long, proceeding in (i) the same direction, (ii) opposite directions, at the rate of 16 miles an hour?

41. A man rides at the rate of 12 miles an hour, but stops 5 minutes to change horses at the end of every 8th mile; how long will it take him to perform a journey of 167 miles?

42. A man rides at the rate of 15 miles an hour, but stops 7 minutes to change horses at the end of every 10th mile; how long will he take to go a distance of 126 miles?

43. A snail creeps 17 in. up a pole during 12 hrs. in the night, and slips down 10 in. during the 12 hrs. in the day. If the pole is 12 ft. high, in how many hours will it get to the top?

44. A monkey, climbing up a greased pole, ascends 13 ft. and slips down 6 ft. in alternate minutes. If the pole is 63 ft. high, how long will it take him to reach the top?

45. A train is 88 yds. long, and is running at 20 miles an hour, in what time will it pass a particular point on a telegraph post standing by the side?

46. A train 66 yds. long passes a particular point on a telegraph post in 6 seconds. Find the speed of the train.

47. Two trains, 88 yds. and 44 yds. long respectively, are running on parallel rails at the rates of 15 and 20 miles an hour. In what time will they pass one another if they are running in opposite directions?

48. Two trains, 66 yds. and 99 yds. long respectively, are running with uniform velocities on parallel rails at the rates of 20 and 25 miles per hour. In what time will they pass one another, if they are running in the same direction?

49. Two trains, 92 ft. and 84 ft. long respectively, are moving with uniform velocities on parallel rails in opposite directions, and are observed to pass each other in $1\frac{1}{2}$ sec.; but when they are moving in the same direction, their velocities being the same as before,

the faster train is observed to pass the other in 6 seconds. Find the rates at which the trains are moving.

50. Two trains running at the rates of 25 and 20 miles an hour respectively on parallel rails in opposite directions are observed to pass each other in 8 seconds, and when they are running in the same direction at the same rates as before, a person sitting on the faster train observes that he passes the other in $31\frac{1}{2}$ seconds. Find the lengths of the trains.

51. A train, 88 yds. in length, overtook a person walking along the line at the rate of 4 miles an hour, and passed him in 10 seconds. Twenty minutes after, the train overtook another person and passed him in 9 seconds. When will the first person overtake the last ?

52. *A* and *B* are two Railway stations. A train 195 yds. long starts from *A* for *B* 26 min. after a man started from *A* to travel in the direction of *B*. The train overtakes the man and passes him in 15 seconds. After an interval of one hour the man is again overtaken by the train returning from *B*, where it has stayed for 5 min. and it passes him in 13 seconds. Find the distance of *B* from *A* and the rates of the man and of the train per hour.

53. A passenger train 130 ft long leaves station *A* for *B* 10 min. before a mail train 110 ft. long, which also leaves *A* for *B*. The mail overtakes and passes the other in 6 sec. After an interval of 45 min. the mail on its return from *B*, where it has stayed for 5 min. again overtakes the passenger and passes it in 3 sec. Find the distance between *A* and *B*, and the rate of each.

54. *A*, *B*, *C* and *D* are four Railway stations. The roads from *A* to *B*, *B* to *C*, and *C* to *D* are inclined planes of different inclinations. A train leaves *A* and passing through *B* and *C*, arrives at *D*, and staying there 30 min. returns to *A*. The down and up-speed of the train along *AB* are 17·5 miles and 8·75 miles per hour respectively, along *BC* are 14 miles and 10 miles per hour respectively, and along *CD* are 12·5 miles and 10·9375 miles per hour respectively. If the whole time occupied by the train be $3\frac{1}{2}$ hours, find the length of $AB+BC+CD$ in miles.

55. A ship 40 miles from the shore springs a leak which admits $3\frac{3}{4}$ tons of water in 12 minutes. 60 tons would suffice to sink her, but the ship's pumps can throw out 12 tons of water in an hour. Find the average rate of sailing so that she may reach the shore just as she begins to sink.

56. I have to attend a meeting at a certain place in a certain time, and I find that, if I walk at the rate of 4 miles per hour, I shall be 40 minutes too late, if at the rate of 6 miles per hour, I shall be 35 minutes too soon. How far have I to go ? 15 miles

57. A train 88 yards long overtook a person walking along the line at the rate of 4 miles an hour and passed him completely in

10 seconds ; it afterwards overtook another person and passed him in 9 seconds. At what rate per hour was this second person walking ?

58. Two trains start from *A* to arrive at a fixed time at *B* travelling at the rates of 25 and 30 miles an hour respectively. The first reaches *B* 40 min. behind time and the second 26 min. before. Find the distance between *A* and *B*.

59. Two trains 88 yds. and 96 yds. long run with uniform velocities at the rates $67\frac{1}{2}$ and $112\frac{1}{2}$ miles per hour respectively. How long will a person sitting in the faster train take to pass the other, when going in (i) opposite directions, (ii) the same direction ?

60. *A* and *B* start from *P* and *Q*, a distance of 60 miles, for *Q* and *P* respectively, at 4 and 5 miles an hour. They meet at *R*, reach *Q* and *P*, return immediately, and meet again at *S*. Find the distance between *R* and *S*.

61. Two boats row a race over a straight course 1 mile 995 yds. long, their rates of speed being 12 miles and $11\frac{23}{29}$ miles an hour respectively. Assuming that sound travels at the rate of 1140 feet in a second, find how much the faster boat will be ahead of the other when the sound of the gun fired at starting is heard at the winning-post.

62. Two boats start to row in a race at 3 o'clock. The winning boat comes in at $6\frac{3}{4}$ min. past 3, 40 yds. ahead of the other. At 4 min. past 3 the losing boat was 1140 yds. from the winning-post. Find the length of the course, and the speed of the winning boat in miles per hour.

63. A railway train having left a terminus at noon is overtaken at 6 P. M. by another train, which left the same terminus at 1 P. M. If the former train had been 10 miles further on the road when the latter started, it would not have been overtaken till 8 P. M. Find the rates of the trains.

64. *A*, *B* and *C* are three stations. *A* and *B* are connected by a railway along an inclined plane, but *B* and *C* by a ferry. A man started from *A* for *B* (by a train) at 4-45 P. M. and 30 min. after his arrival at *B* took a seat in steamer for *C*. Having stayed at *C* for 2 hours, he returned to *A* (through *B*) next day at 8-30 A. M. If the down and up-speed of the train along *AB* be 15 miles and 10 miles per hour respectively, and the rate of the steamer with and against the stream be 18 miles and 9 miles per hour respectively, find the length of *AB*+*BC* in miles.

Examples worked out.

Ex. 1. The products of the sum of two numbers multiplied by each separately are 3825 and 3400 ; find the numbers.

The sum of the products of the sum of two or more numbers × each separately = the square of their sum.

Now $3825 \div 3400 = 7225$; and $\sqrt{(7225)} = 85$;

$\therefore$ one number $= 3825 \div 85 = \underline{45}$, and the other $= 3400 \div 85 = \underline{40}$. *Ans.*

Ex. 2. The product of two numbers is 1215, and the quotient when one is divided by the other is $\frac{5}{3}$. Find the numbers.

The product × the quotient = the square of the dividend.

Now $1215 \times \frac{5}{3} = 2025$; and $\sqrt{(2025)} = 45$;

$\therefore$ one number $= \underline{45}$, and the other $= 1215 \div 45 = \underline{27}$. *Ans.*

Ex. 3. A horse is sold at a gain for *Rs.*450 ; had it been sold for *Rs.*390, the loss would have been three times the gain. What did it cost ?

The difference between *Rs.*450 and *Rs.*390 is $(3+1)$ or 4 times the gain ; therefore the gain $= \frac{1}{4}$ of *Rs* $(450-390) = \frac{1}{4}$ of *Rs.*60 = *Rs.*15.

$\therefore$ the cost pilce = *Rs.*450 − *Rs.*15 = *Rs.*$\underline{435}$. *Ans.*

Ex. 4. A man, walking a distance of 18 miles, finds at the end of 1 hr. 48 min. that the distance which he has walked is $\frac{5}{7}$ths of the remaining distance. Find the average pace.

Here, $(1+\frac{5}{7})$ or $\frac{12}{7}$ of the remaining distance = whole distance = 18 miles.

$\therefore$ the remaining distance $= (18 \times \frac{7}{12})$ or $10\frac{1}{2}$ miles.

and $\frac{5}{7}$ of the remaining distance $= (\frac{5}{7} \times 10\frac{1}{2})$ or $7\frac{1}{2}$ miles.

Now, in 1 hr. 48 min. or $1\frac{4}{5}$ hr. he has walked $7\frac{1}{2}$ miles,

$\therefore$ in 1 hr. he can walk $(7\frac{1}{2} \div 1\frac{4}{5})$ or $\frac{15}{2} \times \frac{5}{9}$ miles $= 4\frac{1}{6}$ miles.

Hence the man's rate per hour $= \underline{4\frac{1}{6}}$ miles. *Ans.*

Ex. 5. The majority was a fifth of the number on the winning side ; if 10 voters change to the losing side the majority would be 1. How many voted on each side ?

Since a transfer of 10 votes would reduce the majority to 1, therefore the majority $= 2 \times 10 + 1 = 21$.

Thus, $\frac{1}{5}$ of the votes on the winning side = 21 ; hence the number of votes on the winning side $= 21 \times 5 = 105$. Therefore the number of votes on the losing side $= 105 - 21 = \underline{84}$. *Ans.*

Ex. 6. *Rs.*49 was divided amongst 150 children ; each girl had 8*a.* and each boy 4*a.* ; how many boys were there ? (*E. E.* 1879.)

Each girl may be paid 4*a.* along with the boys, and 4*a.* *afterwards separately*, so that she may get 8*a.* altogether.

The 150 children would in this way first get 4*a.* × 150 or *Rs.*37. 8*a.*

The remaining *Rs.*49 − *Rs.*37. 8*a.* = *Rs.*11. 8*a.* would have to be distributed amongst the girls at the rate of 4*a.* each. Therefore the no. of girls = *Rs.*11. 8*a* ÷ 4*a.* = 46; and the no. of boys = 150 − 46 = $\underline{104}$. *Ans.*

Ex. 7. A workman was engaged for 28 days at *Re.*1. 4*a.* a day, but instead of receiving anything, was to pay 8*a.* a day on all days on which he was idle ; he received altogether *Rs.*26. 4*a.* How many days was he idle ?

Had he worked for 28 days, he would have received *Re.*1. 4*a.* × 28 or *Rs.*35 ; but as he received only *Rs.*26. 4*a.*, the loss for his idleness is *Rs.*35 − *Rs* 26 4*a.* = *Rs.*8. 12*a.*

But his loss per day for being idle = *Re.*1. 4*a.* + 8*a.* = *Re.*1. 12*a.*

$$\therefore \text{ the no. of days he was idle} = \frac{Rs.8.\ 12a.}{Re.1.\ 12a.} = \frac{140a.}{28a.} = \underline{5}. \quad Ans.$$

Ex. 8. A gentleman wishing to relieve a number of beggars, finds that if he give them 6 pice a head, he will have 5*a.* left ; and that he has not enough by $3\frac{1}{2}a.$ to give them 2*a.* a head. Find the number of beggars, and the money he possesses.

Raising the rate from 6 pice to 8 pice or by 2 pice, increases the money required, by $(5+3\frac{1}{2})$ or $8\frac{1}{2}a.$ (*i. e.*) 34 pice.

Therefore the no. of beggars = 34 ÷ 2 = 17 ;

and the man has (17 × 6)*ps.* + 5*a.* = 122 pice = <u>*Re.*1. 14*a.* 2*ps.*</u> *Ans.*

Ex. 9. In rifle-shooting, a bull's eye counts 4, a centre 3, an outer 2 Eleven men fire five shots each at a target, and score 113 ; 8 misses are made and 3 bull's eyes. Find the number of centres and outers.

The 11 men fire 5 × 11 or 55 shots ; 8 being misses, 47 shots score 113 ; the bull's eyes score 3 × 4 or 12. Therefore the remaining (47 − 3) or 44 shots, which are either centres or outers, score (113 − 12) or 101. If the 44 shots had been all centres, the score would have been 44 × 3 or 132. Therefore (132 − 101) or 31 points are due to outers only.
Hence the no. of outers is <u>31</u>, and the no. of centres (44 − 31) or <u>13</u>. *Ans.*

Ex. 10. *A* has three times as much money as *B*, and *Rs.*10 more than *C* ; the sum of their money is *Rs.*165. Find *A*'s money.

If *C* had *Rs.*10 more, then *A*'s money would be equal to *C*'s and the sum of *A*, *B* and *C*'s money would be *Rs.*(165 + 10) or *Rs.*175. Now representing *B*'s money by 1, *A*'s money would be 3, and *C*'s money 3.

And 3 + 1 + 3 = 7 ; ∴ *A*'s money = $\frac{3}{7}$ of *Rs.*175 = *Rs.*<u>75</u>. *Ans.*

Ex. 11. I have a certain sum of money, wherewith to buy a certain number of nuts, and I find that if I buy at the rate of 40 a penny, I shall spend 5*d.* too much ; if at the rate of 50 a penny, 10*d.* too little. How much money have I spent ?

A nut in the first case costs $\frac{1}{40}$ pence and in the second $\frac{1}{50}$ pence. Therefore the difference in the price of a nut in the two cases is $(\frac{1}{40} - \frac{1}{50})$ or $\frac{1}{200}$ pence. Also the diff. in money = (10 + 5) or 15*d.*

Now $\frac{1}{200}d.$ is the diff. in price for 1 nut.

∴ 1$d.$ .... 200 nuts.

∴ 15$d.$ .... 15×200 nuts.

Then the price of 15×200 nuts at 40 a penny$=(15 \times 200 \div 40)$ or 75$d.$ Hence the money expended$=(75-5)$ or 70$d.$

Ex. 12. A viaduct is made of 3 series of arches built over one another, the spans of the arches being $12\frac{2}{3}$ yds., $8\frac{1}{2}$ yds. and $4\frac{3}{4}$ yds. respectively. The piers on which they stand are in each series 4 yds. wide. Find the least length of the viaduct.

Leaving one of the extreme piers, *i. e.*, 4 yds. the remaining length of the viaduct must be a multiple of $(12\frac{2}{3}+4)$, $(8\frac{1}{2}+4)$ and $(4\frac{3}{4}+4)$. Hence the least length of the viaduct must be the L. C. M. of $16\frac{2}{3}$, $12\frac{1}{2}$ and $8\frac{3}{4}$ increased by 4 yds. Now the L. C. M. of $16\frac{2}{3}$, $12\frac{1}{2}$ and $8\frac{3}{4}=350$. Hence the reqd. length$=354$ yds.

Miscellaneous Examples VI.

1. Given that the sum of the divisor and quotient$=33600$; also that the quotient$=$divisor$\times 15$, and that the remainder$=$divisor$\div 15$; find the dividend.

2. What will it cost to make a gravel walk, 7 ft. wide, along the sides of a square field, containing $2\frac{1}{2}$ ac. at 10*a.* per sq. yd., the walk being part of the field?

3 A piece of cloth, when measured with a yard measure which is ·6 of an inch too short, appears to be $88\frac{1}{4}$ yards long, what is its true length?

4. A person having paid an income-tax of 4$d.$ in the £ during the first half of the year, and $3\frac{1}{2}d$ in the £ during the second half of the year, has £359. 5*s.* $11\frac{1}{2}d.$ left. What was his gross income?

5. A square room 6 yards long or broad and 5 yards high has the ceiling painted at 6*a.* 4*p.* a sq. yd, its 4 walls papered with paper $\frac{3}{4}$ of a yard wide at 6*a.* per yard, and its floor covered with carpet $\frac{3}{4}$ of a yard wide at *Rs.*2. 4*a.* per yard. Find the cost.

6. Of three pipes *A*, *B* and *C*, *A* fills a cubic inch in a second, *B* a cubic foot in a minute, *C* a cubic yard in an hour; if all were running together, in what time would they fill 1069 cubic inches?

7. A rectangular pile is 12 yards high and stands on a base 10 ft. square; find the number of oblong pieces 2 ft. long, 8 in. broad, 4 in. deep, contained in it. Also the cost of covering the pile with matting $\frac{3}{4}$ of a yard wide at 3*a.* 2*p.* per yard.

8. A plate of gold 3 in. square and $\frac{1}{8}$ of an inch thick is extended by hammering so as to cover surface of 7 sq. yds., find its present thickness.

9. A room is 20 ft. long, 16 broad and 12 high. If pure gold be worth £4. 5*s*. per oz. Troy, and a cubic foot of gold weigh 19260 oz. Avoirdupois, what is the value of the gold which will exactly fill the room ?

10. A creditor receives upon a debt of *Rs* 3270 a dividend of 9*a*. 2*p*. in the rupee, and afterwards he receives a further dividend upon the deficiency of 3*a*. 4*p*. in the rupee ; how much does he receive on the whole ?

11. If £1 be worth 25·2 francs, $9\frac{1}{2}$ thalers worth 35 francs, and 60 thalers worth 107 Austrian paper florins, find how many Austrian paper florins should be received for £10.

12. A gentleman has a bowling-green 300 ft. long and 200 ft. broad, which he would raise one foot higher, by means of the earth to be dug out of a ditch that goes round it ; to what depth must the ditch be dug, supposing the breadth to be everywhere 8 ft. ?

13. A room is 57 feet long by 32 ft. wide. How many people can be seated in it on chairs, which are $1\frac{1}{4}$ feet wide, and placed 2 feet apart from back to back ; allowing a clear passage 4 feet wide down the middle of the room, and a clear space 13 feet deep at the end ?

14. A cistern 12 ft. long, 2 ft. wide, and 6 in. deep, contains pulp for making paper. If $\frac{1}{8}$ the volume of the pulp is lost in the process of drying, how many sheets of paper, 8 in. by 6, will be obtained, if 300 sheets in thickness go to the inch.

15. A train consisting of 3 first class, 4 second class and 5 third class carriages, travelled from Calcutta to Jagadispur, a distance of 191 miles. The rates charged per mile were, 1st class 1*a*. 6*p*. ; 2nd class 1*a*. 2*p*. ; 3rd class 8*p*. ; the amount paid by the passengers was *Rs*.4659. 9*a*. 8*p*. ; each 2nd class and each 3rd class carriage contained 32 and 48 passengers respectively. Find the number of passengers in each first class carriage.

16 A heap of cocoanuts can be made up exactly into groups of 25 ; but when made up into groups of 18, 27 and 32, there is always a remainder 11 ; find the least number of cocoanuts such a heap can contain.

17. A person who can walk down a hill at the rate of $3\frac{1}{4}$ and up at the rate of $2\frac{1}{2}$ miles an hour, ascends and comes down to his starting point after walking for 4 hrs. 36 min. How far did he walk ?

18. A man rides at the rate of 11 miles an hour, but stops 5 min. to change horses at the end of every 7th mile ; how long will he take to go a distance of 94 miles ?

19. What least number is that which leaves a remainder 5 when divided either by 9, 99, 999 and 9999 ?

20. Find the least number of seven digits such that if it be divided by 27, 33, 45, 60 and 75, the remainder in each case will be 5

21. Find the value of

$$\frac{2\frac{1}{4}+\frac{3}{4}\text{ of }2\frac{3}{8}-1\frac{1}{4}}{3{\cdot}6+{\cdot}15\times4+{\cdot}24-2{\cdot}1}\times\frac{12{\cdot}3\dot{4}\dot{5}-9{\cdot}4\dot{8}}{3{\cdot}4\dot{7}\dot{2}+{\cdot}6\dot{0}\dot{8}}\times\frac{\frac{1}{3}\text{ of }81\text{ po. }4\frac{1}{2}\text{ yds.}}{\frac{1}{4}\text{ of }101\text{ po. }4\frac{1}{2}\text{ yds.}}\times$$

$$\frac{20{\cdot}1\text{ of }1\text{ md. }30\frac{1}{4}\text{ sr}}{17{\cdot}5625\text{ of }2\text{ mds. }20\frac{1}{2}\text{ sr.}}\times Rs.2.\ 8a.$$

22. A man travels 60 miles in 3 hours, partly by rail and partly by coach. If he had gone all the way by rail he would have arrived at his destination an hour earlier, and would have saved $\frac{2}{5}$ths of the time he was on the coach. How far did he travel by coach ?

23. It costs *Re.*1. 10*a.* 8*p.* less to feed a horse a month when gram is 36 seers for the rupee than when it is 24 ; find the cost when gram is 40 seers for the rupee.

24. The quotient in a division sum equals six times the divisor, and the divisor equals six times the remainder ; the three amount together to 516 ; find the dividend.

25. Three lines of palings run side by side for a distance o 90 feet. The rails are respectively 2, 3 and 5 feet apart. How often will a person walking outside the palings, on looking across them, see three rails in a line ?

26. If a piece of work can be finished in 45 days by 35 men working continuously, and if the men drop off by 7 at the end of every 15 days ; find in what time the work will be finished.

27. A garrison of 100 men had provisions for 27 days ; at the end of 10 days a reinforcement arrives, and there are now only provisions for 5 days. What was the reinforcement ?

28. By what factor less than 1000 must 7983 be multiplied so that the last 3 figures of the product may be 986 ?

29. The G. C. M. of two numbers of 4 digits is 187, and their L. C. M. is 21879. Find the numbers.

30. A riband is 47·3824 yards long ; how many pieces, each 7871 yd., can be cut off ? and how many inches will be left ?

31. A gravel walk 6 ft wide runs round a grass plot 60 ft. long and 40 ft. wide. If gravel be *Re.*1. 8*a.* per cubic yard, find the cost of a coat of gravel on the walk 3 in. deep.

32. A man sells a horse for *Rs.*120 more than he gave for and realises a profit equal to $\frac{2}{7}$ths of its cost price. What was the cost price ?

33. When 52 lbs. of coffee are worth as much as 12 lbs. of tea, 22 lbs. of tea are worth as much as 572 lbs. of sugar, a cask of sugar costs 2 guineas, and 1 cwt. of coffee costs 8 guineas, what is the weight of a cask of sugar ?

34. A square field is bordered by a path 3 yds. wide, the field and path together occupying $2\frac{1}{4}$ acres. Find the cost of covering the path with gravel at 12*a.* per sq. yd.

35. On a stream, *B* is intermediate to and equidistant from *A* and *C*; a boat can go from *A* to *B* and back in 5 hrs. 15 min., from *A* to *C* in 7 hrs. How long would it take to go from *C* to *A*?

36. One man walks 3 miles in 32 min., and another walks $2\frac{1}{2}$ miles in 28 min.; how much start must the slower walker have that in a 7 mile race they may walk a dead heat?

37. If a cubic foot of gold be made to cover uniformly and perfectly 432000000 square inches, find the thickness of the gold.

38. *A* barters some sugar with *B* for flour, which is worth 2*s.* 3*d.* per stone, but uses a false stone-weight of $13\frac{1}{2}$ lbs.; what value should *B* set upon his flour, that the exchange may be fair?

39. *A* and *B* are the termini of a Railway 144 miles long. A fast train starts from *B* at 9 h. 0. m.; another fast train travelling at the same rate, starts from *A* at 10 h. 0. m. A slow train starts from *B* at 10 h. 20 m., the fast train from *A* meets the other fast train at 11 h. 30 m, and the slow train at 12 h. 32 m.; find the rates at which the trains travelled.

40. A crow wishing to quench its thirst came to a vessel which contained 28 cub in. of water. The crow being unable to reach the water picked up several small stones, each three quarters of a cubic inch in size, and let them drop into the vessel until the water came to the top of the vessel. If the size of the vessel was such that it would exactly hold 73 cub. in. of water, find the number of stones dropped in by the crow.

41. A down train usually travels at the rate of 30 miles an hour and meets an up-train 5 miles from the terminus. On one occasion, on account of an accident, it only went at the rate of 20 miles an hour and met the up-train $4\frac{1}{3}$ miles from the terminus. Find the speed of the up-train.

42. *A* works for 6 days at the rate of 8 hours per day; *B* works for 5 hours on the first day and on each of the five subsequent days one hour longer than on the preceding day; *A* does as much in 4 hours as *B* does in 5 hours. If the total sum paid to *A* and *B* a wages for the week be *Rs.*21, how much should each receive?

43. If one watch gain, and another lose, at the rate of a minute a day, and they are both set right at noon on Monday, what time will be indicated by the latter when the former points 6 hours $3\frac{3}{4}$ min. on the following Friday morning? Also what is the correct time?

44. In paving a court-yard 1296 bricks are employed, the exposed surface of each brick measuring $9\frac{1}{4}$ in by $4\frac{1}{2}$ in.; how many tiles 6 in. square would be required for paving a yard one-ninth of the size of the former?

45. How many degrees, &c., must be added to or subtracted from $15^\circ\ 12'\ 2''$ that the sum or difference shall be the same fraction of $22^\circ\ 10'\ 20''$ that $8^\circ\ 10'\ 15''$ is of $12^\circ\ 15'\ 22''\cdot5$?

46. A supply of water would fall short of the calculated time by 12 days if 4 seers leak off every day, but it would fall short by 20 days if 8 seers leak off daily. Find the total quantity of water in the supply.

47. If 5 men working 6 hrs. a day and 8 boys working $7\frac{1}{2}$ hrs. a day can complete a piece of work in 10 days or 6 men working $6\frac{1}{4}$ hrs. a day and 5 boys working 7·2 hrs. a day can complete the same piece of work in 11 days, how many days will it take 8 men working 7 hrs. a day and 17 boys working 8 hrs. a day to finish a piece of work twice as great ?

48. A watch which gains 2·4 min. in 24 hours is set right at 9 P.M. on Monday. What will be the true time when the watch indicates 2 P.M. on Thursday next ?

49. The external dimensions of a box without a lid are, length 4 ft., breadth 3 ft, depth 2 ft. and the thickness of the sides and bottom is the same, namely 1 inch ; if the cost of a cubic yard of the material is *Rs* 4. 8*a.*, and the cost of making the box $=\frac{1}{11}$ of the cost of the material, what will the box cost ?

50. If gold can be beaten out so thin that a grain will form a leaf of 56 square inches, how many of these leaves will be required to make up the thickness of a sheet of paper, the weight of a cubic foot of gold being 1215 lbs. Troy and 400 sheets of paper making a book 1 inch thick ?

51. A viaduct consists of 3 series of arches built upon each other, the breadths of the arches in each being respectively 8 yds. 2 ft., 6 yds. and 5 yds. ; whenever the piers in all the series are vertically above each other, there occurs a mass of masonry 4 yds. wide ; of such there are 3 ; find the length of the viaduct.

52. To complete a piece of work, *B* would take twice as long as *A* and *C* together, and *C* thrice as long as *A* and *B* together ; *A* *B* and *C* by their united exertions can do it in 5 days. In what time could each do it by himself ?

53. *A* can run at the rate of 8 miles an hour, *B* at the rate of $7\frac{1}{2}$ miles an hour ; what is the greatest number of yards start that *A* can give *B* so as to beat him in a race of 440 yards ?

54. The distance between two stations *A* and *B* is 65 miles. A train starts from *A* to go to *B* at the rate of 15 miles an hour, and is delayed 10 min. on the way ; another train starts from *A* two hours after the former at the rate of 25 miles an hour ; find the interval between their times of arrival at *B*.

55. Five men do $\cdot600\dot{6}$ of a piece of work in 2·12 hours, how

long will 6 boys take to finish it, it being known that 3 men and 7 boys have done the whole piece of work in 3 hours ?

56. Two clocks, one of which gains and the other loses one minute in an hour, strike one o'clock together ; shew that the interval between their respective striking 2 will be $2\frac{2}{3599}$ minutes by a correct clock.

57. An express train owing to a defect in the engine goes at $\frac{5}{8}$ths of its proper speed, and arrives at 6-49 P. M. instead of 5-55 P.M. At what hour did it start ?

58. A peon walks from *A* to *B* at the rate of 3 miles an hour and after transacting some business which occupies him an hour, returns to *A* by the tram-way at the rate of 5 miles an hour. He then finds he has been absent 2 hrs. 20 min. Find the distance from *A* to *B*.

59. The products of the sum of two numbers multiplied by each separately are 12400 and 11625. Find the numbers.

60. The products of the sum of three numbers multiplied by each separately are 4674, 4920 and 5535. Find the numbers.

61. I sold an article for *Rs.*450 at a loss . had I sold it for *Rs.*498, the gain would have been 5 times the loss. Find the cost.

62. A number of rupees is divided amongst four people. *A* receives $\frac{2}{3}$rds of the whole, *B* $\frac{3}{4}$ths of the remainder, *C* $\frac{4}{5}$ths of what then remains, and the number of rupees given to *D* is the square root of the whole number to be distributed. What sum does each receive ?

63. The product of two numbers is 1575 and the quotient of the one divided by the other is $\frac{9}{7}$. Find the numbers.

64. *A* is twice as old as *B* and 10 years older than *C* ; the sum of their ages is 105 years ; find *B*'s age.

65. A person meeting a company of beggars gave 4*a.* to each and had *Re.*1 left ; he found that he should have required 12*a.* more to enable him to give the beggars 6*a.* each. How many beggars were there ?

66. A carpenter agreed to work for 60 days on condition that he should receive for each day that he worked *Re.*1. 4*a.* and his board, and pay 4*a.* 8*p.* for his board each day he was idle. At the end of the term he received *Rs.*50. 5*a.* 4*p.* How many days did he work ?

67. A person distributes *Rs.*45 amongst 50 men and women, giving each woman 9*a.* and each man 15*a.* Find the number of men.

68. Bought 12 yards of broad cloth and silk for *Rs.*107. For the silk I paid *Rs.*9. 8*a.* per yard and for the broad cloth *Rs.*8. 8*a.* per yard. How many yards of silk were bought ?

69. An undergraduate rowed down the river a distance of

11 miles in $1\frac{1}{3}$ hours with the stream, and on his return met the same stream and with a uniform stroke throughout he rowed back again in $3\frac{2}{3}$ hours. Find the rate of the current per hour.

70. A train is to arrive in Calcutta from Allahabad at 9 A.M. Had it travelled at 40 miles an hour it would have been 48 minutes late, but if at 45 miles an hour 46 minutes earlier. Find the distance between Allahabad and Calcutta.

71. Which will be the more advantageous to employ to do a piece of work, 6 men who work 10 hrs. a day for 15*s*. or 9 boys who work 8 hrs. a day for 9*s*., it being given that a man can do half as much work again as a boy in an hour ?

72. I want to buy a certain number of mangoes for a certain sum ; if I buy at the rate of 4 for an anna, I shall spend 5*a*. too much ; if at the rate of 5 for an anna, 10*a*. too little ; what is the sum ?

73. A man walking $13\frac{1}{2}$ miles finds that in 1 hr. 15 min. he has walked $\frac{4}{5}$ of the remaining distance ; find his rate of walking.

74. If the hands of a clock coincide every $65\frac{1}{2}$ minutes, how much does the clock gain or lose in a day ?

75. A supply of water would fall short by 10 days if 12 gallons leak off daily, but it would fall short by 14 days if 20 gallons leak off daily. Find how long the supply would last if 5 gallons leak off daily.

76. 36 men can do a piece of work in 24 days. After working for a certain number of days they take 12 men to their help, and then finish the work 4 days sooner. When do these 12 men join ?

77. *A* person leaves *A* for *B* at 4 miles an hour between 4 and 5 P. M., and returns after staying at *B* for 10 minutes, between 8 and 9 P. M. on the same day and finds the hands of his watch have exactly changed places. When does he return and what is the distance from *A* to *B* ?

78. A squad of 11 boys fired 10 shots each at a target, and scored 286 ; 20 bull's eyes were made and 11 misses. How many centres and outers were there ? (A bull's eye scores 4, a centre 3 and an outer 2.)

79. If a snail, on the average, creep 2 ft. 3 in. up a pole during 12 hours in the night and slip down 1 ft. 4 in. during the 12 hours in the day ; how many hours will he be in getting to the top of a pole 25 feet high ?

80. The cost of papering the walls of a room 30 ft. long at 4*s*. a sq. foot is £200 and the cost of matting the floor at *Rs*.3 per 5 sq. ft. is *Rs*.360. Find the height. If this room has a verandah 9 ft. deep all round the outside, what will it cost to pave it at *Rs*.2 a sq. yd., the walls being 18 in. thick ?

81. Find the cost of painting the walls of a square room 14 ft. high and 18 ft. long with two doors 8 ft. by 4 ft., and three win-

dows 10 ft. by 5 ft., the amount saved by each window being *Rs.*28. 2*a.* What additional height would increase the cost by *Rs.*4. 8*a.* ?

82. A viaduct consists of two series of arches built over one another, the spans of the arches are 12 yds. and 11 yds. respectively. The piers on which they stand are 5 and 4 yds. respectively. Find the least length of the viaduct.

CHAPTER XI.

Ratio and Proportion.

I. RATIO.

462. **Ratio** is the relation which one number has to another, or, which one quantity numerically considered bears to another of the *same kind*, the comparison being made by observing what *multiple*, *part* or *parts*, the former is of the latter.

Thus, the ratio of the *abstract numbers* 4 and 2 is *written* 4 : 2, and *read four* is to *two* ; and it will be *expressed* by $\frac{4}{2}=2$; the same, being used to denote the ratio of the *concrete quantities* 4 ft. and 2 ft., provided they be of the *same kind* and of the *same denomination*.

463. Of the numbers or quantities *compared* and called the **terms** of the ratio, the former is styled the **antecedent** and the latter the **consequent** ; also, the ratio is said to be **a ratio of greater** or **less inequality** according as the antecedent is *greater* or *less* than the consequent, and it is a ratio of **equality** when these terms are equal.

Thus, in the ratio £4 to £5, £4 is called the *antecedent* and £5 the *consequent* ; also £4 and £5 are its *terms*. Again, 6 : 5 is a ratio of *greater inequality* ; 4 : 9 is one of *less inequality*, and a ratio of *equality* may be denoted by 1 : 1 or 2 : 2 or 3 : 3, &c., at pleasure.

464. Hence, the **magnitude** of a ratio is expressed by the *vulgar fraction* whereof the antecedent is the *numerator* and the consequent the *denominator*.

Thus, the ratio of £9 and £12, written 9 : 12, will have its magnitude expressed by the fraction $\frac{9}{12}$, or, reduced to its lowest terms by the fraction $\frac{3}{4}$; whereas, the ratio of 9*d.* to 6*s.* will be that of 9*d.* to 72*d.*, which $=\frac{9d.}{72d.}=\frac{9}{72}=\frac{1}{8}$; and this is therefore the same as that of 9 lbs. to 72 lbs.

Also, if the terms of the ratio be vulgar fractions or decimals the fraction expressing its magnitude may be simplified by the rules already given.

465. It appears from the last Article, that the value of a ratio does not depend upon the nature of the quantities involved. Thus, the ratios 4 yds. : 5 yds., 4*s.* : 5*s.*, 4 lbs. : 5 lbs. are all equal, each of these being determined by the fraction $\frac{4}{5}$. Hence, in treating of ratios we usually consider the terms to be numbers; for at any time we can pass from quantities of the same kind to the numbers which measure them, and *vice-versâ*, whenever we find it necessary to do so.

466. The magnitudes of two or more ratios may therefore be compared, by comparing the values of the vulgar fractions which represent them, according to the principle of the last Article.

Thus, if the ratios be 3 : 4 and 5 : 7, their magnitudes will be represented by $\frac{3}{4}$ and $\frac{5}{7}$;

$$\text{but } \frac{3}{4}=\frac{21}{28} \text{ and } \frac{5}{7}=\frac{20}{28};$$

and $\frac{21}{28}$ being greater than $\frac{20}{28}$, it follows that the ratio 3 : 4 is greater than the ratio 5 : 7; in other words, 3 has to 4 a greater ratio than 5 has to 7.

467. One ratio is said to be the **inverse** or **reciprocal** of another, when the antecedent and consequent of the one are respectively the consequent and antecedent of the other.

Thus, the *inverse* ratio of 5 : 7 is the ratio of 7 : 5.

468. *A ratio of greater inequality is diminished, and a ratio of less inequality is increased, by adding the same quantity to both its terms.*

First, let us take the ratio of *greater* inequality 7 : 5, and add 1 to both its terms, so that it becomes 8 : 6;

then the original ratio $=\frac{7}{5}=\frac{42}{30}$, and the new ratio $=\frac{8}{6}=\frac{40}{30}$; therefore the new ratio is *less* than the original one.

Secondly, taking the ratio of *less* inequality 8 : 11, and adding 2 to each term, so as to make it 10 : 13, we have

the original ratio $=\frac{8}{11}=\frac{104}{143}$, and the new ratio $=\frac{10}{13}=\frac{110}{143}$;

the latter of which fractions being greater than the former, the new ratio is the *greater* of the two.

Exactly in the same manner, it may be shewn that a ratio of *greater inequality* is *increased*, and a ratio of *less inequality* is *diminished*, by subtracting the same quantity from each of its terms.

469. *If the terms of a ratio be multiplied or divided by the same quantity, the magnitude of the ratio will not be altered.*

Let the ratio be 3 : 8; then its magnitude is $\frac{3}{8}$ which is equivalent to

$$\frac{6}{16} \text{ or } \frac{9}{24}, \text{ or } \frac{12}{32}, \text{ or } \frac{15}{40}, \text{ \&c.};$$

that is, the ratio 3 : 8 is equal to each of the ratios 6 : 16, 9 : 24, 12 : 32, 15 : 40, &c. which arise from the equal *multiplication* of its terms:

and *conversely*, each of the latter ratios is reducible to the original one by the equal *division* of its terms.

470. If the antecedents of two or more ratios be multiplied together for a new antecedent, and their consequents be multiplie together for a new consequent, the resulting ratio is said to be **Compounded** of the others, and it is called their **compound ratio**.

Thus, if the ratios be 2 : 3, 4 : 7 and 8 : 13, the ratio which arises from their composition will be $2\times4\times8 : 3\times7\times13$, or 64 : 273.

Examples CXLIV.

1. What are the simplest expressions of the magnitudes of the following ratios ?—

(1) 3 : 5 ; 4 : 12 ; 9 : 21 ; 64 : 48 ; 48 : 64 ; 20 : 32.

(2) $7\frac{1}{2} : 37\frac{1}{2}$; $37\frac{1}{2} : 7\frac{1}{2}$; $6\frac{1}{4} : 75$; $16\frac{2}{15} : 60\frac{1}{2}$; $6\frac{5}{18} : 3\frac{8}{9}$; $2\frac{1}{4} : 6$.

(3) 2·25 : $\cdot\dot{7}\dot{5}$; 6·5 : 7·75 ; $\frac{5}{8}$: $\cdot\dot{5}7142\dot{8}$; 3·5 : 3·75 ; 62·5 : ·025.

(4) *Rs.*57. 8*a.* : *Rs.*2. 8*a.* ; *Rs* 2. 8*a.* : *Rs.*57. 8*a* ; £$2\frac{1}{8}$: £$8\frac{1}{2}$.

(5) *Rs.*10 : *Rs.*2. 15*a.* ; 14cwt. 3qrs. 7lbs. : 1 ton ; 3mds. : 5 mds. 10 sr.

(6) 3 yds. : 2 ft. ; 16 hrs. 40 min. 40 sec : 1 day ; 3 cwt. 2 qrs. : 2tons.

2. Which of the following ratios is the greater ?—

(1) 5 : 9 or 7 : 11 ; 10 : 17 or 17 : 23 ; 34 : 27 or 37 : 31.

(2) 17 : 18 or 11 : 12 ; $7\frac{1}{3}$: 10 or 3 : 4 ; $\frac{7}{8} : 3\frac{1}{8}$ or $1\frac{1}{2} : \frac{5}{13}$.

(3) 11 : 12 or 17 : 18 ; $\frac{2}{3} : \frac{5}{8}$ or $\frac{2}{3} : \frac{1}{2}$; $\frac{4}{5} : 2\frac{1}{2}$ or $\frac{3}{8} : 1\frac{1}{3}$.

3. Of the following ratios which is the greatest ?—

(1) 6 : 7, 8 : 9 or 21 : 24 ; $2\frac{3}{8} : 6\frac{5}{7}$, $2\frac{4}{7} : 3\frac{11}{12}$ or $4\frac{5}{9} : 11\frac{3}{4}$.

(2) 7 : 15, $1\frac{3}{4} : 2\frac{5}{8}$ or ·75 : ·96 ; 4 : 7, 8 : 15 or 13 : 24.

(3) 14*s.* : 1gui., 16lbs. : 1qr., 2ft. 1in. : 1yd. 9in. or 3gals. 3qts. : 42pts.

4. Find whether the ratios 7 : 9, 11 : 17 and 10 : 7 are increased or diminished by adding 1, 2, 3, respectively to their terms.

5. Are the ratios 4 : 3, 9 : 13 and 15 : 22 increased or diminished by subtracting 2, 3, 4, respectively from their terms ?

6. What are the ratios arising from the composition of the following ratios ?—

(1) 5 : 12, 8 : 7 and 6 : 25 ; 5 : 7, 7 : 18 and 18 : 35.

(2) 5 : 7, 13 : 15, 21 : 91 and 45 : 52 ; $1\frac{3}{4} : 2\frac{5}{8}$ and $2\frac{8}{21} : 7\frac{7}{8}$.

(3) 7 : 15, 9 : 16 and 24 : 35 ; 4 : 9, 3 : 4, 5 : 6 and 12 : 7.

7. If the consequent be 32, and the value of the ratio $\frac{3}{8}$; what is the antecedent ?

8. If the antecedent be 15·6 and the value of the ratio 3 ; what is the consequent ?

9. If the consequent be 3 acres, and the value of the ratio $\frac{81}{160}$; what is the antecedent ?

10. If $A = 3\frac{1}{3}$ of B, and $C = 5\frac{1}{3}$ of B, find the ratio of A to C.

11. Compare the rates of two trains, one of which travels $397\frac{5}{8}$ miles in $11\frac{3}{8}$ hours, and another which travels $262\frac{4}{15}$ miles in $8\frac{3}{5}$ hours.

12. If, when A makes a profit of *Rs.*2, B makes *Rs.*3; and when B makes a profit of *Rs.*4, C makes *Rs.*5; and when C makes a profit of *Rs.*6, D makes *Rs.*7; compare the profits of A, B, C and D.

II. PROPORTION.

471. Proportion is the *equality* of two ratios.

Thus, the ratios 2 : 3 and 6 : 9, being expressible by the equal fractions $\frac{2}{3}$ and $\frac{6}{9}$, are equal ; and the four numbers 2, 3, 6, 9 form a proportion which is *written* 2 : 3 :: 6 : 9, and is *read* 2 is to 3 as 6 is to 9, or 2 : 3 = 6 : 9 and is then *read* 2 to 3 equals 6 to 9 ; the numbers 2, 3, 6, 9 being its **terms** which taken in order are called **proportionals**.

Hence, in every proportion, the first term is greater than, equal to, or less than the second, according as the third term is greater than, equal to, or less than the fourth.

472. In the proportion above expressed, the numbers 2 and 9 are called the **extremes**, and the numbers 3 and 6 the **means** ; 9 is called **a fourth proportional** to 2, 3, and 6 ; and it follows immediately from the equality of the ratios denoted by

$$\frac{2}{3} = \frac{6}{9}, \text{ and the multiplication of them both by 27, that}$$

$$\frac{2}{3} \times 27 = \frac{6}{9} \times 27, \text{ that is, } 2 \times 9 = 6 \times 3 \text{ ;}$$

in other words, if *four* numbers constitute a proportion, the product of the *extremes* is equal to the product of the *means* (the first and last being the *extremes* and the second and third the *means*).

473. This property of a proportion proves immediately that *either* of the extremes may be obtained by dividing the product of the means by the *other* ; and that either of the means may be had by the division of the product of the extremes by the *other* ; also, these qualities constitute the general practical application of Proportion.

474. The terms of a proportion may be made to undergo changes and modifications in the same way as the *corresponding* terms of the vulgar fractions.

Thus, 3 : 4 :: 9 : 12, gives $3 \times 12 = 4 \times 9$; whence

$$\frac{3}{9} = \frac{4}{12} \text{ ; or } 3 : 9 :: 4 : 12 \text{ ; and } \frac{4}{3} = \frac{12}{9} \text{ ; or } 4 : 3 :: 12 : 9 \text{ ;}$$

and we observe that in each of these the product of the extremes equals that of the means.

Also, if four numbers form a proportion and any equi-multiples whatever of the first and second be taken, and any equi multiples whatever of the third and fourth, the resulting numbers taken in order will still form a proportion.

For, since $5 : 3 = 15 : 9$, or $\frac{5}{3} = \frac{15}{9}$; and also $\frac{2}{2} = \frac{3}{3}$;

we have $\frac{5}{3} \times \frac{2}{2} = \frac{15}{9} \times \frac{3}{3}$, or $\frac{5 \times 2}{3 \times 2} = \frac{15 \times 3}{9 \times 3}$;

whence, $5 \times 2 : 3 \times 2 = 15 \times 3 : 9 \times 3$.

Again, if any equi-multiples whatever of the first and third numbers be taken, and also any equi-multiples whatever of the second and fourth, the numbers thence arising will form a proportion.

Thus, if we take the proportion above, we have

$\frac{5}{3} \times \frac{4}{7} = \frac{15}{9} \times \frac{4}{7}$, or $\frac{5 \times 4}{3 \times 7} = \frac{15 \times 4}{9 \times 7}$;

whence, $5 \times 4 : 3 \times 7 = 15 \times 4 : 9 \times 7$.

The *new* ratios constituting these proportions being *equal* to the *original*, the division of the terms of a proportion, in accordance with this Article, will often facilitate practical computations by diminishing the number of figures employed.

475 If four quantities of the *same* kind taken in order be proportionals, it will be *useful* to recollect that,

(i) The first : the third :: the second : the fourth.

(ii) The second : the first :: the fourth : the third.

(iii) The sum of the first and second : the first :: the sum of the third and fourth : the third.

(iv) The sum of the first and second : the second :: the sum of the third and fourth : the fourth.

(v) The difference of the first and second : the first :: the difference of the third and fourth : the third.

(vi) The difference of the first and second : the second :: the difference of the third and fourth : the fourth.

(vii) The sum of the first and second : the difference of the first and second :: the sum of the third and fourth : the difference of the third and fourth.

These may easily be shown to be correct by any of the proportions hitherto given.

476. Of two or more proportions if the corresponding terms be multiplied together, the numbers thence arising will also form a proportion.

Thus, if the proportions be

$3 : 7 = 6 : 14$ and $4 : 9 = 12 : 27$; then $\frac{3}{7} = \frac{6}{14}$ and $\frac{4}{9} = \frac{12}{27}$,

whence $\frac{3}{7} \times \frac{4}{9} = \frac{6}{14} \times \frac{12}{27}$; or $\frac{3 \times 4}{7 \times 9} = \frac{6 \times 12}{14 \times 27}$;

and $3 \times 4 : 7 \times 9 = 6 \times 12 : 14 \times 27$.

This operation is called the **compounding** of proportions and the resulting proportion is said to be **compounded** of the others.

477. In the above Articles, *abstract* numbers have been considered ; but when the quantities are *concrete*, we must take care to exclude such proportions as *express* ratios between things of *different kinds* : thus, the ratio of 10 lbs. to 15 lbs. being the *same* as that of 2*s*. to 3*s*., we have the proportion

10 lbs. : 15 lbs. = 2*s*. : 3*s*. ;

but we cannot have the proportion

10 lbs. : 2*s*. = 15 lbs. : 3*s*.

as no ratio subsists between 10 lbs. and 2*s*. or between 15 lbs. and 3*s*.

Nor indeed can we even in the first of these forms multiply together the *concrete* quantities so that the product of the extremes equals the product of the means ; but what we do in finding any term in such cases, is to consider merely their *numerical* values, because the ratios being *abstract magnitudes* will remain the same whatever be the *nature* of the quantities they are used to compare.

478. Three quantities of the same kind are said to be in **continued proportion** when the ratio of the first to the second is equal to the ratio of the second to the third. The second quantity is called a **mean proportional** between the first and third ; and the third quantity is called a **third proportional** to the first and second.

Thus, 16, 8 and 4 are in *continued proportion*, for $16 : 8 = 8 : 4$; 8 is *a mean proportional* between 16 and 4 ; and 4 is *a third proportional* to 16 and 8.

479. In a continued proportion expressed as above, it is obvious that the *square of the mean proportional* between two numbers is equal to their *product* ; and consequently the mean proportional between two numbers is equal to the *square root* of their product.

Ex. 1. Find a fourth proportional to 5, 7 and 15.

$5 : 7 = 15$: number required,

$\therefore$ $5 \times$ number required $= 15 \times 7$,

$\therefore$ number required $= 15 \times 7 \div 5) = \underline{21}$. *Ans.*

Ex. 2. What number has the same ratio to 9 that 3 has to 5 ?

$3 : 5 =$ number required : 9,

$\therefore$ $5 \times$ number required $= 3 \times 9$,

$\therefore$ number required $= (3 \times 9 \div 5) = \frac{27}{5} = \underline{5\frac{2}{5}}$. *Ans.*

Ex. 3. Find a mean proportional between 14 and 56.

Square of the required number $= 14 \times 56 = 784$.

$\therefore$ the required number $= \sqrt{(784)} = \underline{28}$. *Ans.*

Ex. 4. One *gowala* to 19 sr. of milk adds 5 sr. of water, and another to 27 sr. of milk adds 7 sr. of water ; compare the amount of milk in the two mixtures.

The first mixture consists of (19+5) or 24 sr., of which $\frac{19}{24}$ is milk ;
......second..................(27+7) or 34 sr., of which $\frac{27}{34}$...... ... ;
∴ the ratio of milk in the two mixtures is $\frac{19}{24} : \frac{27}{34}$ or $\frac{19}{12} : \frac{27}{17}$,

i. e., 19 × 17 : 27 × 12 or 323 : 324. *Ans.*

Ex. 5. A mixture is composed of 9 parts brandy and 1 water ; 4 gallons of water are added, and the mixture contains 6 times as much brandy as water ; how many gallons of brandy does it contain ?

In the original mixture, brandy : water = 9 : 1 or 18 : 2 ;
... ..new....................brandy : water = 6 : 1 or 18 : 3.

Now, brandy remaining the same 18 parts, the water is increased by (3 − 2) or 1 part.

∴ for 1 gal of water added, there are 18 gals. of brandy,
∴ for 4 gals ....(18 × 4) or 72 gals...

Hence the required quantity of brandy = 72 gals. *Ans.*

Examples CXLV.

1. Find the fourth proportional to :—

(1) 15, 27 and 20. (2) 11, 80 and 70. (3) 1590, 53 and 1710.

(4) 9⅓, 7⅛ and 28½. (5) 18$\frac{2}{7}$, 5⅓ and 75. (6) 1·02, 5·1 and 10·3.

(7) 11·1, 16·38 and 17·76. (8) *Rs.*15, *Rs.*17½ and *Rs.*100.

(9) 11$\frac{1}{10}$ in., 55⅓ yds. and *Rs.*2. 4*a.*

(10) 57 mi., 38 mi. and 17 tons 13 cwt. 1 qr.

(11) 19¾ ac., 11 ac. 2 ro. 20 po. and 79 tons.

(12) 13*s.* 7½*d.*, £2. 0*s.* 10½*d.* and 5*s.* 8¼*d.*

2. Find a number which shall have the same ratio to 7 as 27 has to 3 ; also, a magnitude to which 39 has the same relation as 3¼ has to 2⅜.

3. Required the number which has to 40, the ratio of 3·75 to 3 ; and find a fourth proportional to $\frac{2}{7}$, 17 and 1·25.

4. Complete the proportion of which the first, second and fourth terms are $\frac{1}{20}$, 35 and 3¾ ; also, that whose first, third and fourth terms are ·35, 125 and ·0145.

5. Find a mean proportional between :—

(1) 16 and 4. (2) 5 and 125. (3) ·057 and ·513.

(4) $\frac{5}{63}$ and $\frac{35}{81}$. (5) 3⅜ and 10⅔. (6) ·1 and ·001.

6. Find the term not given in each of the following proportions :—

(1) 144 : :: 740 : 370. (2) : 1⅓ :: 3¼ : 2⅓. (3) $\frac{5}{7}$: :: $\frac{10}{21}$: $\frac{16}{27}$.

(4) 1·2 : 3·6 :: : 3·9. (5) ·01 : 7 :: : 5775. (6) : ·8 :: ·79 : 12·64.

7. Find a third proportional to (1) 25 and 400. (2) ⅜ and $\frac{5}{7}$.

8. The ratio of A to B is 2 : 3 ; of B to C is 5 : 6 ; and of C to D is 7 : 8 ; find the continued ratio of A, B, C and D.

9. If 3 men and 11 boys working together, can do 5 times as much work per hour as a man and a boy together, compare the work of a boy with that of a man.

10. A buys 15 cows and 130 sheep for a certain sum, and B buys 9 cows and 175 sheep, at the same rates as A, for the same sum. Compare the values of a sheep and of a cow.

11. A cask of 72 gallons consists of 11 parts brandy and 1 part water ; how much water must be added that it may consist of 9 parts brandy and 1 water ?

12. 270 sheep and 14 horses eat 101 acres of grass in 30 days, and 155 sheep and 21 horses eat 185 acres of grass in 75 days. Compare the amount eaten by a sheep and by a horse in the same time.

13. A greyhound pursues a hare and takes 3 leaps for every 4 leaps of the hare, but 2 leaps of the hound are equal to 3 of the hare ; compare the rates of hound and hare.

14. Six coins of equal weight, made of gold and silver mixed, are melted together and re-cast. In one of them the gold and silver were in the ratio of 2 : 1 ; in two others of 3 : 5, and in the rest 7 : 5. In what ratio will the gold and silver be mixed in the new coins ?

15. The values of gold and silver are as 14·28 to 1, in what proportions must these metals be combined, in order that the compound may be twice as valuable as an alloy of two parts of silver and one of gold ?

III. RULE OF THREE.

480. The object of the **Rule of Three** is, by means of *three* quantities given, to determine a *fourth*, which shall be the same multiple, part or parts of one of them, that one of the remaining quantities is of the other ; and it therefore follows that the operation, by which this may be accomplished, depends on the rule already laid down in Art. 473, that in a proportion if three of the numbers or quantities are given the fourth number or quantity can easily be found.

RULE. *For the Statement.* Of the three quantities proposed, put down as the last on the right hand, that which is of the *same kind*, or under the *same circumstances* as the one required ; and the *greater* or *less* of the two others in the second place, according as the required one ought, from the nature of the case, to be *greater* or *less* than the last ; and the remaining one in the first place.

For the Operation. Reduce, if necessary, the first and second *terms* to the *same* denomination and the third to the *lowest* denomination contained in it ; multiply together the second and third

terms thus reduced, and the quotient arising from division of the product by the first, will be the quantity required, expressed in the *denomination* to which the *last* term was reduced ; and it may be had in other denominations by the proper divisions or multiplications.

481. It is sometimes necessary to consider what *preparation* may be required before the rule is applied ; and when the statement is made, the first, and the second *or* third terms may be divided by any factor common to them, either *before* or *after* the reductions, without affecting the result, inasmuch as no alteration is produced from multiplication and division by the same number.

Ex. 1. If 11 mds. of rice cost *Rs* 41. 7*a.* 8*p.*, what sum must be paid for 45 mds. ?

Here, what is required being *money*, the *last* term of the statement will be *Rs.*41 7*a.* 8*p.*, since it is of the *same kind* and because the price of 45 mds. must manifestly be *greater* than that of 11 mds., the *second* term must be 45 mds. and the *first* will be 11 mds., that is, the statement and operation will be as follows :—

11 mds. : 45 mds. : : *Rs* 41. 7*a.* 8*p.* : the required price,
or 11 mds. : 45 mds. : : 7964*p.* : the required price.

$$\therefore \text{ the required price } = \frac{7964 \times 45}{11}p = (724 \times 45)p.$$
$$= 32580p. = Rs.169.\ 11a. \quad Ans.$$

Ex. 2. If a person can walk a certain distance in 8 days of 7 hrs. 30 min. each ; in how many days can he do the same, when 10 hrs. of each are available for the purpose ?

Since *days* are *required* here, and the number of days is necessarily *less* as the number of hours employed in each is *greater*, we shall have,

10 hrs. : 7 hrs. 30 min. :: 8 days : reqd. time.
or 10 hrs. : $7\frac{1}{2}$ hrs.($=\frac{15}{2}$ hrs.) :: 8 days : reqd. time.

$$\therefore \text{ reqd. time} = \frac{15 \times 8}{10 \times 2} \text{ days} = 6 \text{ days.} \quad Ans.$$

482. The RULE OF THREE is applicable in two different kinds of cases, according to which it is called the **Rule of Three Direct** or the **Rule of Three Inverse** ; but the rules for the *Statement* and *Operation* above given are applicable to both kinds of cases.

The RULE OF THREE DIRECT is that in which *more* requires *more* and *less* requires *less* ; or, in other words, in which a *greater* number requires a *greater* answer, and a *less* number a *less* answer. The first of the above two examples is an instance of the **Rule of Three Direct** ; for 45 mds. being greater than 11 mds , the price of 45 mds. is necessarily greater than that of 11 mds.

The RULE OF THREE INVERSE is that in which *more* requires *less* and *less* requires *more* ; or, in other words, in which a *greater*

number requires a *less* answer, and a *less* number a *greater* answer. The second of the above Examples is an instance of the **Rule of Three Inverse**; for 10 hrs. being greater than 7 hrs. 30 min, it is plain that walking 10 hrs. a day will take a *less* time than walking 7 hrs. 30 min. daily.

483. The following Examples are instances of the RULE OF THREE DIRECT.

Ex. 1. What will 5 cwt. 2 qrs. 24 lbs. cost, if 18 lbs. cost *Rs.*6. 15*a.*?

18 lbs. : 5 cwt. 2 qrs. 24 lbs. :: *Rs.*6. 15*a.* : reqd. cost,
or 18 lbs. : 640 lbs. :: *Rs.*$\frac{111}{16}$: reqd. cost;

$\therefore$ the reqd. cost $= Rs.\frac{111 \times 640}{18 \times 16} = Rs.\frac{740}{3} =$ *Rs.*246. 10*a.* 8*p.* *Ans.*

Ex. 2. If 17 mds. 6 sr. can be bought for *Rs.*142. 14*a.* 8*p.*, how much can be bought for *Rs.*373. 5*a.* 4*p.*?

*Rs.*142. 14*a.* 8*p.* : *Rs.*373. 5*a.* 4*p.* :: 17 mds. 6 sr. : the reqd. weight;
or *Rs.*$\frac{1715}{12}$: *Rs.*$\frac{1120}{3}$ :: $\frac{343}{20}$ mds. : the reqd. weight.

$\therefore$ the reqd. weight $= \frac{343 \times 1120 \times 12}{20 \times 3 \times 1715}$ mds. $= \frac{224}{5}$ mds. $=$ 44 mds. 32 sr.

Ex 3. A bankrupt's debts are *Rs.*5255. 4*a.* and his assets *Rs.*3753. 12*a.*; how much can he pay in the rupee?

Rs 5255. 4*a.* : *Re.*1. :: *Rs* 3753. 12*a.* : what he can pay per *Re.*
or *Rs.*$\frac{21021}{4}$: *Re.*1. :: *Rs.*$\frac{15015}{4}$: what he can pay per *Re.*

$\therefore$ what he can pay per *Re.* $= \frac{15015 \times 4}{2\ 021 \times 4} = Re.\frac{5}{7} =$ 11*a.* 5$\frac{1}{7}$*p.* *Ans.*

Ex. 4. A man's income is *Rs.*2755, and the income-tax is 3*a.* 4*p.* in the rupee; what tax does he pay?

*Re.*1 : *Rs.*2755 :: 3*a.* 4*p.* : reqd. tax
$\therefore$ the reqd. tax $=$ 3*a.* 4*p.* $\times$ 2755 $=$ *Rs.*573. 15*a.* 4*p.* *Ans.*

Ex. 5. If after paying an income tax of 7*d.* in the pound, a person has £776 13*s.* 4*d* remaining, what is his actual gross income?
After paying 7*d.* in the £, he has left (240 − 7) or 233*d.*

233*d.* : £776 13*s.* 4*d.* :: £1 : reqd. income;
or £$\frac{233}{240}$: £$\frac{2330}{3}$ :: £1 : reqd income.
$\therefore$ the reqd. income $= £\frac{2330 \times 240}{3 \times 233} =$ £800. *Ans.*

Ex. 6. If I can travel 198 miles by railway for *Rs.*24. 12*a.*, how far at the same rate of charge ought I to be carried for *Rs.*80. 7*a.*?

*Rs.*24. 12*a.* : *Rs.*80. 7*a* :: 198 miles : reqd. distance;
or *Rs.*$\frac{99}{4}$: *Rs.*$\frac{1287}{16}$ :: 198 miles : reqd. distance;

$\therefore$ the reqd. distance $= \frac{1287 \times 198 \times 4}{16 \times 99}$ miles $=$ 643$\frac{1}{2}$ miles. *Ans.*

Ex. 7. If 5 cows eat as much as 3 horses, and that the charge per annum for pasturing 5 horses is *Rs.*200 ; what will be the cost of the annual pasture of 15 cows ?

Here, 5 cows = 3 horses ; ∴ 15 cows = 9 horses.

5 horses : 9 horses :: *Rs.*200 : cost required.

$$\therefore \text{ the reqd. cost} = Rs. \frac{200 \times 9}{5} = Rs. \underline{\underline{360.}} \quad \textit{Ans.}$$

484. The following are Examples of the RULE OF THREE INVERSE.

Ex. 1. In what time will 45 men do a piece of work, which 36 men can do in 35 days ?

45 men : 36 men :: 35 days : reqd. no. of days.

$$\therefore \text{ the reqd. time} = \frac{35 \times 36}{45} \text{ days} = \underline{\underline{28 \text{ days.}}} \quad \textit{Ans.}$$

Ex. 2. What sum of money must be advanced on loan for 63 days, as an equivalent for the loan of *Rs.*1107. 12*a.* for 125 days ?

63 days : 125 days :: *Rs.*1107. 12*a.* : the reqd. sum ;
or 63 days : 125 days :: $Rs \frac{4431}{4}$: reqd. sum ;

$$\therefore \text{ the reqd. sum} = Rs. \frac{125 \times 4431}{63 \times 4} = \underline{\underline{Rs.2197.\ 14a.\ 8p.}} \quad \textit{Ans.}$$

Ex. 3. What length of carpet 2 ft. 3 in. wide, will be required to cover a room which is 27 ft. 6 in. long, and 22 ft. 6 in. wide ?

2 ft. 3 in. : 22 ft. 6 in. :: 27 ft. 6 in. : the reqd. length ;
or 27 in. : 270 in :: 330 in. : the reqd. length ;

$$\therefore \text{ the reqd. length} = \frac{330 \times 270}{27} \text{ in.} = 3300 \text{ in.} = \underline{\underline{91 \text{ yds. } 2 \text{ ft.}}} \quad \textit{Ans.}$$

Ex. 4. How many yards of cloth at *Re.*1. 13*a.* per yard must be given in exchange for 942½ yards of silk at *Rs.*9. 10*a.* per yard ?

*Re.*1. 13*a.* : *Rs* 9 10*a.* :: 942½ yds. : reqd. cloth ;
or $Rs.\frac{29}{16} : Rs.\frac{77}{8} :: \frac{1885}{2}$ yds. : reqd cloth ;

$$\therefore \text{ the reqd. cloth} = \frac{1885 \times 77 \times 16}{2 \times 8 \times 29} \text{ yds.} = \underline{\underline{5005}} \text{ yds.} \quad \textit{Ans.}$$

Ex. 5. If 2000 men have provisions for 95 days, and if after 15 days 400 men go away, find how long the remaining provisions will serve the number left.

After 15 days there will be provisions left of 2000 men for (95 − 15) or 80 days, while the number of men will be reduced to (2000 − 400) or 1600 men.

1600 men : 2000 men :: 80 days : no. of days reqd. ;

$$\therefore \text{ no. of days reqd.} = \frac{2000 \times 80}{1600} = \underline{\underline{100.}} \quad \textit{Ans.}$$

485. There are certain Examples. in which, at first sight, more than three terms appear to be given, but they, in certain cases, come under this Rule, as in the following instances.

Ex. 1. If the 4*d.* loaf weighs 2 lbs. 3 oz. when wheat is at 7*s.* $1\frac{1}{2}$*d.* a bushel, what should it weigh when wheat is at 7*s.* 11*d.* a bushel ?

Here 4*d.* may be left out of consideration, as being the same in both cases

7*s.* 11*d.* : 7*s* $1\frac{1}{2}$*d.* :: 2 lbs. 3 oz. : reqd. weight ;
or 95*d.* : $85\frac{1}{2}$*d.* :: 35 oz. : reqd. weight.

$\therefore$ the reqd. weight $= \frac{35 \times 171}{2 \times 95}$ oz. $= 31\frac{1}{2}$ oz. $=$ 1 lb. $15\frac{1}{2}$ oz. *Ans.*

Ex. 2. If the carriage of 5 cwt. 7 lbs. for 84 miles cost *Rs.*39, what will it cost to have 21 cwt. 1 qr. 14 lbs. carried the same distance?

Here, 84 miles may be neglected, as it is the same in both cases.
5 cwt. 7 lbs. : 21 cwt. 1 qr. 14 lbs. :: *Rs.*39 : reqd. cost ;
or 567 lbs. : 2394 lbs. :: *Rs.*39 : reqd. cost ;

$\therefore$ the reqd. cost $=$ *Rs.*$\frac{2394 \times 39}{567} =$ *Rs.*$\frac{494}{3} =$ *Rs.*164. 10*a.* 8*p.* *Ans.*

486. Examples such as the following are easily worked by the RULE OF THREE.

Ex. 1. A person gives away annually *Rs.*200 in charity, and his weekly bills amount to *Rs* 75 ; what additional daily expenditure may he incur with an income of *Rs.*5925 ? (A year $=$ 52 weeks.)

For the annual amount of his weekly bills, we have
1 wk. : 52 wks. :: *Rs.*75 : annual weekly bills ;
$\therefore$ annual weekly bills $=$ *Rs.*$(75 \times 52) =$ *Rs.*3900.

Therefore his charity and weekly bills amount to *Rs.*$(3900 + 200)$ or *Rs.*4100 : and he has *Rs.*$(5925 - 4100)$ or *Rs.*1825 left to be expended in 365 days ; whence,

365 days : 1 day :: *Rs.*1825 : additional daily expenses ;

$\therefore$ addl. daily expenses $=$ *Rs.*$\frac{1825}{365} =$ *Rs.*5. *Ans.*

Ex. 2. Two trains 210 ft. and 180 ft. in length respectively are going in opposite directions, the first at the rate of 23 miles per hour and the other, at the rate of 27 miles per hour ; find how long they will take to pass each other.

As the trains are going in opposite directions, they are approaching each other by $(24 + 27)$ or 51 miles per hour, and one shall pass the other, when $(210 + 180)$ or 390 ft. or $\frac{13}{176}$ mile is passed over.

51 miles : $\frac{13}{176}$ mile :: 1 hr. : the reqd. time ;

$\therefore$ the reqd. time $= \frac{13}{51 \times 176}$ hr. $= 5\frac{40}{187}$ sec. *Ans.*

Examples CXLVI.

1. Required the price of 450 lbs., at *Rs.*2. 5*a.* 8*p.* per lb.

2. A person's salary is £191 12*s.* 6*d.* for 365 days; in how many days will he have a claim for £31. 10*s.*?

3. Required the price of 4 cwt. 1 qr. 4 lbs. 8 oz. of sugar, when 1 lb. costs 7*s.* 10½*d.*

4. If an artificer earn *Rs.*190. 8*a.* in 20 days; in what time will he earn *Rs.*238. 2*a.*?

5. If 17 ells. 3 qrs., each ell containing 5 qrs., be bought for *Rs.*68. 12*a.*; how much must be paid for 18 yards?

6. If 1000 sovereigns weigh 21 lbs. 5 oz. 16 dwts. 6 grs., what weight of gold will be contained in 384 sovereigns?

7. How much wheat can be purchased for *Rs.*550. 2*a.*, at the rate of *Rs.*3. 6*a.* 4*p.* per maund?

8. If a farm of 375 bighas be let for *Rs.*4015. 10*a.* a year, what is that for each bigha?

9. If lodgings be let at *Rs.*6. 12*a.* per week, what will the demand amount to for 273 days?

10. Required the price of 36 cwt. 1 qr. of rice, when 2 cwt. 2 qrs. 10 lbs. cost £4. 7*s.* 9¼*d.*

11. If a servant's wages be £30. 0*s.* 8¾*d.* a year, what will be his demand for a service of 338 days?

12. If a person can walk 3 mi. 6 fur. 25 po. in an hour, in what time will he complete a journey of 99 mi. 4 fur. 10 po.?

13. If the carriage of 3 cwt. 2 qrs. 14 lbs. for 51 miles come to *Rs.*9. 3*a.* 6*p.*; what will be the charge for carrying 10 tons 3 cwt., the same distance?

14. At the rate of 11*s.* 7½*d.* in the pound, what is the sum paid by a bankrupt for a debt of £2735 10*s*?

15. If 67 mds. 8 sr. cost *Rs* 746. 10*a.* 8*p.*, how much can be bought for *Rs.*285. 13*a* 4*p.*?

16. If 11 mds. 8 sr. cost *Rs.*2240, what will 4 mds. 11 sr. 8 ch. cost at the same rate?

17. If 3 qrs. 7 lbs. of tobacco cost £17. 13*s.* 6*d.*, what is the value of 5 cwt. 1 qr. 23 lbs.?

18. If 17 cwt. 2 qrs. 14 lbs. can be obtained for £8. 13*s.* 3¾*d.*, what weight can be obtained for £21. 10*s.* 2½*d.*?

19. The clothing of a regiment of 735 men costs *Rs.*13987. 12*a.*, what will the clothing of a regiment of 903 men cost at the same rate?

20 The interest on *Rs.*2719. 2*a*. 8*p*. for 77 days is *Rs.*31. 13*a*. 8*p*. ; find the interest on the same sum for 245 days.

21. A person in 87 days spends *Rs.*389. 11*a*., in how many days will he spend *Rs* 1634. 14*a*. 4*p*. at the same rate ?

22. If 15 workmen can do a piece of work in 25 days, in what time can 25 men do the same ?

23. How much in length, that is 3 ft. 9 in. broad, will be equivalent to 37 ft. 9 in. in length, which is 7 ft. 6 in. broad ?

24. If 69 yds. of carpet 3 qrs. wide, cover a room 8 yds. 2 qrs. 2 nls. long ; find the width of the room.

25. If a person's annual income be 650 guineas, how much will he have saved at the end of the year, after spending £10. 13*s*. 9½*d*. a week ?

26. What may a person, having an income of *Rs.*10000 a year spend daily, so as to lay by *Rs.*4342. 8*a*. yearly ?

27. If I lend a friend *Rs* 2500 for 6 months, how long ought he to lend me *Rs.*1875, to requite the kindness ?

28. What is the tax upon *Rs* 3021. 12*a*. 8*p*., when *Rs* 4294. 2*a*. is rated at *Rs.*6. 12*a*. ?

29. If the rate levied upon a rental of *Rs.*7637. 8*a*. amount to *Rs.*1336. 9*a*., how much is it in the rupee ?

30. A person buys 136 yds. of cloth for *Rs.*1500, and retails it at *Rs.*19 per yard ; what does he gain by the transaction ?

31. A person's daily income is *Rs* 17 8*a*., and his quarterly expenditure is *Rs.*1355 ; how much will he have saved at the end of 9 years ?

32. If a gentleman spend £152. 10*s*. every week ; what must be his daily income that in 15 years he may lay by £7522. 10*s*. ? (a year = 52 weeks.)

33. How many ducats of 4*s*. 11¾*d*. each are equal in value to 55926 rix-dollars of 4*s*. 10½*d*. each ?

34. After payment of an income-tax of 7*d*. in the £, a person has left £249. 19*s*. 9½*d*. ; find his full income.

35. A bankrupt's debts amount to *Rs* 5355. 3*a*. 4*p*. and his assets to *Rs.*3213. 2*a*. How much can he pay in the rupee ?

36. A certain number of reapers can reap 84 ac. 3 ro. 14 po. in 13¾ hours ; in how many hours can they reap 401 ac. 8 po. ?

37. If a person having an income of *Rs.*1855 has to pay an income-tax of *Rs.*54. 1*a*. 8*p*., what is the income of a person who pays *Rs.*306. 4*a*. ?

38. A merchant exchanged 1134 yds. of velvet for 5313 yds. of silk at *Re.*1. 11*a*. per yard ; find the value of the velvet per yard.

39. A bankrupt pays 9*a.* $3\frac{3}{8}$*p.* in the rupee ; what will be lost on a debt of *Rs.*27350 ?

40. A person after paying for the first half of a year an income-tax of $4\frac{1}{2}$*p.* in the rupee, and for the second half one of $7\frac{1}{2}$*p.* in the rupee on his income, has *Rs.*3535. 15*a.* left ; what was the income on which he paid ?

41. A merchant paid *Rs.*187. 8*a.* for a year's income-tax ; but after the tax has been increased to 7·2 pies in the rupee, he paid *Rs* 525 ; what was his income, and at what rate in the rupee was the tax levied at first ?

42. A person bought 180 gallons of wine for *Rs.*1250 ; find the quantity of water to be added that he may retail the mixture at *Rs.*6. 4*a.* a gallon.

43. If an estate produce £1680 a year, and the land-tax be payable upon this sum at 3*s.* 6*d.* in the pound ; what is its clear annual value ?

44. When a bankrupt's effects pay three dividends of 4*s.* $2\frac{1}{4}$*d.*, 3*s.* $2\frac{1}{4}$*d.*, and 2*s.* $4\frac{1}{2}$*d.* in the £ ; what do his creditors lose upon this entire debt, which is £4265 ?

45. A person bought 125 yds. of cloth, at the rate of 2 yds. for *Rs.*2. 8*a.* and 125 yds. at the rate of 3 yds. for *Rs.*2. 8*a.* ; what will he gain or lose by selling the 250 yards at the rate of 5 yds. for *Rs.*5 ?

46. A garrison of 638 men has provisions for 124 days ; how long will the provisions last if the garrison be reinforced by 418 men ?

47. If $1\frac{3}{4}$ yards of silk cost *Rs* 5. 7*a.* 6*p.*, what will be the cost of $75\frac{5}{8}$ yards ? and how many yards can be got for *Rs.*50 ?

48. If $2\frac{3}{8}$ lbs. of tea cost *Rs.*4. 5*a.* 8*p*, how much will $27\frac{8}{11}$ lbs of the same quality cost ?

49. The price of ·0625 lb. of coffee is $\cdot458\dot{3}$*s.*, what is the value of ·075 of a ton ?

50. If when the price of wheat is 55·5*s.* a quarter, the 6*d.* loaf weighs 3·4375 lbs., what is the price of wheat when the loaf weighs 2·8125 lbs. ?

51. If 5 lbs. of sugar cost ·0703125 of *Rs.*8, what will ·0625 cwt. of the same quality of sugar cost ?

52. *A* has 3 lbs. $1\frac{8}{11}$ oz. of ginger worth 10*a.* 2*p.* a lb. which he exchanges with *B* for 3 lbs. 8 oz. of pepper ; what is the value of the pepper per lb. ?

53. If 7 oxen or 11 horses can eat the grass of a field in 37 days, in how many days will 5 oxen and 8 horses eat it ?

54. A clock, set on Friday at 8 P.M. loses $2\frac{1}{4}$ min. daily ; what the correct time when the clock strikes 8 next Tuesday morning ?

55. If 1000 men have provisions for 50 days, and if after

10 days, 150 men go away, find how long the remaining provisions will serve the number left.

56. Find the height of a tower which casts a shadow of 75 ft. 6 in. long, when the length of the shadow of a walking stick 3 ft. 3 in. long is 2 ft. 9 in.

57. If 7 boys earn as much as 4 men, and 48 men assisted by 14 boys earn *Rs.*423. 8*a.*, what number of boys assisting 20 men will earn *Rs.*272. 4*a.* in the same time?

58. A besieged town containing 22400 inhabitants has provisions for 3 weeks, how many must be sent away that they may be able to hold out 7 weeks?

59. Two watches one of which gains 3 min. while the other loses 5 min. a day are set right at 10 o'clock A.M. on Wednesday, the 26th February, 1868; when will there be a difference of 90 minutes?

60. If $\cdot\dot{4}2857\dot{1}$ of a barrel of beer be worth ·72 of £2. 10*s.*, what is the value of ·625 of the remainder?

61. If 4 men working 15 hours, 3 men working 12 hours, and 8 men working 3 hours earn *Rs.*32. 8*a.*, what will a man's wages for 6 days come to, if he works 11 hours a day?

62. If $\frac{1}{128}$ of $3\frac{2}{3}$ of $\frac{7}{9}$ of $5\frac{1}{6}$ of 22 lbs. of sugar cost 5*a.* 6*p.*, how much will 1 ton 11 cwt. 3 qrs. cost?

63. If 4 men or 6 women or 9 boys can perform a piece of work in $27\frac{1}{2}$ days, in what time can (i) 5 men and 9 women perform it? and (ii) 5 men and 8 boys perform it?

64. A clock which was $1\cdot\dot{2}$ min. fast at a quarter to 11 P.M. on Nov. 28, was exactly right at 11-30 P.M. the following day. How many minutes was it slow at a quarter to 2 P.M. on Dec. 7?

65. If 17 men can do a piece of work in 89 days; and if after working for 33 days, 3 men leave; in how many days in *all* will the work be done?

66. If 13 men, 10 women and 17 boys can complete a piece of work in 50 days, how long would 7 men, 13 women and 14 boys be in doing twice as much work, the parts done by each in the same time being as the numbers 3, 2 and 1?

67. If a besieged garrison have 4 months' provisions, at the rate of 18 chataks per man per day, how long would they be able to hold out, if each man were allowed only 12 chataks per day?

68. If the 6*d.* loaf weighs 3 chataks when wheat is at *Rs.*3 per maund, what ought it to weigh when wheat is at *Rs.*3 6*a.* per md.?

69. A hare starts with 25 of her leaps in advance of a hound, and takes 4 leaps to the hound's 3; but 2 of the hound's leaps are equal to 3 of the hare's; how many leaps must the hound take to overtake the hare?

70. A besieged fortress has provisions for 3 weeks, at the

rate of 14 ch. a day for each man ; at what rate per day must the provisions be distributed, so that the place may hold out 5 weeks ?

71. A person contracts to make a road 189 miles long in 15 months. He employs 256 men, but after 3 months finds that he has only finished 28 miles. How many men must he now employ to finish it within the time required ?

72. If 7 gallons of brandy cost as much as 9 gallons of rum ; and 9 gallons of rum as much as 12 gallons of gin, and the cost of 3 gallons of these, taken one of each kind, be £2. 2*s*. 6*d*., what is the value of each per gallon ?

73. A servant's wages are fixed at *Rs*.231 a year ; he enters his situation on the 12th of July, and leaves on Christmas day in the same year ; his master had advanced him *Rs*.36. 12*a*., what sum ought he to receive on leaving ?

74. A man takes 55 min. to walk to Barrackpore by the road, and 58 min. 40 sec. to return by the fields, walking at the same pace. The distance by the road is $3\frac{3}{4}$ miles ; what is it by the fields ?

75. If 4 men and 5 boys earn *Rs*.46. 1*a*. 4*p*. in 7 days, and 3 men and 8 boys earn *Rs*.60. 6*a*. in 9 days ; in what time will 12 men and 12 boys earn *Rs*.388. 8*a*. ?

76. A piece of gold at *Rs*.38. 15*a*. per oz. is worth *Rs*.1500 ; what will be the value of a piece of silver of equal weight at *Rs*.27. 4*a*. per lb. ?

77. A contractor undertook to build a house in 21 days, and engaged 15 men to do the work. But after 10 days he found it necessary to engage 10 men more, and then he accomplished the work one day too soon. How many days behindhand would he have been if he had not engaged the 10 additional men ?

78. A ship leaves port with sufficient provisions to last 14 weeks ; 6 of the crew absconded upon setting sail, and the voyage lasted 16 weeks, at the end of which time the provisions were just exhausted ; find the number of the full crew.

79. A fixed rent of £1170 per annum is converted into a corn-rent of one-half wheat at the average price of 48*s*. per quarter ; and the other half barley at the average price of 30*s*. per quarter ; what will be the rent when wheat has advanced to 56*s*. and barley to 32*s*. per quarter ?

80. A railway train travels 27 miles an hour, including stoppages, and 30 miles an hour when it does not stop ; in what distance will it lose 45 minutes by stopping ?

IV. DOUBLE RULE OF THREE.

487. Double Rule of Three or Compound Rule of Three is a process in which five quantities are given to find a sixth, and

four of the five given quantities form two parts of different kinds, and the fifth and the answer required form a third pair of another kind, also the quantities of the first and second kind are directly or inversely proportional to the given quantity of the third kind.

488. This definition of Double Rule of Three may be extended to cases where seven quantities are given to find an eighth ; nine to find a tenth ; and so on.

489. For the sake of convenience, we may divide each question in Double Rule of Three into two parts, the **supposition** and the **demand** ; the *supposition* being the part which expresses the conditions of the question, and the *demand* the part which mentions the thing demanded or sought.

Thus, in the question—If 16 horses eat 54 mds. of corn in 6 days ; how many horses will eat 135 mds. in 8 days ?—we might arrange the terms thus :

Supposition—16 horses, 54 mds. 6 days.
Demand—horses reqd. 135 mds. 8 days.

Here, we see that four of the given quantities form two pairs of different kinds,—*maunds* and *days* ; and the fifth and the answer form a third pair of another kind—*horses*.

Also the number of horses is inversely proportional to the number of days and directly proportional to the number of maunds.

490. We will now explain the ordinary method of Solution which is by **two** or **more** Rule of Three statements, and by compounding them into one final statement ; hence the name **Double** Rule of Three and **Compound** Rule of Three.

Taking the preceding Example, we can divide it into two parts :—

First.—If 16 horses eat 54 mds. of corn in 6 days, how many horses will eat 135 mds. in the same time ?

Here, since the time is the same in the two hypotheses, the number of horses will depend on, and is *directly* proportional to the number of maunds. Hence we have the following statement :—

(i) 54 mds. : 135 mds. :: 16 horses : no. of horses reqd.
54 : 135 :: 16 : $(16 \times 135) \div 54$ or 40 horses.

Secondly.—If 40 horses eat 135 mds. in 6 days, how many horses will eat the same number of maunds in 8 days ?

Here, the number of maunds being the same in the two hypotheses, the number of horses depends on, and is *inversely* proportional to, the number of days ; hence the statement,

(ii) 8 days : 6 days :: 40 horses : no. of horses reqd.
or 8 : 6 :: 40 : no. of horses reqd.

Compounding the statements (i) and (ii), we have
$54 \times 8 : 135 \times 6 :: 16 \times 40$: $40 \times$ no. of horses reqd.
:: 16 : no. of horses reqd.

This last statement is generally written thus :—

$$\left.\begin{matrix} 54 : 135 \\ 8 : 6 \end{matrix}\right\} = 16 : \text{no. of horses reqd.}$$

$$\therefore \text{ no. of horses reqd.} = \frac{16 \times 135 \times 6}{54 \times 8} = \underline{30.} \quad \textit{Ans.}$$

491. From the above considerations, we deduce the following Rule :—

RULE. *For the Statement.* Write down as the third term that quantity which corresponds to the quantity required. Then take the first pair of given quantities of the same kind, and with them make the Rule of Three statements with reference to the third term *only* ; do the same with the second pair, and with each succeeding pair writing these ratios under one another.

For the Operation. Reduce the first and second terms in each of these statements to the same denomination. Also, reduce the common third term to a single denomination if it be not already in that state, and then treat the terms as abstract numbers. Multiply all the first terms together for a final first term, and all the second terms together for a final second term, and retain the former third term. In this final statement multiply the second and third terms together and divide the product by the first. The quotient will be the answer to the question in the denomination to which the third term was reduced.

Ex. 1. If 5 men earn *Rs.*187. 8*a.* in 12 weeks, how much will 16 men earn in 20 weeks ?

$$\left.\begin{matrix} 5 \text{ men} : 16 \text{ men} \\ 12 \text{ wks.} : 20 \text{ wks.} \end{matrix}\right\} = Rs.187.\ 8a. : Rs. \text{ reqd.} \left\{\begin{matrix} \text{More men, } \textit{more} \text{ earning ;} \\ \text{more wks., } \textit{more} \text{ earning ;} \end{matrix}\right.$$

$$5 \times 12 : 16 \times 20 :: Rs.\tfrac{375}{2} : Rs. \text{ reqd.}$$

$$\therefore \text{ no. of } Rs. \text{ reqd.} = \frac{16 \times 20 \times 375}{5 \times 12 \times 2} = Rs.\underline{1000.} \quad \textit{Ans.}$$

492. Every question in Double Rule of Three may be solved by the *Unitary Method* :—

Taking the preceding Example, we proceed thus :—

Since 5 men in 12 weeks earn $Rs.187\frac{1}{2}$,

$\therefore$ 5 men in 1 week earn $Rs.187\frac{1}{2} \times \frac{1}{12}$,

$\therefore$ 1 man in 1 week earns $Rs.\frac{375}{2} \times \frac{1}{12} \times \frac{1}{5}$,

$\therefore$ 1 man in 20 weeks earns $Rs.\frac{375}{2} \times \frac{20}{12} \times \frac{1}{5}$,

$\therefore$ 16 men in 20 weeks earn $Rs.\frac{375}{2} \times \frac{20}{12} \times \frac{16}{5} = Rs.\underline{1000.}$ *Ans.*

Ex. 2. If 3 tons 16 cwt. can be carried 25 miles for *Rs.*118. 12*a.*, what weight can be carried 52 miles for *Rs.*59. 9*a.* 4*p.* ?

$Rs.118.\ 12a. = Rs.118\frac{3}{4}$; $Rs.59.\ 9a.\ 4p. = Rs.59\frac{7}{12}$; 3 tons 16 cwt. = 76cwt.

$$\left.\begin{matrix} 52 \text{ mi.} : 25 \text{ mi.} \\ Rs.118\frac{3}{4} : Rs.59\frac{7}{12} \end{matrix}\right\} = 76 \text{ cwt.} : \text{wt. reqd.} \left\{\begin{matrix} \text{More miles, } \textit{less} \text{ weight ;} \\ \text{more money, } \textit{more} \text{ weight.} \end{matrix}\right.$$

$$\therefore \text{ weight reqd.} = \frac{76 \times 25 \times 715 \times 4}{52 \times 475 \times 12} = \underline{18\tfrac{1}{3} \text{ cwt.}} \quad \textit{Ans.}$$

Ex. 3. If a penny loaf weigh 6 oz. when wheat is 5*s.* 6*d.* a bushel, what should be the weight of a shilling loaf when wheat is 8*s.* 3*d.* a bushel ?

$$\left.\begin{array}{l} 1d. : 12d. \\ 8\tfrac{1}{4}s. : 5\tfrac{1}{2}s. \end{array}\right\} = 6 \text{ oz. : weight reqd.} \left\{\begin{array}{l} \text{More price of loaf, } \textit{more} \text{ weight;} \\ \text{more price of wheat, } \textit{less} \text{ weight.} \end{array}\right.$$

$$\therefore \text{ weight reqd.} = \frac{6 \times 12 \times 11 \times 4}{2 \times 33} \text{ oz.} = 48 \text{ oz.} = \underline{3 \text{ lbs.}} \; \textit{Ans.}$$

Ex. 4. If 144 men in 5 days of 11 hours each can dig a trench 132 yds. long, 5 ft. wide and 2 ft. deep, in how many days of 9 hours each can 56 men dig a trench 210 yds. long, 8 ft. wide and 3 ft. deep ?

$$\left.\begin{array}{lcl} 56 \text{ men} & : & 144 \text{ men.} \\ 9 \text{ hrs.} & : & 11 \text{ hrs.} \\ 132 \text{ yds.} & : & 210 \text{ yds.} \\ 5 \text{ ft.} & : & 8 \text{ ft.} \\ 2 \text{ ft.} & : & 3 \text{ ft.} \end{array}\right\} = 5 \text{ days : days reqd.} \left\{\begin{array}{ll} \text{More men,} & \textit{less} \text{ days;} \\ \text{more hours,} & \textit{less} \text{ days;} \\ \text{more length,} & \textit{more} \text{ days;} \\ \text{more breadth,} & \textit{more} \text{ days;} \\ \text{more depth,} & \textit{more} \text{ days.} \end{array}\right.$$

$$\therefore \text{ days reqd.} = \frac{5 \times 144 \times 11 \times 210 \times 8 \times 3}{56 \times 9 \times 132 \times 5 \times 2} = \underline{60}. \; \textit{Ans.}$$

Ex. 5. A garrison of 4500 men is supplied with provisions for 15 months at the rate of 13 chataks per day per man ; how many must leave, that the same provisions may supply those that remain 27 months at 10 chataks per day per man ?

$$\left.\begin{array}{l} 27 \text{ mo. : } 15 \text{ mo.} \\ 10 \text{ ch. : } 13 \text{ ch.} \end{array}\right\} = 4500 \text{ men : men reqd.} \left\{\begin{array}{l} \text{More months, } \textit{less} \text{ men;} \\ \text{more chataks, } \textit{less} \text{ men.} \end{array}\right.$$

$$\therefore \text{ men reqd.} = \frac{4500 \times 15 \times 13}{27 \times 10} = 3250.$$

$$\therefore \text{ the no. of men that must leave} = 4500 - 3250 = \underline{1250}. \; \textit{Ans.}$$

Ex. 6. If 48 cannon firing 4 rounds in 7 minutes kill 540 men in 1½ hours, how many cannon firing 8 rounds in 9 minutes will kill 840 men in 40 minutes ?

$$\left.\begin{array}{ll} 8 \text{ rounds} & : 4 \text{ rounds.} \\ 7 \text{ min.} & : 9 \text{ min.} \\ 540 \text{ men} & : 840 \text{ men.} \\ \tfrac{2}{3} \text{ hr.} & : 1\tfrac{1}{2} \text{ hrs.} \end{array}\right\} = 48 \text{ cannon : cann. reqd.} \left\{\begin{array}{ll} \text{More rounds,} & \textit{less} \\ \text{more interval,} & \textit{more} \\ \text{more men,} & \textit{more} \\ \text{more hours,} & \textit{less} \end{array}\right\} \text{cannon.}$$

$$\therefore \text{ cannon reqd.} = \frac{48 \times 4 \times 9 \times 840 \times 3 \times 3}{8 \times 7 \times 540 \times 2 \times 2} = \underline{108}. \; \textit{Ans.}$$

Examples CXLVII.

1. If 6 men can mow 48 bighas in 4 days, how long will it take 10 men to mow 120 bighas, at the same rate ?

2. If 67 maunds carried 87 miles cost *Rs.* 242. 14*a.*, what will 73 maunds carried 93 miles cost ?

3. If 939 men consume 364 maunds of wheat in 7 months, how many will consume 1404 maunds in 13½ months ?

4. If a tradesman with a capital of *Rs.*10000 gains *Rs.*900 in 7 months, how long will he be in gaining *Rs.*202. 8*a.* with a capital of *Rs.*3150 ?

5. If 37 tons carried 57 miles cost *Rs.*87. 14*a.* ; what weight can be carried 83 miles for *Rs.*217. 14*a.* ?

6. If 29 mds. 16 sr. be carried 40 miles for *Rs.*5. 13*a.* 4*p.*, how far ought 9 mds. 32 sr. to be carried for *Rs.*8. 7*a.* 4*p.* ?

7. If a regiment of 1878 soldiers consume 245 mds. 28 sr. of wheat in 336 days, how many maunds will an army of 22356 men consume in 112 days ?

8. If the penny loaf weigh 6 oz. when wheat is at *Rs.*2. 12*a.* per maund, what should be the price of a loaf weighing $4\frac{1}{2}$ lbs. when wheat is at *Rs.*4. 2*a.* per maund ?

9. If the carriage of 5 cwt. 1 qr. 12 lbs. for 39 miles be £2. 8*s.* 6*d.*, what must be paid for the carriage of 7 cwt. 16 lbs. for $48\frac{3}{4}$ miles ?

10. If the wages of 3 men be *Rs.*150 for 20 days, how many men will earn *Rs.*157. 8*a.* in 9 days at the same rate ?

11. If a quantity of provisions will serve a besieged garrison of 1500 men for 12 weeks at the rate of 20 oz. a day for each man, how many men would the same provisions maintain for 20 weeks at the rate of 8 oz. a day for each man ?

12. If 5 men can reap a rectangular field whose length is 800 feet and breadth 700 feet in $3\frac{1}{2}$ days of 14 hours each, in how many days of 12 hours each can 7 men reap a field 1800 feet long and 960 feet broad ?

13. If 21 horses and 217 sheep can be kept 10 days for *Rs.*564. 2*a.* 8*p.*, what sum will keep 9 horses and 60 sheep for 27 days, supposing that 3 horses eat as much as 50 sheep ?

14. If the 4*d.* loaf weigh 1 lb. $9\frac{1}{2}$ oz. when wheat is at 9*s.* 3*d.* per bushel, how much bread can be got for 5*s.* $7\frac{1}{2}$*d.* when wheat is at 76*s.* per quarter ?

15. If 30 cannon firing 4 rounds in 5 min. kill 640 men in an hour, how many men would be killed in an hour and a half by 10 cannon firing at the rate of 3 rounds in 4 min. ?

16. If 60 guns firing 5 rounds in 6 min. kill 350 men in $1\frac{1}{4}$ hrs. how many guns firing 7 rounds in 9 min. will kill 980 men in 25 min. at the same rate ?

17. If 5 horses require as much corn as 8 ponies, and 15 quarters of corn last 12 ponies for 64 days, how long may 25 horses be kept for £41. 5*s*, when corn is 22*s.* a quarter ?

18. If 20 men could perform a piece of work in 12 days, find the number of men who would perform another work three times as great in one-fifth of the time.

19. If 9 men can reap 15 ac. 1 ro. 28 po. in 5 days of $10\frac{1}{2}$ hours each, how many men will reap 401 ac. 8 po. in 7 days of $11\frac{1}{4}$ hrs. each ?

20. If when copper is at £7. 14*s*. 4¾*d*. per cwt. I can get 3 cwt. 2 qrs. 14 lbs. of brass for £27. 0*s*. 3¾*d*., how much brass shall I get for £153. 17*s*. 6*d*. when copper is at £9$\frac{1}{16}$ per cwt.?

21. A person is able to perform a journey of 142·2 miles in 4½ days when the day is 10·164 hours long; how many days will he be in travelling 505·6 miles when the days are 8·4 hours long?

22. If the 6*d*. loaf weigh 4·35 lbs. when wheat is at 5·75*s*. per bushel, what ought to be paid for 49·3 lbs. of bread when wheat is at 9·2*s*. per bushel?

23. If 240 men working 10 hours a day can dig a trench 300 yds. long, 3½ ft. deep and 2¼ ft. wide in 6 days; in how many days would 80 men, working 9 hours a day, dig a trench 500 yds. long, 3 ft. wide and 2½ ft. deep?

24. If 30 men, 6 women and 5 boys can dig a trench 120 yds. long, 5 wide and 2 deep in 16 days of 10 hours each; in how many days of 8 hours each will 24 men, 8 women and 4 boys dig a trench 144 yds. long, 6 wide and 4 deep; supposing that 5 women can do the work of 3 men, and 2 boys that of 1 man?

25. Two cogged wheels, of which one has 15 cogs and the other 28, work in each other. If the first turn 16 times in 7½ seconds, how often will the other turn in 21 seconds?

26. Two gangs of 6 and 9 men are set to reap two fields of 35 and 45 bighas respectively. The first gang works 7 hours in the day, and the latter 8 hours. If the first gang complete their work in 12 days, in how many days will the second complete theirs?

27. If 10 compositors, in 16 days of 14 hrs. each, can compose 20 sheets of 24 pages in each sheet, 50 lines in a page, and 40 letters on the average in a line; in how many days of 7 hours each can 20 compositors compose a volume, to be printed in the same letter, containing 40 sheets, 16 pages in a sheet, 60 lines in a page and 50 letters in a line?

28. A piece of work is to be done in 36 days; 15 men work at it 15 hours a day, but after 24 days only ⅗ of it is done; if 3 more men are put on, how many hours a day must all work to finish it in the given time?

29. If 1000 men, besieged in a fort and supplied with provisions for 5 weeks at the rate of 10 oz. a day for each man, be reinforced by 500 men, to what daily amount must the provisions be reduced for each man if it be desired to make the original supply last for 8 weeks?

30. If 7 women earn as much as 4 men, and 48 men assisted by 14 women earn 121 guineas in 17 days, what number of women assisting 20 men will earn £21. 3*s*. 6*d*. in one-third of the time?

31. Two sets of men perform the same amount of work. Each man in the first set is stronger than each one in the second in the

ratio of 7 to 6 ; the first set works 6 days a week for 10 weeks, and the second set, 5 days a week for 7 weeks. If there are 9 men in the first set, how many are there in the second ?

32. If 48 pioneers in 5 days of 12½ hours each can dig a trench 139·75 yards long, 4·5 yds. wide and 2·5 yds. deep, how many hours per day must 90 pioneers work during 42 days in order to dig a trench 1636·6875 yds. long, 4·875 yds. wide and 3·2 yds. deep ?

33. If the rent of 77 bi. 10 kat. 4 ch. of land be *Rs.*1572 . 8*a.* for 1 year 20 days, of how much land will the rent be *Rs.*1258 for 44 days ?

34. If the rent of a farm of 53 ac. 1 ro. 6 po. be *Rs.*1008. 12*a.*; what would be the rent of another farm containing 17 ac. 3 ro. 2 po., if 6 acres of the latter be worth 7 acres of the former ?

35. If the wages of 25 men amount to *Rs.*766. 10*a.* 8*p.* in 16 days, how many men must work 24 days to receive *Rs.*1035, the daily wages of the latter being one-half of those of the former ?

36. The cost of paving a yard with tiles 9 in. square is *Rs.*181. 8*a.* ; what would have been the cost had the tiles been 9 in. long and 6 in. broad, their price being to that of the former as 5 : 8, and the cost of labour being $\frac{1}{10}$ and $\frac{1}{11}$ of the price of each tile in the former and the latter case respectively ?

37. If 17 men by working 8 hours a day made an excavation 121 ft. 6 in. long, 25 ft. 6 in. broad and 24 ft. deep in 54 days, how many hours daily must 18 men work during 51 days in order that they may make an excavation whose length and breadth are 1 ft. 6 in. less and depth 1 ft. 6 in. greater than the preceding one, supposing that 9 men of the latter do as much as 10 men of the former ?

38. A contract is to be finished in 200 days and 50 men are put on to work at once ; at the end of $\frac{2}{5}$ths of this time, it is found that only $\frac{1}{4}$th of the work is done. What extra number of men will be required to complete the contract in the given time ; the last employed men to work 12 hours per day, whilst the first 50 men work, until the contract is finished, only 10 hours per day ?

39. If 38 men working 6 hours a day can do a piece of work in 12 days, find in what time 57 men working 8 hrs. a day can do a piece of work twice as great, supposing 2 men of the first set to do as much work in 1 hour, as 3 men of the second set can do in 1½ hours.

40. A contractor agrees to execute a certain piece of work in a certain time. He employs 55 men who work 9 hrs. daily. When $\frac{3}{4}$ths of the time expired, he finds that only $\frac{3}{8}$ths of the work is done. How many men must he employ during the remaining part of the time, working 11 hrs. daily, in order that he may fulfil his contract ?

41. If 5 pumps, each having a length of stroke of 3 feet, working 15 hours a day for 5 days, empty the water out of a mine ; what must be the length of stroke of each of 15 pumps which, working 10

hours a day for 12 days, would empty the same mine, the strokes of the former set of pumps being performed 4 times as fast as those of the latter ?

42. If 10 men reap 4 acres of corn in 4 days, working 10 hours a day, how many men would be required to reap 21 acres in 12 days, working 12 hours a day, supposing that a reaper who works 10 hours a day does $\frac{1}{85}$th part more work in an hour than one who works 12 hours a day ?

43. If 20 men can perform a piece of work in 12 days, how many men will perform a piece of work half as large again in a fifth part of the time, if they work the same number of hours per day, supposing that 2 of the second set can do as much work in an hour as 3 of the first set ?

44. If it require 4199 bricks, each 9 in. long, 4 in. wide and 3 in. thick to build a certain wall ; how many will be wanted to build a wall of double the length, half the height, and half the thickness, each brick being $9\frac{1}{2}$ in. long, $4\frac{1}{2}$ in. wide and $3\frac{1}{2}$ in. thick ?

45. A town is garrisoned with 10000 troops and has provisions sufficient for that number for 6 months, allowing a ration of $3\frac{1}{2}$ lbs. daily to each man. How many must be sent away so that by giving $\frac{1}{2}$ lb. less to each man, the provisions may last for 8 months ?

46. A boat is propelled by 8 oars which take 10 strokes per minute, and it goes at the rate of 9 miles an hour. Find the rate of a boat propelled by 6 oars which take 8 strokes per minute, when 5 strokes of each man in the latter boat are equivalent to 6 strokes of each man in the former.

47. If a person drink 5 dozen bottles of wine in 3 months when the wine is *Rs.*2 a bottle, how many bottles of wine may he drink in 3 months, without increasing the expense, when the price is *Rs.*2. 8*a.* a bottle ?

48. If a wall 660 yds. long, 10 ft. high and $22\frac{1}{2}$ in. thick, be built by 27 men in $31\frac{1}{4}$ days of 11 hours each ; in how many days of 12 hours each could 60 men build a wall 12 ft. high and $2\frac{1}{4}$ ft. thick round a park $4\frac{1}{2}$ miles in circumference ?

49. If it cost *Rs.*118. 2*a.* to supply a family of 12 persons for 5 weeks, when rice is at *Rs.*7. 8*a.* per maund, how much will it cost to supply a family of 18 persons for 7 weeks, when rice is at *Rs.*10 per maund ?

50. If a family of 9 people in Calcutta spend *Rs.*16380 in a year, what must be the expenses of a family of 8 people to live in Jessore in the same style for 7 months, the prices there being $\frac{3}{4}$ of what they are in Calcutta ?

51. A farmer engages 30 men and 45 women to cut down his crop in 20 days of 12 hours each ; but after 12 days' reaping, he wishes the remainder of the crop to be cut down in 4 days of 10

hours each ; how many additional men must he employ, supposing 2 men to do as much work as 3 women?

52. If 8*s.* 2*d.* worth of bread be consumed in 7 days, by a family consisting of 6 adults and 5 children, when the price of the quartern loaf is 7½*d.* ; how many adults, along with 3 children, will consume 12*s.* 9*d.* worth of bread in 15 days, when the price of the quartern loaf is 8½*d.* ; supposing that 3 children consume as much per day as 2 adults?

53. If *Rs.*50 is sufficient to maintain 8 coolies for a fortnight, when rice is 2½ seers for 6*a.* ; how much will be required to maintain 6 coolies for 25 days, when rice is 2 maunds for *Rs.* 7?

54. If 5 steam engines of ; horse-power in 3 weeks (when employed 3 days in the week and 10 hours a day) raise through a certain altitude, 25 three-bushel sacks of wheat, weighing 60 lbs. a bushel ; in what time will 9 engines of 8 horse-power (when employed 5 days in the week and 9 hours a day) raise through 15 times the former altitude, 75 three-bushel sacks of wheat, weighing 63 lbs. a bushel?

55. A canal, which was 14⅛ miles long, 21 yds. wide and 19½ ft. deep, was dug by 1200 men in 8 months. What was the depth of another canal, which was dug by 1500 men in a year, it being 17½ miles long and 72 ft. wide, remembering that the time employed for completing a mile of the second canal, with a given number of men, was (owing to greater difficulties in cutting the rock) to the time employed over a mile of the first as 13 : 8?

V. THE RULE OF PROPORTION.

493. As has been observed in **The Rule of Three** of which this is only another name, we have here *three* quantities either simple or compound given, to find a *fourth* which shall complete the proportion ; and this is a *fourth-proportional* to the three quantities proposed.

494. Assuming as an *Axiom*, that **effects** have the same *relation* or *ratio* to each other as the **causes** which produce them under the same circumstances, it is evident that in any two causes of the same kind we shall have the following proportion :—

First Cause : **Second Cause** :: **First Effect** : **Second Effect** ; and then, what was said in Articles 472 and 473 will enable us to find any *one* term if the *three* others be supposed to be given.

495. To avoid the trouble of writing the name of the *required* term or quantity at length, we shall always denote it by the simple *symbol* x which must be treated in the same way as any other number ; and it may occupy *any* place in the proportion either by *itself* or as a *factor* either *integral* or *fractional* with given numbers, as in the following Examples.

Ex. 1. If 5 men can mow 12 acres of grass in a certain time :

how many acres will 16 men be able to mow in the same or an equal time?

Here, 5 men, 16 men } are the first and second { *Causes*;
12 acres, x acres } are the first and second { *Effects*;

whence, we have the following proportion :—

5 men : 16 men :: 12 ac. : x ac.;

and therefore by the Articles just referred to, we find

$5 \times x = 16 \times 12 = 192$;

whence $x = \frac{192}{5}$ ac = 38 ac. 1 ro. 24 po. *Ans.*

Ex. 2. If 8 oz. of bread be sold for 6*d.* when wheat is at £15 a load; what should be the price of wheat when 12 oz. are sold for 4*d.*?

If the price of a load of wheat be *regulated* by, so as to be *proportional* to, the price of an ounce of bread, since,

in the former case the price of 1 oz. $= \frac{6}{8}d. = \frac{3}{4}d.$,

and in the latter the price of 1 oz. $= \frac{4}{12}d. = \frac{1}{3}d.$,

we must have the following proportion,

$\frac{3}{4}d. : \frac{1}{3}d. :: £15 : £x$;

whence $x = (\frac{1}{3} \times 15) \div \frac{3}{4} = £\frac{20}{3} =$ £6. 13*s.* 4*d.*, which is the reqd. price.

These Examples, the causes in which are simple terms being dependent upon only *one* magnitude, are instances of what is called **Direct Proportion**, because the effect is *greater* or *less* in the same proportion as the cause is *greater* or *less*.

Ex. 3. If 10 men can perform a piece of work in 12 days; how many days will it take 8 men to do the same?

Here, the causes will evidently be to each other as 10×12 to $8 \times x$; and the effects are the *same*, and may therefore be represented by 1, or any other symbol;

whence, $10 \times 12 : 8 \times x :: 1 : 1$;
therefore $8 \times x = 10 \times 12 = 120$, and $x = 120 \div 8 = 15$ days. *Ans.*

Ex. 4. How much in length, that is 3 ft. 9 in. broad, will be equal to what is 37 ft. 9 in. long and 7 ft. 6 in. broad?

Here, the first cause = 45 in. × x in.; the second cause = 90 in. × 453 in.; and the effects are to be equal:

therefore $45 \times x : 90 \times 453 :: 1 : 1$; whence $45 \times x = 90 \times 453$,
and $x = (90 \times 453) \div 45 = 906$ in. = 75 ft. 6 in. *Ans.*

In these two Examples, the *entire* causes are *compound* quantities depending upon two *subordinate* causes; and because the effect is the *same*, each subordinate cause is *less* or *greater* according as

the other is *greater* or *less*, constituting what is called **Inverse Proportion**.

Ex. 5. If a person can perform a journey of 100 miles in 12 days of 8 hours each ; how far will he be able to travel in 15 days of 9 hours each ?

Here, 12×8 and 15×9 are the causes, and the distances travelled 100 and x are the effects ; whence

$$12 \times 8 : 15 \times 9 :: 100 : x ;$$

$$\text{and } x = \frac{15 \times 9 \times 100}{12 \times 8} = \underline{140\tfrac{5}{8}} \text{ miles. } \textit{Ans.}$$

Ex. 6. If 60 maunds of corn feed 6 horses for 50 days ; in how many days will 15 horses consume 75 maunds ?

The causes are 6×50 and $15 \times x$, and the effects are 60 and 75 maunds ; therefore

$$6 \times 50 : 15 \times x :: 60 : 75, \text{ or } 2 \times 10 : x :: 4 : 5 ;$$

$$\text{whence, } x = \frac{2 \times 10 \times 5}{4} = \underline{25} \text{ days. } \textit{Ans.}$$

In the former of these Examples, the distance travelled are in the *compound* ratio of the numbers of days and their lengths ; and in the latter, the numbers of maunds have the same ratio as that which is *compounded* of the numbers of horses and days.

Ex. 7. If 25 labourers can dig a trench 220 yards long, 3 ft. 4 in. wide and 2 ft. 6 in. deep, in 32 days of 9 hours each ; how many would it require to dig a trench half a mile long, 2 ft. 4 in. deep and 3 ft. 6 in. wide, in 36 days of 8 hours each ?

$$\left.\begin{array}{l}\text{First cause} = 25 \times 32 \times 9 \\ \text{Second cause} = x \times 36 \times 8\end{array}\right\} \text{being the products of the } \textit{subordinate} \text{ causes ;}$$

$$\left.\begin{array}{l}\text{First effect} = 220 \times \frac{10}{9} \times \frac{5}{6} \\ \text{Second effect} = 880 \times \frac{7}{9} \times \frac{7}{6}\end{array}\right\} \text{the } \textit{mixed} \text{ quantities being reduced to fractions of 1 yard.}$$

Hence, we have the following proportion :—

$$25 \times 32 \times 9 : x \times 36 \times 8 :: 200 \times \tfrac{10}{9} \times \tfrac{5}{6} : 880 \times \tfrac{7}{9} \times \tfrac{7}{6} ;$$

$$\text{or, } 25 : x :: 1 \times 10 \times 5 : 4 \times 7 \times 7 ;$$

$$\text{whence, } x = \frac{25 \times 4 \times 7 \times 7}{1 \times 10 \times 5} = \underline{98} \text{ labourers. } \textit{Ans.}$$

These Examples, the causes and effects being simple and compound quantities consisting of their respective subordinate *partial* causes and effects, are instances of **Compound Proportion** in its fullest meaning.

Ex. 8. If 10 excavators can dig 12 loads of earth in 16 hours, whilst 12 others can dig 12 loads in 15 hours ; find the time in which they will jointly dig 108 loads.

Since, the ratio $10 \times 16 : 12 \times 15$ is *not* equal to the [ratio 12 : 9,

it follows that the individuals of the two sets do not work at the *same rate;* but the rate of one of the *first* set being represented by $\frac{12}{10\times16}=\frac{3}{40}$; that of one of the second set will be equal to $\frac{9}{12\times15}=\frac{1}{20}$;

whence, $\{10\times\frac{3}{40}+12\times\frac{1}{20}\}\times$ the required time $=108$;
or $(\frac{3}{4}+\frac{3}{5})$ of the required time $=108$ hours;
that is, the required time $=\frac{20}{27}$ of 108 hrs. $=\underline{80}$ hrs. *Ans.*

496. In practice, when the *partial* causes and effects consist of compound quantities, it is most convenient to express them by vulgar fractions or decimals; and when the *entire* causes and effects are compound quantities, to proceed as in *Sections* III. and IV. (all the Examples of which are instances of this Rule) shortening the operation as much as possible by means of Article 474.

Examples CXLVIII.

1. If when malt costs 63*s.* a quarter the price of a quart of ale be $4\frac{1}{3}$*d.*, what should its price be when malt is at 66*s.* 6*d.* per quarter?

2. If a person can perform a journey in 24 days of $10\frac{1}{2}$ hours each; what time will it take him to do the same when the days are $12\frac{3}{5}$ hours long?

3. If the expenses of 7 persons for 3 months amount to *Rs.*735; what will be the expenditure of 10 persons for 12 months at the same rate?

4. If 10 horses consume 7 mds. 20 sr. of oats in 7 days; in what time will 28 horses consume 30 maunds at the same rate?

5. If 10 men reap 20 acres of corn in 4 days: how many men can reap 70 acres in 10 days, at the same rate of labour?

6. If 7 lbs. of sugar be sold for 4*s.* 8*d.* when the cost of a cwt. is £3. 7*s.* 8*d.*; what should be the cost of a cwt. when 11 lbs. is sold for 7*s.* $1\frac{1}{4}$*d.*?

7. If the 4*a.* loaf weighs 1 lb. 11 oz. 12 drs. when wheat is at *Rs.*3. 12*a.* per maund, what ought the 6*a.* loaf to weigh when wheat is at *Rs.*2. 10*a.* per maund?

8. If 21 maunds be carried 40 miles for *Rs.*5. 13*a.* 4*p.*, how far ought 7 maunds to be carried for *Rs.*8. 7*a.* 4*p.*?

9. If 6664 men consume 357 maunds of wheat in 57 days, how many maunds of wheat will 1596 men consume in 119 days?

10. If the carriage of 13 cwt. 2 qrs. 19 lbs. for 35 miles come to *Rs.*48. 12*a.*; what must be paid for the carriage of 41 cwt. 1 lb. for 49 miles?

11. If 12 men can perform a piece of work in 20 days; required the number of men who could perform another piece of work four times as great in a fifth part of the time.

12. If with a capital of *Rs.*10000, a tradesman gain *Rs.*1000, in 7 months, in what time will he gain *Rs.*605, with a capital of *Rs.*3850?

13. If the 4*d.* loaf weighs 3·35 lbs. when wheat is 4·75*s.* a bushel, what ought to be the price of wheat per bushel, when 47·5 lbs. of bread cost 13·$\dot{3}$*s.*?

14. If 7 men can build a wall 245 yds. long, 8 ft. high and 18 in. thick, in 35 days of 12 hours each; what length of wall, 10 ft. high and 27 in. thick, could 12 men build in 43 days of 10 hours each?

15. If 27 men can do a piece of work in 14 days, working 10 hrs. a day; how many hours a day must 24 boys work, in order to complete the same in 45 days, the work of a boy being half that of a man?

16. If 4 artillery men can fire a gun 48 times and 5 men 52 times in an hour; how much more time will be required for firing 21216 shots from 26 guns, when there are 4 men to a gun than when there are 5 men?

17. If 10 cannon which fire 3 rounds in 5 minutes, kill 270 men in 1½ hours; how many cannon which fire 5 rounds in 6 minutes, will kill 500 men in 1 hour, at the same rate?

18. If 120 men in 3 days of 12 hours each, can dig a trench 30 yds. long, 2 ft. broad and 4 ft. deep; how many men would be required to dig a trench 50 yds. long, 6 ft. deep and 1½ yds. broad in 9 days of 15 hours each?

19. If 6 men can reap 15 acres in 3 days of 14 hrs. each and 10 boys can reap 10⅝ acres in 5 days of 9 hours each, find the ratio of the work of a man to that of a boy; and determine what number of acres 4 men and 7 boys together reap in a day.

20. If beer which is brewed with 3 bushels of malt to the barrel cost 1*s.* 3*d.* per gallon, when malt is at 62*s.* 8*d.* the quarter; how much will beer cost per gallon, which is brewed with 5 bushels of malt to the barrel, when a quarter of malt costs 50*s.*?

21. A town is garrisoned with 50000 troops and has provisions sufficient for that number for 3½ months, allowing a ration of 2 lbs. daily to each man; how many must be sent away so that by giving an additional ½ lb. to each man the provisions may last 14 months?

22. A wall 700 yds. long was to be built in 29 days. At the end of 11 days 18 men had built 220 yds. of it; how many additional men was it then necessary to engage to work at the same rate, in order that the wall might be completed in the given time?

23. If 6 horses eat 3 tons of hay in 15 days, how long will it take 18 sheep to eat 5 acres of grass, the voracity of 2 horses being equal to that of 7 sheep; and one acre producing 3 tons of grass, each ton being as satisfying as 1½ tons of hay?

24. If 5 men, 4 women, and 7 boys can complete a piece of work in 60 days, how long will 9 men, 15 women and 18 boys take to complete a piece of work 5 times as great ; the parts done by each man, woman, and boy respectively, in the same time, being as the numbers 3, 2, 1 ?

25. If 4 men and 2 boys can paper a room 23 ft. long, 19 broad and 16 high, with paper 2 ft. 8 in. wide, in 7 days, working 12 hours a day ; in how many days of 9 hours each can 4 men and 4 boys paper 3 rooms, each 32 ft. long, 27 broad and 15 high, with paper 3 ft. broad ; a boy's working being 25 per cent. of a man's ?

CHAPTER XII.

Miscellaneous Propositions.

497. Percentage. The term **per centum** or **per cent** means *for a hundred.*

If a man has 75 sheep and if 3 of them are lost, we say that 4 per cent. of the man's sheep are lost ; meaning thereby that if the man had possessed 100 sheep, 4 would have been lost according to this rate of loss. The 4 is called the **rate per cent.**

The symbol % or *p. c.* is used as an abbreviation for the words *per cent.*

498. It is not usual to calculate gain, loss, increase, decrease, &c., at so much per **unit,** but at so much **per cent.** or per 100 units, whatever the *unit* may be. When, however, the rate per unit or, more briefly, the rate is known, it is obvious that the **rate per cent.** is found by multiplying the *rate per unit* by 100. It is worth noting that 5 per cent. $=\frac{5}{100}$ of the whole, $7\frac{1}{2}$ per cent. $=\dfrac{7\frac{1}{2}}{100}$ of the whole, 8 per cent. $=\frac{8}{100}$ of the whole, whatever may be the whole or whatever may be the equal units constituting the whole ; and so on with other percentages.

Ex. 1. Find how much per cent. 15 is of 96, *i.e.*, find what number bears the same ratio to 100, that 15 bears to 96.

$$96 : 100 :: 15 : \text{rate per cent.} ;$$

$$\therefore \text{ rate per cent.} = \frac{100 \times 15}{96} = \frac{125}{8} = \underline{15{\cdot}625.} \ \textit{Ans.}$$

Ex. 2. What rate per cent. does the fraction $\frac{7}{30}$ denote ?

Rate per cent. $=\frac{7}{30} \times 100 = \frac{70}{3} = \underline{23\frac{1}{3}}.$ *Ans.*

Ex. 3. The number of boys in a school increases in a certain period from 125 to 180 ; what is the increase per cent. ?

On 125 the increase is (180 − 125) or 55 ;

∴ 125 : 100 :: 55 : increase per cent. ;

∴ increase per cent. $= \frac{55 \times 100}{125} = 44$. *Ans.*

Ex. 4. The population of a town increased 8 per cent. from 1875 to 1885, and its population in the latter year was 9720 ; find its population in 1875.

Every 100 men in 1875 increased to 108 men in 1885 ;

∴ 108 : 9720 :: 100 : pop. in 1875 ;

∴ pop. in 1875 $= \frac{100 \times 9720}{108} = 9000$. *Ans.*

Ex. 5. Of the gross profits of a Railway Company in a certain year 41 per cent. are spent to pay the working expenses, 56 per cent. are paid to the shareholders, and the remainder *Rs.*150000 is reserved. Find the total profits.

100 − (41 + 56) or 3 per cent. is reserved ;

∴ *Rs.*3 : *Rs.*150000 :: *Rs.*100 : total profits ;

∴ total profits = *Rs.*5000000. *Ans.*

Ex. 6. A house depreciates in value each year at the rate of 10 per cent. of its value at the beginning of the year, and its value at the end of 3 years is *Rs.*14580. What was its original value ?

At the end of each year the value of the house is $\frac{90}{100}$ or $\frac{9}{10}$ of what it was at the beginning of the year ;

∴ its value at the end of 1st year $= \frac{9}{10}$ of its original value,
its value at the end of 2nd year $= \frac{9}{10}$ of $\frac{9}{10}$ of its original value, and
its value at the end of 3rd year $= \frac{9}{10}$ of $\frac{9}{10}$ of $\frac{9}{10}$ of its original value
$= \frac{729}{1000}$ of its original value.

∴ $\frac{729}{1000}$ of its original value = *Rs.*14580 ;

∴ its original value = *Rs.* $\frac{14580 \times 1000}{729}$ = *Rs.*20000. *Ans.*

Examples CXLIX.

1. What fractions are denoted by the following rates per cent. ?

(1) $12\frac{1}{2}$. (2) 15. (3) $\frac{3}{4}$. (4) $76\frac{1}{4}$. (5) 130. (6) $16\frac{2}{3}$.

2. Find the values of :—

(1) 6 per cent. of 360. (2) $7\frac{1}{2}$ p. c. of *Rs.*240.

(3) 18% of 325 mds. (4) $\frac{4}{5}$% of *Rs.*13675.

(5) 75% of 250 bi. 10 kat. (6) $9\frac{1}{2}$ p. c. of 1800 yds.

(7) $62\frac{1}{2}$% of *Rs.*720. 10*a.* (8) 50 per cent. of 4 hrs. 25 min. 12 sec.

3. What percentage is :—

(1) 7 of $24\frac{1}{2}$? (2) 37 of 75 ? (3) 23 of 256 ?
(4) $43\frac{3}{4}$ of 165 ? (5) $185\frac{1}{4}$ of 7321·75 ? (6) ·57 men of 600 men ?
(7) *Rs.*782·8 of *Rs.*760 ? (8) 90·848 of 801·6 ?
(9) £34. 17*s.* $9\frac{3}{4}$*d.* of £607. 1*s.* $11\frac{1}{4}$*d.* ?

4. What rates per cent. do the following fractions denote ?

(1) $\frac{3}{8}$. (2) $\frac{1}{6}$. (3) $4\frac{3}{8}$. (4) $1\frac{1}{4}$. (5) $3\frac{3}{8}$. (6) $5\frac{9}{10}$.

5. The population of a city increased from 15635 to 18762 in five years ; what was the rate per cent. of the increase ?

6. The population of a village decreased in a certain time from 1916 to 1437 ; find the rate per cent. of decrease.

7. The population of York in 1861 was 40433, and it increased 8·32 per cent. between 1861 and 1871 ; find the population in 1871.

8. The population of a city decreased 33·11 per cent. between 1861 and 1871. ; in 1861 it was 113387 ; find what it was in 1871.

9. The population of a certain country increased 5 per cent. from 1875 to 1885, and its population in the latter year was 40841010 ; find its population in 1875.

10. The population of a town increased 35 per cent. between 1851 and 1861, and 19 per cent. between 1861 and 1871 ; the population in 1871 was 93177, find the population in 1851.

11. A farmer gave 100 bighas of land to each of his sons. After three years the eldest lost 50 per cent., and the second increased as much ; what per cent. was the eldest son's land now of that of the second ?

12. A man took from a bank *Rs.*393, which was $13\frac{1}{10}$ per cent. of what he had deposited ; how much then remained ?

13. A man spends $\frac{3}{25}$ of his money, and then 5 per cent. of what remains ; what percentage of his money has he left ?

14. Of what number is 57 men, $9\frac{1}{2}$ per cent. ?

15. A regiment which lost in an engagement $31\frac{1}{4}$ per cent. of its men, had 440 men left ; how many had it at first ?

16. The population of a certain village is 4059, which is $12\frac{1}{2}$ per cent. more than it was 5 years ago ; what was it then ?

17. A High School has 4 classes. There are 20 per cent. of the whole number in the first class, 26 per cent. in the second, 29 per cent. in the third, and the remaining 150 are in the fourth class. Find the number of boys in each class.

18. Archimedes discovered that the crown made for king Hiero consisted of gold and silver in the ratio of 2 : 1 ; how much per cent. was gold and how much per cent. silver ?

19. *A* bought goods to the value of *Rs.*3457. 8*a*. and sold them to *B* at a gain of 15 per cent. on his outlay, and *B* sold them to *C* at a loss of 15 per cent. on his outlay ; how much did *C* give for them ?

20. A man's capital increased 20 per cent every year ; at the end of 4 years it was *Rs.*5184 ; what was his capital at first ?

21. A house depreciates in value each year at the rate of 10 per cent. of its value at the beginning of the year, and its value at the end of 3 years is *Rs.*10935 ; find its original value.

22. After spending 50 per cent. of his income and then 20 per cent. of the remainder, a man has *Rs.*600 left ; find his income.

23. A person lays out *Rs.*4800 and *Rs.*3600 respectively in two different speculations ; in the first he loses 10 per cent. and in the second he gains 15 per cent. Find his gain or loss per cent. on the sum invested.

24. A person loses in his first year 10 per cent. of his capital, but in the next year he gains 20 per cent. of what he had at the end of the first year, and his capital is now *Rs.*1440 more than it was at first ; find his original capital.

25. 90 per cent. of the boys of a school pass in spelling, and 85 per cent. in arithmetic, 150 pass in both subjects and no boy fails in both. How many boys are there in the school ?

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499. Average. The **Average or mean value** of two or more numbers is found by adding the numbers together and dividing the *sum* by the *number* of those given numbers.

Hence, the sum of a number of given numbers is their average value multiplied by the number of them.

Ex. 1. Find the average of 14, 26, 9, 18, 13, 24, 27 and 39.

Here, the sum of the numbers = 170 ; and as the number of the given numbers is 8, we get the average as 170 ÷ 8 or 21·25. *Ans.*

Ex. 2. The average age of a school of 750 boys is 15·4 years ; 50 boys leave, thereby diminishing the average age of the school to 15·3 ; find the average age of those who leave.

The sum of the ages of the 750 boys = 750 × 15·4 or 11550 years.

The sum of the ages of the remaining (750 − 50) or 700 boys = 700 × 15·3 or 10710 years.

∴ the sum of the ages of the 50 boys who leave is (11550 − 10710) or 840 years.

∴ the average age reqd. = (840 ÷ 50) or 16·8 years. *Ans.*

Examples CL.

1. Find the average value of the following numbers :—

(1) 13, 15, 74, 23, 6 and 31. (2) 1600, 276, 974, 0, 236, 845 and 1239.

(3) $9\frac{3}{4}$, $12\frac{1}{8}$, 14, 0, $28\frac{1}{2}$ and 19 (4) 5·063, 7·00285, 12, 8·7396 and 5·69352.

(5) $15\frac{1}{3}$, $36\frac{3}{4}$, $17\frac{5}{8}$, 0, $10\frac{3}{8}$, $74\frac{1}{3}$, $28\frac{1}{4}$ and 33.

(6) 12·48, 21, 7·75, ·034, 3·125, 0, 24·5 and 12·35.

2. At a competitive examination there were 7 candidates of the age of 19, 12 of 18, 4 of 22, 9 of 20, and 15 of 17. Find the average age.

3. On Sunday I spent no money, on Monday *Rs.*43. 14*a*, on Tuesday *Rs.*51. 12*a*. 8*p*., on Wednesday *Rs.*46. 14*a*. 6*p*., on Thursday *Rs.*52 8*a*., on Friday *Rs.*32. 15*a*. 6*p*., on Saturday *Rs.*26. 4*a*. ; find my average daily expenditure during the week.

4. The weights of a boat's crew are respectively 9 st. 5 lbs., 10 st., 10 st. 5 lbs., 11 st. 1 lb., 12 st., 11 st. 6 lbs., 11 st. 4 lbs, and 10 st. 7 lbs. ; what is the average weight of the crew? If the average weight of the crew be diminished by 2 lbs., when the weight of the coxswain is included, find the weight of the coxswain.

5. The population of five parishes being 1236, 452, 364, 516 and 3430 respectively, find what the population of a sixth parish must be, in order that the average population of the six may be 1256·5.

6. The populations of three towns in the year 1881 were 21326, 42324, and 6700 ; and in the year 1891 it was found that the first two had increased 12 and 10 per cent. respectively, and the last had decreased 18 per cent. ; find the average population of the three towns in the year 1891.

7. In a school, 17 children average 6 years ; 26, $7\frac{1}{2}$ years ; 35, $9\frac{1}{4}$ years ; 20, 10 years ; and 8, $12\frac{1}{2}$ years. Find the average age of all the children.

8. The average age of 27 men is 57 years ; that of the first eleven is 53 years, and that of the last eight $59\frac{1}{4}$ years. Find the average age of the rest.

9. The average of 21 results is 61, that of the first eight being 64, and of the next eleven 59. Required the average of the last two.

10. The average age of a school of 200 boys is 14·75 years ; what will be the average age, if 10 new boys come whose average age is 12·3 years?

11. The average weight of 8 men is increased by 2 lbs, when one of them who weighs 12 stones is replaced by a fresh man ; what is the weight of the new man?

12. The average temperature for Monday, Tuesday and Wednesday was 53° ; the average for Monday, Wednesday and

Thursday was 56°, that for Thursday being 60°; what was the temperature on Tuesday?

13. The average salary of *A*, *B* and *C* is *Rs.*40 per month. That of *B*, *C* and *D* is *Rs.*50 per month, and *D*'s salary is *Rs.*60. What is the salary of *A*?

14. The average of 25 results is 18; that of the first 12 being 14, and that of the last 12 being 17. Find the 13th result.

15. In a class there are 40 boys, and their average age is 15 years. One boy, however, aged 18, leaves the school, and another joins, and then the average age of the class is 14·875. Find the age of the new boy.

16. The average of ten results was $17\frac{1}{2}$; that of the first three was $16\frac{1}{3}$, and of the next four $16\frac{1}{2}$; the eighth was 3 less than the ninth, and 4 less than the tenth. What was the last result?

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500. Profit and Loss. All questions which relate to gain or loss in mercantile transactions fall under the head of **Profit and Loss.**

We have already given the student some idea of *Gain* or *Loss* (See Art. 181). Under the head of *Profit and Loss*, we estimate a profit or a loss not absolutely, but **in relation to the cost price.** Men of business adopt 100 as a **standard cost price,** and reduce the gain or loss on a particular cost price to the corresponding gain or loss on 100; that is, to a gain or loss of so much per cent.

Again, when the cost price is represented by 100, the selling price is represented by 100+gain per cent., or 100 − loss per cent., according as a gain or loss has been made.

501. But although questions in Profit and Loss can always be solved by the Rule of Three, yet it is often useful to remember that since a gain of 18 per cent. means a gain of 18 on 100, where 100 represents the cost price, it is a gain of $\frac{18}{100}$ of the cost price. And in like manner a loss of 12 per cent. means a loss of $\frac{12}{100}$ of the cost price.

Ex. 1. A house, bought for £4250, is sold at a profit of 12 per cent.; how much was gained, and for what was the house sold?

Here, the selling price of £100 goods is £(100+12) or £112.

∴ £100 : £4250 :: £112 : selling price;

∴ selling price = £112 × 42·5 = £4760.
Hence gain = £(4760 − 4250) = £510. } *Ans.*

Ex. 2. Goods were sold for *Rs.*216, at a loss of 10 per cent.; what did they cost?

Here, the selling price of *Rs.*100 goods is *Rs.*(100 − 10) or *Rs.*90.

∴ *Rs.*90 : *Rs.*216 :: *Rs.*100 : cost price;

∴ cost price = *Rs.*100 × $\frac{216}{90}$ = *Rs.*240. *Ans.*

Ex. 3. A watch which costs *Rs*.40 was sold for *Rs*.49; find the gain per cent.

Here, gain on *Rs*.40 is *Rs*.(49 − 40) or *Rs*.9.

∴ *Rs*.40 : *Rs*.100 :: *Rs*.9 : gain per cent.;

∴ gain per cent. $=\frac{10}{4}\times$ *Rs*.9 = *Rs*.$22\frac{1}{2}$. *Ans.*

Ex. 4. I buy a horse for *Rs*.120, what do I sell it at, to lose 10%?

Here, the selling price of *Rs*.100 goods is *Rs*.(100 − 10) or *Rs*.90.

∴ *Rs*.100 : *Rs*.120 :: *Rs*.90 : selling price;

∴ selling price $=\frac{12}{10}\times$ *Rs*.90 = *Rs*.108. *Ans.*

Ex. 5. If I gain 5 per cent. by selling an article for *Rs*.6. 9*a*., how much shall I gain or lose per cent. by selling it for *Rs*.6?

Here, the selling price of *Rs*.100 goods is *Rs*.(100 + 5) or *Rs*.105.

∴ *Rs*.105 : *Rs*.$6\frac{9}{16}$ :: *Rs*.100 : cost price;

∴ cost price $= Rs.\frac{105\times 100}{105\times 16} = Rs.\frac{25}{4} = Rs.6\frac{1}{4}$.

Hence, loss on every *Rs*.$6\frac{1}{4}$ is *Rs*.$(6\frac{1}{4}-6)$ = *Re*.$\frac{1}{4}$.

∴ *Rs*.$6\frac{1}{4}$: *Rs*.100 :: *Re*.$\frac{1}{4}$: loss per cent.;

∴ the loss per cent. $= Rs.\frac{100\times 4}{4\times 25} = Rs.4$. *Ans.*

Ex. 6. A merchant buys 4000 mds. of rice, one-fifth of which he sells at a gain of 5 p. c., one-fourth at a gain of 10 p. c., one-half at a gain of 12 p. c., and the remainder at a gain of 16 p. c. If he had sold the whole at a gain of 11 p. c., he would have made *Rs*.728 more. What was the cost of the rice per maund?

$\frac{1}{5}$ of 4000 mds. = 800 mds.; $\frac{1}{4}$ of 4000 mds. = 1000 mds.,
$\frac{1}{2}$ of 4000 mds. = 2000 mds.; ∴ remainder = 200 mds.

Gain on 800 mds. at 5% = $\frac{5}{100}$ of 800 mds. at *cost* price; and so on.

∴ total actual gain = $\frac{5}{100}$ of 800 mds. + $\frac{10}{100}$ of 1000 mds. + $\frac{12}{100}$ of 2000 mds. + $\frac{16}{100}$ of 200 mds.
= 40 mds. + 100 mds. + 240 mds. + 32 mds.
= 412 mds. at *cost* price.

If the whole be sold at a gain of 11%, the gain would be $\frac{11}{100}$ of 4000 mds. at *cost* price = 440 mds. at *cost* price.

∴ difference of the two gains = (440 − 412) or 28 mds. at *cost* price.

∴ cost price of 28 mds. = *Rs*.728;

∴ cost price of 1 md. = *Rs*.728 ÷ 28 = *Rs*.26. *Ans.*

Examples CLI.

1. If goods are bought at *Rs*.22. 14*a*. 8*p*. per cwt. and sold at *Rs*.25. 10*a*. 8*p*. per cwt., what is the gain per cent.?

2. A piano is bought for *Rs.*631. 4*a.*; what must it be sold for that a gain of 4 per cent. may be made?

3. If I pay *Rs.*122. 8*a.* for 84 lbs. of tea, what price per lb. must I sell it at to gain 15 per cent.?

4. A horse was bought for *Rs.*312. 8*a.* and sold at a loss of 12 per cent.; at what price was he sold?

5. Cloth is sold for 6*s.* $1\frac{3}{4}$*d.* per yard at a loss of $12\frac{1}{2}$ per cent.; find the prime cost.

6. By selling an article for *Re.*1. 14*a.* a person gains 16 per cent.; find the cost of the article.

7. By selling a horse for *Rs.*722. 8*a.*, I lose 15 per cent.; find the cost price of the horse.

8. By selling an article for 3*s.* 9*d.* a person loses 5 per cent.; at what price must he sell it to gain $4\frac{1}{2}$ per cent.?

9. If by selling wine at *Rs.*7. 8*a.* per gallon I lose 10 per cent., at what price must I sell it to gain 15 per cent.?

10. The cost of a 38 gallon cask of wine was *Rs.*250, and 8 gallons are lost by leakage; at what price per gallon must the remainder be sold to realize 10 per cent. on the outlay?

11. If $5\frac{1}{2}$ per cent. be gained by selling butter at *Rs.*52. 12*a.*, per cwt., how much per cent. will be gained by selling it at 10*a.* per lb.?

12. By selling a horse for *Rs.*1168. 8*a.* a person lost 5 per cent.; what would have been his gain or loss per cent. had he sold him for *Rs.*1322. 4*a.*?

13. If a tradesman gains 4*s.* $10\frac{1}{2}$*d* on an article which he sells for 16*s.* 3*d.*, what is his gain per cent.?

14. A contractor bought 250 sheep and sold them for *Rs.*5322. 14*a.* 8*p.* at a gain of $16\frac{2}{3}$ per cent.; what was the cost price of each sheep?

15. If by selling an article for *Rs.*2. 12*a.*, I gain $\frac{3}{8}$ of my outlay, what should I gain per cent. by selling it for *Rs.*3. 4*a.*?

16. A stationer sold quills at 11*s.* a thousand, clearing $\frac{3}{8}$ of the money; what would he clear per cent. by selling them at 13*s.* 6*d.* a thousand?

17. By selling goods for *Rs.*800, I lost $\frac{1}{21}$ of their cost; for what should I have sold them to gain $\frac{3}{20}$ of their cost?

18. A person, having bought goods for *Rs.*400, sells half of them at a gain of 5 per cent.; for how much must he sell the remainder so as to gain 20 per cent. on the whole?

19. I bought 100 articles for *Rs.*4 9*a.* 4*p.*, and sold them so as to gain $\frac{3}{8}$ of the selling price; find the selling price and the gain per cent.

20. A draper bought 3672 yards of linen at *Re.*1. 9*a.* 4*p.* per yard. He sells $\frac{1}{3}$ of it at *Re.*1. 12*a.* per yard, and $\frac{1}{2}$ of the remainder at *Re.*1. 15*a.* 4*p.* ; at what price per yard must he sell the rest to gain on the whole $12\frac{1}{2}$ per cent. ?

21. A tradesman's prices are 20 per cent. above cost price ; if he allows a customer $12\frac{1}{2}$ per cent. on his bill, what profit per cent. does he make ?

22. A merchant sells to a retailer at 40 per cent. profit ; but the latter failing, the former receives only 4*a.* in the rupee ; find his gain or loss per cent.

23. By selling tea at *Rs.*2. 10*a.* 8*p.* per lb. a grocer clears $\frac{1}{8}$ of his outlay ; if he raises the price to *Rs.*3. 1*a.* 4*p.*, what does he clear per cent. ?

24. Bought eggs at 10*a.* per score ; how many may be sold for *Re.*1. 2*a.* so as to gain $12\frac{1}{2}$ per cent. ?

25. If a trader uses a light weight of $13\frac{1}{2}$ lbs. instead of a stone, how much per cent. does he gain fraudulently ?

26. A person sold 55 yards of silk for *Rs.*257. 13*a.*, thereby gaining the cost of $13\frac{3}{4}$ yards ; find the prime cost and the gain per cent.

27. A person sold 20 horses for *Rs.*1750 thereby losing the cost of 6 horses ; find the prime cost of the horses and the loss per cent.

28. If oranges are bought at 20 for a half-rupee, how many should be sold for *Rs.*14 to gain 40 per cent. ?

29. A grocer buys 1 cwt. of tea at 4*s.* 2*d.* per lb. and mixes it with tea at 2*s.* 11*d.* per lb. ; how much of the latter must he add to the former that by selling the mixture at 3*s.* 8*d.* per lb. he may gain 20 per cent. on his outlay ?

30. A person sold 72 yards of cloth for *Rs* 87, his profit being the cost of 11·52 yards ; how much did he gain per cent. ?

31. A grocer buys coffee at the rate of *Rs.*85 per cwt. and chicory at *Rs.*25 per cwt. and mixes them in the proportion of 5 parts chicory to 7 parts coffee ; at what rate per lb. must he sell the mixture so as to gain $16\frac{2}{3}$ per cent. on his outlay ?

32. A watch is bought for 25 guineas ; at what price must it be sold to secure a clear profit of 30 per cent. after allowing a discount of $2\frac{1}{2}$ per cent. to the purchaser ?

33. How much tea at *Re.*1. 5*a.* 4*p.* per lb. must be mixed with 4 cwt. 47 lbs. at *Re.*1. 6*a.* per lb., so that by selling the mixture at *Re.*1. 8*a.* per lb., 10 per cent. may be gained ?

34. *Rs.*61. 4*a.* was spent in buying apples at *Re.*1. 7*a.* 4*p.* a score. When they came to be sold part of them were worthless

but the rest, on being sold at a profit of 30 per cent., realized *Rs.*68. 4*a.* ; how many scores were there of worthless ones ?

35. A merchant buys 1260 mds. of rice, one-fifth of which he sells at a gain of 5 per cent., one-third at a gain of 8 per cent. and the remainder at a gain of 12 per cent. ; if he had sold the whole at a gain of 10 per cent. he would have obtained *Rs.*28. 14*a.* more ; what was the prime cost per maund ?

36. A merchant buys 3150 yards of cloth. He sells $\frac{1}{3}$ of it at a gain of 6 per cent., $\frac{1}{5}$ at a gain of 8 per cent., $\frac{1}{7}$ at a gain of 12 per cent. and the remainder at a loss of 3 per cent. Had he sold the whole at a gain of 5 per cent. he would have received *Rs.*120. 12*a.* more than he did ; what was the prime cost of 1 yard ?

37. A grocer had 150 lbs. of tea, of which he sold 50 lbs. at *Rs.*4. 8*a.* per lb., and found that he was thereby gaining $7\frac{1}{2}$ per cent. At what rate must he sell the remaining 100 lbs. so as to clear 10 per cent. upon the whole ?

38. An article when sold at a profit of $7\frac{1}{2}$ per cent. yields 2*s.* 1*d.* more than when it is sold at a loss of $12\frac{1}{2}$ per cent. ; find its prime cost.

39. If 3 per cent. more be gained by selling a horse for *Rs.*832. 8*a.* than by selling him for *Rs.*810, what was the original cost ?

40. An article when sold at a profit of 5 per cent. yields 12*a.* more than when sold at a loss of 5 per cent. ; find its prime cost.

41. A person by selling an article, which costs *Rs.*140 per cwt. at *Re.*1. 6*a.* 6*p.* per lb., makes 5 per cent. more profit than he would do if he sold the whole for *Rs.*557. 10*a.* 6*p.* ; what was the quantity sold ?

42. A person sells an article at 5 per cent. profit ; if he had bought it at 5 per cent. less, and sold it for 8*a.* less, he would have gained 10 per cent. ; what was the cost price of the article ?

43. An article passes successively through the hands of three dealers, each of whom in selling adds as his profit 10 per cent. of the price at which he bought it ; if the third dealer sells the article for *Rs.*332. 12*a*, what did the first dealer pay for it ?

44. The cost of freight and insurance on a certain quantity of goods was 15 per cent., and that of duty 10 per cent on the original outlay : the goods were sold at a loss of 5 per cent., but if they had brought *Rs.*30 more there would have been a gain of 1 per cent. ; find how much they cost.

45. A man sells a horse, at a loss, for 40 guineas ; had he sold it for 50 guineas his gain would have been $\frac{3}{4}$ of his former loss ; find the cost price.

46. A man having bought a quantity of goods for *Rs.*1500, sells $\frac{1}{3}$ at a loss of 4 per cent. ; by what increase per cent must he

raise the selling price that by selling the remainder at the increased rate he may gain 4 per cent. on the whole transaction ?

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502. Division into Proportional Parts. A given quantity is said to be divided into **Proportional Parts**, when it is divided into parts which have the same *ratio* to each other that certain given numbers have.

503. *To divide a given quantity into parts which shall be proportional to certain given numbers.*

RULE. (i) Divide the given quantity by the sum of the given numbers expressing the ratios of the parts ; multiply the quotient by each of these numbers, and the products will give the parts required. (Rule of **Unitary Method**).

(ii) The sum of the given parts : any one of them :: the given quantity to be divided : the corresponding part of it. (Rule of **Proportion**).

This proportion must be repeated for each of the parts, or at all events for all but the last part, which may be found either by this proportion or by subtracting the sum of the values of the other parts from the given quantity to be divided.

Ex. 1. Divide *Rs.*837 among *A*, *B* and *C*, so that their shares may be as 5, 9 and 13 respectively.

(i) Here $5+9+13=27$; *Rs.*$837 \div 27=$ *Rs.*31.

∴ *A*'s share = *Rs.*$31 \times 5=$ *Rs.*155,
B's share = *Rs.*$31 \times 9=$ *Rs.*279,
and *C*'s share = *Rs.*$31 \times 13=$ *Rs.*403 } *Ans.*

(ii) Here $5+9+13=27$.

∴ 27 : 5 :: *Rs.*837 : *A*'s share ; ∴ *A*'s share = *Rs.*155.
27 : 9 :: *Rs.*837 : *B*'s share ; ∴ *B*'s share = *Rs.*279.
27 : 13 :: *Rs.*837 : *C*'s share ; ∴ *C*'s share = *Rs.*403. } *Ans.*

or, *C*'s share = *Rs.*837 − *Rs.*$(155+279)=$ *Rs.*403.

504. If the given numbers are fractions we may follow the same method ; but it will be more convenient to find integral numbers proportional to the given fractions, by multiplying each fraction by the L. C. M. of their denominators.

Ex. 2. Divide 3925 nuts among three persons *A*, *B* and *C*, so that their shares may be to each other in the ratio of $\frac{2}{3}$, $\frac{3}{8}$ and $\frac{4}{15}$.

(i) Here, $\frac{2}{3}+\frac{3}{8}+\frac{4}{15}=\frac{157}{120}$; $3925 \div \frac{157}{120}=3000$.

∴ *A*'s share $=3000 \times \frac{2}{3}=2000$,
B's share $=3000 \times \frac{3}{8}=1125$,
and *C*'s share $=3000 \times \frac{4}{15}=800$. } *Ans.*

(ii) The L. C. M. of 3, 8 and 15 is 120; multiply each fraction by this; then the shares are in the ratios 80 : 45 : 32.

Now, $80+45+32=157$; and $3925 \div 157=25$.

$\therefore$ *A's* share $=25 \times 80=2000$; *B's* share $=25 \times 45=1125$; and *C's* share $=25 \times 32=800$.

Ex. 3. Divide *Rs.*1050 among *A*, *B*, *C* and *D* so that *A*'s share : *B*'s share :: 2 : 3, *B*'s : *C*'s :: 4 : 5, and *C*'s : *D*'s :: 6 : 7.

A's share : *B*'s $=2:3=16:24$,
B's share : *C*'s $=4:5=24:30$,
C's share : *D*'s $=6:7=30:35$,

$\therefore$ *A*'s : *B*'s : *C*'s : *D*'s $=16:24:30:35$.

But $16+24+30+35=105$, and *Rs.*$1050 \div 105=$ *Rs.*10;

$\therefore$ *A*'s share $=$ *Rs.*$10 \times 16=$ *Rs.*160; *B*'s share $=$ *Rs.*$10 \times 24=$ *Rs.*240;
C's share $=$ *Rs.*$10 \times 30=$ *Rs.*300; *D*'s share $=$ *Rs.*$10 \times 35=$ *Rs.*350.

Ex. 4. Divide *Rs.*288. 12*a.* among *A*, *B* and *C*, so that *B*'s share may be half as much again as *A*'s, and *C*'s share one-third as much again as both *A*'s and *B*'s.

B's share $=\frac{3}{2}$ of *A*'s share, and $\therefore$ *A*'s share $+$ *B*'s share $=\frac{5}{2}$ of *A*'s share, and *C*'s share $=\frac{4}{3}$ of $\frac{5}{2}$ of *A*'s share $=\frac{10}{3}$ of *A*'s share;

$\therefore$ *A*'s share : *B*'s : *C*'s $=1:\frac{3}{2}:\frac{10}{3}=6:9:20$.

But $6+9+20=35$; and *Rs.*288. 12*a.* $\div 35=$ *Rs.*8. 4*a.*

$\therefore$ *A*'s share $=$ *Rs* 8. 4*a.* $\times 6=$ *Rs.*49. 8*a.*,
B's share $=$ *Rs* 8. 4*a.* $\times 9=$ *Rs.*74. 4*a.*,
and *C*'s share $=$ *Rs.*8. 4*a.* $\times 20=$ *Rs.*165. *Ans.*

Ex. 5. Gunpowder is composed of nitre, charcoal and sulphur in the proportion of 33, 7 and 5. How many lbs. of sulphur are there in 180 lbs. of powder?

$33+7+5=45$; also 180 lbs. $\div 45=4$ lbs.

$\therefore$ quantity of sulphur $=4$ lbs. $\times 5=$ 20 lbs. *Ans.*

Ex. 6. A debt of *Rs.*28. 14*a.* is paid in rupees, 8-anna pieces, and 2-anna pieces. How many coins of each kind were there if the numbers were proportional respectively to 3, 2, 1?

The values of the three groups of coins (rupees, 8-anna pieces, and 2-anna pieces) with *two-anna* as unit, are as $3 \times 8 : 2 \times 4 : 1 \times 1$ or $24:8:1$.

Now $24+8+1=33$; and also *Rs.*28. 14*a.* $\div 33=14$*a.*

Hence the groups of coins are worth 14*a.* $\times 24$, 14*a.* $\times 8$, and 14*a.* $\times 1$ or 336*a.*, 112*a.*, and 14*a.* respectively.

$\therefore$ there are $336 \div 16$ or 21 rupees, $112 \div 8$ or 14 eight-anna pieces, and $14 \div 2$ or 7 two-anna pieces. *Ans.*

Ex. 7. 600 coins consist of guineas, half-sovereigns and half-crowns; the values of the guineas, the half-sovereigns and half-crowns are as 14 : 8 : 3; find the number of guineas.

Suppose the values to be all expressed in shillings.

The numbers of guineas, half-sovereigns and half-crowns are as

$14 \div 21 : 8 \div 10 : 3 \div 2\frac{1}{2}$, or as $\frac{2}{3} : \frac{4}{5} : \frac{6}{5}$.

Now $\frac{2}{3} + \frac{4}{5} + \frac{6}{5} = \frac{8}{3}$; and $600 \div \frac{8}{3} = 225$.

$\therefore$ the number of guineas $= \frac{2}{3} \times 225 = \underline{150}$. *Ans.*

Examples CLII.

1. Divide :—

(1) 1008 into three parts proportional to 2, 3, 4.

(2) *Rs.*25000 into parts proportional to 2, 3, 7, 8.

(3) *Rs.*84700 ... ... $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$.

(4) £1064 ... ... $2, 2\frac{1}{4}, 2\frac{2}{5}$.

(5) 694 bi. 13 kat. 12 ch. ... ... 13, 17, 20.

(6) *Rs.*71. 10*a.* 8*p.* ... ... $10, 2\frac{1}{2}, 1, \frac{1}{2}, \frac{1}{3}$.

(7) 360 lbs. ... ... 3·3, ·7, ·5.

(8) £593. 8*s.* 6*d.* ... ... $2\cdot\dot{3}, 3\cdot\dot{4}\dot{5}, 5\cdot\dot{6}2\dot{9}$.

2. The proportions used in making English gunpowder are saltpetre 75 parts, sulphur 10 parts, charcoal 15 parts How many seers of each material is required to manufacture 11 mds. 8 sr. of gunpowder?

3. In the alloy of which brass cannon are made, there are 11 parts of tin to 100 of copper. Calculate the weight of tin in a piece weighing 33 mds. 12 sr.

4. Gunpowder is composed of nitre 33 parts, charcoal 7 parts and sulphur 5 parts. How many pounds of powder can be made with 30 lbs. of sulphur, and how much nitre is required?

5. In England, gunpowder is made of 75 parts of nitre, 10 of sulphur and 15 of charcoal; in France 77 parts of nitre, 9 of sulphur and 14 of charcoal. If 14 maunds of each be mixed, what weight of nitre, sulphur and charcoal will there be in the compound?

6. Divide *Rs.*26 between 5 men, 7 women and 14 boys, so that each woman may have $\frac{3}{7}$ of each man's share, and each boy $\frac{3}{5}$ of each woman's share.

7. Divide *Rs.*19089 among *A*, *B* and *C*, so that *A*'s share : *B*'s = 3 : 5, and *B*'s : *C*s = 10 : 11.

8. £1630 is divided among *A*, *B* and *C*; *A*'s share : *B*'s share is as $\frac{1}{2} : \frac{3}{4}$, and *B*'s : *C*'s as $\frac{5}{6} : \frac{7}{8}$; find the share of each.

9. Divide *Rs.*14787. 8*a.* among *A*, *B*, *C* and *D*, so that *A*'s share : *B*'s = 3 : 8, *B*'s : *C*'s = 4 : 9, and *C*'s : *D*'s = 15 : 4.

10. A certain sum is divided among *A*, *B* and *C*, so that *A* gets three times as much as *B*, and *B* three times as much as *C*; *B*'s share is *Rs.*100; what is the sum divided?

11. An examiner wishes to mark three questions in the proportion of the numbers 2, 3·2, 4·8. The sum of the marks is to be 150. How must he distribute them?

12. Divide *Rs.*390 among *A*, *B* and *C*, so that as often as *A* receives *Rs.*3, *B* may receive *Rs.*4, and as often as *B* receives *Rs.*6, *C* may receive *Rs.*9.

13. 6270 persons meet together, of whom the number of women : number of men :: 1 : 8, boys : women :: 2 : 9 and girls : boys :: 3 : 10. Find how many were there of each description.

14. A mixed metal consists of 87 parts silver and 13 parts copper; what is the value of 29 lbs. 2 oz. Troy of this mixture, when pure silver is at 5*s.* 6*p.* per oz. Troy, and copper at 1*s.* per lb. Avoir.

15. Of 2180 mds., *A*'s share : *B*'s share :: 2 : 3, *B*'s : *C*'s :: 4 : 7, and *C*'s : *D*'s :: 5 : 11; find the share of each.

16. A person has £12. 4*s.* in half-crowns, florins, and shillings, and the number of coins of each kind are respectively as the numbers 7, 5 and 3. Find the number of coins of each kind.

17. A purse contains £42. 8*s.* made up of pence, shillings, half-crowns, and sovereigns; the half-crowns, pence, sovereigns and shillings are as the numbers 1, 2, 3 and 8 respectively; find how many of each coin are in the purse.

18. 310 coins consist of sovereigns, half sovereigns and florins, whose values are as 5 : 3 : 2. Find the number of each.

19. Divide *Rs.*9510 among *A*, *B*, *C* and *D*, so that $\frac{1}{2}$ of *A*'s share may be equal to $\frac{2}{3}$ of *B*'s, and $\frac{3}{4}$ of *B*'s may be equal to $\frac{4}{5}$ of *C*'s and $\frac{4}{5}$ of *C*'s may be equal to $\frac{1}{8}$ of *D*'s.

20 378 coins consist of rupees, half-rupees and quarter-rupees, whose values are as 13 : 11 : 7. Find the number of each.

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505. When two or more men are partners in a common concern, they each contribute a certain sum of money called the **Capital**, to carry on business.

Fellowship or **Partnership** is a method by which gains or losses are distributed among partners in any mercantile transaction, in proportion to the capital which each has contributed.

506. Fellowship is either **Simple** or **Compound**.

In *Simple Fellowship*, the sums of money put in by the several partners continue in the business for the *same* time; in *Compound Fellowship*, for *different periods* of time.

507. Simple Fellowship. In Simple Fellowship, the gain or loss arising at the end of any given time is divided among the partners in *proportion* to the capital subscribed by each of them.

Ex. A, B and C from a partnership ; A subscribes $Rs.1750$, B $Rs.2100$ and C $Rs.2650$. At the end of 9 months they dissolve, and share the profits amounting to $Rs.422$. $8a.$; what will be the share of each ?

Here, we have to divide $Rs.422\frac{1}{2}$ in the proportion of $Rs.1750$, $Rs.2100$, and $Rs.2650$; or in the proportion of the numbers 35,42,53.

Now, $35+42+53=130$; and $Rs.422\frac{1}{2} \div 130 = Rs.\frac{13}{4}$.

$\therefore$ A's share $= Rs.\frac{13}{4} \times 35 = Rs.113$. $12a.$;
B's share $= Rs.\frac{13}{4} \times 42 = Rs.136$. $8a.$;
and C's share $= Rs.\frac{13}{4} \times 53 = Rs.172$. $4a.$ } *Ans.*

508. Compound Fellowship. In Compound Fellowship the gain or loss is divided among the partners not only in *proportion* to the *capital* subscribed by each, but also to the *time* for which it has been subscribed. Hence, we obtain the following Rule :—

RULE. Reduce all the times into the same denomination, and multiply each man's stock by the time of its continuance, and then state thus :—

The sum of all the products : each particular product :: the whole quantity to be divided : the corresponding share.

Ex. 1. A, B and C are partners ; A puts in $Rs.5000$ for 7 months, B $Rs.6000$ for 8 months, and C $Rs.9000$ for 9 months. The profit is $Rs.4100$; what is the share of each ?

$Rs.5000 \times 7 = Rs.35000$; $Rs.6000 \times 8 = Rs.48000$;
$Rs.9000 \times 9 = Rs.81000$.

Also $Rs.35000 + Rs.48000 + Rs.81000 = Rs.164000$.

$\therefore$ $Rs.164000 : Rs.35000 :: Rs.4100$: A's share $= Rs.875$;
$Rs.164000 : Rs.48000 :: Rs.4100$: B's share $= Rs.1200$;
$Rs.164000 : Rs.81000 :: Rs.4100$: C's share $= Rs.2025$. } *Ans.*

Ex. 2. There were at a feast 20 men, 30 women, and 15 servants ; for every $10a.$ that a man paid, a woman paid $6a.$ and a servant $2a.$; the bill amounted to $Rs.410$; how much did each man, woman, and servant pay ?

$10a. \times 20 = 200a.$; $6a. \times 30 = 180a.$; and $2a. \times 15 = 30a.$
Also $200a. + 180a. + 30a. = 410a.$

$\therefore$ $410a. : 200a. :: Rs.410$: 20 men's share $= Rs.200$;
$410a. : 180a. :: Rs.410$: 30 women's share $= Rs.180$;
$410a. : 30a. :: Rs.410$: 15 servants' share $= Rs.30$.

$\therefore$ a man pays $Rs.10$; a woman $Rs.6$; and a servant $Rs.2$. *Ans.*

Ex. 3. A and B begin to trade in partnership. A puts in £400 at first, and £500 at the end of 2 months; B puts in £300 at first and £600 at the end of 3 months. The profit at the end of the year is £470. How should this be divided?

A's equivalent capital = £400 × 12 + £500 × 10 = £9800;
B's .... = £300 × 12 + £600 × 9 = £9000.

Also £9800 + £9000 = £18800.

∴ £18800 : £9800 :: £470 : A's share; ∴ A's share = £245; } *Ans.*
£18800 : £9000 :: £470 : B's share; ∴ B's share = £225 }

Examples CLIII.

1. Three partners put into business the sums of *Rs.*300, *Rs.*400, and *Rs.*500, and at the end of a certain time they gained *Rs.*600; find the share of each.

2. Three persons forming a joint stock of *Rs.*45000, gain by trading *Rs* 15000; and of this their shares are *Rs.*7500, *Rs.*5000 and *Rs.*2500; find the portion of stock contributed by each.

3. Three partners A, B and C invest *Rs.*2000, *Rs.*3500, and *Rs.*4750 for 6, 3 and 2 months respectively; how should a profit of *Rs.*4000 be divided among them?

4. If A contribute *Rs.*6000 for 5 months, B *Rs.*5000 for 6 months, C *Rs.*4000 for $7\frac{1}{2}$ months and D *Rs.*2500 for 12 months, in the formation of a joint stock; divide a profit of *Rs.*4760 equitably among them.

5. A is a working, B a sleeping partner in a bookseller's business. Their capital amounts to £6400; of which £2400 belongs to A, the rest to B. Their profits, at the end of the first year, amounted to £1600. A receives 10 per cent. of the profits for managing the business. How ought the remaining part of the profits to be divided?

6. Two traders A and B form a partnership for a year; A contributes *Rs.*5000, and at the end of 4 months he adds *Rs.*3000; B contributes *Rs.*8000 but at the end of 9 months withdraws *Rs.*2500. At the end of the year a profit of *Rs.*4600 is to be divided; how much should each partner get?

7. A and B enter into partnership for three years: A contributes *Rs.*5000, at the end of the first year he adds another *Rs.*5000, and at the end of the second year *Rs.*4500; B contributes *Rs.*8000, at the end of the first year and a half he adds *Rs.*5000, and six months before the partnership concludes he withdraws *Rs.*3500. Eight months after the commencement of the partnership C is admitted with *Rs.*10000, which he leaves in till the end; how should a profit of *Rs.*40135. 8*a.* be divided among the partners?

8. *A*, *B* and *C* rent a field for *Rs.*710 : *A* puts in 12 oxen for 4 months, *B* 15 oxen for 3 months, and *C* 20 oxen for 6 months. How much of the rent should each pay ?

9. *A*, *B* and *C* are partners ; *A*'s money has been in the business 3 months, and he claims $\frac{1}{18}$ of the gain ; *B*'s money has been in the business 9 months and *C* has had £756 in the business 4 months, and he claims half the gain : how much money did *A* and *B* contribute to the capital ?

10. Four merchants *A*, *B*, *C* and *D*, trading with a capital of *Rs.*238000, find after a certain time their respective shares increased by *Rs.*265. 13*a.* 4*p.*, *Rs.*372. 2*a.* 8*p.*, *Rs.*531. 10*a.* 8*p.* and *Rs.*638. How much did they respectively subscribe to the original capital ?

11. *A* and *B* enter into partnership with capitals as 4 : 5. At the end of 3 months they withdraw respectively $\frac{1}{4}$ and $\frac{2}{5}$ of their capitals. When the year closes they find their profit to be *Rs.*4364. 12*a.* ; how must it be divided between them ?

12. *A* and *B* rent a field for 21 guineas. *A* puts into 10 horses for $1\frac{1}{3}$ months, 30 oxen for 2 months and 100 sheep for $3\frac{1}{4}$ months ; *B* 40 horses for $2\frac{1}{3}$ months, 50 oxen for $1\frac{1}{4}$ months and 115 sheep for 3 months. If the food consumed in the same time by a horse, an ox and a sheep be in the ratio 3 : 2 : 1, what portion of the rent must each pay ?

13. Three persons with a joint stock gain £3650 ; the first advances $\frac{1}{3}$ of the capital for $\frac{1}{4}$ of the time, the second $\frac{1}{4}$ of the capital for $\frac{1}{3}$ of the time and the third the remainder of the capital for the whole time ; find their shares.

14. *A* employs his capital of *Rs.*2000 in a trade for 8 months. *B*, who joins later on, keeps his capital for 6 months, and receives n the end *Rs.*150 out of a total profit of *Rs.*400. How much capital does *B* put in the trade ?

15. The capitals contributed by *A* and *B* to a joint stock are in the proportion of 4 : 5, and the profits received by them are in the proportion of 5 : 4. If *B*'s capital is in the joint stock for 15 months, how long is *A*'s ?

16. *A* and *B* hired a pasture for *Rs.*125 for 50 days. *A* put in 25 oxen for a certain number of days, and paid *Rs.*75, *B* put in 20 oxen for the remaining days, and paid the remaining sum. How long had *B* put in his oxen ?

17. The profits received by *A* and *B* by a partnership are in the proportion of 2 : 3, and their capitals are in the proportion of 6 : 7. If *A*'s capital is in the joint stock for 8 months, how long is *B*'s ?

18. *A* and *B* rent a pasture for £200 for 80 days. *A* puts in 60 oxen for a certain number of days ; *B* puts in 40 for the remaining days, and pays $\frac{1}{3}$ of the total rent. Find the time for which *A* puts in his oxen.

509. Equation of Payments. The Equation of Payments is the finding of a proper time, called the **equated time**, at which two or more debts due at *different* times should be discharged at *one* payment ; and it is here *assumed* that the interests of *all* the debts for their respective periods are together equal to the interest of their *sum* for the *equated time*.

510. *To find the equated time of different payments.*

RULE. Multiply each debt into the time which will elapse before it becomes due, and then divide the sum of the products by the sum of the debts ; the quotient will be the equated time required.

Ex. 1. If *Rs.*100 be due in 3 months, *Rs.*210 in 2 months and *Rs.*160 in 5 months, find the equated time.

Here, $100\times3+210\times2+160\times5=300+420+800=1520$; and $100+210+160=470$.

$\therefore$ the equated time $=\frac{1520}{470}$ months $=\underline{3\frac{11}{47}}$ months. *Ans.*

Ex. 2. *A* owed *B* £100, to be paid at the end of 9 months ; he pays however £20 at the end of 3 months, and £30 at the end of 8 months ; when ought the remainder to be paid ?

The remainder $=$ £$(100-20-30)=$ £50.

Here, $20\times3+30\times8+50\times$ no. of mo. reqd. $=100\times9$;

or, $60+240+50\times$ no. of mo. reqd. $=900$;

$\therefore$ $50\times$ no. of mo. reqd. $=900-60-240=600$;

$\therefore$ no. of mo. reqd. $=600\div50=\underline{12}.$ *Ans.*

Examples CLIV.

1. If *Rs.*75 be due in 4 months, *Rs.*125 in 5 months and *Rs.*150 in 7 months ; what is the equated time ?

2. What will be the equated time of payment of £200 due at 3 months, £300 at 8 months and £500 at 12 months ?

3. Find the equated time of payment, when $\frac{1}{2}$ of a sum of money is due in 3 months, $\frac{1}{5}$ in 8 months and the remainder in 15 months.

4. *A* finds on the 1st of March that he owes *B* the following sums : *Rs.*1400 due on the 20th of April, *Rs.*1200 due on the 14th of May, and *Rs* 3800 due on the 15th of June. On what day may *A* pay these debts together ?

5. *A* owes *B* £4600, whereof £1000 is to be paid in 50 days, £1300 in 40 days and the remainder in 140 days ; find the equated time.

6. *A* owes *B* *Rs.*7300 to be paid in $4\frac{44}{73}$ months ; he pays however *Rs.*1500 at the end of 3 months, and *Rs.*2100 at the end of 5 months ; when was the remainder due ?

7. Of a sum of money due 15 months hence, $\frac{1}{3}$ was paid at 4 months, $\frac{1}{4}$ at 6 months, and $\frac{1}{8}$th at 12 months; how many months may the payment of the residue be deferred ?

8. At the beginning of the year *A* finds that he owes *B* four debts, *viz.*, *Rs* 1200 payable on the 3rd February ; *Rs.*350, payable on the 17th March ; *Rs.*1000, payable on the 5th May, and *Rs.*850, payable on the 18th August. To simplify accounts he proposes to pay *B* *Rs* 3400 in one sum ; on what day should the payment be made ?

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511. Alligation is the rule by means of which the rate of quality of a composition or mixture is found from the rates or qualities of the ingredients of which it is made up.

512. Alligation is of two kinds, **Medial** and **Alternate**. *Alligation Medial* is that in which the price, and quantity, of each of the things composing the mixture are given, to find the price of the mixture. This is equivalent to finding an *average* or *mean* price. (See Arts. 179 and 184).

RULE. Multiply the number of each quantity expressed in the same denomination by its price, and divide the sum of these products by the sum of the numbers. The quotient will be the rate of the mixture.

Ex. A wine merchant mixes 60 gals. of sherry at *Rs.*12 per gal. 50 gals. at *Rs.*13 per gal., and 70 gals. at *Rs.*16 per gal. ; find the price of a gallon of the mixture.

60 gals. at *Rs.*12 a gal. = *Rs.* 720	∴ 1 gal. of the mixture
50 ...... at *Rs.*13 ...... = *Rs.* 650	= *Rs.* $\frac{2490}{180}$
70 ...... at *Rs.*16 ...... = *Rs.*1120	= *Rs.* 13. 13*a.* 4*p.* *Ans.*
∴ 180 gals. of the mixture = *Rs.*2490	

513. *Alligation Alternate* is the method of finding from the price of each of the things which compose a mixture, what quantity must be taken of each, in order that the mixture may be of a given price. (Examples of alligation alternate have *generally* an indefinite number of solutions).

RULE. Place the given prices under one another in order, and to the left place the mean price. Link all the prices, so that one *under* and one *above* the mean price shall always be together ; and write against each price the difference between the price with which it is linked and the mean price :—these differences, or any equimultiples of them, will give the quantities required.

Ex. 1. How must a grocer mix tea at *Re.*1. 2*a.* per lb. and *Re.*1. 7*a.* per lb. to make a mixture worth *Re.*1. 5*a.* a lb. ?

*Re.*1. 2*a.* = 18*a.* ; *Re.*1. 7*a* = 23*a.* ; *Re.*1. 5*a.* = 21*a.*

21 18) 2 lbs. at *Re.*1. 2*a.*
23) 3 lbs. at *Re.*1. 7*a.*

∴ the mixture must be made in the ratio of 2 : 3. *Ans.*

To make the mixture at *Re.*1. 5*a.* per lb. 1 lb. at *Re.*1. 2*a.* brings a gain of 3*a.* and 1 lb. at *Re.*1. 7*a*......loss of 2*a.*

In order therefore, that the gain in using the former may be equal to the loss in using the latter, for every 2 lbs. of the former we must take 3 lbs. of the latter, for then the gain would be 2 × 3*a.* and the loss 3 × 2*a.* We must therefore take the quantities in the ratio of 2 and 3 ; that is, *in the inverse ratio of the differences of the two prices and the mean price.*

Ex. 2. How must a grocer mix teas at *Re.*1. 5*a.* 4*p.*, *Re.*1. 7*a.* 4*p.* and *Re.*1. 10*a.* per lb., to make a mixture worth *Re.*1. 8*a.* a lb. ?

*Re.*1. 5*a.* 4*p.* = $21\frac{1}{3}$*a.* ; *Re.*1. 7*a.* 4*p.* = $23\frac{1}{3}$*a.* ; *Re.*1. 10*a.* = 26*a.* ; *Re.*1. 8*a.* = 24*a.*

24	$21\frac{1}{3}$	2	lbs. at *Re.*1. 5*a.* 4*p.*
	$23\frac{1}{3}$	2	lbs. at *Re.*1. 7*a.* 4*p.*
	26	$2\frac{2}{3}+\frac{2}{3}$	lbs. at *Re.*1. 10*a.*

Hence, mixing at all the three prices, he must take the kinds in the following proportion :—

2 lbs. : 2 lbs. : ($2\frac{2}{3}+\frac{2}{3}$) or $3\frac{1}{3}$ lbs. ; or 1 : 1 : $1\frac{2}{3}$ or 3 : 3 : 5. *Ans.*

Ex. 3. How may a grocer mix teas at 2*s.* 6*d.*, 2*s.* 9*d.*, 3*s.* 1*d.* and 3*s.* 4*d.* per lb., to form a mixture worth 2*s.* 10*d.* per lb. ?

2*s.* 6*d.* = 30*d.* ; 2*s.* 9*d.* = 33*d.* ; 3*s.* 1*d.* = 37*d.* ; 3*s.* 4*d.* = 40*d.* ; 2*s.* 10*d.* = 34*d.*

34	30	3 lbs. at 2*s.* 6*d.*
	33	6 lbs. at 2*s.* 9*d.*
	37	4 lbs. at 3*s.* 1*d*
	40	1 lb. at 3*s.* 4*d.*

Hence a mixture may be made by mixing them in the proportion of 3 : 6 : 4 : 1. *Ans.*

or

34	30	6 lbs. at 2*s.* 6*d.*
	33	3 lbs. at 2*s.* 9*d.*
	37	1 lb. at 3*s.* 1*d.*
	40	4 lbs. at 3*s.* 4*d.*

Hence a mixture may be made by mixing them in the proportion of 6 : 3 : 1 : 4. *Ans.*

Ex. 4. How much coffee at 13*a.* per lb. and chicory at 3*a.* 10*p.* per lb. must a person take to make a mixture of 33 lbs. worth 9*a.* 8*p.* per lb. ?

13*a.* = 156*p.* ; 3*a.* 10*p.* = 46*p.* ; 9*a.* 8*p.* = 116*p.*

116 156) 70 lbs. at 13*a.* per lb.
46) 40 lbs. at 3*a.* 10*p.* per lb.

∴ coffee must be to chicory as 70 : 40 or 7 : 4 ; and 7 + 4 = 11 ;

∴ coffee in the mixture = $\frac{7}{11}$ of 33 lbs. = 21 lbs.
and chicory ... ... = $\frac{4}{11}$ of 33 lbs. = 12 lbs. } *Ans.*

Ex. 5. I buy one kind of tea at *Re* 1. 2*a* per lb., and another kind at *Re*.1. 12*a*. per lb. ; in what ratio must I mix them in order that by selling the mixture at *Rs*.2 per lb. I may gain 20 per cent. ?

Rs.120 : *Rs*.2 :: *Rs*.100 : cost price of the mixture per lb. ;

∴ cost price of the mixture per lb. = *Re*.1. 10*a*. 8*p*.

Re.1. 2*a*. = 18*a*. ; *Re*.1. 12*a*. = 28*a*. ; *Re*.1. 10*a*. 8*p*. = $26\frac{2}{3}$*a*.

$26\frac{2}{3}$ 18) $1\frac{1}{3}$ lbs. at *Re*.1. 2*a*. per lb.
28) $8\frac{2}{3}$ lbs. at *Re*.1. 12*a*. per lb.

Hence, the proportion of cheaper to dearer is $1\frac{1}{3}$: $8\frac{2}{3}$ or 2 : 13. *Ans.*

514. If there be a lump of two ingredients, and if an equal quantity of another lump with the proportion of the ingredients interchanged be added to it, the quantities of the ingredients become equal in the fresh compound.

Ex. The price of gold is *Rs* 38. 15*a*. per oz A composition of gold and silver weighing 18 lbs is worth *Rs* 7246. 8*a*. ; but if the weights of the gold and silver in the composition were interchanged, it would be worth only *Rs*.1717. 8*a*. Find the proportion of gold and silver in the composition and the price of silver per oz.

If the two lumps were added together, there would be 18 lbs. of gold and 18 lbs. of silver, and the price of the two lumps together would be *Rs*.7246. 8*a*. + *Rs*.1717. 8*a*. or *Rs*.8964.

Since 18 lbs. of gold + 18 lbs. of silver is worth *Rs*.8964,
and 18 lbs. of gold is worth *Rs*.38. 15*a*. × 18 × 12 = *Rs*.8410. 8*a*.
∴ 18 lbs of silver is worth *Rs*.553. 8*a*.
∴ 1 oz. of silver is worth *Rs*.553. 8*a*. ÷ (18 × 12) = *Rs*.2. 9*a*. *Ans.*

Again, cost of 1 oz. of the mixture = *Rs*.7246. 8*a*. ÷ (18 × 12) = *Rs*.33. $8\frac{7}{9}$*a*.
cost of 1 oz. of gold = *Rs*.38. 15*a*. = 623*a*., = $536\frac{7}{9}$*a*.
and cost of 1 oz. of silver = *Rs*.2. 9*a*. = 41*a*.

$536\frac{7}{9}$ 623) $495\frac{7}{9}$ oz. of gold.
41) $86\frac{2}{9}$ oz. of silver.

Hence gold : silver = $495\frac{7}{9}$: $86\frac{2}{9}$
= 4462 : 776
= 23 : 4. *Ans.*

Examples CLV.

1. A wine-merchant mixes 20 gals. of wine at *Rs*.6 per gal., 25 gals. at *Rs*.7 per gal., and 36 gals. at *Rs*.8 per gal. ; what will be the price of a gallon of the mixture ?

2. A grocer mixes 47 lbs. of tea at *Re*.1. 1*a*. 2*p*. per lb., 25 lbs. at *Re*.1. 2*a*. 8*p*. per lb., and 20 lbs. at *Re*.1. 6*a* 10*p*. per lb., what is the price of a lb. of the mixture ? If he had also added 8 lbs. of sloe-leaves at 2*a*. 2*p*. per lb., what then would be the price ?

3. In what ratio must tea worth 2*s*. 5*d*. per lb. be mixed with tea worth 3*s*. 4*d*. per lb. to make a mixture worth 2*s*. 9*d*. per lb. ?

4 How must a person mix teas worth *Re.*1. 8*a.* 8*p.*, *Re.*1. 6*a.* and *Re.*1. 12*a.* per ℔. respectively, to make a mixture worth *Re.*1. 8*a.* per ℔. ?

5. In what proportions must spirits worth 8*s.* 3*d.*, 7*s.* 9*d.*, 6*s.* 6*d.* and 8*s.* 4*d.* per gallon respectively be mixed, so that the compound may be worth 8*s.* per gallon ?

6. I buy some tea at *Re.*1. 2*a.* per ℔. and some at *Re.*1. 12*a.* per ℔. ; in what ratio must they be mixed so that by selling the mixture at *Rs.*2. 2*a.* 8*p.* per ℔., I may gain 30 per cent. ?

7. A green-grocer sells potatoes at *Re.*1. 5*a.* 4*p.*, *Re.*1. 7*a.* 4*p.* and *Re.*1. 10*a.* per maund ; what quantities of each kind must he sell that the average price obtained shall be *Re.*1. 8*a.* per maund ?

8. A merchant buys wheat at *Rs.*19. 8*a.* per quarter, and another kind at *Rs.*3 per bushel ; in what ratio must he mix them to gain 25 per cent. by selling the mixture at *Rs.*28. 12*a.* per quarter ?

9. A druggist makes from ingredients worth *Rs.*2. 8*a.*, *Rs.*3, *Rs.*4 and *Rs.*4. 8*a.* per ℔. respectively, $1\frac{1}{2}$ cwt. of a mixture worth *Rs.*3. 8*a.* per ℔. ; how much of each ingredient does he use ?

10. A grocer buys teas at *Re.*1. 5*a.* 4*p.*, *Re.*1. 8*a.* and *Re.*1. 14*a.* per ℔. respectively ; how is he to mix them that by selling the mixture at *Rs.*2. 0*a.* 8*p.* per ℔., he may gain $16\frac{2}{3}$ per cent. ?

11. I mix wines at 12*s.*, 13*s.*, 14*s.* and 14*s.* 6*d.* per gallon respectively ; if the mixture is worth 13*s.* 6*d.* a gallon, how much of each kind of wine is there in 63 gallons of it ?

12. How much tea at *Rs* 3 per ℔, must I mix with 12 ℔s. at *Re.*1. 13*a.* 4*p.* per ℔, to make a mixture worth *Rs.*2. 2*a.* 8*p.* a ℔. ?

13. It is required to mix teas at *Re.*1. 7*a.*, *Re.*1. 4*a.* and *Re.*1. 2*a.* per ℔. with sloe-leaves at 2*a.* per ℔., so that the mixture being sold at *Re.*1. 6*a.* 8*p.* per ℔. one-fourth of the receipts may be clear profit.

14. I have 50 ℔s. of copper worth 10*a.* 8*p.* per ℔. and with this I wish to melt pewter worth 7*a.* per ℔. and brass worth 9*a.* 4*p.* per ℔. ; find how much brass and pewter I must use to make a mixture worth 8*a.* per ℔.

15. How many ℔s. of tea at 2*s.* 8*d.* per ℔. must be mixed with 495 ℔s. at 2*s.* 9*d.* per ℔. so that a profit of 10 per cent. may be made by selling the mixture at 3*s.* per ℔. ?

16. A silversmith gave *Rs.*485. 6*a.* 8*p.* for 16 ℔s. 8 oz. of silver, giving *Rs.*2. 9*a.* 8*p.* an oz. for one part and *Rs.*2. 3*a.* for the rest ; how many oz. of each kind did he buy ?

17. How much gold at £4. 5*s.* per oz., silver at 5*s.* an oz. and copper considered as of no value comparatively, may be melted together that the compound may be worth £2. 15*s.* per oz. ?

18. The price of gold is £3. 17*s.* 10½*d.* per oz.; a composition of gold and silver weighing 18 lbs. is worth £637. 7*s.*, but if the proportions of gold and silver were interchanged, it would be worth only £259. 1*s.* Find the proportion of gold and silver in the composition, and the price of silver per oz.

19. A person bought apples and pears for *Rs.*2. 3*a.* If the numbers of apples and pears be interchanged, it would have cost him only *Re.*1. 9*a.* If apples cost 9*a.* and pears 3*a.* a dozen, how many of each did he buy?

20. A mass of gold and silver weighing 9 lbs. is worth £318 13*s.* 6*d.*; if the proportions of gold and silver in it were interchanged, it would be worth £129. 10*s.* 6*d.*; it is known that 1 oz. of gold and 2 oz. of silver are worth £4. 8*s.* 1½*d.*; what is the price of gold and silver per ounce?

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515. Pasture with Growing Grass. The following Solutions will illustrate the method of operation to be adopted in such cases.

Ex. 1. A meadow of 15 acres, grass growing uniformly, is consumed by 20 oxen in 96 days, but by 30 oxen in 60 days. How many oxen will eat it in 24 days?

Original grass + 96 days' growth keeps 20 oxen for 96 days,

∴ .... 1 ox for 1920 days.

Also, original grass + 60 days' growth keeps 30 oxen for 60 days,

∴ .... 1 ox for 1800 days.

Hence, by subtraction, we get

36 days' growth keeps 1 ox for 120 days,

∴ (36 × 16) days' .... 1 ox for 1920 days;

but original grass + 96 days' growth keeps 1 ox for 1920 days;

∴ original grass + 96 days' growth = (36 × 16) or 576 days' growth;

∴ original grass + 24 days' .. ... = (576 − 72) or 504 days' ......

Now, 36 days' growth keeps 1 ox for 120 days,

∴ 36 days' .... 5 oxen for 24 days,

∴ 504 days' .... (5 × 14) oxen for 24 days.

Hence the reqd. no. of oxen = 5 × 14 or 70. *Ans.*

Ex. 2. If 133 oxen consume the grass of a meadow in 13 days, and 112 of the oxen could consume the grass of the same meadow in 16 days—the grass growing uniformly; in what time could 125 of the oxen do it?

Original grass + 13 days' growth maintains 133 oxen for 13 days,

∴ .... 1729 oxen for 1 day.

Also original grass + 16 days' growth maintains 112 oxen for 16 days,
$\therefore$ ....1792 oxen for 1 day.

Hence, by subtraction, we have

3 days' growth maintains 63 oxen for 1 day,
$\therefore$ 1 day's...........................21 oxen for 1 day,
$\therefore$ 13 days'............21×13 or 273 oxen for 1 day.

Hence, original grass of the field is eaten by $(1729 - 273)$ or 1456 oxen in 1 day.

Now, in 1 day 21 oxen consume 1 day's growth of grass and $\therefore$ there remain $(125 - 21)$ or 104 oxen, to consume the original grass of the field.

$\therefore$ 104 oxen : 1456 oxen :: 1 day : no. of days required ;
$\therefore$ no. of days reqd. $= 1456 \div 104 = \underline{14}$. *Ans.*

Ex. 3. If 25 oxen consume 10 acres of pasture, grass growing uniformly, in 30 days, and 30 oxen consume 8 acres of it in 18 days ; how many acres will be consumed by 80 oxen in 60 days ?

Let g denote the daily growth of grass per acre.

Since 25 oxen in 30 days consume 10 ac. of pasture + what grows in it in 30 days, and 30 oxen in 18 days consume 8 ac. of pasture + what grows in it in 18 days,

$\therefore$ 1 ox in 1 day consumes $\dfrac{10 \text{ ac.} + 30 \times 10g}{25 \times 30}$, in 1st case ;

and 1 ox in 1 day consumes $\dfrac{8 \text{ ac.} + 18 \times 8g}{30 \times 18}$, in 2nd case.

Hence, $\dfrac{10 \text{ ac.} + 30 \times 10g}{25 \times 30} = \dfrac{8 \text{ ac.} + 18 \times 8g}{30 \times 18}$, or $\dfrac{2 \text{ ac.} + 60g}{5} = \dfrac{4 \text{ ac.} + 72g}{9}$.

$\therefore$ 18 ac. + $540g$ = 20 ac. + $360g$, $\therefore 180g$ = 2 ac., and $g = \frac{2}{180}$ ac. $= \frac{1}{90}$ ac.

Now, 10 ac. + $30 \times 10g$ = 10 ac. + $30 \times \frac{1}{9}$ ac. $= \frac{40}{3}$ ac.

$\therefore 25 \times 30 : 80 \times 60 :: \frac{40}{3}$ ac. : no. of ac. reqd. + what grows in it in 60 da.

$\therefore$ no. of acres reqd. + what grows in it in 60 days $= \dfrac{80 \times 60 \times 40}{25 \times 30 \times 3} = \frac{256}{3}$.

But no. of acres reqd. + what grows in it in 60 days
= no. of acres reqd. + $60 \times$ no. of acres reqd. $\times \frac{1}{90}$
= no. of acres reqd. + $\frac{2}{3} \times$ no. of acres reqd.
$= \frac{5}{3} \times$ no. of acres reqd.

$\therefore \frac{5}{3} \times$ no. of acres reqd. $= \frac{256}{3}$, $\therefore$ no. of acres reqd. $= \frac{256}{5} = \underline{51\frac{1}{5}}$ *Ans.*

Examples CLVI.

1. If 29 oxen would eat up a field of grass in 7 weeks, or 25 oxen would eat up the same field in 9 weeks,—the grass growing uniformly; how many oxen would eat it up in 6 weeks ?

2. Suppose that in a certain meadow the grass is of uniform quality and growth, and that 20 oxen would exhaust the grass in $12\frac{3}{4}$ days, or 21 oxen would do so in 12 days; in what time would 26 oxen do it?

3. A meadow of 50 acres, grass growing uniformly, is consumed by 80 oxen in 60 days, but by 60 oxen in 90 days. How many oxen will eat half of it in 30 days?

4. In a pasture ground wherein grass grows uniformly and which contains several days' growth of grass, 29 oxen eat all the grass in 7 days. If 25 oxen be kept on the same pasture ground, the grass would last for 9 days. How many oxen should be allowed to graze so that all the grass may be eaten up in 6 days?

5. A field of uniformly growing pasture is consumed by 120 oxen in 20 days or by 150 oxen in 15 days. How many oxen will consume it in 12 days, if at the end of 8 days 30 of them are removed?

6. If 23 oxen consume 8 acres of pasture in 26 days, and 25 oxen consume 7 acres of the same in 20 days,—the grass growing uniformly; how many acres of it would 33 oxen consume in $5\frac{7}{9}$ days?

7. Suppose that 17 oxen in 30 days, or 19 oxen in 24 days, could consume a field of uniformly growing pasture; find what number of oxen, diminished by the removal of 4 at the end of 6 days, would eat up the same field in 8 days.

8. Suppose that in a meadow of 20 acres the grass grows at a uniform rate, and that 133 oxen could consume the whole grass in 13 days, or that 28 of the oxen could eat up 5 acres of it in 16 days; how many of the oxen could eat up 4 acres of it in 14 days?

9. Suppose that a tank receives a regular and continual supply of water, and that when it contains a certain quantity, 12 equal taps being set open would empty it in $7\frac{1}{2}$ minutes, or 7 of the same taps would empty it in 16 minutes; how many of the taps would empty it in 50 minutes?

10. In a certain meadow there is a crop of 91 mds. 35 sr. of grass which grows uniformly. If 11 oxen would consume all the grass in 48 days, but 6 oxen would require 98 days, what weight of grass would each ox eat in a day?

11. A cistern constantly flowing is emptied by a number of waste pipes. If 10 of these be opened the cistern is emptied in 15 minutes, but if 25 be opened it is emptied in 5 minutes. In how many minutes will it be emptied if 15 are opened?

12. In a field in which grass grows uniformly, suppose that 31 oxen can consume $8\frac{3}{4}$ acres in $\frac{3}{4}$ of the time in which 15 oxen would consume $5\frac{1}{4}$ acres, and that 22 oxen would require 3 days longer to consume $7\frac{1}{2}$ acres than 20 oxen would require for $6\frac{1}{4}$ acres:—In what time would the 31 oxen eat up the $8\frac{3}{4}$ acres?

Examples worked out.

Ex. 1. A fruiterer buys a certain number of mangoes at 20 for 3*a*. and an equal number at 30 for 3*a*. He mixes and sells them at 25 for 3*a*. What is his gain or loss per cent. ?
Cost price of one of each of the two kinds is $\frac{3}{20}a$. and $\frac{3}{30}a$. respectively.

$\therefore$ average cost of a mango $=\frac{1}{2}(\frac{3}{20}+\frac{1}{10})a.=\frac{1}{8}a.$;

and selling price of a mango $=\frac{3}{25}a.$;

$\therefore$ loss on 1 mango, *i. e.*, on $\frac{1}{8}a.=(\frac{1}{8}-\frac{3}{25})a.=\frac{1}{200}a.$

Hence $\frac{1}{8}$: 100 :: $\frac{1}{200}$: percentage required ;

$\therefore$ percentage required $=800\div200=4$. *Ans.*

Ex. 2. One lb. of tea and 3 lbs. of sugar cost *Rs.*3 ; but, if sugar were to rise 50 per cent. and tea 10 per cent., they would cost *Rs.*3. 8*a.* ; find the prices per lb. of tea and sugar.

If both tea and sugar were to rise 50 per cent.,

the cost of 1 lb. of tea and 3 lbs. of sugar would be $=\frac{150}{100}$ of *Rs.*3 $=$ *Rs.*4. 8*a.*

But tea rises only 10 per cent. ;

$\therefore$ 40 p. c. of the cost of a lb. of tea $=$ *Rs.*4. 8*a.* $-$ *Rs.*3. 8*a.* $=$ *Re.*1.

$\therefore$ the cost of a lb. of tea $=$ *Re.*1 $\div\frac{40}{100}=$ *Rs.*2 8*a.* *Ans.*

Hence the cost of 3 lbs. of sugar $=$ *Rs.*3 $-$ *Rs.*2. 8*a.* $=$ 8*a.*

$\therefore$ the cost of a lb. of sugar $=\frac{8}{3}a.=2a.\ 8p.$ *Ans.*

Ex 3. Four parcels of gold, weighing respectively 10, 4, 2 and 4 oz and of 13, 12, 11 and 10 carats fineness, being mixed, what is the fineness of the compound ?

Here, $10\times13+4\times12+2\times11+4\times10=130+48+22+40=240$.

The weight of the compound in oz. $=10+4+2+4=20$.

$\therefore$ the fineness of the compound $=(240\div20)$ or 12 carats. *Ans.*

Ex. 4. A man engages a servant on the understanding that he would get *Rs.*60, and a broad cloth after a year's service. The servant wishing to go away after 8 months' service, gets the broad cloth and *Rs.*36. Find the value of the broad cloth.

Since 8 months is $\frac{2}{3}$ of a year, therefore the servant ought to get $\frac{2}{3}$ of the value of the broad cloth and $\frac{2}{3}$ of *Rs.*60 or *Rs.*40.

But he is given $\frac{1}{3}$ of the value of the broad cloth more, and for that *Rs.*(40 − 36) or *Rs* 4 are deducted from cash payment.

Hence $\frac{1}{3}$ of the value of the broad cloth $=$ *Rs.*4.

$\therefore$ the value of the broad cloth $=$ *Rs.*$4\times3=$ *Rs.*12. *Ans.*

Ex. 5. The sum of the ages of *A*, *B* and *C* is 150 years. Ten

years ago, their ages were in the proportion of 7 : 8 : 9 ; find their ages.

Ten years ago, each was 10 years younger, and therefore the sum of their ages was 30 years less, *i e.*, was (150 − 30) or 120 years. Also 7+8+9 = 24.

∴ the age of *A* was $\frac{7}{24}$ of 120 yrs. = 35 yrs. ; the age of *B* was $\frac{8}{24}$ of 120 yrs. = 40 yrs. and the age of *C* $\frac{9}{24}$ of 120 yrs. = 45 yrs.

∴ the present age of *A* = (35 + 10) years = 45 years,
... .*B* = (40 + 10) years = 50 years,   *Ans.*
....*C* = (45 + 10) years = 55 years.

Ex. 6. Two passengers have together 9 mds. of luggage and are charged for the excess above the weight allowed *Rs.*2. 15*a.* 4*p.*, and *Rs.*4. 5*a.* 8*p.* respectively ; but if the luggage had all belonged to one of them he would have been charged *Rs.*7. 14*a.* Find the quantity of luggage allowed free, as also the charge per maund.

A man is allowed to carry a certain weight of his luggage free of charge, so two men are allowed twice that weight.

Also *Rs.*2. 15*a.* 4*p.* + *Rs.*4. 5*a.* 8*p.* = *Rs.*7. 5*a.*

∴ charge on 9 mds. = *Rs.*7. 14*a.* + charge on free allowance,

and .... = *Rs.*7. 5*a.* + twice... .... ;

∴ charge on free allowance = *Rs.*7. 14*a.* − *Rs.*7. 5*a.* = 9*a.*

∴ charge on 9 mds. = *Rs.*7. 14*a.* + 9*a.* = *Rs.*8. 7*a.*

∴ proper charge per maund = *Rs.*8. 7*a* ÷ 9 = 15*a* *Ans.*

Again charge on 1 md. = 15*a.*, and that on free allowance 9*a.*

∴ 15*a.* : 9*a.* :: 1 md. : the free allowance ;

∴ the free allowance = $\frac{9}{15}$ md. = 24 sr. *Ans.*

Ex. 7. A certain number is divided into two parts, such that 5 times one part added to 18 times the other may give 7 times the whole. Find the ratio of the parts.

Since 5 times the 1st part + 18 times the 2nd part = 7 times the whole ;
and 5 times the 1st part + 5 times the 2nd part = 5 times the whole,

∴ by subtraction, 13 times the 2nd part = twice the whole.
Hence the 2nd part = $\frac{2}{13}$ of the whole ; ∴ the 1st part = $\frac{11}{13}$ of the whole.

∴ 1st part : 2nd part :: $\frac{11}{13}$: $\frac{2}{13}$ or 11 : 2. *Ans.*

Ex. 8. A person bought 20 railway tickets for *Rs.*71. Each first-class ticket costs *Rs.*6, and each second-class ticket costs *Rs.*2. 8*a.* What will another lot of 20 tickets, in which the present numbers of first and second-class tickets, are interchanged, cost ?

Had all the 20 tickets been second-class, he should have to pay *Rs.*2½ × 20 = *Rs.*50. Thus he shall have to pay *Rs.*(71 − 50) or *Rs.* 21

more than he pays at present. But he has to pay $Rs.(6-2\frac{1}{2})$ or $Rs.3\frac{1}{2}$ more for a first-class than for a second-class ticket : therefore the no. of first-class tickets $= Rs.21 \div Rs.3\frac{1}{2} = 6$, and the no. of second-class tickets $= 20 - 6 = 14$.

Now, on the interchange of tickets, the cost $= Rs.6 \times 14 + Rs\ 2\frac{1}{2} \times 6 = Rs.(84+15) = \underline{Rs.99}$. *Ans.*

Ex. 9. A person bought 30 animals, consisting of oxen and cows for *Rs.*860. If he had bought as many oxen, as he had bought cows, and as many cows as he had bought oxen, the whole lot would have cost him *Rs.*790 If the cost of an ox and two cows together be *Rs.*65, find the cost of an ox and of a cow.

Since the original lot of 30 animals cost *Rs.*860,

and the second lot of 30 animals cost *Rs.*790 ;

∴ 30 oxen + 30 cows cost *Rs.*1650 (Art. 514.)

∴ 1 ox and 1 cow cost *Rs* 55.

But 1 ox and 2 cows cost *Rs.*65.

∴ by subtraction, 1 cow costs *Rs.*10.
Hence the cost of 1 ox $= Rs.(55-10) = Rs.45$. } *Ans.*

Ex. 10. In a vessel water and milk are mixed in the proportion of 2 : 7. In another they are mixed in the proportion of 2 : 9. In what proportion should quantities be taken from the vessels to form a mixture in which milk and water will be in the proportion of 4 : 1 ?

Since $2+7=9$; ∴ in 9 sr. of the first vessel, we have 2 sr. water and 7 sr. milk. If, therefore, 1 sr. be taken out of the first vessel, it will contain $\frac{7}{9}$ sr. of milk. Similarly, 1 sr. from the second vessel contains $\frac{9}{11}$ sr. of milk ; and 1 sr. of the final mixture contains $\frac{4}{5}$ sr. of milk.

$\frac{4}{5}$ $\left.\begin{matrix}\frac{7}{9} \\ \frac{9}{11}\end{matrix}\right)$ $\frac{1}{55}$ sr. of 1st.
$\frac{1}{45}$ sr. of 2nd. | Hence the reqd. proportion is
$\frac{1}{55} : \frac{1}{45}$ or $45 : 55$ or $9 : 11$.

Miscellaneous Examples VII.

1. A bill of 37 lbs. of sugar and 24 lbs. of tea amounted to *Rs.*21. 7*a.* 2*p.* ; the tea cost 1*a.* 2*p.* more per lb. than the sugar. Find the price of the sugar per lb.

2. A merchant's average rate of profit for five years was 5 per cent. on his capital, and for the first four years his average profit was 4 per cent. What was his rate of profit in the fifth year ?

3. If mangoes be bought at the rate of seven for an anna ; how must they be sold to gain 33 per cent. ?

4. Fifty coolies are employed for a day and receive at the end of the day for the whole work they have done *Rs.*11. 2*a.*, the men

being paid at the rate of 4*a.*, and the women at the rate of 3*a.* per day. How many of them were women ?

5. *A*, *B* and *C* are in partnership, and $\frac{2}{3}$ of *A*'s share is equal to $\frac{1}{2}$ of *B*'s share or $\frac{2}{3}$ of *C*'s and is *Rs.*250 less than *B*'s. What is each man's share ?

6. The price of rice being raised 50 per cent., how much per cent. must a house-holder reduce his consumption of that article so as not to increase his expenditure ?

7. One-third of a population can read ; of the remainder 45 per cent. can read and write ; of what still remains 9 per cent. can read, write and count ; the rest is 500500 who can neither read, write nor count. Find the total population.

8. A merchant buys some cloth at such a price that by selling it at *Rs.*4. 6*a.* per yard he will gain 5 per cent. on his outlay. What percentage will he gain or lose if the cloth be sold at *Rs.*3. 14*a.* per yd. ?

9. In a company of 100 people, of whom some are rich and some poor, the rich subscribe and give 1*a.* 3*p.* to each poor man ; this costs the rich men 7*a.* 1*p.* each ; how many rich and how many poor men are there ?

10. In sending 1000 cheroots to England I paid freight $\frac{2}{3}$rds of their prime cost ; landing charges $\frac{1}{3}$rd of their cost, including freight ; and duty $2\frac{1}{2}$ times their cost including freight and landing charges. Altogether the cheroots, duty paid, in London cost me £7. What did I give for them at Calcutta ?

11. Divide 1800 into two parts such that 8 times the first added to 12 times the second may become 17600.

12. The average weight of students in a class of 30 boys is 100 lbs. If, however, the weight of the teacher be included, the average weight will increase by 1 lb. Find the weight of the teacher.

13. *A*'s salary is 40 per cent. more than *B*'s. How much per cent. is *B*'s less than *A*'s ?

14. In an examination, a candidate must get 40 per cent. marks to pass. A candidate who gets 210 marks fails by 40 marks. Find the maximum number of marks.

15. An oz. of gold is worth £4 and an oz. of silver is worth 8*s.*, and a mass of gold and silver weighing 80 oz. is worth £104. What is the worth of another mass, in which the number representing gold and silver in oz. in the first mass are interchanged ?

16. If 10 oz. of gold 13 carats fine, 14 oz. of gold 12 carats fine, 12 oz. of gold 11 carats fine, and 24 oz. of gold 10 carats fine be mixed together, find the fineness of the compound.

17. A person bought 400 chairs, some at *Rs.*4 each and others at *Rs.*3 each. The total price that he paid was *Rs.*1350. How many of each sort did he buy ?

18. 2 lbs. of tea and 5 lbs. of sugar cost 7*s.* 6*d.*; but, if sugar were to rise $33\frac{1}{3}$ per cent., and tea 20 per cent., they would cost 9*s.* 4*d.*; find the cost of the tea and the sugar per lb.

19. 12 lbs. of tea and 25 lbs. of coffee together cost *Rs.*43. 5*a.* 4*p.*; but, if the tea were to rise $2\frac{1}{2}$ per cent., and the coffee to fall $4\frac{1}{6}$ per cent., the same quantities would cost *Rs.*42. 15*a.* 4*p.*; find the price of the tea and the coffee per lb.

20. There are two compound metals, the one consisting of a mixture of copper and gold, and the other of a mixture of copper and silver. The value of an ounce of gold, silver and copper are £5, 5*s.* and 5*d* respectively. Find how much copper must be mixed with the first mixture in order that the value of a given quantity of the first mixture may be 15 times the value of an equal weight of the second mixture containing 80 per cent. of pure silver.

21. Gold costs £3. 17*s.* $10\frac{1}{2}$*d.* per oz. and silver 5*s.* 6*d.* per oz.; in what proportion must these metals be mixed that a lb. of the mixture may be worth £32. 5*s.*?

22. The external length, breadth and height of a wooden box are 18, 10 and 6 in. respectively, and the thickness of the wood is half an inch; when the box is empty it weighs 15 lbs. and when filled with sand 100 lbs. Compare the weights of equal bulks of sand and wood.

23. A well is fed by a spring which flows continuously and uniformly in it. When there are 10000 cub. ft. of water in the well, 7 men can empty it in 20 days; and when there are 15000 cub. ft. of water in the well, 5 men can empty it in 50 days. How many cub. ft. of water flow into the well in one day?

24. There are two fields whose lengths are 150 yds. and 244 yds. and the breadth of the second half as great again as that of the first; these are respectively dug by men and boys who each takes a strip of one given breadth. If the men be paid £49. 9*s.* 7*d.*, what will be due to the boys, the workmanship of each of these being $\frac{4}{5}$ths as good as a man's?

25. A merchant buys 5000 mds. of rice, one-fifth of which he sells at a profit of 5 per cent, one-fourth at a profit of 10 per cent. and the remainder at a profit of 16 per cent. If he had sold the whole at a profit of 15 per cent. he would have made *Rs.*438. 12*a.* more; what was the cost of the rice per maund?

26. A person buys 5 shares in a company, and sells three of them at a gain of 10 per cent. and the remaining two at a gain of $16\frac{2}{3}$ per cent. The gain on the latter sale is £2. 19*s.* $7\frac{1}{2}$*d.* more than on the former. Find the price of a single share.

27. Divide 320 nuts among three boys *A*, *B* and *C*, in such a way that as often as *A* receives 8, *B* shall receive 5, and for every 6 *B* receives, *C* shall receive 10.

28. A shareholder in a commercial company receives one year a dividend of 5 per cent. on his shares. The next year he receives a dividend of $7\frac{1}{2}$ per cent., and finds that he is *Rs.*412. 8*a.* richer. Find the amount of his shares.

29. A person bought 600 animals—horses and oxen—for *Rs.*40000; each horse cost *Rs.*80 and each ox *Rs.*60. Find the number of horses.

30. Divide *Rs.*50 between *A* and *B*, so that $\frac{3}{8}$ of *A*'s share may exceed $\frac{3}{4}$ of *B*'s share by *Rs.*5.

31. A sum of money is divided between *A* and *B* so that *A* gets 10 per cent. more than *B*. If *A*'s share exceed *B*'s by *Rs.*4, find the sum distributed and the share of each.

32. Divide *Rs.*240 among 8 men, 12 women and 16 boys, so that each man will receive *Rs.*2 more than each woman, and each woman *Rs.*2 more than each boy.

33. A certain number is divided into two parts, such that 5 times the first part added to 11 times the second makes 7 times the whole. Find the ratio of the parts.

34. A gentleman engaged a servant on the understanding that he was to receive a coat and *Rs.*60 after serving for 6 months. He served only for 4 months and received the coat and *Rs.*30. Find the price of the coat.

35. A boy buys a number of apples at 4 for 3*d.* and a third of the number at 5 for 4*d.* He sells the whole lot at 6 for 5*d.* How much does he gain per cent? If his total gain is 7*s.* 1*d.*, how many did he buy?

36. A market woman buys a certain number of mangoes at 20 a rupee, and an equal number at 25 a rupee. She mixes them together and sells them at 21 a rupee. What does she gain or lose per cent.?

37. *A* contributes a certain capital for 4 months, and *B* contributes *Rs.*400 for 5 months. If their profits be in the proportion of 3 : 4, what was the capital contributed by *A*?

38. The times for which *A* and *B* had contributed their capitals are in the proportion of 1 : 2, and the profits received by them are in the proportion of 3 : 4. Compare their capitals.

39. The sum of the ages of three men is 108 years. Twelve years before, their ages were in the proportion of 5 : 4 : 3; find their ages.

40. A lump composed of gold and silver measures 6 cub. in. and weighs 100 oz.: if a cubic inch of gold weighs 20 oz. and an equal bulk of silver 12 oz., find the weight of gold in the mixture.

41. In a certain school, 85 per cent. of the boys passed in English, 80 per cent. passed in Arithmetic, 5 per cent. failed in both subjects and 210 passed in both subjects. Find the number of boys in the school.

42. *A*, *B* and *C* rent a field for *Rs*.2878. *A* puts in 12 horses for 5 months and 45 sheep for 3 months; *B* puts in 15 oxen for 6 months and 54 sheep for 2 months; *C* puts in 6 horses and 48 oxen for 3 months. Now, 4 horses and 3 sheep together eat as much as 5 oxen and 1 horse, and 2 oxen eat as much as 7 sheep; how much of the rent should *A*, *B* and *C* pay respectively?

43. A mass of copper and brass weighs 11 seers, and is worth *Rs*.7. 2*a*. If the proportion of copper and brass be interchanged, the mass would be worth *Rs*.7. 5*a*. If 1 seer of copper and 2 seers of brass cost *Re*.1. 14*a*., find the cost of copper and brass per seer and the proportion of the mixture.

44. Divide the number 540 into two parts such that 25 times the greater may exceed 11 times the less by 15 times the whole number.

45. A person buys a certain number of animals for *Rs*.8000, consisting of horses at *Rs*.150 each and oxen at *Rs*.80 each. If he interchanges the numbers of horses and oxen, he requires *Rs*.1050 less. Find the number of horses and also that of oxen.

46. 8 oz. of gold 10 carats fine, and 2 oz. of gold 11 carats fine, are mixed with 6 oz. of gold of unknown fineness. If the fineness of the compound be 12 carats, find the fineness of the 6 oz. gold.

47. A person bought 60 apples, some at 3 for 2*ps*. and others at 4 for 3*ps*., and gave 10*a*. 3*ps*. in all. How many were bought at the first rate?

48 Divide 50 into two parts such that 4 times the first part added to 8 times the second may exceed 5 times the whole by 10

49. A person bought 36 fruits, consisting of mangoes and plantains, for 10*a*. 2*ps*. Had he bought as many plantains as he had bought mangoes, and as many mangoes as he had bought plantains, they would have cost him 12*a*. If the cost of one plantain and two mangoes be 4*ps*., how many of each sort did he buy?

50. The ages of *A*, *B* and *C* are in the proportion of 3 : 7 : 11; but 7 years hence the sum of their ages will be 105 years. Find their present ages.

51. Two passengers are charged for excess of luggage *Rs*.2. 10*a*. and *Rs*.5. 8*a*. respectively; had the luggage all belonged to one of them he would have been charged for excess *Rs*.8. 12*a*.: how much would they have been charged if none had been allowed free?

52. Two passengers have together 8 mds. 25 sr. of luggage and are charged for the excess *Rs*.3. 4*a*. and *Rs*.4. 6*a*. respectively. If all the luggage had belonged to one of them he would have been charged *Rs*.8. 2*a*. Find the amount of luggage owned by each.

53. A footman who contracted for *Rs*.80 a year and a livery suit, was turned away at the end of 7 months and received only *Rs*.21. 10*a*. 8*p*. and his livery. What was its value?

54. In a jail, there are 260 male criminals and 120 female criminals. The former in the course of a year increased 5 per cent., while the latter decreased 10 per cent. Find the increase or decrease per cent. in the whole number of criminals.

55. If 40 oxen consume the grass of a meadow growing uniformly in 12 days, and if 25 oxen consume the grass of the same meadow in 20 days, find in how many days 30 oxen will consume it.

56. A market woman bought 300 oranges at 3 for *2ps.*, and some more at 2 a pice. She mixed them together, and sold them at 4 for *3ps.* and gained 25 per cent. How many of the second sort did she buy ?

57. A sum of money is distributed among *A*, *B*, *C* and *D* in the proportion of 1, 2, 3 and 4 so that *D* gets *Rs.*20 more than *C*. Find the whole sum distributed and the share of each.

58. A sum of money is divided among *A*, *B* and *C*, so that *A* has 20 per cent. more than *B*, and *B* has 25 per cent. more than *C*. If *A*'s share be *Rs.*50, find the whole sum distributed and the share of each.

59. Two casks, *A* and *B* are filled with different mixtures of wine and water ; in the cask *A* the mixture is in the proportion of 2 : 5 and in the cask *B* it is in the proportion of 1 : 3. What quantities must be taken from the two casks to have a new mixture consisting of 8 gallons of wine and 21 gallons of water ?

60. One vessel contains a mixture of milk and water in the proportion of 3 : 5, in another they are mixed in the proportion of 6 : 1. In what proportion should quantities be taken from the two vessels so as to form a mixture in which milk and water will be in the proportion of 7 : 3 ?

CHAPTER XIII.

Interest, Present Worth and Discount.

516. Interest is the payment made for the *use* of money lent for any length of time at a fixed rate.

The money lent is called the **Principal.** The sum lent or principal together with its interest is termed the **Amount.** The **Rate** is the money paid for the use of a certain sum for a certain time. The interest of *Rs* 100 or £100 for a year is called the **rate per cent. per annum** (*per annum* means *for a year*). Unless otherwise stated, by rate per cent. is always to be understood the rate of interest *per annum.*

Thus, if a man borrows a sum of money on the condition that for the use of every *Rs.*100 in the loan for one year he shall pay

an interest of *Rs*.8, he is said to borrow *at the rate of 8 per cent. per annum*. Again, if the sum of money borrowed be *Rs*.900, and the interest upon it in a certain time is *Rs* 70, *Rs*.900 is called the *Principal*, *Rs*.70 the *Interest* and *Rs* (900+70) or *Rs*.970, the *Amount*.

517. Interest is of two kinds, **Simple and Compound.**

It is called *Simple Interest*, when the money advanced only pays interest for the whole time it is lent ; and *Compound Interest*, when, at the end of any *assigned* period, as a year for instance, the interest which has accrued is added to the principal, and the whole then bears interest at the same rate for another *equal* period, and so on.

SIMPLE INTEREST.

518. *To find the Simple Interest on a given sum of money for a given time, when the rate of interest per Re. or £ for one month is given.*

RULE. Multiply together the principal, the given time, and the rate of interest; the product will give the required interest in the same denomination as the given rate of interest. Or apply *Rule of Three*.

Ex. Find the Simple Interest on *Rs*.76 for 9 mo. at $\frac{1}{2}a$. per *Re*.a mo.

The reqd. int. $= 76 \times 9 \times \frac{1}{2}a$.
$= 342a$.
$= Rs.21.\ 6a$.
or, 1 : 76 :: $\frac{9}{2}a$. : reqd. int.
∵ $\frac{9}{2}a$. = int. on *Re*.1 for 9 mo.

Since the interest on *Re*.1 for 1 mo. is $\frac{1}{2}a$., ∴ the interest on *Rs*.76 for 1 mo. is $76 \times \frac{1}{2}a$., and ∴ the interest on *Rs*.76 for 9 mo. is $76 \times 9 \times \frac{1}{2}a$. or $342a. =$ *Rs*.21. 6*a*.

Examples CLVII.

1. Find the Simple Interest on :—

(1) *Rs*.58 for 6 months at 6*p*. per rupee per month.
(2) ... 86 ... 8 ... ... 2*ps*. ... ...
(3) ... 465 ... 9 ... ... $\frac{1}{2}a$. ... ...
(4) ... 370 ... 18 ... ... 3*p*. ... ...
(5) ... 860 ... 2½ years ... $\frac{3}{4}a$. ... ...
(6) ... 550 ... 2 years 4 mo. 6*p*. ... ...
(7) £735 ... 14 months... $\frac{3}{8}d$. per pound per month.
(8) ... 975 ... 4½ years ... $\frac{1}{2}d$. per pound ... ...

2. If a man gets 3*d*. for the loan of 2*s*. 6*d*. for 1 year, what is the rate per cent. per annum ?

3. What is the rate of interest per cent. per annum, if I pay *Rs*.25. 5*a*. for the use of *Rs*.3000 for 1 month ?

4. A man gets 4*a*. for the loan of *Rs*.2. 8*a*. for 2 months ; find the rate of interest per cent. per annum.

5. If 1*a.* per week be paid for the loan of *Rs.*6. 8*a.*, what is the rate per cent. per annum ?

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519. *To find the Interest on a given sum of money at a given rate per cent. for a given time.*

RULE. Multiply the principal by the rate per cent., the product by the time in years, and divide the result by 100. Or apply *Rule of Three.*

Ex. Find the Simple Interest on *Rs.*2500 for 4 years at 5 per cent.

The reqd. int. $= Rs. \frac{2500 \times 5 \times 4}{100}$	Since the int. on *Rs.*100 for 1 yr. is *Rs.*5.
$= Rs.500.$	∴ .... *Re.*1... . ... $Re. \frac{5}{100}$;
or 100 : 2500	∴ .... *Rs.*2500........... $Rs. \frac{2500 \times 5}{100}$
:: *Rs.*20 : reqd. int.	∴ ....*Rs.*2500 for 4 yrs. is $Rs. \frac{2500 \times 5 \times 4}{100}$
	or *Rs.*500

520. In practice, however, we adopt the following *forms* :—

Ex. Find the Simple Interest on £240. 12*s.* 6*d.* at 2½ per cent. for 8¾ years, and also the Amount.

(i)	(ii) $2\frac{1}{2} \times 8\frac{3}{4} \div 100 = 7 \div (8 \times 4)$
£. s. d.	£. s. d.
240 12 6 Principal	240 12 6
2½ rate p. c.	7
481 5 0	8 | 1684 7 6
120 6 3	4 | 210 10 11¼
601 11 3	£52. 12*s.* 8¾*d.* ¼*q.*
8¾ no. of years.	
4812 10 0	
451 3 5¼	∴ interest = £52. 12*s.* 8¾*d.* ¼*q.*
£52·63 13 5¼	
20	and principal = £240. 12*s.* 6*d.*
s. 12·73	
12	
d. 8·81	∴ the amount = £293. 5*s.* 2¾*d.* ¼*q.* *Ans.*
4	
q. 3·25	

521. When the interest for *months* and *days* is required, it is found by *Practice* and the *Rule of Three* respectively, reckoning 12 months and 365 days to a year, and 30 days to a month ; but if *calendar* months be specified, and the interest has to be calculated from one given day to another, it is customary to include *one only* of the days named in counting the period of time.

Ex. 1. Find to the *nearest penny* the Simple Interest on £512.

16*s*. 8*d*. at $4\frac{5}{8}$ per cent. for 3 years 7 months 21 days ; and also the amount.

		£.	s.	d.
		512	16	8
				$4\frac{5}{8}$
		2051	6	8
		320	10	5
6 mo.	$\frac{1}{2}$	2371	17	1
				3
		7115	11	3
1 mo.	$\frac{1}{6}$	1185	18	$6\frac{1}{2}$
15 days.	$\frac{1}{2}$	197	15	$1\frac{1}{12}$
6 days.	$\frac{1}{5}$	98	16	$6\frac{13}{24}$
		39	10	$7\frac{5}{12}$
		£86·37	10	$0\frac{13}{24}$
		20		
		s. 7·50		
		12		
		d. 6·00		

£512. 16*s*. 8*d*. × $\frac{5}{8}$

		5
8)2564	3	4
£320	10	5

∴ Interest = £86. 7*s*. 6*d*. *nearly*

and principal = £512. 16*s*. 8*d*.

∴ Amount = £599. 4*s*. 2*d*. *Ans*.

Ex. 2. Find the amount of *Rs*.13765. 10*a*. at $4\frac{3}{4}$ per cent. from April 6th to August 30th.

Rs.	a.	p.
13765	10	0
		$4\frac{3}{4}$
55062	8	0
10324	3	6
Rs.653·86	11	6
16		
a. 13·87		
12		
p. 10·50		

Rs.	a.	p.
13765	10	0 × $\frac{3}{4}$
		3
4)41296	14	0
Rs.10324	3	6

∴ Interest for 1 year = *Rs*.653. 13*a*. 10·5*p*.
Now, the number of days from April 6th to August 30th = 24 + 31 + 30 + 31 + 30 = 146.

∴ 365 days : 146 days :: *Rs*.653. 13*a*. 10·5*p*. : int. reqd.

or 5 : 2 :: *Rs*.653. 13*a*. 10·5*p*. : int. reqd.

∴ int. reqd. = $\frac{2}{5}$ of *Rs*.653. 13*a*. 10·5*p*. = *Rs*.261. 8*a*. 9*p*.

∴ Amount reqd. = *Rs*.13765. 10*a*. + *Rs*.261. 8*a*. 9*p*. = *Rs*.14027. 2*a*. 9*p*.

Examples CLVIII.

1. Find the simple interest for one year on :—

(1) *Rs*.6400 at $4\frac{1}{4}$ per cent.

(2) „984. 12*a*. at 5 per cent

(3) „1762. 4*a*. at 5 per cent.

(4) *Rs*.7150. 13*a*. 4*p*. at $2\frac{3}{8}$ per cent.

(5) „578. 8*a*. at 5 per cent.

(6) £2055 at $5\frac{3}{4}$ per cent.

(7) £3197. 5*s.* at 5 per cent (9) £2814. 7*s.* 6*d.* at 5 per cent.
(8) ... 572 at $4\frac{1}{4}$ per cent. (10) ...55. 16*s.* 8*d.* at $3\frac{7}{8}$ per cent.

2. Find the Simple Interest on :—

(1) *Rs.*525 for 5 yrs. at $3\frac{1}{2}$ p. c. (7) £650 for 8 yrs. at $4\frac{1}{4}$ p. c.
(2) ...9513. 12*a.* for $2\frac{1}{4}$ yrs. at $6\frac{3}{4}$%. (8) ...450. 10*s.* for 7 yrs. at $3\frac{1}{4}$%
(3) ...3406. 4*a.* for $4\frac{3}{4}$ yrs. at 4 p. c. (9) ...237. 10*s.* for 15 mo. at 4%
(4) 12500 for 18 mo. at $4\frac{3}{4}$ p. c. (10) ...476. 18*s.* 6*d.* for $4\frac{3}{4}$ yrs. at $3\frac{1}{8}$%
(5) 4336. 10*a.* 8*p* for 219 dys. at $4\frac{1}{2}$% (11) ...3450. 12*s.* 7*d.* for $8\frac{1}{2}$ yrs. at $4\frac{5}{8}$%
(6) ...5438. 12*a.* for $2\frac{1}{4}$ yrs. at $3\frac{7}{8}$% (12) ...1923. 15*s.* for 2 yrs. 8 mo. at $5\frac{1}{8}$%

3. Find to the *nearest pie* or *penny* the Simple Interest on :—

(1) *Rs.*5438. 12*a.* for 43 days at $3\frac{1}{2}$ per cent.
(2) ...568. 5*a.* for 3 yrs. 143 days at $5\frac{3}{8}$ per cent.
(3) £317. 10*s.* $2\frac{3}{4}$*d.* for 3 yrs. 73 days. at $3\frac{5}{8}$ per cent.
(4) ...550. 14*s.* 8*d.* for 2 yrs. 9 mo. 25 days at $4\frac{3}{4}$ per cent.
(5) *Rs.*52605. 1*a.* 4*p.* for 6 yrs. 5 mo. 21 days at $2\frac{7}{8}$ per cent.
(6) £460. 3*s.* 6*d.* for 3 yrs. $8\frac{3}{4}$ mo. at $4\frac{3}{4}$ per cent.
(7) *Rs.*1841. 14*a.* from July 17th to Dec. 5th at $5\frac{1}{2}$ per cent.
(8) ...48655. 11*a.* 4*p.* from Jan. 1st to Aug. 28th 1876, at $5\frac{3}{8}$ per cent.
(9) £473. 3*s.* 6*d.* from April 14th to July 6th at $3\frac{1}{2}$ per cent.
(10) ...164. 15*s.* 11*d.* from 9 Nov. 1867 to 3 Mar. 1868, at $5\frac{3}{4}$ p. c.

4. Find the amount of :—

(1) *Rs.*5378. 5*a.* 4*p.* for 4 yrs. at $2\frac{1}{3}$% (2) *Rs.*3458. 12*a.* for 3 yrs. at 4%.
(3) £825. 13*s.* 8*d.* for 3 yrs. 5 mo. at $4\frac{3}{4}$ per cent.
(4) *Rs.*2375 for 2 yrs. 8 mo. 29 days at 5 per cent.
(5) ...18354. 2*a.* 8*p*, from March 11th to Aug. 4th at $4\frac{3}{4}$ per cent.

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522. Inverse Questions on Simple Interest.

Every question in Simple Interest involves the consideration of **Principal, Rate per cent, Time** and **Interest** or **Amount**; and three of these quantities are always given, to find the fourth. There are then *four* cases according as the quantity to be found is (1) *Interest* or *Amount*; (2) *Principal*; (3) *Rate per cent.*; (4) *Time.*

The RULES for finding the Interest or amount have already been explained. The next three cases will be considered in order.

Case II. *Having given the interest or amount, rate per cent. and time, to find the principal.*

(i) Let the *interest* be given.

RULE. Find the interest of *Rs.*100 or £100 at the given rate per cent. for the given time ; then, state thus :—

this interest : given interest :: *Rs.*100 or £100 : principal reqd.

Ex. What principal will produce *Rs.*438. 6*a.* interest in 5 years at 3½ per cent.

*Rs.*100 produce, in 5 yrs. at 3½ p. c., $Rs.5 \times 3\frac{1}{2} = Rs.17\frac{1}{2}$ interest.
∴ $Rs.17\frac{1}{2} : Rs.438\frac{3}{8} :: Rs.100$: principal reqd.

$$\therefore \text{ required principal} = Rs.\frac{100 \times 3507 \times 2}{8 \times 35} = \underline{Rs.2505.} \quad \textit{Ans.}$$

(ii) Let the *amount* be given.

RULE. Find the amount of *Rs.*100 or £100 at the given rate per cent. for the given time ; then, state thus :—

this amount : given amount :: *Rs.*100 or £100 : principal reqd.

Ex. What sum will amount to *Rs.*4053. 7*a.* in 4¾ yrs. at 4 p.c. ?

*Rs.*100 in 4¾ yrs. at 4 p. c. amounts to $Rs.(100 + 4\frac{3}{4} \times 4)$ or *Rs.*119.
∴ $Rs.119 : Rs.4053\frac{7}{16} :: Rs.100$: principal reqd.

$$\therefore \text{ reqd. principal} = Rs.\frac{100 \times 64855}{16 \times 119} = \underline{Rs.3406.\ 4a.} \quad \textit{Ans.}$$

Case III. *Having given the principal, time, interest or amount, to find the rate per cent.*

RULE. Find the interest on the given principal for the given time at 1 per cent. ; then, state thus :—
this interest : given interest :: 1 : rate per cent. reqd.

that is,— rate per cent. required is found *by dividing the given interest by the interest at 1 per cent.*

Ex. 1. At what rate per cent. will £33. 6*s.* 8*d.* amount to £38. 4*s.* 2*d.* in 4½ years ?

Given Int. = £38. 4*s.* 2*d.* — £33. 6*s.* 8*d.* = £4. 17*s.* 6*d.*

Int. on £33. 6*s.* 8*d.* at 1 p. c. for 4½ yrs. = £1. 10*s.*
∴ rate per cent. reqd. = £4. 17*s.* 6*d.* ÷ £1. 10*s.* = $\underline{3\frac{1}{4}}$. *Ans.*

Ex. 2. At what rate per cent. will a sum of money double itself in 12½ years ?

In 12½ years the interest is equal to the principal,
∴ interest on £100 for 12½ years = £100 ;
but interest on £100 at 1 per cent. for 12½ years = £12½,
∴ rate per cent. reqd. = £100 ÷ £12½ = $\underline{8}$. *Ans.*

Case IV. *Having given the principal, rate per cent., and interest or amount, to find the time.*

RULE. Find the interest on the given principal for one year ; then, state thus :—
one year's int. : given int. :: 1 : no. of years reqd.

that is,—the number of years is found *by dividing the given interest by the interest for 1 year.*

Ex. 1. In what time will *Rs.*4250 amount to *Rs.*6353. 12*a.* at $5\frac{1}{2}$ per cent.?

Given interest $=$ *Rs.*6353. 12*a.* $-$ *Rs.*4250 $=$ *Rs.*2103. 12*a.*

Int. on *Rs.*4250 for 1 year $=$ *Rs.*$4250 \times 5\frac{1}{2} \div 100$.

$$\therefore \text{ no. of years reqd.} = \frac{Rs.2103\frac{3}{4} \times 100}{Rs.4250 \times 5\frac{1}{2}} = 9. \quad Ans.$$

Ex. 2. In what time will a sum of money treble itself at 8 p.c.?

The time will be the same whatever sum of money be taken as the principal; suppose, the principal to be *Rs.*100,

$\therefore$ given interest $= 2 \times$ principal $=$ *Rs.*200,

and interest on *Rs.*100 for 1 year $=$ *Rs.*8.

$\therefore$ no. of years required $=$ *Rs.*200 $\div$ *Rs.*8 $= 25$. *Ans.*

523. The following *formulæ*, if committed to memory, will be of great use.

If P, I, M, n, r, be respectively the Principal, Interest, Amount, time and rate,

$$I = \frac{Pnr}{100}\ ;\ M = P + I = P + \frac{Pnr}{100} = P\left(1 + \frac{nr}{100}\right)\ ;\ P = \frac{100 \times I}{nr} = \frac{100 \times M}{100 + nr}\ ;$$

$$r = \frac{(M - P) \times 100}{Pn} = \frac{100 \times I}{Pn}\ ;\ n = \frac{(M - P) \times 100}{Pr} = \frac{100 \times I}{Pr}.$$

Examples CLIX.

1. What principal will produce *Rs.*366. 4*a.* interest in $2\frac{1}{2}$ years at $3\frac{1}{3}$ per cent.?

2. What principal must be put out for $2\frac{1}{2}$ years at 4 per cent. to amount to *Rs.*1325. 8*a.*?

3. What sum will amount to *Rs.*3761. 14*a.* in $3\frac{1}{4}$ years at $4\frac{1}{2}$ per cent. per annum?

4. At what rate per cent. will *Rs.*1368. 12*a.* amount to *Rs.*1642. 8*a.* in $6\frac{1}{4}$ years?

5. At what rate per cent. will the interest on *Rs.*357. 8*a.* amount to *Rs.*40. 3*a.* 6*p.* in $4\frac{1}{2}$ years?

6. At what rate per cent. will the interest on 500 guineas amount to £103. 9*s.* $4\frac{1}{2}$*d.* in 3 years 7 months?

7. What principal will amount to *Rs.*13577. 2*a.* in 2 years 7 months at $4\frac{1}{2}$ per cent.?

8. What principal will amount to £725. 12*s.* 6*d.* in 2 years 9 months 18 days at $2\frac{1}{2}$ per cent.?

9. Find the principal whose interest amounts to *Rs.*578. 5*a.* 4*p.* in 1 year 9 months 24 days at $3\frac{3}{8}$ per cent.

10. In how many years will the interest on *Rs.*357. 8*a.* amoun to *Rs.*40. 3*a.* 6*p.* at $2\frac{1}{2}$ per cent.?

11. In what time will £563. 13*s*. $4\frac{1}{2}$*d*. amount to £901. 17*s*. $4\frac{4}{5}$*d*. at $3\frac{3}{4}$ per cent. ?

12. In what time will the interest on *Rs.*8125. 6*a*. 8*p*. amount to *Rs.*7719. 2*a*. 4*p*. at $4\frac{3}{4}$ per cent. ?

13. At what rate per cent. will £7433. 6*s*. 8*d*. amount to £9942. 1*s*. 8*d*. in $7\frac{1}{2}$ years ?

14. The interest of a sum of money at the end of $6\frac{1}{4}$ years is $\frac{3}{8}$ths of the sum itself ; what rate per cent. was charged ?

15. In how many years will a sum of money amount to half as much again as itself at $7\frac{1}{2}$ per cent. ?

16. If *Rs.*79. 12*a*. be charged for the loan of *Rs.*7435 for 87 days ; what is the rate per cent. ?

17. In how many years will a sum of money double itself at $6\frac{1}{4}$ per cent. per annum ?

18. At what rate per cent. will the interest on *Rs.*13687. 8*a*. become *Rs.*142. 5*a*. from July 5th to Nov. 20th ?

19. What sum of money laid out at 4 per cent. will give 2*a*. interest a day ?

20. What principal in 15 years at 4 per cent. will amount to the same sum as *Rs.*45000 in 9 years at 6 per cent. ?

21. In what time will *Rs.*10755 amount to *Rs.*15594. 12*a*., if in 4 years *Rs.*1762. 8*a*. amount to *Rs.*1974 ?

22. At what rate will *Rs.*500 amount to *Rs.*700 in a time in which £120 produces £15 at 4 per cent. Simple Interest ?

23. In what time will £140. 12*s*. 6*d*. amount to £175. 10*s*. at a rate of interest at which £638 doubles itself in 25 years ?

24. What sum will amount to £678. 8*s*. in $1\frac{1}{3}$ years at a rate in which £502. 13*s*. 4*d*. amounts to £578. 1*s*. 4*d*. in $3\frac{1}{3}$ years ?

25. What will *Rs.*4906. 4*a*. amount to in 8 years, the rate of interest being that at which *Rs.*50000 amount to *Rs.*54020 in 1 year 219 days ?

26. What is the rate of Simple Interest, if in 8 years the amount of £425 becomes the same as the amount of £502. 18*s*. 4*d*. in $3\frac{3}{4}$ years at $5\frac{1}{3}$ per cent. ?

27. What will *Rs.*18375 amount to at $4\frac{1}{2}$ per cent. in a time in which £1033. 6*s*. 8*d*. amounts to £1103. 1*s*. 8*d*. at $2\frac{1}{4}$ per cent. ?

28. A certain sum amounted to *Rs.*3666. 10*a*. 8*p*. at 4 per cent. Simple Interest in a time in which *Rs.*9120 amounted to *Rs.*10488 at 6 per cent. What was the sum ?

29. What sum at $3\frac{1}{2}$ per cent. will give a guinea interest per day ?

30. The sum of *Rs.*3270 is borrowed at the beginning of the year at a certain rate of interest and after 9 months *Rs.*4000 more is borrowed at double the previous rate. At the end of the year the

interest on both loans is *Rs*.131. 12*a*. What is the rate of interest a which the first sum was borrowed?

31. *A* lends *Rs*.300 to *B* for 2 years, and *Rs*.75 to *C* for 4 years, and receives altogether from both *Rs*.60 as interest. Find the rate of interest, Simple Interest being calculated.

32. The interest on *Rs*.800 at 4 per cent. for a certain time, and that on *Rs*.1000 for 2 years more at 5 per cent. are together *Rs*.346. For what periods are the interests calculated?

33. *A* received from *B* *Rs*.500 on the 8th of April, at 10 per cent. per annum, Simple Interest. Finding, however, that it was a bad debt, *B* agreed to accept *Rs*.480 on the 1st of September of the same year in repayment of the debt. How much did *B* lose?

34. *A* lends *Rs*.500 to *B*, and a certain sum to *C*, at the same time, at 8 per cent. Simple Interest. If in 4 years he altogether receives *Rs*.210 as interest from the two, find the sum lent to *C*.

35. *A* lends a certain sum to *B*, and a sum larger than the first by *Rs*.800 to *C*; *B* agrees to pay 5 per cent. interest and *C* 7 per cent. Both return the sums with interests at the end of 5 years. If *C*'s amount exceeds *B*'s by *Rs*.1240, what sum did each borrow?

II. COMPOUND INTEREST.

524. In Compound Interest, the interest of each period is added to its principal and the amount forms a new principal for the next period. The period is always understood to be a year, unless the contrary is stated.

525. *To find Compound Interest on a given sum of money at a given rate per cent. for any number of years.*

RULE. At the end of each year add the interest of that year, found by (Art. 519), to the principal at the beginning of it; this will be the principal for the next year, and continue the process in the same way as far as may be required by the question. Add together the interests so arising in the several years, and the result will be the compound interest for the given period.

Ex. 1. Find the Compound Interest on *Rs*.4508. 6*a*. for 3 years at 4¼ per cent.

	Rs.	*a.*	*p.*
1st Principal	4508	6	0
			4¼
	18033	8	0
	1127	1	6
	Rs.191·60	9	6
	16		
	a.9·69		
	12		
	p.8·34		

	Rs.	*a.*	*p.*
principal	4508	6	0
1st year's int.	191	9	8·34

	Rs.	*a.*	*p.*		*Rs.*	*a.*	*p.*
2nd Principal	4699	15	8·34	2nd year's int.	199	11	11·84445
			$4\frac{1}{4}$				
	18799	14	9·36				
	1174	15	11·085				
	*Rs.*199·74	14	8·445				
	16						
	a. 11·98						
	12						
	p. 11·84445						

	Rs.	*a.*	*p.*		*Rs.*	*a.*	*p.*
3rd Principal	4899	11	8·18445	3rd year's int.	208	3	9·797839125
			$4\frac{1}{4}$				
	19598	14	8·7378				
	1224	14	11·0461125				
	*Rs.*208·23	13	7·7839125				
	16						
	a. 3·81			∴ Total int. *Rs.*599. 9*a.* 6*p.* *nearly.*			
	12						
	p. 9·797839125						

Or thus, by Decimals.—It is not really necessary to take more than *four* decimal places in the calculation.

Now, *Rs.*4508. 6*a.* = *Rs.*4508·375, and $4\frac{1}{4}$ per cent. $=\frac{4}{100}+\frac{1}{400}$.

4 p. c. $=\frac{4}{100}$	*Rs.*4508·3750	1st Principal.
$\frac{1}{4}$ p. c. $=\frac{1}{400}$	180·3350	} 1st year's interest.
	11·2709	
	4699·9809	2nd Principal.
	187·9992	} 2nd year's interest.
	11·7500	
	4899·7301	3rd Principal.
	195·9892	} 3rd year's interest.
	12·2493	
	*Rs.*5107·9686	Amount.
	4508·375	1st Principal.

Interest reqd. *Rs.*599·5936 = *Rs.*599. 9*a.* 6*p.* nearly.

526. When the Compound Interest is required for any number of **entire** years and a **part** of a year, it is done either by considering the fractional part as a new period and finding the interest at a rate equal to the same fraction of the given rate or by finding the interest of the next full period and taking the required part of it.

Ex. Find the amount of £45. 12*s.* 6*d.* for $3\frac{1}{2}$ years at $3\frac{1}{2}$ per cent., Compound Interest.

Now, £45. 12*s.* 6*d.* = £45·625, and $\frac{1}{2}$ of $3\frac{1}{2}$ p. c. = $1\frac{3}{4}$ p. c.

(i)	3 p. c. = $\frac{3}{100}$	£45·6250	Principal.
		1·3688	1st year's interest.
	$\frac{1}{2}$ p. c. = $\frac{1}{200}$	·2281	
		£47·2219	Amount in 1 year.
		1·4166	2nd year's interest.
		·2361	
		£48·8746	Amount in 2 years.
		1·4662	3rd year's interest.
		·2443	
	1 p. c. = $\frac{1}{100}$	£50·5851	Amount in 3 years.
	$\frac{1}{2}$ p. c. = $\frac{1}{200}$	·5058	last $\frac{1}{2}$ year's interest.
	$\frac{1}{4}$ p. c. = $\frac{1}{400}$	·2529	
		·1264	
		£51·4702	Amount in $3\frac{1}{2}$ years.
		= £51. 9*s*. $4\frac{7}{8}$*d*.	***Ans.***
		£50·5851	Amount in 3 years.
		1·5176	4th year's interest.
		·2529	
		£52·3556	Amount in 4 years.
		·8852	last $\frac{1}{2}$ year's interest.
		£51·4704	Amount in $3\frac{1}{2}$ years.

527. If the interest be payable **half-yearly**, the result may be obtained by finding the interest for double the number of years at half the given rate per cent, or if **quarterly**, by finding the interest for four times the number of years at one-fourth of the given rate per cent.

Ex. Find the amount at Compound Interest on *Rs.*871. 12*a.* for $1\frac{1}{2}$ years at $4\frac{3}{4}$ per cent. per annum, payable half-yearly.

Here, there are 3 periods of half-a-year each, and the rate per cent. *per period* is $\frac{1}{2}$ of $4\frac{3}{4}$ or $2\frac{3}{8}$.

2 p. c. = $\frac{2}{100}$ *Rs.*871. 12*a.* = *Rs.*871·7500		Principal.
$\frac{1}{4}$ p. c. = $\frac{1}{400}$	17·4350	int. for 1st period.
$\frac{1}{8}$ p. c. = $\frac{1}{800}$	2·1794	
	1·0897	
	*Rs.*892·4541	Amount in 1st period.
	17·8491	int. for 2nd period
	2·2311	
	1·1156	
	*Rs.*913·6499	Amount in 2 periods.
	18·2730	int. for 3rd period.
	2·2841	
	1·1421	
	*Rs.*935·3491	Amount in 3 periods. or $1\frac{1}{2}$ years.

= *Rs.*935. 5*a.* 7*p.* *nearly.* ***Ans.***

528. The following method of finding the amount at compound interest may be used with advantage.

Amount of *Rs.*100 at the end of *one* year $=$ *Rs.*104 ; (rate $=4$ p. c.)

∴..............*Re.*1.......................................$=$ *Rs.*1·04 ;

∴.............any sum ....$=$ 1·04 of the sum

∴Amt. of any sum at the end of 1st year $=$ 1·04 of that sum

.... 2nd......$= 1\cdot04 \times 1\cdot04 = (1\cdot04)^2$ ....

.... 3rd......$= (1\cdot04)^2 \times 1\cdot04 = (1\cdot04)^3$ ......

and so on.

Hence, if P represent the principal, M the amount, r the rate per cent., and n the number of years, then $M = P \times \left(1 + \frac{r}{100}\right)^n$.

Ex. Find the Compound Interest on *Rs.*2000 in $2\frac{1}{4}$ years at 4 p.c.

4 per cent. for $\frac{1}{4}$ year $=$ 1 per cent. for 1 year.

Amt. at the end of 2 years $=$ *Rs.*$2000 \times (1\cdot04)^2$

.... $2\frac{1}{4}$ years $=$ *Rs.*$2000 \times (1\cdot04)^2 \times (1\cdot01) =$ *Rs.*2148·832.

∴ reqd. interest $=$ *Rs.*2184·832 $-$ *Rs.*2000 $=$ *Rs.*184·832

$=$ *Rs.*184. 13*a.* 4*p. nearly.*

Examples CLX.

1. Find the Compound Interest, to the nearest *pie* or *penny*, on:—

(1) *Rs.*3252 for 3 yrs. at 4 p. c. (2) *Rs.*2250 for 3 yrs. at $3\frac{3}{8}$ p. c.

(3) *Rs.*11500 for $2\frac{1}{2}$ yrs. at 5 p. c. (4) *Rs.*$16277\frac{3}{4}$ for 3 yrs. at $6\frac{5}{8}$ p.c.

(5) ...$975\frac{1}{4}$ for 4 yrs. at $4\frac{1}{8}$ p. c. (6) £8000 in 3 yrs. at $3\frac{3}{4}$ p. c.

(7) £7853. 16*s.* 8*d.* for 3 years at 5 per cent.

(8) £2554. 12*s.* 9*d.* for 3 years at $6\frac{3}{4}$ per cent.

(9) *Rs.*15639. 13*a.* 4*p.* for $2\frac{3}{4}$ years at $3\frac{1}{8}$ per cent.

2. Find the Compound Interest, (payable half-yearly) to the nearest *pie* or *penny*, on :—

(1) *Rs.*7500 for $1\frac{1}{2}$ yrs. at 4 p. c. (2) *Rs.*1867. 6*a.* for $2\frac{1}{2}$ yrs. at 6 p.c.

(3) *Rs.*8501. 4*a.* 6*p.* for 3 yrs. at $4\frac{3}{4}$ p. c.

(4) £550. 10*s.* for $1\frac{1}{2}$ years at 4 p. c.

3. Find the Compound Interest, (payable quarterly) to the nearest *pie* or *penny*, on :—

(1) *Rs.*4000 for $1\frac{1}{4}$ yrs. at 4 p. c. (2) *Rs.*8750 for 15 mo. at 4 p. c.

(3) *Rs.*6469. 1*a.* 8*p.* for $1\frac{3}{4}$ yrs. at 8 p. c.

4. Find the Amount at Compound Interest of :—

(1) *Rs.*7205 in 3 yrs. at 4 p. c. (2) *Rs.*2880 in 3 yrs. at $3\frac{1}{2}$ p. c.

(3) *Rs.*6500 in $3\frac{1}{4}$ yrs. at 3 p. c. (4) £4321. 10*s.* in $2\frac{3}{4}$ yrs. at $6\frac{1}{4}$ p.c.

(5) *Rs.*17605 in $2\frac{1}{2}$ yrs. at 4 p. c. (6) £2533. 6*s.* 8*d.* in $2\frac{1}{4}$ yrs. at 3 p.c.

5. Find the difference between the Simple and the Compound Interest on *Rs.*17505 for 3 years at $5\frac{1}{4}$ per cent.

6. Find the difference between the Simple and the Compound Interest on £3333. 6*s.* 8*d.* for $3\frac{1}{2}$ years at $3\frac{1}{2}$ per cent.

7. The population of a city is 765240 and its annual increase is at the rate of 2·7 per cent. ; what will be the number of its inhabitants at the end of 5 years ?

8. A person at the beginning of each year lays aside *Rs.*2800, and employs the money at $3\frac{1}{2}$ per cent. Compound Interest ; how much will he be worth at the end of 5 years ?

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529. Inverse Questions on Compound Interest.

Case I. *Having given the amount or interest, time and rate per cent., to find the principal.*

RULE. Find the amount or interest of *Re.*1 or £1 for the given time, and then state thus :—

$$\left.\begin{matrix}\text{amount or interest}\\ \text{of } Re.1 \text{ or } £1.\end{matrix}\right\} : \left\{\begin{matrix}\text{given amount}\\ \text{or interest}\end{matrix}\right\} :: Re.1 \text{ or } £1 : \text{principal reqd.}$$

that is,——the principal is found *by dividing the given amount by the amount of* 1 *or the given interest by the interest on 1.*

Ex. What sum of money will amount to *Rs.*4134. 6*a.* in 2 years at 5 per cent. per annum, Compound Interest ?

Amount of *Re.*1 for 2 years at 5 p. c. $= Rs.(1·05)^2 = Rs.1·1025$,
and *Rs.*4134. 6*a.* = *Rs.*4134·375.

∴ Principal reqd. $= Rs.(4134·375 \div 1·1025) =$ <u>*Rs.*3750.</u> *Ans.*

Case II. *Having given the principal, amount and time, to find the rate per cent.*

RULE. Divide the given amount by the given principal, and then extract that root of the quotient which is denoted by the number of years ; this will be the amount of *Re.*1 or £1 for 1 year ; whence the rate per cent. will be known.

Ex. At what rate per cent. Compound Interest, will *Rs.*3750 produce as interest *Rs* 1740. 6*a.* in 4 years ?

Amount = *Rs.*3750 + *Rs.*1740. 6*a.* = *Rs.*5490. 6*a.* = *Rs.*5490·375 and *Rs.*5490·375 ÷ *Rs.*3750 = 1·4641 ; also $\sqrt[4]{(1·4641)} = 1·1$.

∴ the amount of *Re.*1 for 1 year = 1·1 ;

∴ the interest of *Re.*1 for 1 year = ·1, and ∴ rate = <u>10</u> per cent.

Note. It is obvious that this method can be adopted only when the number of years is some *power* of 2, or of 3, or the product of some *power* of 2 and some *power* of 3.

Case III. *Having given the principal, amount and rate per cent., to find the time.*

RULE. Divide the amount of *Re.*1 or £1 in the required number of years, (which may be obtained by dividing the amount by the principal), by the amount of *Re.*1 or £1 for 1 year. If the quotient be greater than the divisor, divide it by that divisor, and repeat the operation till a quotient smaller than the divisor is obtained. If the last quotient be 1, the number of times the operation of division is performed will represent the number of years; but if not, add to this the time in which *Re.*1 or £1 rises to the quantity represented by the last quotient. This will give the required number of years.

Ex. 1. The amount of *Rs.*10000 put out at Compound Interest for a certain number of years at 4 per cent. per annum is *Rs.*11248. 10*a.* 2·88*p.*; find the time.

*Rs.*11248. 10*a.* 2·88*p.* = *Rs.*11248·64.

∴ Amt. of *Re.*1 in the reqd. no. of years = *Rs.*(11248·64 ÷ 10000)
= *Rs.*1·124864.

Amount of *Re.*1 for 1 year = *Rs.*1·04.

Now, 1·124864 ÷ 1·04 = 1·0816; 1·0816 ÷ 1·04 = 1·04 and 1·04 ÷ 1·04 = 1.

Hence the reqd. time = 3 years. *Ans.*

Ex. 2. In what time will £12500 amount to £15185. 9*s.* 0·96*d.* at 6 per cent. Compound Interest?

£15185. 9*s.* 0·96*d.* = £15185·454.

∴ Amt. of £1 in the reqd. no. of years = £(15185·454 ÷ 12500)
= £1·21483632.

Amount of £1 for 1 year = £1·06.

Now, 1·21483632 ÷ 1·06 = 1·146072; 1·146072 ÷ 1·06 = 1·0812 and 1·0812 ÷ 1·06 = 1·02.

Here, the last quotient 1·02 is less than the divisor; therefore the reqd. no. of years = 3 years + a fraction of a year.

Now, to find the fractional part of a year in which £1 rises to £1·02, at 6 per cent. Simple Interest.

The interest on £1 for reqd. time = £1·02 − £1 = £·02 = £$\frac{1}{50}$.

The Simple Interest on £1 for 1 year = £$\frac{6}{100}$ = £$\frac{3}{50}$.

and $\frac{1}{50}$ is $\frac{1}{3}$ of $\frac{3}{50}$; ∴ the reqd. time = $3\frac{1}{3}$ years. *Ans.*

Ex. 3. The difference between the Simple and the Compound Interest on a certain sum of money for 3 years at 5 per cent. per annum is *Rs.*133. 7*a.*; find the sum.

Simple Int. of *Re.*1 for 3 yrs. at 5 p. c. $= Re.\frac{5 \times 3}{100}$ or *Rs.*15.

Amount of *Re.*1 at the end of 3rd year at Comp. Int. $= Rs.(1\cdot05)^3$
$= Rs.1\cdot157625$.

∴ the int. of *Re.* 1 ... ... $= Re.\cdot157625$.

∴ the diff. between the Simple and the Compound Interest of *Re.*1.
$= Re.(\cdot157625 - \cdot15) = Re.\cdot007625$; and *Rs.*133. 7*a.* $= Rs.133\cdot4375$.

∴ *Re.*·007625 : *Rs.*133·4375 :: *Re.*1 : sum reqd.

∴ sum reqd. $= Rs.(133\cdot4375 \div \cdot007625) = Rs.17500.$ *Ans.*

Examples CLXI.

1. What sum of money lent at Compound Interest at $7\frac{1}{4}$ per cent. per annum will amount to *Rs.*49691. 14*a.* in 3 years?

2. What sum placed out at 3 per cent. Compound Interest amounts in 3 years to £364242. 6*s.* 8*d.* ?

3. Find the principal which will produce £21. 0*s.* 4*d.* in 3 years at 5 per cent. Compound Interest.

4. What sum of money will amount to *Rs.*48027. 10*a.* 6*p.* in $2\frac{1}{2}$ years at 5 per cent. Compound Interest?

5. At what rate per cent. will £100000 amount to £108243. 4*s.* $3\frac{2}{5}\frac{1}{5}$*d.* in 4 years at Compound Interest?

6. At what rate per cent. will £500 amount to £607. 15*s.* $0\frac{3}{8}$*d.* in 4 years at Compound Interest?

7. The amount of *Rs.*4000 for a certain time at 5 per cent. Compound Interest is *Rs.*4520. 4*a.* ; find the time.

8. Required the time in which £20000 amounts to £24261. 2*s.* $4\frac{1}{8}$*d.* at 8 per cent. Compound Interest.

9. In what time will £15000 gain £1390. 18*s.* $1\frac{1}{8}$*d.* by Compound Interest at 3 per cent. per annum ?

10. The difference between the Simple and the Compound Interest on a certain sum for 3 years at $4\frac{1}{2}$ per cent. is *Rs.*86. 13*a.* 2*p.* ; find the sum.

11. The difference between the Simple and the Compound Interest on a certain sum for 3 years at $3\frac{3}{4}$ per cent. per annum is £3. 8*s.* $4\frac{1}{8}$*d.* ; find the sum.

12. What will £400 amount to at 3 per cent. Compound Interest in a time in which *Rs.*600 amounts to *Rs.*678. 0*a.* $7\frac{1}{5}$*p.* at 5 per cent. Compound Interest ?

13. A sum of money placed out at Compound Interest amounts to *Rs.*2420 in 2 years and to *Rs.*2662 in 3 years. Find the sum and the rate of interest.

14. *A* placed out a sum of money for 3 years at 5 per cent. Simple Interest, and *B* placed out an equal sum at the same rate and for the same time, but at Compound Interest, and thereby gained £26. 13*s*. 4*d*. more than *A*. What money was placed out by each?

15. A banker borrows money at $3\frac{1}{2}$ per cent. per annum, and pays the interest, at the end of the year; he lends it out at 5 per cent. per annum payable quarterly, and receives the interest at the end of the year; by this means he gains *Rs.*2000 a year; how much money does he borrow?

III. PRESENT WORTH AND DISCOUNT.

530. Suppose *A* sold a house to *B* for *Rs.*1025 payable 6 months hence; the debt here cannot be claimed *at present*, for it will fall due only 6 months hence. But when such a debt is paid before it is due, a sum smaller than the actual debt must be paid by the debtor and will be accepted by the creditor as payment in full, with no loss to either party. In the present case, let us see what is that smaller sum of money which *B* pays and *A* accepts as his full due. Suppose the interest at which money is lent to be 5 per cent. per annum. *Rs.*1000, when put out to interest at 5 per cent. will in 6 months amount to *Rs.*1025. Therefore *B* may pay *Rs.*1000 at present, and *A* may receive it as his full due instead of *Rs.*1025, which he is entitled to, at the end of 6 months. For, if *A* at once puts out to interest *Rs.*1000 at 5 per cent., it will in 6 months amount to *Rs.*1025 which is the actual debt. We call *Rs.*1000, the smaller sum accepted as the present payment the **Present Worth**, and *Rs.*25 the money deducted the **Discount**. Hence,

531. The **Present Worth or Present Value** of a sum of money due at the end of a given time is that sum which with its interest for the given time amounts to the sum due.

Thus, if *Rs.*350 in 6 months at 4 per cent. amounts to *Rs.*357, it follows that *Rs.*350 paid now is, equivalent to *Rs.*357 paid at the end of 6 months; that is, the *Present Worth* of *Rs.*357 due at the end of 6 months is *Rs.*350.

532. **Discount** is the abatement or allowance made when a sum of money is paid before it is due. But a sum of money due at the end of a given time is discharged now by the payment of its present worth, true discount therefore is the difference between the sum due and its present worth.

Thus, in the above example, *Rs.*(357 − 350) or *Rs.*7 is the *Discount* on *Rs.*357 due 6 months hence.

533. Since Present Worth + Discount = Sum due; (Art. 532), and Present Worth + Int. of Present Worth = Sum due; (Art. 531).

∴ Discount on sum due = Interest of its Present Worth.

534. *Discount on any sum is always less than the interest on the same sum for the same time.*

Since the Present Worth is always less than the sum due, therefore the interest on the Present Worth is always less than the interest on the sum due. But, by *Def.*—the interest on the Present Worth is the Discount. Therefore the Discount is always less than the interest on the sum due.

535. *To find the Present Worth and Discount of a given sum of money due at the end of a given time at a given rate per cent.*, we should first find what *Rs.*100 or £100 amounts to for the given time at the given rate; and then state thus :—

(i) Amount of *Rs.*100 : given sum :: *Rs.*100 : Present Worth; and

(ii) Amount of *Rs.*100 : given sum :: Interest of *Rs.*100 : Discount;
or Discount = Sum − Present Worth.

Ex. Find the Present Worth and Discount of *Rs.*3552. 8*a.* due at the end of 4 months at $4\frac{1}{2}$ per cent.

Int. at $4\frac{1}{2}$ p. c. on *Rs.*100 for 4 mo. $= Rs.\frac{1}{3}$ of $4\frac{1}{2} = Rs.1\frac{1}{2}$.

∴ Amount of $Rs.100 = Rs.(100 + 1\frac{1}{2}) = Rs.101\frac{1}{2}$.

(i) ∴ $Rs.101\frac{1}{2} : Rs.3552\frac{1}{2} :: Rs.100$: Present Worth.

∴ Present Worth $= Rs.\frac{7105}{2} \times \frac{2}{203} \times 100 = Rs.3500$. *Ans.*

(ii) $Rs.101\frac{1}{2} : Rs.3552\frac{1}{2} :: Rs.1\frac{1}{2}$: Discount.

∴ Discount $= Rs.\frac{7105}{2} \times \frac{2}{203} \times \frac{3}{2} = Rs.52.$ 8*a* *Ans.*

or Discount = (*Rs.*3552. 8*a.* − *Rs.*3500) = *Rs.*52. 8*a.*

536. When Present Worth and Discount are reckoned at **Compound Interest,** we shall find the amount of *Rs.*100 or £100 at Compound Interest for the given time at the given rate, and proceed as before.

Ex. Find the Present Worth and Discount of *Rs.*2112. 8*a.* due at the end of 2 years, at 4 per cent. per annum. Compound Interest.

At 4 per cent. Compound Interest for 2 years *Re.*1 amounts to $Rs.(1{\cdot}04)^2$ or *Rs.*1·0816.

∴ *Rs.*1·0816 : *Rs.*2112·5 :: *Re.*1 : Present Worth reqd.

∴ Present Worth = *Rs.*(2112·5 ÷ 1·0816) = *Rs.*1953·125
= *Rs.*1953. 2*a.*
Hence Discount = *Rs.*2112. 8*a.* − *Rs.*1953. 2*a.* = *Rs.*159. 6*a.* } *Ans.*

Examples CLXII.

1. Find the Present Worth of :—

(1) *Rs.*9265 due 2 years hence at $4\frac{1}{2}$ per cent. Simple Interest

(2) ... 1321. 8*a.* ... $2\frac{1}{2}$ years ... ... $4\frac{1}{2}$ ... ...

(3) *Rs.*7933. 12*a.* due 3yrs. 4mo. hence at 3 per cent. Simple Interest.
(4) ...3223. 8*a.* ... 8 months ... ... $3\frac{1}{2}$ ... ...
(5) £46. 16*s.* 8*d.* ... 9 months ... ... $3\frac{3}{4}$ ... ...
(6) ...370. 4*s.* $8\frac{1}{4}$*d.* ... 15 months ... ... $4\frac{5}{8}$ ... ...
(7) ...437. 14*s.* 9*d.* ... $5\frac{1}{2}$ years ... ... $3\frac{1}{3}$ ... ...
(8) *Rs.*11444. 0*a.* 8*p.*... 4 yrs. 90 days ... $2\frac{1}{4}$ ... ...
(9) £241. 12*s.* 4*d.* ...146 days ... ... $4\frac{1}{3}$ ... ...
(10) £9724. 1*s.* ... 4 years ... ... 5 ... Compound Interest.

2. Find the Discount on :—

(1) *Rs.*41204. 4*a.* 8*p.* due 9 months hence at 4 per cent. Simple Int
(2) ...5600 ... 16 months ... ... 5 ... ...
(3) £355. 5*s.* ... 4 months ... ... $4\frac{1}{2}$ ... ...
(4) ...690. 3*s.* 9*d* ... 9 months ... ... 3 ... ...
(5) ...520. 17*s* 6*d.* ... $3\frac{1}{2}$ years ... ... $4\frac{1}{2}$ ... ...
(6) *Rs.*2516. 4*a.* ... 3 yrs. 9mo. 18 days $6\frac{1}{4}$ ... ...
(7) £621. 3*s.* 4*d.* ...245 days ... ...$3\frac{1}{7}$ ... ...
(8) ...298. 0*s.* 10*d.* ... 11 months ... .. 4 ... ...
(9) *Rs* 32457 8*a.* ...136 days ... ...$5\frac{1}{4}$ ... ...
(10) £2450. 18*s.* 9*d.* ... $3\frac{1}{2}$ years ... ...$3\frac{3}{4}$ Compound Int.

537. Inverse Questions on P. W. and Discount.

When the *Sum due, its Present Worth or Discount* and the *Time* are given, to find the *Rate per cent.* allowed, proceed precisely as in Interest (Art. 529, *Case* II.); and so too, when the other quantities are given, to find the time (Art. 529, *Case* III.). In such cases, consider the sum due as Amount, the Present Worth as Principal and the Discount as Interest.

Ex. 1. The discount on *Rs.*2957. 8*a* due at the end of 2 yrs. 8mo. is found to be *Rs.*332. 8*a.*; at what rate per cent. is the interest allowed?

The Present Worth of *Rs.*$2957\frac{1}{2}$ is *Rs.*$(2957\frac{1}{2} - 332\frac{1}{2})$ = *Rs.*2625. Therefore the interest on *Rs.*2625 for $2\frac{2}{3}$ yrs. is *Rs.*$332\frac{1}{2}$.

Now the Int. on *Rs.*2625 at 1% for $2\frac{2}{3}$ yrs. = *Rs.* $(2625 \times 2\frac{2}{3} + 100)$;
∴ Rate per cent. reqd. = $332\frac{1}{2} \div (2625 \times 2\frac{2}{3} + 100) = \underline{4\frac{3}{4}}$. *Ans.*

Ex. 2. If the discount on £1321. 10*s.* at $4\frac{1}{2}$ per cent. be £121. 10*s.*, how long was the sum paid before it was due?

£$1321\frac{1}{2}$ − £$121\frac{1}{2}$ = £1200 is the Present Worth of £$1321\frac{1}{2}$.

∴ £$121\frac{1}{2}$ is the interest on £1200.

Now the Int. on £1200 for 1 year = £$(1200 \times 4\frac{1}{2} \div 100)$ = £54.

∴ the time reqd. = $(121\frac{1}{2} + 54)$ yrs. = $2\frac{1}{4}$ years. *Ans.*

Examples CLXIII.

1. What is the rate of interest in the following cases?—

(1) When the Disc. on *Rs.*1556. 10*a.* 8*p.* due in 3 mo., is *Rs.*23. 5*a.* 4*p.*

(2) When the Disc. on *Rs.*784. 14*a.* due 8 mo. hence, is *Rs.*30. 3*a.*

(3) When the Disc. on *Rs.*3888. 14*a.* due 18 mo. hence, is *Rs.*220. 2*a.*

(4) When the Disc. on £574. 3*s.* 4*d.* due 2 yrs. 3 mo. hence, is £32. 10*s.*

(5) When the P.W. of *Rs.*3286. 11*a.* 4*p.* due in 3 mo. is *Rs.*3254. 2*a.* 8*p.*

(6) When the P. W. of £1336. 11*s.* 3*d.* due in $3\frac{1}{2}$ yrs. is £1137. 10*s.*

2. When is the sum due, if the—

(1) Discount on *Rs.*13735 at $3\frac{3}{4}$ per cent. be *Rs.*335?

(2) Discount on *Rs.*13371. 8*a.* at $4\frac{3}{5}$ per cent. be *Rs.*1621. 8*a.*?

(3) Discount on *Rs.*238. 14*a.* 8*p.* at 5 per cent. be *Rs.*35. 9*a.* 4*p.*?

(4) Discount on £481. 8*s.* $2\frac{1}{4}$*d.* at $3\frac{1}{4}$ per cent. be £5. 3*s.* $2\frac{1}{4}$*d.*?

(5) P. W. of £668. 5*s.* $7\frac{1}{2}$*d.* at 5 per cent. be £568. 15*s.*?

(6) P. W. of *Rs.*2753. 5*a.* 4*p.* at 4 per cent. be *Rs.* 2581. 4*a.*?

3. The Discount allowed on a bill for £17192. 8*s.* paid $8\frac{1}{3}$ years before due, is £3438. 9*s.* $7\frac{1}{2}$*d.*; what is the money worth?

4. Find the difference between the discount on £196. 4*s.* $4\frac{1}{2}$*d.* due 6 months hence at 8 per cent., and the interest on the same sum for the same time at the same rate.

5. Find the difference between the interest on *Rs.*2466. 10*a.* 8*p.* for $2\frac{3}{4}$ years at $5\frac{1}{2}$ per cent., and the discount on *Rs.*2839. 12*a.* due $2\frac{3}{4}$ years hence at the same rate. Explain the result.

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538. Miscellaneous Questions on P. W. and Discount.

Ex. 1. The discount on a sum of money due 1 yr. 4 mo. hence at $4\frac{3}{4}$ per cent. is £48. 9*s.* Find the sum due, and also its present worth.

Int. on £100 for $1\frac{1}{3}$ years at $4\frac{3}{4}$ per cent. = £$4\frac{3}{4} \times 1\frac{1}{3}$ or £$6\frac{1}{3}$.
∴ Discount on £$(100+6\frac{1}{3})$ due $1\frac{1}{3}$ yrs. hence at $4\frac{3}{4}$ p. c. = £$6\frac{1}{3}$.

Hence £$6\frac{1}{3}$: £48. 9*s.* :: £$106\frac{1}{3}$: sum due;

∴ sum due = £$(106\frac{1}{3} \times 48\frac{9}{20} \div 6\frac{1}{3})$ = £813. 9*s.*
and Present Worth = £813. 9*s.* − £48. 9*s.* = £765. } *Ans.*

Ex. 2. If the interest on *Rs.*2531. 4*a.* at 5 per cent. be equal to the discount on *Rs.*2573. 7*a.* for the same time at the same rate, when is the latter sum due?

Interest on *Rs.*2531. 4*a.* = Discount on *Rs.*2573. 7*a.*

∴ *Rs.*2531. 4*a.* is the *P. W.* of *Rs.*2573. 7*a.*

∴ Interest on *Rs.*2331. 4*a.* = *Rs.*2573. 7*a.* − *Rs.*2531. 4*a.* = *Rs.*42. 3*a.*, but Interest on *Rs.*$2531\frac{1}{4}$ for 1 year = *Rs.*$2531\frac{1}{4} \times \frac{5}{100}$ = *Rs.*$\frac{2025}{16}$.

∴ the reqd. time = $(42\frac{3}{16} \div \frac{2025}{16})$ yr. = $\frac{1}{3}$ yr. = 4 months. *Ans.*

Ex. 3. If the difference between the interest and the discount on a sum of money for 8 months at 4 per cent. is 12*a.* Find the sum.

Interest on *Rs.*100 for 8 mo. at 4 p. c. = *Rs.*$\frac{8}{3}$.
∴ ....*Rs.*300 .... = *Rs.*8.
∴ Discount on *Rs.*308 .... = *Rs.*8.
∴ ....*Rs.*300 .... = *Rs.*$\frac{300}{308} \times 8$.
∴ the diff. bet. Int. and Dis. on *Rs.*300 = *Re.*$(8 - \frac{300}{308} \times 8)$ = *Re.*$\frac{64}{308}$.
Hence *Re.*$\frac{64}{308}$: 12*a.* :: *Rs.*300 : sum reqd.
∴ sum reqd. = *Rs.*$300 \times \frac{12}{16} \times \frac{308}{64}$ = *Rs.*1082. 13*a* *Ans.*

539. *The difference between the interest and the discount on a certain sum of money for a given time at a given rate per cent. is the interest on the discount for the same time at the given rate.*

Since the sum due = the *P. W.* + the Discount ;
∴ Int. on the sum due = Int. on the *P. W.* + Int. on the Disc.
But Int. on the *P. W.* = the Discount on the sum due ;

∴ Int. on the sum due = the Disc. on the Sum due + Int. on the Disc. ;
∴ Int. on the sum due − the Disc. on the Sum due = Int. on the Disc.

Ex. 1. The interest on a certain sum of money is *Rs.*280 for a certain time, and the discount on the same sum for the same time and at the same rate is *Rs.*218. 12*a* ; find the sum.

*Rs.*280 − *Rs.*218. 12*a.* or *Rs.*61. 4*a.* is the interest on *Rs.*$218\frac{3}{4}$.
Hence *Rs.*$61\frac{1}{4}$: *Rs.*280 :: *Rs.*$218\frac{3}{4}$: required sum.
∴ reqd. sum = *Rs.*$(218\frac{3}{4} \times 280 \div 61\frac{1}{4})$ = *Rs.*1000. *Ans.*

Ex. 2. The discount on a certain sum of money due 9 months hence is *Rs.*200 and the interest on the same sum for the same time is *Rs.*207. 8*a.* ; find the sum and the rate per cent. per annum.

Rs.$(207\frac{1}{2} - 200)$ or *Rs.*$7\frac{1}{2}$ is the interest on *Rs.*200 for 9 mo.
∴ *Rs.*$7\frac{1}{2} \times \frac{4}{3}$ .... for 1 year.

∴ Int. on *Rs.*100 or the rate reqd. = *Rs.*$\frac{1}{2} \times \frac{15}{2} \times \frac{4}{3}$ = *Rs.*5. *Ans.*

Again, Int. on the sum for 9 mo. at 5 p. c. = *Rs.*207. 8*a.*
... ... for 1 year ... = *Rs.*$207\frac{1}{2} \times \frac{4}{3}$ or *Rs.*$\frac{830}{3}$.

Hence *Rs.*5 : *Rs.*$\frac{830}{3}$ :: *Rs.*100 : sum reqd..
∴ sum reqd. = *Rs.*$100 \times \frac{830}{3} \times \frac{1}{5}$ = *Rs.*5533. 5*a.* 4*p.* *Ans.*

Ex. 3. If *Rs.*8 be allowed as discount off a bill of *Rs.*80 due 6 months hence, how much should be allowed off a bill of the same amount due 15 months hence?

*Rs.*8 is 6 months' discount off *Rs.*80,
∴ *Rs.*8 is 6 months' interest on *Rs.*(80 − 8) or *Rs.*72 ;
∴ *Rs.*4 is 3 months' ...*Rs.*72 ;

$\therefore$ *Rs.*20 is 15 months' interest on *Rs.*72 ;
$\therefore$ *Rs.*20 ... ... discount *Rs.*72 + 20 or *Rs.*92.

Hence *Rs.*92 : *Rs.*80 :: *Rs.*20 : discount reqd.
$\therefore$ discount reqd. = *Rs.*(80 × 20 ÷ 92) = *Rs.*17. 6*a.* $3\frac{3}{23}$*p.* *Ans.*

Ex. 4. If *Rs.*24 be allowed as discount off a bill of *Rs.*132 due 6 months hence, what should be the bill from which the same sum is allowed as 3 months' discount ?

*Rs.*24 is 6 months' discount off *Rs.*132 ;
$\therefore$ *Rs.*24 .... interest on *Rs.*(132 − 24) or *Rs.*108 ;
$\therefore$ *Rs.*24 is 3 months' ... ... *Rs.*108 × 2 or *Rs.*216 ;
$\therefore$ *Rs* 24............. ...discount on *Rs.*(216 + 24) or *Rs.*240.
Hence the amount reqd. = *Rs.*240. *Ans.*

Ex. 5. If *Rs.*12 be allowed as 6 months' discount off a bill of *Rs.*132, and at the same rate of interest *Rs.*40 be allowed off a bill of *Rs.*240, for how long a period had the little bill to run ?

*Rs.*12 is 6 months' discount off *Rs.*132 ;
$\therefore$ *Rs.*12...................interest on *Rs.*(132 − 12) or *Rs.*120 ;
$\therefore$ *Rs.*20................ .... *Rs.*200 or *Rs.*(240 − 40) ;
$\therefore$ *Rs.*40 is 12 months'............ *Rs.*(240 − 40) ;
$\therefore$ *Rs.*40 is 12 months' discount on *Rs.*240.
Hence the time reqd. is 12 months. *Ans.*

Examples CLXIV.

1. On what sum of money due at the end of 1 year 4 months does the discount at $4\frac{3}{4}$ per cent. amount to *Rs.*484. 8*a.* ?

2. On what sum of money due at the end of 3 years 9 months does the discount at 4 per cent. amount to £105. 8*s.* ?

3. The discount on a bill due 9 months hence at 4 per cent. per annum is £6. 15*s.* ; what is the amount of the bill ?

4. If the interest on *Rs.*8825 at 4 per cent. be equal to the discount on *Rs.*11119. 8*a.*, when is the latter sum due ?

5. If the interest on *Rs.*5333. 5*a.* 4*p.* for 2 years is equal to the discount at the same rate on *Rs.*5600 due 2 years hence, what is the rate of discount ?

6. If the interest on £4550 at 3 per cent. be equal to the discount on £5573. 15*s.* at the same rate, when is the latter sum due ?

7. At what rate per cent. will the interest on £3729. 7*s.* 6*d.* in 4 years, be equal to the discount on £4661. 14*s.* $4\frac{1}{2}$*d.* for the same time ?

8. The difference between the interest on a certain sum for 3 years at 5 per cent. and the discount on the same sum due 3 years hence at the same rate is *Rs.*152. Find the sum.

9. The interest on a certain sum of money for a certain time is *Rs.*36, and the discount for the same time is *Rs.*30 ; find the sum.

10. The interest on a certain sum for a certain time is *Rs.*70 and the discount for the same time is *Rs.*54. 11*a.*, find the sum.

11. The discount on a certain sum due 2 years hence is *Rs.*638. 8*a.*, and the interest on the same sum for the same time is *Rs.*718. 5*a.* ; find the sum, and the rate per cent. per annum.

12. On what sum will the difference between the interest and discount for 6 months at 4 per cent. be 2*a.* 1·44*p.* ?

13. The interest on a certain sum for 4 years is £35, and the discount on the same sum due 4 years hence is 15*s.* less. Find the sum and the rate per cent.

14. A tradesman marks his goods with two prices, one for ready money and the other for credit of 6 months ; what ratio should the two prices bear to each other, allowing interest at $7\frac{1}{2}$ per cent. per annum ? If the credit price of an article be *Rs.*332, what is the cash price ?

15. The discount on *Rs.*275 for a certain length of time is *Rs.*25 ; what is the discount on the same sum (i) for twice that length of time, and (ii) for half that length of time ?

16. The interest on *Rs.*5225 for a certain time is *Rs.*343. 12*a.* ; find the discount on the same sum for the same time.

17. If 13*s.* 4*d.* be allowed as discount off a bill of £10 due 6 months hence, how much should be allowed off a bill for the same amount due 8 months hence ?

18. If *Rs.*10 be allowed as discount off a bill of *Rs.*50, and at the same rate of interest *Rs.*25. be allowed as discount off a bill for *Rs.*150, due at the end of 8 months ; for how long a period had the first bill to run ?

19. If £6 be allowed as discount off a bill of £56 for 8 months, find the amount of another bill, off which £30 is allowed as discount for 20 months.

20. If *Rs.*5 be allowed as discount off a bill of *Rs.*125 due a certain time hence, what would be the discount allowed off, if the bill had twice as long to run ?

21. If £2652. 5*s.* be due 3 years hence, what sum will be due at the end of 1 year, if Compound Interest be allowed at 3 per cent. ?

22. Five volumes of a work can be bought for a certain sum payable at the end of a year, and six volumes of the same work can be bought for the same sum in ready money ; what are the rates of discount and interest ?

23. A bookseller sells 20 copies of a book for a certain sum. How many will he give for the same sum, allowing credit for 5 years, if money is worth 5 per cent. ?

24. The discount on a sum due 3 years hence is $\frac{1}{4}$ of the

interest on the same sum for the same period. Find the rate of interest.

25. If *Rs.*10 be allowed as discount off a bill for *Rs.*70, due sometime hence, what should be the present worth of a bill for *Rs.*1300 which has only half the time to run ?

26. A man bought a horse for 30 guineas and sold him immediately for £38. 10*s.* payable at the end of 6 months. If the use of money be reckoned at $6\frac{1}{4}$ per cent. per annum, what is now his gain per cent. ?

27. *A* offers for an estate *Rs.*37800, and *B* offers *Rs.*45400 to be paid at the end of 4 years Which is *now* the better offer and by how much, allowing 5 per cent. interest ?

28. What sum must be paid now in order that a person may receive *Rs.*2500 at the end of every year for the next three years, the rate of interest being $3\frac{1}{2}$ per cent. ?

29. *Rs.*1250 is due at the end of 3 months and *Rs.*900 at the end of 7 months ; what sum at the present time is equivalent to both these sums, calculating interest at $4\frac{1}{2}$ per cent. ? In what time will the result amount to *Rs.*1250+*Rs.*900 at the same rate of interest ?

30. I buy a garden for *Rs.*35000, and sell it the same day for *Rs.*42000 to be paid in two equal instalments at the end of 3 and 6 months respectively. How much do I gain by the transaction, reckoning interest at 6 per cent. per annum ?

CHAPTER XIV.

Mercantile Transactions.

540. Commercial Allowances. It is customary with merchants to allow both in buying and selling certain deductions (such as *Tare* and *Tret*) from the weight of goods that are sold by weight for the casks, bags, &c., which contain the goods.

Tare is the allowance of a certain weight which the seller makes to the buyer on account of the weight of casks, bags, &c., which contain the goods.

Tret is the allowance made to purchasers for waste or refuse matter after the tare is deducted.

The total weight of goods without deductions for *tare*, *tret* or *waste* is called the **gross** weight ; the weight free from all deductions is called the **net** weight.

Ex. Required the net weight of 18 chests of tea, weighing 28 cwt. 0 qr. 2 lbs., tare being allowed at 4 lbs. per chest and tret 16 lbs. per cwt.

	28 cwt.	0 qr.	2 lbs.	gross
4 lbs. × 18		2	16	tare
16 lbs. = $\frac{1}{7}$ of 1 cwt.	27	1	14	
	3	3	18	tret
	23	1	4	net weight.

541. Discounting Bills. A **Bill** is a discount of agreement to pay a particular sum of money at the end of a certain time.

(i) A **Bill of Exchange** or **Hundi** is a written instrument in which one person orders another to pay to him, or to some other person, a sum of money at a specified time. Thus :—

*Rs.*500. *Calcutta, 30th May 1897.*

Two months after date pay me or order Five hundred Rupees, value received.

Accepted X. Y.

A. B.

To X. Y.
Lucknow, N.-W. P.

Here the Bill is drawn by *A. B.* and sent to *X. Y.*, who on **accepting** it, writes his name across the Bill, and engages to pay to *A. B.* or order *Rs.*500 in 2 months after 30th May, 1897.

(ii) A **Promissory Note** or **Note of Hand**, is a written instrument in which one person promises to pay another or his order a sum of money at a specified time. Thus :—

*Rs.*800. *Calcutta, 30th May, 1897.*

Three months after date, I promise to pay *A. B.* or order Eight hundred rupees, value received.

X. Y.

Here, *X. Y.* engages to pay *A. B.* or his order *Rs.*800 at the end of 3 months from 30th May, 1897.

When a bill is due 2 months hence, it is called payable **at two months' Sight.**

542. If now the holder of the Bill or the Note wishes to cash it before it is due, he takes it to a *banker* or *bill broker* ; and if he be satisfied of the credit of the parties to the bill, he **discounts** it ; that is, he pays the sum specified on the bill or the note, deducting discount for the time it has still to run. But the practice prevailing among the bankers is to deduct the *Interest* of the sum specified, instead of the *interest of the Present Worth* of that sum, which is the **True discount** (Art. 533). And as the Present Worth of a sum, due at some future time is less than the sum itself, the *true discount* is less than the *banker's* or *mercantile* discount, called the **Commercial** or **Practical Discount** Hence the difference between the true and mercantile discount is the banker's gain.

Thus, it has already been seen that the *True Discount* on *Rs.*357 due at the end of 6 months at 4 per cent. interest is *Rs.*7. The *Commercial Discount*, however, is the interest on *Rs.*357 for 6 months at 4 per cent., and is therefore *Rs.*$357 \times \frac{4}{100} \times \frac{6}{12}$ = *Rs.*7. 2*a.* 3*p. nearly.*

543. A Bill of Exchange or a Promissory Note always runs 3 days beyond the time specified in it, and these three days are called the **Three days of Grace.** Although originally a mere custom, it has now all the force of law.

Thus, a Bill drawn on 5th January at 3 months **nominally** due on 5th April, but **legally** on 8th April. Moreover, **calendar months** are always reckoned, so that a Bill at 3 months, whether drawn on 30th or 31st Jan. is *nominally* due on the 30th April, and *legally* on the 3rd May. In calculating the number of days, it is usual to omit the day on which the Bill is drawn.

Ex. 1. Find the banker's disc. on *Rs.*2505 due in 5 yrs. at $3\frac{1}{2}\%$.

The Banker's discount is the interest on the given sum.

Interest or Banker's discount on *Rs.*100 for 5 yrs. at $3\frac{1}{2}$ p.c. $=$ *Rs* $\frac{35}{2}$.

$\therefore$ interest on *Rs.*2505 $=$ *Rs.* $\frac{35}{2}\times\frac{2505}{100}=$ *Rs.*438. 6*a.* *Ans.*

Ex. 2. A bill of *Rs.*770 is drawn on 8th March at 6 months, and discounted on 3rd June at 5 per cent. Find the banker's gain.

The Bill is nominally due on the 8th of September, and therefore legally due on the 11th. Also it is discounted on the 3rd June, and the number of days between 3rd June and 11th Sept. is 100.

Interest on *Rs.*100 for 100 days at 5 p. c. $=$ *Rs.* $5\times\frac{100}{365}=$ *Rs.* $1\frac{27}{73}$.

$\therefore$ *Rs.* $101\frac{27}{73}$: *Rs.*770 :: *Rs.* $1\frac{27}{73}$: true discount.

$$\therefore \text{ true discount} = Rs.\ \frac{770\times73\times100}{73\times7400} = Rs.10.\ 6a.\ 5{\cdot}8p.$$

and *Rs.*100 : *Rs.*770 :: *Rs.* $1\frac{27}{73}$: banker's discount.

$\therefore$ banker's discount $=$ *Rs.* $770\times\frac{100}{73}\times\frac{1}{100}=$ *Rs.*10. 8*a.* 9·2*p.*

$\therefore$ banker's gain $=$ *Rs.*10. 8*a.* 9·2*p.* $-$ *Rs.*10. 6*a.* 5·8*p.*,

$=$ 2*a.* 3·4*p.* *Ans.*

Examples CLXV.

1. Find the banker's discount on *Rs.*4635, due in $2\frac{1}{2}$ years, at $2\frac{3}{4}$ per cent.

2. A bill of *Rs.*12800, due in 1 year 10 months, is discounted by a banker at $4\frac{3}{4}$ per cent. ; what will the holder receive ?

3. Find the difference between the banker's and the true discount on *Rs.*7249. 6*a.* due in 9 months, at $4\frac{3}{4}$ per cent.

4. Find the difference between the banker's and the true discount on £300. 2*s.* 6*d.*, due 4 months hence, at $6\frac{1}{4}$ per cent.

5. How much less than the true present worth will a banker give for a bill of *Rs.*9504, due in $7\frac{1}{2}$ months ; interest at 5 p. c. ?

6. What will a banker gain by discounting a bill of *Rs* 7310. 4*a.*, due $2\frac{1}{2}$ years hence, at $5\frac{1}{2}$ per cent. ?

7. What will a banker retain on discounting a bill of *Rs.*12750 drawn on the 4th of March at 10 months, and discounted on the 14th of August at 5 per cent. ?

8. A bill is drawn for *Rs*.337. 8*a*. on July 17th at 2 months, and discounted Aug. 11th at $3\frac{3}{4}$ per cent.; how much does the holder receive?

9. What deduction does a banker make in discounting a bill for *Rs*.77163. 6*a*. drawn Oct. 10th at 9 months and discounted March 15th at $6\frac{1}{4}$ per cent?

10. What does a banker give as the present worth of a bill for £562 2*s*. 6*d*. drawn on Sept. 4th at 5 months and discounted the same day at $6\frac{1}{4}$ per cent.? How much is the result less than the true present worth?

11. How much does a banker give as the present worth of a bill for *Rs*.52534. 4*a* drawn Nov. 6th at 10 months, and discounted by him on Feb. 21st at $4\frac{1}{2}$ per cent?

12. On 31st Oct. a bill was drawn at 6 months for *Rs*.3097. 10*a*. 8*p*. and discounted Jan. 27th at 7 per cent; what was charged for discount, and how much did the banker gain?

13. Required the true discount on £2454. 2*s*. 10*d*. drawn on May 10, 1884, at 1 year, and discounted on April 3, 1885, at $9\frac{1}{8}$ per cent.

14. A bill was drawn on May 14th at 2 months, and was discounted on July 2nd at $8\frac{1}{3}$ per cent. If the banker's gain was 1*s*. 4*d*., for what sum was the bill drawn?

15. A person, discounting a bill 8 months before it is due, at 6 per cent. interest, receives £2. 7*s*. 6*d*. less than the amount of the bill. What was the bill drawn for?

16. A bill was drawn on March 11 for 30 days, and was discounted on March 18 at 4 per cent; and the banker's gain was *Rs*.5. What was the bill drawn for?

17. A bill for £126. 5*s*. was drawn on March 9 at 5 months, interest being calculated at 5 per cent; the discount on the bill was £1. 5*s*. On what date was the bill discounted?

18. A banker discounted a bill 9 months before due, and found, that he would have lost $\frac{1}{18}$ of what he deducted as discount had he reckoned true discount. Find the rate at which interest was calculated.

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544. Commission and Brokerage Commission is the charge made by an **Agent** for buying or selling goods, property, &c. for another, and is usually a percentage on the value of the goods or property bought or sold.

Brokerage is the charge made by a **Broker** for buying or selling goods, shares, &c., for another; and is usually a percentage on the full amount of the transaction.

545. Cash Discount. Discount, without reference to time, is an allowance which merchants and tradesmen make to such of their

customers as are willing to pay ready money. This allowance is usually percentage on the amount of the account.

546. Since Commission, Brokerage, &c. is a percentage on the given sum of money, to find their amount proceed thus :—

Multiply the sum by the number expressing the rate per cent. and divide by 100. Or apply *Rule of Three.*

Ex. 1. An agent sells goods to the value of *Rs.*5835, on which he receives a commission of $3\frac{3}{4}$ per cent.; how much does his commission amount to?

The commission $= Rs.(5835 \times 3\frac{3}{4} \div 100) = Rs.218{\cdot}8125 = Rs.\underline{218.\ 13a}.$

Ex. 2. Find the brokerage on £5000 at $1\frac{1}{2}$ per cent.

The brokerage $= £5000 \times 1\frac{1}{2} \times \frac{1}{100} = \underline{£75}.$ *Ans.*

547. Insurance. Insurance is a contract by which one party undertakes to pay a specified sum, at the death of a person, or against any loss to houses or goods by fire, or to ships or their cargoes at sea, in consideration of the protected party paying year by year, or once for all, a certain percentage of that sum.

The parties who take the risk are called the **Insurers** or **Underwriters**, and the protected party is called the **Insured.** The consideration money paid by the insured is called the **Premium**, the instrument containing the contract is called the **Policy of Insurance**; and the stamp duty on the policy is called the **Policy duty.**

548. When a man insures, so as to recover not only his property, but the premium and all other expenses connected with its insurance, it is said to be **covered.**

549. *Insurance* is calculated in the same way as Commission.

Ex. 1. What sum should be paid for insuring a vessel and cargo worth *Rs.*22250, at $3\frac{1}{4}$ per cent.?

The premium reqd. $= Rs.22250 \times 3\frac{1}{4} \times \frac{1}{100} = Rs\ \underline{723.\ 2a.}$ *Ans.*

Ex. 2. A cargo is valued at £5270. 6*s.*; the premium on insurance is at the rate of 5 guineas per cent., policy duty at 4*s.* per cent; and commission $\frac{7}{16}$ per cent.; what sum must be insured to cover the cargo and the expenses of insurance, and what premium must be paid?

Deduct from		£100
Premium	£5. 5*s.* 0*d.*	
Policy duty	4*s.* 0*d.*	
Commission	8*s.* 9*d.*	£5. 17*s.* 9*d.*
	£5. 17*s.* 9*d.*	£94. 2*s.* 3*d.*

In case of loss, for every £100 received from the underwriters, £5. 17*s.* 9*d.* is for expenses of insurance, and the remaining £94. 2*s.* 3*d.* is for cargo; hence to recover both cargo and expenses of insurance we must insure £100 for every £94. 2*s.* 3*d.* of cargo; therefore

£94. 2*s.* 3*d.* : £5270. 6*s.* :: £100 : sum to be insured,

or $£94\frac{9}{80}$: $£5270\frac{3}{10}$ :: £100 : sum to be insured.

$\therefore$ sum to be insured $= £\frac{52703}{10} \times 100 \times \frac{80}{7529} = \underline{£5600}.$ *Ans.*

Also, the expenses of insurance are at the rate of £5. 17*s.* 9*d.* for every £94. 2*s.* 3*d.* of cargo ; therefore

£94. 2*s.* 3*d.* : £5270. 6*s.* :: £5. 17*s.* 9*d.* : expenses of insurance ;
∴ expenses reqd. = £($5270\frac{3}{10} \times 5\frac{71}{80} \div 94\frac{9}{80}$) = £329. 14*s.*

And £94. 2*s.* 3*d.* : £5270. 6*s.* :: £5. 5*s.* : premium to be paid.
∴ premium reqd. = £($5270\frac{3}{10} \times 5\frac{1}{4} \div 94\frac{9}{80}$) = £294. *Ans.*

Examples CLXVI.

1. What does a factor receive for selling goods to the amount of *Rs.*3758. 5*a.* 4*p.* at a commission of $1\frac{3}{4}$ per cent. ?

2. What is the ready money payment of an account amounting to *Rs.*3597. 6*a.*, allowing a discount of $2\frac{1}{2}$ per cent. ?

3. What is the brokerage upon a money transaction of £273. 15*s.* at 3*s.* 4*d.* per cent. ?

4. What is the brokerage on *Rs.*7681. 4*a.* at *Re.*1. 10*a.* 8*p.* per cent. ?

5. For what sum should goods, worth £4384. 0*s.* 3*d.* be insured at £2. 6*s.* 8*d* per cent. that the owner may recover, in case of loss the value of both goods and premium ?

6. A commission agent sells 1436 barrels of flour at *Rs.*15 per barrel ; what commission does he receive at $4\frac{1}{2}$ per cent ?

7. The brokerage on a certain sum at 3*s.* 4*d.* per cent. amounts to £1. 5*s.* $7\frac{1}{4}$*d.* ; find the sum.

8. A man insured his house for *Rs.*16800 at *Rs.*3. 12*a.* per cent., so that in case of fire he might recover both the value of the house and the premium. Find the value of the house.

9. What sum must be paid to insure a cargo worth *Rs.*25850, the premium being *Rs* 17. 8*a* , policy duty *Re.*1, and brokerage *Re.*1. 4*a.* per cent. respectively ?

10. A ship is insured for $\frac{5}{8}$ of its value at $\frac{3}{8}$ per cent., and the insurance amounts to *Rs.*1560. What is the value of the ship ?

11. At what rate per cent. is discount allowed when a tradesman deducts *Rs.*40. 6*a.* 6*p.* from a bill of *Rs.*897. 14*a.* 8*p.* ?

12. An agent sells goods to the value of *Rs* 796536. 4*a.* on which he receives a commission of $3\frac{3}{4}$ per cent., while his office and other expenses amount to $22\frac{1}{2}$ per cent. of his commission. How much clear profit does he make, and how much does he remit to his principal ?

13. A broker at the public sales buys 5 chests of indigo weighing 18 cwt. 3 qrs. 22 lbs. net, at *Rs.*2. 14*a.* 8*p.* per lb. ; find the brokerage at $\frac{3}{8}$ per cent.

14. A ship worth £15325 is to be insured, so that its value

and all the expenses connected with its insurance may be covered. The premium is 2½ guineas per cent., policy duty 4*s.* per cent., and brokerage ¼ per cent ; what is the amount of the whole expenses paid on insurance ?

15. What sum must be paid on the insurance of a cargo of the value of *Rs.*34575. 4*a.*, so that in case of loss the cargo and all expenses of insurance may be recovered ? The premium is at the rate of *Rs.*47. 4*a.* per cent., policy *Rs.*2 per cent., and agent's commission ½ per cent.

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550 Invoice, Account-Sales, &c.

An Account-Sale is a statement drawn out by a Commission Agent or Broker shewing the sales he has made of goods in behalf of another party. It is of the following form :—

ACCOUNT-SALE of 15 chests of tea, per S. S. *San Salvador* from London, on account of Messrs. Hare and Blackett, Canton.

		cwt.	qr.	lbs.		cwt.	qr.	lbs.		£.	s.	d.
H. B.	3 chests	23	2	15	gross.	2	0	12	tare.			
No.	5 ...	38	1	24	...	3	1	15	...			
	4 ...	32	3	6	...	2	3	18	...			
1 to 15	3 ...	24	0	18	...	2	2	11	..			
		119	0	7		0	3	17	tret.			
	Deduct	11	3	7		11	3	7				
	Net	107	1	0	at £11.	12*s.*	4*d.*	per cwt...		1245	17	9
						cwt.	qr.	lbs.				
	Overtaker	7	2	19		1	1	19	tare.			
	Deduct	1	3	5		0	1	14	tret.			
	Net	5	3	14	at £11.	10*s.* per cwt...... ..				67	11	3
						Gross proceeds...				1313	9	0
	CHARGES.											
						£.	s	d.				
	Insurance on £1320 at 2 per cent. ...					26	8	0				
	Policy Duty					3	18	6				
	Freight on 120 cwt. at 11½*d.* per cwt. ...					5	15	0				
	Primage and dock dues					40	6	6				
	Other charges					2	17	3				
	Insurance of fire					2	1	5				
	Sale charges					3	9	7				
	Commission on £1313. 9*s.* 0*d.* at 2½ *p.c.*					32	16	9				
	... on £1320. 0*s.* 0*d.* at ½ *p.c.*					6	12	0		124	5	0
	Net proceeds due to Messrs. Hare and Blackett									1189	4	0
	London : *The 31st July, 1897.* } R. S. BUDGETT & Co.											

When a merchant (Henry Smith) of London ships goods, &c., on account and risk of a merchant (Charles Brown) of Calcutta, he sends with the goods a paper containing a memorandum of goods

with prices to the buyer. This is called an **Invoice**. It is of the following form, and differs in many respects from the simpler Invoices given in Page 305 :—

INVOICE of 36 boxes of coffee shipped on board the S. S. *Golconda*, William Burkett, master, for Calcutta on account and risk of Charles Brown.

C. B.	No.	cwt.	qrs.	lbs.		cwt.	qr.	lbs.		£.	s.	d
	1	24	3	18	gross	1	2	25	tare.			
1 to 4	2	18	2	25	......	1	0	9	...			
	3	36	1	7	... ..	2	0	27	...			
	4	30	2	13	......	1	3	14	...			
		120	2	7		6	3	19	...			
		6	3	19								
		103	2	16 net at 10½d. per lb.						507	17	0

CHARGES.

	£.	s.	d.	£.	s.	d
Bond and Custom-house entry.........	0	18	6			
Cost of empty boxes....................	3	4	0			
Small Charges..............................	1	15	6			
Bills of Lading.............	0	3	4			
Brokerage on £508 0 0 at 1½ p. c.	7	12	5	13	13	9
Commission on £521 10 9 at 2½ p.c.	13	0	10			
Insurance on £525 ... 2 p.c.	10	10	0			
Commission on £525 ... ½ p.c.	2	12	6			
Policy duty	1	3	4	27	6	8
Errors Excepted............				548	17	5

London : *The 10th Jan., 1896.* } HENRY SMITH.

551. Bill of Lading, Bill of Exchange, &c.

When a merchant in London ships goods to a merchant in Calcutta, he takes from the captain of the ship a receipt of the goods made over to his charge. This is called a **Bill of Lading** ; and the Captain will not deliver the goods to any one who does not hold the Bill of Lading.

The **Shipper** or **Consignor** also draws a **Bill of Exchange** on the **Consignee** ordering him to pay to his Agent or to a certain person in his behalf the value of the shipment. It is of the following form :—

No. 512. *Exchange for £548. 17s. 5d.*

London, Jan. 10, 1896.

At Thirty-days after sight of this our First of Exchange (Second and Third of the same tenor and date unpaid) pay to the Mercantile Bank of India, Limited, the sum of Five hundred and forty-eight

pounds, seventeen shillings, and five pence, value received against shipment per S. S. *Golconda*, which place to account of

HENRY SMITH.

TO MESSRS. CHARLES BROWN & CO.
Calcutta.

This is accompanied with an advice containing the rate at which Exchange is to be calculated.

Now, the Shipper may, if he chooses, send this Bill, together with the Bill of Lading and Policy of Insurance to his Agent in Calcutta for presentation to the *Drawee* ; but instead of doing this, as it will incur some delay in getting the money, the usual custom is that he goes to a Banker and asks him to **discount** the Bill, *i.e.*, pay over the value of the Bill (less commission) on the strength of the documents. If the banker agrees he takes the Bill and pays cash to the Shipper, at a rate of exchange to be agreed upon between them. The banker now becomes the master of the goods.

The Banker forthwith despatches the Bill by the first and faster mail steamer to his Agent in Calcutta (sending also two duplicates called the Second and Third of Exchange, by other steamers in case of accident). On its arrival in Calcutta, the Banker's Agent presents the Bill to the Drawee for acceptance. The Bill is then said to be **sighted**, and if the business is all in due form, the Drawee **accepts** it, *i.e.*, signs it and writes on it the date at which it **matures**, *i.e*, becomes due. The Bill is then returned to the Banker's Agent and the Drawee binds himself to pay the amount of the Bill on due date. When the Bill is attached to the Bill of Lading and Policy of Insurance, it is called a **Documentary Bill** ; if not, a **Clean Bill**.

The Bill, which one merchant draws on another for certain goods sold to him is called a **Draft**, and it becomes a **Bill** when the buyer **accepts** or puts his signature to it.

552. Exchange. **Exchange** is the rule by means of which it is ascertained what sum of money of one country is equivalent to a *given* sum of another, according to some *settled* rate of commutation.

Thus, by *Exchange* we find the value of £5 in *Rs.*, and *vice-versa.*

553. The **Course of Exchange** is used to express the sum of money of any place given in exchange for a *fixed* sum of that of another ; and the **Par of Exchange** denotes the sum of money of any place, which is of the same *intrinsic* and *real* value as that fixed sum.

Thus, in the *Par of Exchange*, £1 is equal to *Rs.*10 ; but in the *Course of Exchange* £1 sterling, a fixed sum is equivalent to a variable number of Indian Rupees, more or less, according to circumstances.

The **Arbitration** or **Comparison of Exchanges** is the

method of determining upon the rate of Exchange, called the **Par of Arbitration**, between the first and last of a given number of places, where the Course of Exchange between the first and second, the second and third, &c., of these places is known. It is called **Simple** or **Compound** Arbitration, as three or more places are concerned.

554. Exchanges between merchants are effected by means of written instruments, called **Foreign Bills of Exchange** or briefly **Foreign Bill**; and a Bill on London entitles the holder to obtain gold in London for the value of the amount mentioned in the Bill.

Foreign Money Table.

(*a*) GOLD CURRENCY.

France, Belgium, Switzerland.	...1 franc	= 100 centimes	= $9\frac{3}{8}$*d.* nearly.
Italy	...1 lira	= 100 centesimi	
Spain	...1 peseta	= 100 centesimos	
Greece	...1 drachme	= 100 lepta	
Servia	...1 dinar	= 100 paras	
Bulgaria ...	...1 leva	= 100 stotinkis	
Romania ...	...1 ley	= 100 banis	
Austria ...	...1 florin	= 100 kreutzers	= 1*s.* $11\frac{3}{4}$*d.* ...
German Empire	...1 marc	= 100 pfennige	= $11\frac{3}{4}$*d.* ...
Holland ...	...1 florin	= 100 cents or 20 stivers	= 1*s.* 8*d.* ...
Norway, Sweden, Denmark	...1 krone	= 100 ore	= 1*s.* 1*d.* ...
Portugal	...1 milreis	= 1000 reis	= 4*s.* 6*d.* ...
United States, Canada	...1 dollar $	= 100 cents	= 4*s.* 2*d.* ...

(*b*) SILVER CURRENCY.

India, Ceylon ...	...1 rupee	= 16 annas = 100 cents	= 1*s.* 4*d.* ...
China ...	...1 tael	= 1000 cash	= 6*s.* 6*d.* ...
Russia ...	...1 rouble	= 100 copecks	= 3*s.* $1\frac{1}{2}$*d.* ...
Turkey ...	...1 pound	= 100 piastres	= 18*s.* $0\frac{3}{4}$*d.* ...
Japan ...	...1 yen	= 100 sen	= 4*s.* 1*d.* ...

555. From the nature of Exchange we see that the operations necessary to calculate it, are only applications of the *Rule of Proportion*; but the easiest method is the application of the **Chain Rule**.

Ex. 1. What Indian money must be paid for £6643. 17*s.* 6*d.*, the course of exchange being 1*s.* $10\frac{1}{2}$*d.* per rupee ?

(i) 1*s.* $10\frac{1}{2}$*d.* : £6643. 17*s.* 6*d.* :: *Re.*1 : sum in Indian money ;
or $22\frac{1}{2}$*d.* : 1594530*d.* :: *Re.*1 : sum reqd.
∴ sum reqd. = *Rs.*(1594530 ÷ $22\frac{1}{2}$) = *Rs.*70868. *Ans.*

(ii) Reqd. no. of *Rs.* = £6643. 17*s.* 6*d.* or £$6643\frac{7}{8}$.
1*s.* $10\frac{1}{2}$*d.* or £$\frac{3}{32}$ = *Re.*1.
∴ no. of *Rs.* reqd. = $6643\frac{7}{8} + \frac{3}{32} = \frac{53151}{8} \times \frac{32}{3}$ = 70868.

Ex. 2. If the exchange between Amsterdam and Paris be 54*d.* for 1 crown, and between Amsterdam and London be 33*s.* 9*d.* Flemish for £1; what is the par of exchange or the arbitrated price between Paris and London?

Let x be the exchange value of the crown in *d.*

Reqd. no. of *d.* = 1 crown,
1 crown = 54*d.*

∴ $x = \frac{54 \times 240}{405}d. = 32d.$

33*s.* 9*d.* or 405*d.* = £1 or 240*d.* Hence 1 crown at Paris = 32*d.* in London.

Ex. 3. Convert *Rs.*23000 into English money, when English money is at a premium of 15 per cent.; the par of exchange being 2*s.* per rupee.

At par, 2*s.* = *Re.*1; ∴ at 15 p. c. premium, 2*s.* = *Re.*1. + *Re.*$\frac{15}{100}$ = *Rs.*$\frac{23}{20}$.
∴ *Re.*1 = $\frac{20}{23} \times 2s. = \frac{40}{23}s.$; and ∴ *Rs.*23000 = $\frac{40}{23} \times 23000$ = £2000. *Ans.*

Ex. 4. A New York merchant remits 27940 florins to Amsterdam by way of London and Paris, at a time when the exchange of New York on London is 4·885 dollars for £1, of London on Paris is 25 fr. 40 c. for £1, and of Paris on Amsterdam is 212 francs for 100 florins; $\frac{1}{8}$ per cent. brokerage being paid in London and in Paris.

Dollars reqd. = 27940 florins,
100 = 212 francs,
100 = $100\frac{1}{8}$ fr., with brokerage,
2540 = $100\frac{1}{8}$ £, ...
1 = 4·885 dollars;

∴ Dollars reqd.
$= \frac{27940 \times 212 \times 801 \times 801 \times 4885}{100 \times 100 \times 2540 \times 8 \times 8 \times 1000}$
= 11420·317 ....
= 11420 dollars 32 c. *Ans.*

Ex. 5. Find the par of exchange between the U. S. gold eagle, weighing 258 grains $\frac{9}{10}$ fine, and the sovereign of which 1869 weigh 40 lbs. of gold $\frac{11}{12}$ fine (4 eagle = 10 dollars.)

£'s reqd. = 1 eagle,
1 = 258 grs. $ standard,
10 = 9 grs. fine,
11 = 12 grs. Brit. standard,
5760 = 1 lb.
40 = £1869;

∴ £'s reqd. $= \frac{258 \times 9 \times 12 \times 1869}{10 \times 11 \times 5760 \times 40}$
= 2·054838
i. e., 1 eagle = £2·054838...
1 dollar gold = £·2054838...
and £1 = $\frac{10000000}{2054838}$ $ = 4·8665...$.

Ex. 6. Find the relation between the sovereign and the Napoleon, as determined from the intrinsic value of the two coins:— 40 lbs. British standard gold, $\frac{11}{12}$ fine, is coined into 1869 sovereigns; and 15432 grains French standard gold, $\frac{9}{10}$ fine, is coined into 155 Napoleons, (1 Napoleon = 20 fr.)

Napoleons reqd. = £1,
1869 = 40 lbs. British standard,
12 = 11 lbs. fine,
1 = 5760 grs...
9 = 10 grs. French standard,
15432 = 155 Napoleons;

∴ Napoleons reqd.

$$= \frac{40 \times 11 \times 5760 \times 10 \times 155}{1869 \times 12 \times 9 \times 15432}$$

$= 1 \cdot 26110\dot{6}\ldots\ldots$

∴ £1 = 1·261106 Napoleons
= 25·22212 francs
= 25 fr. 22 c. *Ans.*

Examples CLXVII.

1. Reduce £1857. 14*s*. 3*d*. to rupees, &c. at the rate of 1*s*. 11¾*d*. for 1 rupee.

2. How many francs will be given in Paris for £688. 14*s*. 8*d*, when the course of exchange is 25 fr. 42½ c. for £1 ?

3. If £1 = 24 fr. 57 c., express in francs and centimes £107. 16*s*. 0½*d*., and £256. 3*s*. 4½*d*.

4. How many dollars must be given for a letter of credit on London for £2346. 10*s*., when the exchange is 489 cents for £1 ?

5. What is the arbitrated rate of exchange between Hamburgh and Paris in francs per 100 marcs, when the course of exchange between London and Paris is 25·45 francs for £1, and between London and Hamburgh 2048 pfennige for £1 ?

6. A person in London owes another at Petrograd 500 roubles, exchange at 40*d*. sterling per rouble, but remits to Paris at 24 francs per pound sterling ; thence to Lisbon at 500 reis for 3 francs ; thence to Amsterdam at 20 stivers per crusado of 400 reis and thence to Petrograd at 25 stivers per rouble ; find the arbitrated rate between London and Petrograd and the gain or loss by the circuitous mode of remittance.

7. The rate of exchange between London and Petrograd is 31½*d*. for one rouble, between Vienna and Petrograd is 95⅝ florins for 60 roubles, and between Paris and Vienna is 93½ florins for 200 francs ; find the arbitrated rate between London and Paris in francs for £1 sterling.

8. If London exchanges with Holland at a gain of 6¼ per cent. when the rate of exchange is at 35*s*. 6*d*. per £ sterling ; what is the par of exchange ?

9. A person on leaving England exchanged his English money for French at the rate of 25 francs for a sovereign, and on arriving at Munich received 134 Bavarian gulden for 300 francs ; what was his loss (i) in English money, (ii) in French money, supposing a gulden to be worth 1*s*. 8½*d*. ?

10. The rate of exchange being £1 = 25·4 francs, 3 75 francs = 105 kreutzers, 60 kreutzers = 1 florin and the cost of travelling in

Germany being $1\frac{1}{2}$ florins per German mile which is equal to $4\frac{1}{2}$ English miles ; find the expense, in English money, of travelling 381 English miles in Germany.

11. If £1 be worth 11·75 Dutch guilders, 101 thalers worth 175 guilders, and 2 thalers worth 7·35 francs, how many francs should be received for £40 ?

12. Exchange 5220 dollars for English money when it is at a premium of $7\frac{1}{2}$ per cent., given that at par 1 dollar = 4*s.* 2*d.*

13. Exchange *Rs.*19000 for English money when it is at a discount of 5 per cent., given that at par *Re.*1 = 2*s.*

14. A merchant in Calcutta wishes to remit *Rs.*4100 to London a rupee being equivalent to 1*s.* 3*d.* For what sum in English money must he *draw* his bill when bills on London are at a premium of $2\frac{1}{2}$ per cent. ?

15. Calculate the par of exchange between the dollar and the shilling, when British standard silver is valued at $60\frac{5}{8}$*d.* per oz., having given that 1 dollar weighs $412\frac{1}{2}$ grains, and is $\frac{9}{10}$ fine ; and 1 lb. Troy standard silver, $\frac{37}{40}$ fine, is coined into 66 shillings.

16. When British standard silver is valued at $61\frac{5}{8}$*d.* per oz., find how many francs are equal to 20*s.*, having given that 1 lb. Troy standard silver, $\frac{37}{40}$ fine, is coined into 66 shillings and 15432 grains French standard silver, $\frac{9}{10}$ fine, is coined into 200 francs.

17. A person in London owes another at Petrograd 2460 roubles 50 copecks, which must be sent to him through Paris. He pays the needful sum to his broker at a time when the exchange between London and Paris is 25 fr. 35 c. for £1, and between Paris and Petrograd 339 centimes for 1 rouble. The broker delays remitting until the rates are 25 fr. $62\frac{1}{2}$ c. for £1, and 337 centimes for one rouble. What does the broker gain or lose by the delay ?

18. Find the value of £1 in marcs and pfennige of Germany, having given that 15432 grains of fine gold is coined into $139\frac{1}{2}$ 20 marc pieces, that 1 lb. of standard gold is coined into $46\frac{29}{40}$ sovereigns and that standard gold is $\frac{11}{12}$ fine.

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556. Annuities. An **Annuity** is a fixed sum paid periodically under certain stated conditions ; the payment may be made either once a year or at more frequent intervals.

Annuities are called **certain** when they are payable for a fixed number of years ; when they are payable during the lifetime of a person, or of the survivor of a number of persons, they are called **contingent** or **life** annuities ; and when they continue for ever, they are called **perpetual** annuities. An annuity is said to be **in possession**, when it is payable at present ; but when the payment is not to begin until after the lapse of a certain number of years, it is called a **deferred annuity** or **reversion**.

If the annuity is to continue for ever, it is called a **perpetuity**. An annuity is said to be worth as many **years' purchase** as there are pounds in the value of the annuity of £1.

557. A freehold estate is an estate which yields a perpetual annuity called the *rent*, and thus the value of the estate is equal to *so many years' purchase* or so many years' rent.

Ex. 1. If a freehold estate be worth 20 years' purchase, find the rate of interest.

If the annual rent be £1, the value of the estate is £20.

∴ rate of interest $=(\frac{1}{20}\times 100)$ or 5 per cent. *Ans.*

Ex. 2. If the rate of interest be 4 per cent., how many years' purchase is an estate worth?

The interest would amount to £100 in (£100 ÷ £4) or 25 years.

∴ the estate is worth 25 years' purchase. *Ans.*

Ex. 3. An estate is bought at 25 years' purchase for £15000, two-thirds of the purchase money remaining on mortgage at 3 per cent. The cost of repairs averages £100 per annum. What interest does the purchaser make on his investment?

$\frac{2}{3}$ of £15000 or £10000 remains on mortgage; so he invests only £(15000 − 10000) or £5000. Also the estate being bought at 25 years' purchase, he gets yearly $\frac{1}{25}$ of £15000 = £600.

Now he has to pay $\frac{3}{100}\times$ £10000 or £300 for interest on mortgage.
∴ his total expenses are £300 + £100 = £400.

Hence, on £5000 he gets an annual income of £(600 − 400) or £200.
∴ £5000 : £100 :: £200 : rate per cent. reqd.
∴ rate per cent. reqd. = 4. *Ans.*

Examples CLXVIII.

1. A freehold estate is sold at $28\frac{1}{2}$ years' purchase. What rate of interest is received on the investment?

2. How many years' purchase, *i.e.*, how many years' rental should be paid for freehold property to clear $3\frac{1}{3}$ per cent.?

3. A person purchased a freehold estate for *Rs.*40000; what is the annual rent, if it is worth 25 years' purchase?

4. A freehold estate worth *Rs.*2800 a year is sold for *Rs.*70000; find the rate of interest.

5. An estate is bought at 20 years' purchase for *Rs.*200000, three-quarters of the purchase money remaining on mortgage at 4 per cent. The cost of repairs averages *Rs.*1500 per annum. What interest does the purchaser make on his investment?

6. A College purchases 47 ac. 3 ro. 9 po. of land for £2824; and 33 ac. 1 ro. 20 po. of land for £1974; the incidental expenses of

transfer, &c. amount on the first lot to £60, and on the 2nd lot to £40; the first lot realizes a yearly rent of £2. 4*s.* an acre; the second lot a yearly rent of 50*s.* an acre. How many years' purchase was given for all the land?

7. An estate is bought at $25\frac{1}{2}$ years' purchase for £3400; three-fifths of the purchase money remains on mortgage at $4\frac{1}{2}$ per cent. The cost of repairs, &c. averages £9. 6*s.* 8*d.* What interest does the purchaser make on his investment?

8. The annual rent of a freehold estate bought for £8817. 5*s.* 3*d.*, is £345. 15*s.* 6*d.*; at how many years' purchase was it bought?

CHAPTER XV.

Stocks and Shares.

558. **Stocks.** **Stock** is the term applied to money lent to the Government of a country, or to a Trading Company, at some specified rate of interest.

When the Government of a country wants money for any purpose, such as to carry on any costly war or to construct a large railway, it generally *borrows* or *contracts a Loan.* If the money is borrowed from the nation itself, it is called the **National Debt**; but if borrowed from people of other nations, it is called a **Foreign Loan**, or more briefly, a **Loan**. The borrowing of money by the Government is effected by giving to the *Lenders* in exchange for their money, **Government Bonds** or **Acknowledgments,** implying that the Nation is indebted to them for the sums advanced, whilst it reserves to itself the option of the *Time* of paying off the *Principal* on the express condition to pay the interest on it regularly at fixed periods in the mean time.

Thus, if the Government of India were to borrow to the amount of 5 crores of rupees at 4 per cent., and *A* had lent *Rs.*1000 of this sum, *A* would be said to have *Rs.*1000, 4 per cent. Stock, and would receive a document entitling him to receive the Interest, (*viz.*, *Rs.*40) upon this stock from year to year, until the Government chose to repay the Principal and put an end to the debt.

559. The source from which the Interest is paid is called the **Public Funds**, or simply the **Funds**, being however, only an imaginary property, representing the credit of the country itself, which is pledged to the payment of the debts contracted by its Government; the Interest is paid *half yearly*, and the document, entitling the possessor to receive it, may be *sold* and *transferred* from one party to another, just as any other kind of property.

560. The debts of the British and the Indian Governments are

in the form of **Bonds** or **Promissory Notes**. Money lent to the Government of India is said to be invested in **Government of India Securities**; but when lent to the Government of England, it is said to be invested in the **Funds**.

561. The following are the most important of the English Stocks :—

(1) The **Consols** (abbreviated from *Consolidated Annuities*) are so called from the consolidation or amalgamation of several loans into one loan, bearing the uniform interest of 3 per cent., now reduced to $2\frac{3}{4}$ per cent.

(2) The **Reduced Consols** are so called because their rate of interest was *reduced* from a higher to a lower rate at a subsequent date.

(3) The **New 3 per cent. Stock** has originated from the conversion of a higher stock to a 3 per cent. stock.

Besides these, there is a small amount of New $2\frac{1}{2}$ per cent. Stock, and still smaller amount of New $3\frac{1}{2}$ per cent., and New 5 per cent. stock.

562. It is usual to name the several kinds of stocks according to the annual rates of interest they bear; as 'the $2\frac{1}{2}$ per cent. consols,' 'the Turkish 5 per cents.,' 'the 4 per cents.,' 'the 3 per cents.,' 'the 4 per cent. promissory note,' and so on. The prices of the various stocks published in the money-market column in Newspapers, are said to be **quoted**.

Thus, if the 4 per cents. be *quoted* in the money market at 96, the meaning of this is, that for *Rs.*96 or £96 of money a person can purchase *Rs.*100 or £100 stock, which will entitle the owner to receive a half-yearly dividend of *Rs.*2 or £2 from Government.

563. The fluctuations in the price of stock are not caused by any variation in the rate of interest which is paid; for it is fixed once for all by the Government at the time the money is borrowed. But commercial or political changes or expectations at home and abroad constantly disturb the price of stock, even two or three times in the *same* day, according to circumstances Hence, if a person *sell out* his stock from the Funds, he will be able to obtain more or less sterling money for each of his bonds, according to the interest it bears and also according to the circumstances of the times, which may influence the *stability* of the national credit; and if he *buy into* or *invest capital in* the Funds, the sum of ready money advanced by him for each bond will be regulated by the same circumstances.

Thus, if at the time *A* wished to sell his stock, money was elsewhere making 5 per cent., it is plain that no one would give him £100 for the right to receive 4; but since £80 of common or *sterling* money (as it is called) would now bring £4 interest, he would be able to sell his £100 stock for £80; and the 4 per cents. would be said to be selling at £80.

564. The students should most carefully note the difference *between the paper or nominal value of stock and the cash or actual price*, and also *between the sum invested and the amount of stock held*. Thus, if the 4 per cents. are at 96, a man who invests *Rs.*19200 will be able to purchase *Rs.*20000 stock. And, therefore while the amount of stock held by the man in this case is *Rs.*20000, the actual or cash value of that stock is only *Rs.*19200. It is also worthy to note the expression *so many in the so many per cents.*—for instance '£5000 in the 4 per cents.' The meaning of this expression is *not that the man invested* £5000 in the 4 per cents., but that he holds *stock of the nominal value* of £5000 in the 4 per cents. If we suppose these 4 per cents. to be at 98, the sum invested at the time of purchase would be £4900.

565. Shares. Trading Companies raise money in a different way. Suppose several persons interested in a particular line of business, such as the construction of a railway, meet together and propose to start a company. These men, called **Promoters**, first decide what amount of money or **Capital** will be required for the purpose, and elect among themselves a few who are most fit to manage, and direct the affairs of the company. These men are called the **Directors** and the association a **Joint stock company**. The directors then issue a prospectus, stating the amount of capital required and the rate of profit expected if the project succeeds; they divide the capital into a large number of equal parts or **shares** and invite the public to **subscribe** towards them. A man can take as many shares as he pleases, and thus become a **Shareholder**. Generally the whole amount of a share is not paid up at once. The directors make several **Calls** upon the shareholders for paying up each time a part of the amount. Some times the whole capital is not required but only a part of it, and each shareholder has to pay a proportionate part of his share. The part which is thus paid is called the **Paid-up Capital**. When all the shares are fully paid up, and when, therefore, the capital considered necessary for the project is raised, the Company usually *converts its shares into stock*, because, in the case of Stock, transactions can be carried on with reference to *any portions of it*, whereas in the case of Shares, fractional parts of those Shares cannot be transferred. When the concern begins to yield profits, they are divided among the shareholders at so much per share or generally at so much per cent, at regular intervals usually six months, after all the expenses are paid and a certain amount is set aside to meet contingencies. This is called the **Reserve Fund**. What each shareholder receives as his portion is called his **Dividend**. Hence it appears that the rise or fall in the market value of shares depends very much on the dividend which is declared.

Suppose a company's subscribed Capital is 20 lacs divided into 20,000 shares of *Rs.*100 each, and the paid-up capital 16 lacs or *Rs* 80 per share. If the *net* profits at the end of six months be *Rs.*25000, the dividend would be *Rs.*25000 for 20000 shares or *Re.*1.

4*a*. per share ; and the percentage on the paid-up capital would be *Re*.1. 4*a*. on *Rs*.80 in the half year, *i. e.*, $3\frac{1}{8}$ per cent. per annum.

566. Preference Stock. When the originally subscribed capital of a Company proves insufficient to carry on the project, it does not, in such cases, issue more shares of the kind issued at first, but borrows money at a fixed rate of interest promising to pay this interest before the profits are divided among the shareholders. Capital raised in this way is called **Preference Stock**, as distinguished from the capital subscribed at the beginning, which is called **Ordinary stock**.

567. Debentures. The deeds of mortgage or *bonds* given by Joint-stock Companies, Municipalities and similar other Corporated bodies for *borrowed money*, are called **Debentures.**

568. Par, Premium, &c. When the market values of stocks are the same as their paper or nominal values, they are said to be **at par** ; when greater, **at premium** or **above par** ; when less, **at Discount** or **below par.**

Thus, if *Rs*.100 stock sells for *Rs*.100, it is said to be *at par* ; if at *Rs*.105, *at 5 premium* or *above par* ; and if at 96, *at 4 Discount* or *below par.*

569. Brokerage. Purchases and sales of Stocks, Shares, &c. are usually made through *Agents*, called **Stock-Brokers** or **Brokers,** who generally charge $\frac{1}{8}$ per cent. (*i. e.*, 2*a*. on *Rs*.100, or 2*s*. 6*d*. on £100) upon the amount of Stock bought or sold. Hence, the *brokerage* must be *added* to the price of Stock which is *bought* and *subtracted* from the price of that which is *sold* through them.

Thus, if the market value of *Rs*.100 stock be *Rs*.102. 8*a*., the buyer will have to pay *Rs*.102. 10*a*., while the seller will get only *Rs*.102. 6*a*. Unless the brokerage is mentioned, it need not be noticed in working Examples in Stocks.

570. All Examples in Stocks depend upon the *Rule of Proportion* and may therefore be solved by the *Rule of Three.*

Ex. 1. What sum of money will purchase *Rs*.24000 stock in the 3 per cents. at $89\frac{1}{2}$?

Here, *Rs*.100 stock costs *Rs*.$89\frac{1}{2}$ in money ;

$\therefore$ *Rs*.100 stock : *Rs*.24000 stock :: *Rs*.$89\frac{1}{2}$: reqd. sum of money ;
$\therefore$ the reqd. sum of money = *Rs*.$(240 \times 89\frac{1}{2})$ = *Rs*.21480. *Ans.*

Ex. 2. What sum must be invested to purchase £2300 stock in the 3 per cents. at $90\frac{3}{8}$, brokerage $\frac{1}{8}$ per cent. ?

Here, £100 stock costs, with brokerage, £$(90\frac{3}{8}+\frac{1}{8})$ or £$90\frac{1}{2}$;
$\therefore$ £100 stock : £2300 stock :: £$90\frac{1}{2}$: reqd. sum of money ;
$\therefore$ reqd. sum of money = £$(23 \times 90\frac{1}{2})$ = £2081. 10*s*. *Ans.*

Ex. 3. A person has one lac of rupees stock in a Tea Company,

whose shares are at 135 per cent. premium. He sells out; what amount of money will he receive, brokerage being $\frac{1}{8}$ per cent.?

Here, *Rs.*100 stock sells for *Rs.*$(235-\frac{1}{8})$ or *Rs.*$234\frac{7}{8}$ in money;
∴ *Rs.*100 stock : *Rs.*100000 stock :: *Rs.*$234\frac{7}{8}$: reqd. sum of money;
∴ reqd. sum of money $=$ *Rs.*$(1000\times 234\frac{7}{8})=$ *Rs.*234875. *Ans.*

Ex. 4. How much will be received from the sale of £2450. 10*s.* stock in the $3\frac{1}{2}$ per cents. at $96\frac{1}{2}$, brokerage $\frac{1}{8}$ per cent.?

Here, £100 stock realises £$(96\frac{1}{2}-\frac{1}{8})$ or £$96\frac{3}{8}$, in money;
∴ £100 stock : £$2450\frac{1}{2}$ stock :: £$96\frac{3}{8}$: reqd. sum of money;
∴ reqd. sum of money $=$ £$(24\cdot505\times 96\cdot375)=$ £2361. 13*s.* $4\frac{1}{2}$*d.* $\frac{3}{5}$*q.*

Ex. 5. How much stock can be purchased by investing *Rs.*28500 in the 3 per cents. at 75?

Here, *Rs.*75 in money will purchase *Rs.*100 stock;
∴ *Rs.*75 : *Rs.*28500 :: *Rs.*100 : reqd. amount of stock;
∴ reqd. stock $=$ *Rs.*$(28500\times 100\div 75)=$ *Rs.*38000. *Ans.*

Ex. 6. Find the quantity of stock purchased by investing £2353 in the 3 per cents. at $90\frac{3}{8}$, brokerage $\frac{1}{8}$ per cent.

Here, £$(90\frac{3}{8}+\frac{1}{8})$ or £$90\frac{1}{2}$ will be required to purchase £100 stock.
∴ £$90\frac{1}{2}$: £2353 :: £100 stock : stock reqd.
∴ reqd. stock $=$ £$(2353\times 100\div 90\frac{1}{2})=$ £2600. *Ans.*

Ex. 7. What amount of India 5 per cent. stock at $111\frac{5}{8}$ must be sold to realise *Rs.*17728. 8*a.* through the agency of a broker?

By selling *Rs.*100 stock, *Rs.*$(111\frac{5}{8}-\frac{1}{8})$ or *Rs.*$111\frac{1}{2}$ can be realised;
∴ *Rs.*$111\frac{1}{2}$: *Rs.*$17728\frac{1}{2}$ :: *Rs.*100 stock : reqd. quantity of stock;
∴ reqd. stock $=$ *Rs.*$(17728\frac{1}{2}\times 100\div 111\frac{1}{2})=$ *Rs.*15900. *Ans.*

Ex. 8. A person invests £14340 in consols when they are at $89\frac{1}{2}$ and sells out when they are at $93\frac{3}{8}$; what is his gain? (brokerage as usual).

Here, the stock which costs £$(89\frac{1}{2}+\frac{1}{8})$ or £$89\frac{5}{8}$ is sold for £$(93\frac{3}{8}-\frac{1}{8})$ or £$93\frac{1}{4}$; therefore on every £$89\frac{5}{8}$ there is a gain of £$(93\frac{1}{4}-89\frac{5}{8})$ or £$3\frac{5}{8}$.

∴ £$89\frac{5}{8}$: £14340 :: £$3\frac{5}{8}$: gain required;
∴ gain reqd. $=$ £$(14340\times 3\frac{5}{8}\div 89\frac{5}{8})=$ £580. *Ans.*

Ex. 9. A person buys Railway Stock at $89\frac{3}{4}$, and sells out at $103\frac{1}{2}$, and clears *Rs.*3850; how much money did he invest?

Here, what cost him *Rs.*$89\frac{3}{4}$ he sells for *Rs.*$103\frac{1}{2}$, and therefore, on every *Rs.*$89\frac{3}{4}$ invested, he gains *Rs.*$(103\frac{1}{2}-89\frac{3}{4})$ or *Rs.*$13\frac{3}{4}$.

∴ *Rs.*$13\frac{3}{4}$: *Rs.*3850 :: *Rs.*$89\frac{3}{4}$: money invested;
∴ reqd. money $=$ *Rs.*$(3850\times 89\frac{3}{4}\div 13\frac{3}{4})=$ *Rs.*25130. *Ans.*

Note. In Examples like the above, the rate of interest is not considered, for no question of **Income** enters in the statement.

Examples CLXIX.

1. How much money must be given for the purchase of :—

(1) *Rs.*4500 in the $3\frac{1}{2}$ per cents. at 88 ?

(2) £5550 in the 4 per cents. at $97\frac{5}{8}$?

(3) £439. 12*s.* 5*d.* in the $3\frac{1}{2}$ per cents. at $92\frac{7}{8}$?

(4) *Rs.*46494 in the 4 per cents. at 81 ?

(5) £650 in the 3 per cents. at $90\frac{3}{8}$, brokerage as usual ?

(6) *Rs.*32577. 12*a.* in the $3\frac{1}{2}$ per cents. at $91\frac{3}{8}$, brokerage $\frac{1}{8}$ per cent. ?

(7) *Rs.*8833. 5*a.* 4*p.* in the 5 per cents. at $3\frac{3}{4}$ premium, brokerage $\frac{1}{8}$ p.c. ?

(8) £1229. 3*s.* 4*d.* in the 4 per cents. at $\frac{3}{8}$ discount, brokerage $\frac{1}{8}$ p.c ?

2. How much money can be obtained from the sale of :—

(1) *Rs.*16000 in the $3\frac{1}{2}$ per cents. at $88\frac{1}{8}$?

(2) *Rs.*12505. 4*a.* in the 3 per cents. at $64\frac{1}{2}$?

(3) *Rs.*7569. 6*a.* in the 5 per cents. at $6\frac{3}{8}$ premium ?

(4) £15850. 16*s.* 8*d.* in the $3\frac{1}{2}$ per cents. at $98\frac{3}{8}$, brokerage as usual

(5) *Rs.*73515 in the 4 per cents. at $96\frac{1}{2}$, brokerage $\frac{1}{8}$ per cent. ?

3. Find the quantity of stock purchased by investing :—

(1) *Rs.*8280 in the $3\frac{1}{2}$ per cents. at 92.

(2) £821. 5*s.* in the 4 per cents. at $82\frac{1}{8}$.

(3) £6451. 3*s.* 6*d.* in Bank Stock at $217\frac{1}{2}$.

(4) *Rs.*125466. 13*a.* 4*p.* in India Stock at 252.

(5) *Rs.*23172. 6*a.* in the 3 per cents. at $95\frac{5}{8}$, brokerage as usual.

(6) £1101. 6*s.* 8*d.* in M. Railway Stock at $8\frac{1}{2}$ above par.

(7) *Rs.*6307. 12*a.* in R. Railway Stock at $27\frac{1}{2}$ below par.

(8) £506. 9*s.* 2*d.* in the $3\frac{1}{4}$ per cents. at $89\frac{1}{4}$, brokerage $\frac{1}{8}$ per cent.

4. What amount of stock must be sold, when the quotation is $12\frac{3}{8}$ above par, to realise £11306. 11*s.* ?

5. If I lay out *Rs.*13597. 8*a.* in the purchase of the 3 per cents. at $92\frac{3}{8}$ and afterwards sell it at $94\frac{1}{4}$, what profit shall I make ? (brokerage as usual.)

6. A person expended £2653. 10*s.* in the purchase of the New $3\frac{1}{2}$ per cents. at $97\frac{3}{4}$, and after a time sold out at $96\frac{1}{8}$; find his loss, the usual brokerage being charged on each transaction.

7. A person bought some $2\frac{1}{2}$ per cent. stock at $65\frac{1}{2}$, and sold it when the price had risen to $69\frac{1}{8}$, thereby gaining *Rs.*1256. 4*a.* ; how much money did he lay out ?

8. If a person invest *Rs.*126540 in the 4 per cents. at $99\frac{3}{4}$, at what price must he sell to gain *Rs.*15817. 8*a.* ?

9. How much stock in the 4 per cents. must be bought at 96, in order that by selling out at par *Rs.*250 may be gained?

10. How much must a person invest in the 3 per cents. when they are at 10 per cent. below par, that by selling out when they are at 5 per cent. premium he may gain *Rs.*1250?

11. I buy £5000 stock in the 3 per cents. at 85, and sell out when the funds have fallen $3\frac{1}{8}$ per cent. What do I lose by the transaction?

12. A person sold *Rs.*60000 3 per cent. stock at 95, thereby realising *Rs.*800 more than his investment. At what price had he bought in?

13. A person invests *Rs.*15000 in the $3\frac{1}{2}$ per cents. at 98. He sells his stock when the funds rise 4 per cent., and invests the proceeds in the same stock when they sink again to 98. How much more stock does he now hold?

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571. When a **transfer** of capital is made from one kind of stock to another, it is evident that there will be an equitable claim for *more* or *fewer* bonds of the second stock, according as the rate of interest of such bonds is *less* or *greater* than that of the first; thus, a number of bonds or *quantity of stock* in the 4 per cents., will produce the same interest as a *greater* quantity of stock in the 3 per cents., and consequently be of the same value to the possessor in point of income. Hence, all questions on the transfer of stock from one kind to another belong to the Rule of Three Inverse.

Ex. 1. What annual income will be derived from *Rs.*42750 of 4 per cent. paper?

Here, *Rs.*100 paper yieds *Rs.*4 interest per annum;

∴ *Rs.*100 : *Rs.*42750 :: *Rs.*4 : annual income reqd.

∴ income reqd. = *Rs.*(427·5 × 4) = *Rs.*1710. *Ans.*

Ex. 2. What income will be derived from investing £3220 in the $3\frac{1}{2}$ per cent. stock at $80\frac{1}{2}$?

Here, for every £$80\frac{1}{2}$ we get £100 stock, and the int. on £100 stock is £$3\frac{1}{2}$; therefore for every £$80\frac{1}{2}$ of money we get £$3\frac{1}{2}$ interest;

∴ £$80\frac{1}{2}$: £3220 :: £$3\frac{1}{2}$: reqd. annual income;

∴ reqd. income = £$(3220 \times 3\frac{1}{2} \div 80\frac{1}{2})$ = £140 *Ans.*

Ex. 3. What sum must a person invest in the 3 per cent. stock at $94\frac{5}{8}$ to have an annual income of *Rs.*240 brokerage $\frac{1}{8}$ per cent.?

To get *Rs.*3 annually he will have to invest *Rs.*$(94\frac{5}{8} + \frac{1}{8})$ or *Rs.*$94\frac{3}{4}$

∴ *Rs.*3 : *Rs.*240 :: *Rs.*$94\frac{1}{2}$: reqd. investment;

∴ reqd. investment = *Rs.*$(80 \times 94\frac{1}{2})$ = *Rs.*7560. *Ans.*

Ex. 4. Find the alteration in income occasioned by transferring *Rs.*32000 stock from the 3 per cent. stock at $86\frac{5}{8}$ to the 4 per cent. stock at $114\frac{7}{8}$, the brokerage being $\frac{1}{8}$ per cent. on each transaction.

The price of *Rs.*100 stock (excluding brokerage) in the 3 *p. c.* is *Rs.*$(86\frac{5}{8}-\frac{1}{8})$ or *Rs.*$86\frac{1}{2}$; and the price of *Rs* 100 stock (including brokerage) in the 4 *p. c.* is *Rs.*$(114\frac{7}{8}+\frac{1}{8})$ or *Rs* 115.

∴ *Rs.*100 : *Rs.*32000 :: *Rs.*$86\frac{1}{4}$: proceeds of sale of the 3 *p. c.*

∴ proceeds of sale = *Rs.*$(320 \times 86\frac{1}{4})$ = *Rs.*27600.

Now, *Rs.*115 : *Rs.*27600 :: *Rs.*4 : income from the 4 per cents.

∴ income from the 4 per cents. = *Rs.*(27600 × 4 ÷ 115 = *Rs.*960.

Also *Rs.*100 : *Rs.*32000 :: *Rs.*3 : income from the 3 per cents.

∴ income from the 3 per cents. = *Rs.*(3 × 320) = *Rs.*960.

Hence, the income remains the same. ***Ans.***

Ex. 5. Find the price of the 4 per cent. stock, that an investment of £4680 may produce an income of £180, brokerage as usual.

£180 : £4 :: £4680 : cost of £100 stock (including brokerage) ;

∴ cost (including brokerage) = £(4680 × 4 ÷ 180) = £104.

Hence the price of £100 stock = £$103\frac{7}{8}$. ***Ans.***

Examples CLXX.

1. What annual income will be derived from the following ?—

(1) £3300 of 5 p. c. stock. (2) *Rs.*90150 of $3\frac{1}{2}$ p. c. stock.

(3) £4326. 10*s.* of $2\frac{3}{4}$ p. c. stock. (4) *Rs* 82250 of $3\frac{1}{2}$ p. c. paper.

2. What yearly income will arise from the following investments ?—

(1) *Rs.*55800 in the 4 p c. at 93. (2) £4788 in the $3\frac{1}{2}$ p. c. at 105.

(3) *Rs.*35190 in the $3\frac{1}{2}$ p. c. at 90. (4) *Rs.*21755 in the $4\frac{1}{4}$ p. c. at $89\frac{1}{2}$.

(5) *Rs.*23500 in the $3\frac{1}{4}$ per cents. at $97\frac{3}{8}$, brokerage as usual.

(6) *Rs.*276438. 12*a.* in the 3 per cents at $91\frac{5}{8}$, brokerage $\frac{1}{8}$ per cent.

3. How much money must a person invest in the under-mentioned stocks in order to secure the following incomes ?—

(1) *Rs* 600 in the 3 per cents. at 85.

(2) £73. 10*s.* in the $3\frac{1}{2}$ per cents. at 85.

(3) *Rs.*1465 in the 3 per cents. at $90\frac{1}{8}$, brokerage $\frac{1}{8}$ per cent.

(4) 200 guineas in the $3\frac{1}{4}$ per cents. at $96\frac{5}{8}$, brokerage as usual.

4. What half-yearly dividend is derived from an investment of *Rs.*3000 in $3\frac{1}{4}$ per cent. stock at $98\frac{3}{4}$, after deducting income-tax at the rate of 3*p.* in the rupee ?

5. What half-yearly dividend is due upon an investment of

£5000 in 3 per cent. stock at $87\frac{3}{8}$, after deducting 7*d.* in the pound for income-tax ?

6. A man invests £4031. 10*s.* in the 3 per cents. at $94\frac{5}{8}$; what will be his net income after an income-tax of 10*d.* in the pound has been deducted, $\frac{1}{8}$ p. c. brokerage being allowed ?

7. What must be the price of the 3 per cents., so that by investing £16425 a man may have a clear income of £515. 5*s.* after an income-tax of 11*d.* in the pound has been deducted ?

8. What sum must a person invest in the 4 per cents. at $91\frac{1}{8}$, in order to have a clear income of *Rs.*2300 after paying an income-tax of 6*p.* in the rupee ?

9. A person having £10000 in the 3 per cents. sells out at 65 and invests the produce in the 4 per cents. at $82\frac{1}{2}$: find the change in his income.

10. A person transfers *Rs.*11000 from the 4 per cents. at 92 to the 5 per cents. at 110 ; what is the difference in his income ?

11. A person invested £9075 in the 3 per cents. at $90\frac{3}{4}$, and on the stock rising to 91 transferred it to the $3\frac{1}{2}$ per cents. at $97\frac{1}{2}$. What increase does he thereby make in his annual income ?

12. Find the alteration in income occasioned by shifting £11000 from the 3 per cents. at $101\frac{3}{8}$ to $4\frac{1}{2}$ per cent. debentures at $137\frac{5}{8}$, the usual brokerage being charged on each transaction.

13. £5151 is invested in 5 per cent. stock at 101 ; the stock rising to 105, it is sold out, and the proceeds invested in stock at 102, which gives $4\frac{3}{4}$ per cent. interest. Find the change in income.

14. By selling out *Rs.*45000 in the India 5 per cent. stock at $112\frac{1}{2}$, and investing the proceeds in the Egyptian 7 per cent. stock, a person found his income increased by *Rs.*1687. 8*a.* What was the price of the Egyptian stock ?

15. The difference between the incomes derived from investing a certain sum in 6 per cent. stock at 126, and in 9 per cent. stock at 210, is *Rs.*225. What is the amount invested ?

16. A person laid out *Rs.*25500 in a $3\frac{1}{2}$ per cent. stock at 91 and after receiving the half-year's dividend he sold out at $90\frac{7}{8}$; how much did he gain ?

17. An income of £126 is obtained by investing £3591 in the 3 per cents. ; what is the price of the stock ?

18. A person invests *Rs.*4095 in 4 per cent. stock, and obtains an income of *Rs.*173. 5*a.* 4*p.* ; what is the price of the stock ?

19. A person bought the M. Railway Stock at $88\frac{3}{4}$, and after receiving the half-year's dividend at the rate of $4\frac{1}{2}$ per cent. per annum sold out at $93\frac{5}{8}$ and made a profit of *Rs.*1425 ; how much stock did he buy ?

20. If a person invest £25350 in the 3 per cent. consols at 92⅝, at what price must he sell out after receiving the dividend to make a profit of £250?

21. A person invests *Rs.*9625 in the 3 per cents. at 77, and when the funds have fallen 1 per cent. he transfers his capital to the 4 per cents. at 95; find the alteration in his income.

22. A person invested £5330 in the 3 per cents. at 91, and when they had risen 1¾ per cent. he sold out and invested the money in the India stock at 102½; how much India stock does he hold?

23. A person laid out *Rs.*7492. 8*a.* in the purchase of 5 per cent. stock at par, and after receiving the half-yearly dividend he sells out at 4 premium and invests the proceeds in the C. Railway shares at 87⅞; how much Railway stock does he hold?

24. A man invests £4297. 10*s.* in the 3 per cents. at 95½. He sells out one-third when the funds have fallen to 94, £1600 stock when they have risen to 96¼, and the remainder at par. What sum does he gain? If he invests the proceeds in the shares of a gold mine paying 3 per cent. at 67·5, what would be the difference in his income?

572. Miscellaneous Questions on Stocks.

Ex. 1. At what rate will a person receive interest, who invests his capital in the 4 per cents. when they are at 103⅞? brokerage ⅛ %

Since £(103⅞ + ⅛) or £104 produces an interest of £4 annually,

∴ £104 : £100 :: £4 : rate per cent. required.

∴ rate per cent. required = £(100 × 4 ÷ 104) = £3$\frac{11}{13}$. *Ans.*

Ex. 2. Find the price of the 3 per cents. when *Rs.*2600 stock can be purchased for *Rs.*2353, brokerage ⅛ per cent.

*Rs.*2600 : *Rs.*100 :: *Rs.*2353 : price of *Rs.*100 stock (including brokerage);

∴ price (including brokerage) = *Rs.*(2353 ÷ 26) = *Rs.*90½.

∴ price of *Rs.*100 stock = *Rs.*(90½ − ⅛) = *Rs.*90⅜ or *Rs.*90. 6*a.* *Ans.*

Ex. 3. A person investing in the 3¼ per cents. pays ⅛ per cent. for brokerage, and obtains 4 per cent. on his money. At what price does he buy in?

£4 : £3¼ :: £100 : price of 3¼ per cent. stock (including brokerage)

∴ the price (including brokerage) = £(100 × 3¼ ÷ 4) = £81¼.

∴ the price of £100 stock = £(81¼ − ⅛) = £81⅛. *Ans.*

Ex. 4. Which is the better stock to invest, £10000 in the 3 per cents. at 90⅜, or the 4 per cents. at 101?

From the 1st investment, income on £90⅜ = £3, or on £1 = £$\frac{8}{241}$.

... ... 2nd ... ... £101 = £4, or on £1 = £$\frac{4}{101}$.

Now, comparing the fractions $\frac{6}{181}$ and $\frac{4}{101}$,

since 4×181 is $> 6 \times 101$,

the 2nd fraction $>$ the 1st, and $\therefore$ the 2nd investment is the better. *Ans.*

Otherwise thus :—

£$90\frac{1}{2}$ invested in the 3 per cents. yields yearly £3,
and £$90\frac{1}{2}$ ... ... 4 per cents. at 101 yields

(£101 : £$90\frac{1}{2}$ :: £4 : *Ans.*) or £$(90\frac{1}{2} \times 4 \div 101)$ = £$3\frac{59}{101}$.

$\therefore$ the 4 per cents. is the better investment. *Ans.*

Ex. 5. At what price must I invest in the 4 per cents. so that after paying 6*d.* in the pound income-tax I may receive $4\frac{1}{2}$ per cent. on my money?

The tax being 6*d.* in the £, it is $\frac{6}{240}$ or $\frac{1}{40}$ of the gross income, and $\therefore$ the net income $= (1 - \frac{1}{40})$ or $\frac{39}{40}$ of the gross income.

$\therefore$ £$\frac{39}{40}$: £$4\frac{1}{2}$ :: £1 : gross income ; $\therefore$ gross income = £$\frac{60}{13}$.

Now, £$\frac{60}{13}$: £4 :: £100 : price of 4 per cent. stock ;

$\therefore$ price reqd. = £$(100 \times 4 \div \frac{60}{13})$ = £$86\frac{2}{3}$. *Ans.*

Ex. 6. A person finds that if he invests his money in the 4 per cents. at 92 his income will be less by £21 than if he invests it in the $4\frac{1}{2}$ per cents. at par ; find the sum to be invested.

From the 1st he gets £$\frac{4}{92}$ or £$\frac{1}{23}$ per £,

and from the 2nd $\frac{£4\frac{1}{2}}{100}$ or £$\frac{9}{200}$ per £.

$\therefore$ from the 2nd investment he gets £$(\frac{9}{200} - \frac{1}{23})$ or £$\frac{7}{4600}$ more per £.

$\therefore$ £$\frac{7}{4600}$: £21 :: £1 : sum to be invested ;

$\therefore$ sum reqd. = £$21 \times \frac{4600}{7}$ = £13800 *Ans.*

Examples CLXXI.

1. What interest per cent. per annum is obtained from investing money in the following stocks?—

(1) The 5 per cents. at $105\frac{3}{4}$. (2) The 3 per cents. at 91.

(3) ... 3 per cents. at $91\frac{3}{4}$ (B. $\frac{1}{8}$). (4)...$6\frac{1}{2}$ per cents. at $129\frac{7}{8}$ (B. $\frac{1}{8}$).

2. At what price must a person purchase—

(1) The 3 per cent. consols to obtain $3\frac{3}{4}$ per cent. for his money?

(2) $3\frac{1}{2}$ per cents. to get 4 per cent. on his money?

(3) ...$3\frac{1}{2}$ per cents. to get $4\frac{1}{2}$ per cent. on his capital?

3. Find the price of the 3_2 per cents. when *Rs.*23437. 8*a.* stock can be purchased for *Rs* 22500, the usual brokerage being charged.

4. Find the price of the $3\frac{1}{2}$ per cents. when £5050 stock can be purchased for £4311. 8*s*. 9*d*.

5. A man invested in the 3 per cents.; if, after deducting an income-tax of 6*d*. in the pound, he obtained $3\frac{1}{2}$ per cent. interest on the money invested, at what price did he buy?

6. Which stock is the better to invest in, the 4 per cents. at 102, or the $3\frac{1}{2}$ per cents. at 96?

7. Whether is it better to invest in the 3 per cents. at $89\frac{1}{2}$, or in the $3\frac{1}{2}$ per cents. at 95? (Brokerage $\frac{1}{8}$ p. c.)

8. How much 3 per cent. stock has been sold out at 89, if the owner's income is increased by *Rs.*120 a year by investing the proceeds in 4 per cent. stock at 92?

9. What was the price of consols (3 per cent.), when, after paying an income-tax of 5*d*. in the pound, a man received a profit of $3\frac{1}{2}$ per cent. on his money?

10. How much 3 per cent. stock at par must a man sell in order to purchase enough 4 per cent. stock at $124\frac{7}{8}$ to produce an income of *Rs.*3995, the usual brokerage being charged on each transaction?

11. What amount of stock must be sold out of the 3 per cents. at $87\frac{1}{2}$ to pay the present worth of *Rs.* 16458. 12*a*, due 10 months hence at $3\frac{3}{4}$ per cent.?

12. A person has an annual income of £191. 5*s*. from stock in the 3 per cents.; if he were to sell out at $92\frac{3}{4}$ and invest the money in the 5 per cents. at 105, how much of the latter stock would he hold, and what would be the alteration in his income?

13. A person holds *Rs.*46750 stock in the 5 per cents.; what sum must he lay out in the purchase of the $4\frac{1}{2}$ per cents. at $102\frac{1}{4}$ so that his income from both sources may be *Rs.*8435?

14. When the $2\frac{1}{2}$ per cents. are at $83\frac{3}{4}$, what ought to be the price of the $3\frac{1}{2}$ per cents. to give the same rate of interest?

15. Which is the better investment,—3 per cent. stock at $87\frac{1}{8}$, or shares at £233 each, on each of which a dividend of £7. 13*s*. 4*d*. is paid annually? How much more money must be invested in one rather than in the other to produce an annual income of £460?

16. A man sells out his stock in the $3\frac{1}{2}$ per cents. at $97\frac{5}{8}$, and re-invests the money in $4\frac{3}{4}$ per cent. Debentures at $124\frac{7}{8}$, thereby increasing his income by £6. 3*s*.; find the amount of his stock in the $3\frac{1}{2}$ per cents. the usual brokerage being charged on each transaction.

17. If the 4 per cents. give 4 per cent. clear, after paying an income-tax of 7 pies in the rupee, what must be the price of the 4 per cents.?

18. By selling out £3000 in the 4 per cents. at 90, and investing the proceeds in the India 5 per cent. stock, a person finds that his income is increased by £6. 13*s*. 4*d*. What is the price of the India stock?

19. A person invests *Rs*.4000 in the 4½ per cents. at 94½ and a certain sum in the 3½ per cents. at 77. If his total income is *Rs*.800, how much stock does he hold in the 3½ per cents. and how much has he invested therein?

20. A person laid out £1001 in the 3 per cents. at 89⅜, and having received a half-year's dividend, he sold out, and then found that his capital had increased by 72 guineas; find the price at which the stock was sold.

21. What sum must I have invested in the 3½ per cents. at 91, if, after investing £4000 in the 3 per cents. at 75, and paying an income-tax of 7*d*. in the pound on my total gross receipts, I find my net income to be £524. 5*s*.?

22. A person invests a certain sum in the 4 per cents. at 96 and an equal sum in the 3½ per cents. at 75. His total income from both these sources is *Rs*.125o. How much does he invest in each stock?

23. If I invest my money in shares paying *Rs*.7 per share when the *Rs*.100 share is at 122½, I find that I get *Rs*.355 a year more than if I invest it in the 5¼ per cent. bonds at 105; find my capital.

24. A person sells out of the 3½ per cents. at 92¾ and realises *Rs*.18550 If he invests ⅗ths of the produce in the 4 per cents. at 96, and the remainder in the 3 per cents. at 90, find the alteration in his income.

25. By investing a certain sum of money in the 3½ per cents. at 72 a man gets *Rs*.35 less in income than he would get by investing the same sum in the 4½ per cents. at 90; find the sum invested.

26. What sum must a person invest in the 3 per cents. at 90 in order that by selling out *Rs*.20000 stock, when they have risen to 93½, and the remainder when they have fallen to 84¼, he may gain *Rs*.125 by the transaction? If he invest the proceeds in the 4 per cents. at par, what will be the difference in his income?

27. I invest *Rs*.40000 partly in the 3 per cents. at 80, and partly in the 4 per cents. at 96; and then I find that on the whole I receive 4 per cent. interest on the sum invested. What sums have I invested in the two stocks?

28. *A* and *B* invest an equal sum in the 3 per cents. at 90, and the 4 per cents. at 125. *A* purchases an equal amount of each kind of stock, and *B* divides his money equally between the two stocks. If the difference between their incomes is *Rs*.70, what did each invest?

29. A person transferring his stock from the 3 per cents. at 90 to the 4 per cents. increases his annual income by 10 per cent. Find the price of the 4 per cents.

30. Railway Stock is sold at 108 and with the proceeds Government Securities is bought at $91\frac{1}{8}$; after a time the Government Securities is sold at $95\frac{3}{8}$ and the original stock re-purchased at 109, leaving a profit of *Rs.*1090 on the transaction; find the amount of Railway stock sold.

31. A person possesses £3200 3 per cents. which he sells at $99\frac{3}{4}$; he invests the proceeds in Railway Shares at £56 per share, which shares pay 5 per cent. interest on £45, the amount paid on each share. How much is his income altered by the transaction?

32. A person after paying an income-tax of *7d.* in the pound has a clear income of £262. *2s. 6d.* derived from stock in the $4\frac{1}{2}$ per cents.; he sells out $\frac{2}{3}$rds of this stock at $93\frac{3}{4}$ and invests the money in the I. Railway Stock at $112\frac{1}{2}$ which pays $5\frac{1}{4}$ per cent. per annum; what is now his clear income after paying the income-tax as before?

33. When an income is less than £400, income-tax is not chargeable upon the first £120; a man having £13300 in the 3 per cents. sells out £300 at par and invests it in a mortgage returning 4 per cent.; his income is now *1s. 6d.* less than formerly, what is the amount of the income-tax?

34. *A* invests a certain sum of money in the 3 per cents. at 90, and *B* invests an equal sum in the $3\frac{1}{2}$ per cents. at 104. If *A*'s income exceeds *B*'s by *Rs.*400, what is the investment of *A*?

35. *A* invests a certain sum in the 3 per cents. at 80, and *B* invests half the sum in the 4 per cents. If *A*'s income is to *B*'s income, as 8 is to 5, find the price of the 4 per cents.

36. A person invests *Rs.*10000 partly in the 4 per cents. at 96, and partly in the $4\frac{1}{2}$ per cents. at 117. What amount does he invest in each stock, if he receive the same income from each?

37. A man invests *Rs.*16000 partly in the 3 per cents. at 75 and partly in the 4 per cents. at 80. If his total income is *Rs* 760, how much does he invest in each stock?

38. A Railway Company could declare a dividend of 6 per cent. if there were no Preference shares. But *Rs.*80,00,000 being Preference shares, guaranteed at 7 per cent., the ordinary shareholders receive only $5\frac{1}{2}$ per cent. What is the amount of Ordinary Stock?

39. A person invested *Rs.*6000 in the 3 per cents. at 75, and after a certain number of years sold his stock, and found that during that time his sum had amounted to *Rs.*10000 at Simple Interest. If the price of stock was 80 when he sold, find how many years he was a stock-holder.

40. A person invests *Rs.*12000 partly in the 3 per cents. at 72,

and partly in the 4 per cents. at 84. He sells the former at 64 and the latter at 96, and thus realises the sum invested. How much does he invest in each stock?

41. A person invests *Rs.*86666. 10*a.* 8*p.* partly in the 3 per cents. at 80, and partly in the $4\frac{1}{2}$ per cents at 102. If he receive *Rs.*360 more a year from the second stock than from the first, how much does he invest in each kind of stock?

42. A person holds £5000 partly in the 3 per cents. at 60, and partly in the 4 per cents. at 75. If his income from both the sources is £160, what sum does he hold in the 4 per cents.?

43. A person invests £2000 partly in the 4 per cents. at 96, and partly in the 3 per cents at 90. If his total income be £10 less than what it would have been had he invested the whole in the 5 per cents. at 125, find how much he has invested in each stock.

44. A person holds £5000 partly in the 3 per cents. at 90, and partly in the 4 per cents. at par. He sells the former at 80, and the latter at 20 per cent. premium, and thereby realises £100 more than the sum invested. How much does he hold in the 3 per cents.?

45. If the 3 per cents. be at 95, and the Government offer to receive tenders for a loan of £5,000,000, the lender to receive £5,000,000 stock in the 3 per cents together with a certain sum in the $3\frac{1}{4}$ per cents., what sum in the $3\frac{1}{4}$ per cents ought the lender to accept?

CHAPTER XVI.

Evolution and Surds.

I. EVOLUTION.

573. Application of Geometry to Arithmetic. By Euclid I. 47, we know that in a right-angled triangle the square *on* the side opposite the right angle is equal to the sum of the squares *on* the sides containing the right angle. Hence, the square *of* the measure of the side opposite the right angle is equal to the sum of the squares *of* the measures of the sides containing the right angle.

Therefore, we have

$$(\text{Hypotenuse})^2 = (\text{Perpendicular})^2 + (\text{Base})^2; \quad \text{(i)}$$

$$(\text{Perpendicular})^2 = (\text{Hypotenuse})^2 - (\text{Base})^2,$$
$$= (\text{Hyp.} + \text{Base})(\text{Hyp.} - \text{Base}); \quad \text{(ii)}$$

Similarly, $(\text{Base})^2 = (\text{Hyp.} + \text{Perp.})(\text{Hyp.} - \text{Perp.})$. (iii).

Thus, if any two of the three sides in a right-angled triangle be given, we can easily determine the third.

Ex. 1. The sides of a right-angled triangle are 56 yds. and 42 yds., find the hypotenuse.

$$56^2+42^2=3136+1764=4900 \text{ ; and } \sqrt{(4900)}=70.$$

$\therefore$ the required hypotenuse $=$ 70 yds. *Ans.*

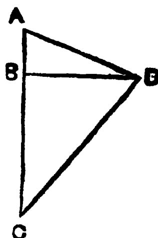
Ex. 2. The diagonal of a rectangular space is 890 ft., and its length 792 ft.; find its width.

The length and breadth form with the diagonal a right-angled triangle, of which the diagonal is the hypotenuse and the length the base, to find the perpendicular.

$\therefore$ $(\text{Perp.})^2=(890+792)\ (890-792)=1682\times98=3364\times49.$

Hence perpendicular or width $=58\times7$ ft. $=$ 406 ft. *Ans.*

Ex. 3. The tip of a reed was 8 inches above the surface of a lake; but forced by the wind, it gradually advanced, and was submerged at a distance of 28 in. Find the depth of water.



Let $CA=CD$ represent the reed, BD the surface, CB the depth; $AB=8$, $BD=28$; to find CB.

In the right-angled triangle CBD, we have $BD=28$, and the difference of CD and $CB=8$.

Since $BD^2=CD^2-CB^2=(CD+CB)\ (CD-CB)=8(CD+CB)$, $\therefore$ $CD+CB=28^2\div8=98$ and $CD-CB=8$; hence $CB=\frac{1}{2}(98-8)=45$. Thus the depth of water $=$ 45 in. *Ans.*

574 Application of Algebra to Arithmetic. The student would do well to remember the following formulæ:—

(i) $a^2-b^2=(a+b)(a-b)$. (ii) $a^3+b^3=(a+b)(a^2-ab+b^2)$.

(iii) $a^3-b^3=(a-b)(a^2+ab+b^2)$.

Ex. 1. Find the value of

$$\frac{\cdot687\times\cdot687-\cdot313\times\cdot313}{\cdot687-\cdot313}.$$

$$\text{Exp.}=\frac{(\cdot687)^2-(\cdot313)^2}{\cdot687-\cdot313}=\frac{(\cdot687+\cdot313)(\cdot687-\cdot313)}{\cdot687-\cdot313}=\cdot687+\cdot313=1.$$

Ex. 2. Find the value of $\dfrac{(\cdot07)^2+(\cdot05)^2}{(\cdot007)^2+(\cdot005)^2}$.

$$\text{Exp.}=\frac{(70)^2+(50)^2}{(7)^2+(5)^2}=\frac{10^2(7^2+5^2)}{7^2+5^2}=10^2=100. \quad \textit{Ans.}$$

Ex. 3. Simplify $\dfrac{3\frac{1}{4}\times 3\frac{1}{4}\times 3\frac{1}{4}-1}{3\frac{1}{4}\times 3\frac{1}{4}-1}$.

$$\text{Exp.}=\frac{(3\frac{1}{4})^3-1}{(3\frac{1}{4})^2-1}=\frac{(3\frac{1}{4}-1)\{(3\frac{1}{4})^2+3\frac{1}{4}+1\}}{(3\frac{1}{4}-1)(3\frac{1}{4}+1)}=\frac{(3\frac{1}{4})^2+3\frac{1}{4}+1}{3\frac{1}{4}+1}$$

$$=\frac{3\frac{1}{4}(3\frac{1}{4}+1)+1}{3\frac{1}{4}+1}=3\frac{1}{4}+\frac{1}{4\frac{1}{4}}=3\frac{1}{4}+\frac{4}{17}=3\frac{33}{68}. \quad \textit{Ans.}$$

Ex. 4. Simplify $\dfrac{2\frac{1}{3}\times 2\frac{1}{3}\times 2\frac{1}{3}+1}{2\frac{1}{3}\times 2\frac{1}{3}-1}$.

$$\text{Exp.}=\frac{(2\frac{1}{3})^3+1}{(2\frac{1}{3})^2-1}=\frac{(2\frac{1}{3}+1)\{(2\frac{1}{3})^2-2\frac{1}{3}+1\}}{(2\frac{1}{3}+1)(2\frac{1}{3}-1)}=\frac{2\frac{1}{3}(2\frac{1}{3}-1)+1}{2\frac{1}{3}-1}$$

$$=2\frac{1}{3}+\frac{1}{1\frac{1}{3}}=2\frac{1}{3}+\frac{3}{4}=3\frac{1}{12}. \quad \textit{Ans.}$$

Examples CLXXII.

1. The two sides of a right-angled triangle are 28 and 195 feet respectively; what is the length of the hypotenuse?

2. The hypotenuse of a right-angled triangle is 75·15 ft., and one of the sides is 60·12 ft.; what is the length of the remaining side?

3. Find the side, and also the diagonal of a square having the same area as a rectangle 43 ft. 5 in. long and 34 ft. 7 in. broad.

4. One side of a rectangle is 8076 yds. and the diagonal is 8749 yds.; find the other side.

5. If the perpendicular sides of a right-angled triangle are 13·02 and 5·2 feet, what is the third side?

6. If the town *A* is 72 miles West of *B* and 135 South of *C*, what is the distance from *B* to *C*?

7. Two men travelled from the same town—the one North, 28 miles per day; the other West, 36 miles per day; how far were they distant from each other after travelling 6 days?

8. Close by the side of a river rises a precipice to the height of 261 feet, and a line, reaching from its top to the opposite bank of the river, measures 582 feet; what is the breadth of the river?

9. Suppose the top of a straight ladder, $18\frac{1}{4}$ ft. long, to rest against a building at the height of $13\frac{3}{4}$ ft. from the ground; at what horizontal distance from the bottom of the building is the foot of the ladder placed?

10. A certain number of boys spent *Rs.*90. 4*a*. each spending as many four-anna pieces as there were boys; what was the number of boys?

11. A ladder 25 ft. long has its foot placed in a street, and its top resting against a wall, on one side of the street, at a height

of 15 ft. from the ground. If the ladder be turned over to the other side, its top reaches to a point 20 ft. high on the opposite wall. Find the breadth of the street.

12. *M* and *N* start together at *B*, to walk to another point *C*, 1332 yds. north from *B*; *M* takes the direct road *BC*, *N* goes first to *A*, a point west of *B*, and then straight to *C*, his journey being 2738 yds. How far is the distance from *B* to *A*?

13. Find the values of :—

(1) $\dfrac{\cdot874\times\cdot874-\cdot126\times\cdot126}{\cdot874-\cdot126}$. (2) $\dfrac{\cdot7654\times\cdot7654-\cdot2346\times\cdot2346}{\cdot7654-\cdot2346}$.

(3) $\dfrac{\cdot425\times\cdot425-\cdot175\times\cdot175}{\cdot425+\cdot175}$. (4) $\dfrac{(\cdot014)^2-(\cdot008)^2}{(\cdot0014)^2-(\cdot0008)^2}$.

(5) $\dfrac{(\cdot54)^4-(\cdot53)^4}{(\cdot0054)^2-(\cdot0053)^2}$. (6) $\dfrac{1\frac{1}{4}\times1\frac{1}{4}\times1\frac{1}{4}-1}{1\frac{1}{4}\times1\frac{1}{4}-1}$.

(7) $\dfrac{4\frac{1}{5}\times4\frac{1}{5}\times4\frac{1}{5}-1}{4\frac{1}{5}\times4\frac{1}{5}-1}$. (8) $\dfrac{3\frac{1}{3}\times3\frac{1}{3}\times3\frac{1}{3}+1}{3\frac{1}{3}\times3\frac{1}{3}-1}$.

14. In extracting the square root of 0·003 you have by mistake "pointed" thus 0·0̇03̇00̇, &c.; and proceeded with the operation and marked off the decimals accordingly. Without extracting the root of 0·003 over again, there is a certain quantity, which if multiplied into your erroneous result will give a correct value of $\sqrt{(\cdot003)}$; find the first three decimal places of this multiplier.

15. In a certain lake the tip of a bud of lotus was seen a span above the surface of the water. Forced by the wind it gradually advanced and was submerged at a distance of two cubits. Compute the depth of the water.

II. SURDS.

575. When the quantity whose root is to be extracted is not *complete* square, cube, &c., we have seen that there will be a remainder left however far we may continue the operation, and the root can therefore be found only *approximately*; that is, such a quantity has no *exact* root, and its representation is termed a **Surd** or **Irrational Quantity.**

For instance, the square root of 2 expressed by $\sqrt{2}$, is evidently not a whole number, because the square of no *whole number* whatever is 2; neither can it be a *vulgar fraction*, because the square of every vulgar fraction properly so called is itself a vulgar fraction, and it cannot be a *recurring decimal*, because all such quantities are equivalent to finite vulgar fractions; in other words, the square root of 2 may be found as nearly as we please, but not exactly; and it is termed an **incommensurable** quantity because it admits of no exact measure which is any *finite* quantity whatever either integral or fractional.

576. The surds of most frequent occurrence are those designated by the sign $\sqrt[2]{}$ or $\sqrt{}$, or by the index $\frac{1}{2}$, and termed **Quadratic surds** : and in general, when any *quantity* is represented in the form of a surd by means of a *fractional index* it is always understood that the numerator of the index denotes the power to which the *number* is intended to be raised, and that the denominator expresses the root afterwards to be extracted.

Thus, $27^{\frac{2}{3}}$ will represent the cube root of the square of 27, and is therefore equivalent to the cube root of 729, which is 9 : that is $27^{\frac{2}{3}}$, though expressed in the *form* of a surd, is in reality a rational quantity : and conversely.

577. Hence, the fundamental operations on surds must be performed upon their approximate values obtained as before ; but these operations may frequently be shortened, as will appear in the following instances :—

Since $\sqrt{8}=\sqrt{(4\times2)}=\sqrt{4}\times\sqrt{2}=2\times\sqrt{2}$, or $=2\sqrt{2}$;

we have, in Addition, $\sqrt{8}+\sqrt{2}=2\sqrt{2}+\sqrt{2}=3\sqrt{2}$;

in Subtraction, $\sqrt{8}-\sqrt{2}=2\sqrt{2}-\sqrt{2}=\sqrt{2}$;

in Multiplication, $\sqrt{8}\times\sqrt{2}=2\sqrt{2}\times\sqrt{2}=4$;

in Division, $\sqrt{8}\div\sqrt{2}=2\sqrt{2}\div\sqrt{2}=2$;

where the extraction of only *one* root is sufficient for the operations of Addition and Subtraction, and the product and quotient are rational quantities.

578. The Involution and the Evolution of such quantities may frequently be effected in the same way.

Thus, the square of $2\sqrt{5}$ = the product of the square of 2 and the square of $\sqrt{5}=4\times5=20$, which is a rational number : and conversely.

Again, by multiplying each of the terms of the numerator and denominator by $\sqrt[3]{(100)}$, we have,

$$\frac{\sqrt[3]{(5\cdot12)}+\sqrt[3]{(\cdot03375)}}{\sqrt[3]{(80)}-\sqrt[3]{(\cdot01)}}=\frac{\sqrt[3]{(512)}+\sqrt[3]{(3\cdot375)}}{\sqrt[3]{(8000)}-\sqrt[3]{1}}=\frac{8+1\cdot5}{20-1}$$

$$=\frac{9\cdot5}{19}=\cdot5=\tfrac{1}{2}\text{, a rational quantity.}$$

579. It has been said that the *values* of surds may be found as nearly as we please ; and this will clearly be done by continuing the extraction to the number of places of decimals in the root which we may find necessary for the purpose.

Thus, since $\sqrt{2}=1\cdot41421$ &c., we have,

$\sqrt{2}=1\cdot4$ nearly :

$=1\cdot41$ more nearly :

$=1\cdot414$ still more nearly :

$=1\cdot4142$ still more nearly :

$=$&c. ....

and consequently its magnitude may be *compared* with that of any other numerical quantity either rational or irrational, although its *absolute* magnitude can never be exactly ascertained.

580. To find the value of such expressions as

$$\frac{1}{\sqrt{2}-1},\ \frac{4}{\sqrt{3}-\sqrt{2}},\ \frac{\sqrt{5}}{\sqrt{5}+\sqrt{2}},\ \frac{\sqrt{3}+\sqrt{2}}{\sqrt{5}-\sqrt{2}},\ \&c.$$

the student must notice the following statements, which are true whatever numbers the letters may stand for :—

(i) $\sqrt{x}\times\sqrt{x}=x$. (ii) $\sqrt{x}\times\sqrt{y}=\sqrt{(x\times y)}$.

(iii) $\sqrt{(x^2\times y)}=x\times\sqrt{y}$. (iv) $(x+y)(x-y)=x^2-y^2$.

(v) $(m+n\ (x+y)=m\times x+m\times y+n\times x+n\times y$.

Ex. 1. Find the value of $\frac{1}{\sqrt{2}-1}$.

The first thing to be done is always to free the denominator from surds. If we multiply both numerator and denominator by $\sqrt{2}+1$, the denominator becomes $2-1$ by (iv.)

$$\text{Thus, } \frac{1}{\sqrt{2}-1}=\frac{(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}=\frac{\sqrt{2}+1}{2-1}=\frac{\sqrt{2}+1}{1}=1+\sqrt{2}$$
$$=1+1\cdot4142\ldots=\underline{2\cdot4142.}\quad \textit{Ans.}$$

Ex. 2. Find the value of $\frac{4}{\sqrt{3}-\sqrt{2}}$.

Multiply both numerator and denominator by $\sqrt{3}+\sqrt{2}$, and we get

$$\frac{4}{\sqrt{3}-\sqrt{2}}=\frac{4(\sqrt{3}+\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}=\frac{4(\sqrt{3}+\sqrt{2})}{3-2}=4(\sqrt{3}+\sqrt{2})$$
$$=4(1\cdot732\ldots+1\cdot414\ldots)=4\times3\cdot146\ldots=\underline{12\cdot584}\ldots \textit{Ans.}$$

Ex. 3. Find the value of $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$.

$$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}=\frac{(\sqrt{3}+\sqrt{2})(\sqrt{5}+\sqrt{2})}{5-2}=\frac{\sqrt{(15)}+\sqrt{6}+\sqrt{(10)}+2}{3}$$
$$=\tfrac{1}{3}(3\cdot873\ldots+2\cdot445\ldots+3\cdot162\ldots+2)=\tfrac{1}{3}\times11\cdot480\ldots$$
$$=\underline{3\cdot826\ldots}\quad \textit{Ans.}$$

Ex. 4. Find the value of $\frac{2+\sqrt{3}}{\sqrt{3}+1}$.

$$\frac{2+\sqrt{3}}{\sqrt{3}+1}=\frac{\frac{1}{2}(4+2\sqrt{3})}{\sqrt{3}+1}=\frac{\frac{1}{2}(\sqrt{3}+1)^2}{\sqrt{3}+1}=\frac{\frac{1}{2}(\sqrt{3}+1)(\sqrt{3}+1)}{\sqrt{3}+1}=\tfrac{1}{2}(\sqrt{3}+1)$$
$$=\tfrac{1}{2}(1\cdot732\ldots+1)=\tfrac{1}{2}\times2\cdot732\ldots=\underline{1\cdot366\ldots}\textit{Ans.}$$

Examples CLXXIII.

1. Find the value (to 5 places of decimals) of :—

(1) $\frac{1}{\sqrt{3}-1}$. (2) $\frac{3}{\sqrt{6}-2}$. (3) $\frac{2}{\sqrt{5}+\sqrt{3}}$. (4) $\frac{\sqrt{5}}{\sqrt{2}+\sqrt{3}}$. (5) $\frac{3}{\sqrt{5}+\sqrt{2}}$.

(6) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-1}$. (7) $\frac{\sqrt{3}+\sqrt{5}}{\sqrt{5}-\sqrt{2}}$. (8) $\frac{\sqrt{3}+\sqrt{7}}{\sqrt{11}-\sqrt{5}}$. (9) $\frac{\sqrt{5}+1}{\sqrt{5}-1}$.

(10) $(\sqrt{6}+\sqrt{3}+\sqrt{2}+2)(\sqrt{6}-\sqrt{3}+\sqrt{2}-2)$. (11) $(\sqrt{3}+2)^2+(\sqrt{3}-2)^2$.

(12) $\frac{\sqrt{(\cdot 05-\cdot 005)}}{\sqrt{(\cdot 05)}-\sqrt{(\cdot 005)}}$. (13) $\frac{(\sqrt{5}+\sqrt{3})(\sqrt{5}+\sqrt{2})(\sqrt{3}+\sqrt{2})}{(\sqrt{5}+\sqrt{3}+\sqrt{2})^2}$.

(14) $\sqrt{\left\{\frac{2+\sqrt{(12)}-\sqrt{(27)}}{2+\sqrt{(48)}-\sqrt{(27)}}\right\}}$. (15) $\frac{3\sqrt{2}}{\sqrt{6}+\sqrt{3}}-\frac{4\sqrt{3}}{\sqrt{6}+\sqrt{2}}+\frac{\sqrt{6}}{\sqrt{3}+\sqrt{2}}$.

2. Which is the greater quantity :—

(1) $\sqrt{2}$ or $\sqrt[3]{3}$? (2) $\sqrt[3]{9}$ or $\sqrt[4]{(19)}$? (3) $\sqrt{3}$ or $\sqrt[4]{(15)}$?

3. What fraction of $\{\sqrt{(4050)}\times\cdot 0\dot{0}\dot{2}+\cdot\dot{2}\dot{0}+\sqrt{(1458)}\}\div\sqrt{(\cdot 02)}$ is $\sqrt{(6\cdot 00\dot{8}+\cdot 3042)}+\sqrt{(116\cdot\dot{6}\times\cdot 04\dot{6})}$?

4. Find the value of $\frac{15+\sqrt{(\cdot 009)}}{1-\sqrt{(\cdot 9)}}$ correct to 3 places of decimals.

CHAPTER XVII.

The Metric System and Decimal Coinage.

I. THE METRIC SYSTEM.

581. Soon after the French Revolution of 1789, a Commission was appointed consisting of several eminent mathematicians, for the purpose of preparing a new system of weights and measures.

The system they recommended was established in France, under the name of **The Metric System** Since then it has been introduced to a greater or less extent into almost all the countries of Europe.

582. In the formation of the multiples and sub-multiples the decimal system is followed exclusively ; the Greek prefixes to any unit denoting multiples and the Latin prefixes denoting sub-multiples ; thus,

Greek prefixes.				Latin prefixes.			
Deka	means	10	times	**Deci**	means	10th	part of
Hecto	...	100	...	**Centi**	...	100th	...
Kilo	...	1000	...	**Milli**	...	1000th	...
Myria	...	10000	...				

583. The unit of **length** is the **Metre**. It is also the *fundamental* unit, because from it every other unit of weight or measure is derived; and hence the name **Metric System**. A metre was defined to be the *ten-millionth* part of the distance from the Equator to the Pole measured along the surface of the ocean; but subsequent calculations have proved this to be incorrect. 1 Metre = 39·371 English inches nearly.

10 Millimetres (m.m.) = 1 Centimetre.	10 **Metres** = 1 Dekametre.
10 Centimetres (c.m.) = 1 Decimetre.	10 Dekametres = 1 Hectometre.
10 Decimetres (d.m.) = 1 **Metre**.	10 Hectometres = 1 Kilometre.
	10 Kilometres = 1 Myriametre.

Note. Long distances are generally quoted in kilometres; 1 kilometre = ⅝ English mile nearly.

584. The unit of **surface** is the **Square Metre**. In measuring land it is usual to take for unit a *square dekametre*. This is called an **Are**. It is equal to 119·6046 English sq. yds. nearly.

10 Centiares (sq.m.) = 1 Deciare.	10 **Ares** = 1 Dekare.
10 Deciares = 1 **Are** = 1 sq. Dekametre.	10 Dekares = 1 Hectare = 1 sq. Hectometre.

Note. Large surfaces are generally quoted in Hectares; 1 Hectare = 2½ English acres nearly.

585. The unit of **volume** is the **Cubic Metre**. In measuring wood this is called a **Stere**. It is equal to 35·317 English cub. feet nearly.

10 Decisteres = 1 **Stere**.	10 **Steres** = 1 Dekastere.

586. The unit of **Capacity** is the **Litre**. It is equal to a cubic decimetre = 61·028 English cub. inches nearly and contains about 22 gallons.

10 Centilitres = 1 Decilitre.	10 **Litres** = 1 Dekalitre.
10 Decilitres = 1 **Litre** = 1 Cub. Decimetre.	10 Dekalitres = 1 Hectolitre.
	10 Hectolitres = 1 Kilolitre = 1 Cub. Metre.

Note. 1 Litre = 1¾ English pint nearly.

587. The unit of **Weight** is the **Gramme**. It is the weight of a cubic centimetre of distilled water at its greatest density, *i.e.*, when the temperature is about 4°C. 1 Gramme = 15·434 grains English, nearly.

10 Milligrams = 1 Centigram.	10 Hectograms = 1 Kilogram.
10 Centigrams = 1 Decigram.	10 Kilograms = 1 Myriagram.
10 Decigrams = 1 **Gram**.	10 Myriagrams = 1 Quintal.
10 **Grams** = 1 Dekagram.	10 Quintals = 1 Millier.
10 Dekagrams = 1 Hectogram.	

Note. 1 Kilogramme = 2⅕ English lbs. Avoir. nearly. 1 Quintal = 100 Kilogrammes = 1·97... English cwt. 1 Millier = 1000 Kilogrammes = 19·7... English cwt.

588. The unit of **money** is the **Franc**. It is about $9\frac{3}{8}d.$ English. The sub-multiples of the franc are decime (seldom used) and the centime, which are respectively the tenth and hundredth part of the franc ; so that

10 Centimes (*c*)	= 1 Decime.
10 Decimes or 100 Centimes	= 1 **Franc** (fr.)

Accounts however, are kept in France in francs and centimes only ; thus 58·76 francs is read 58 francs 76 centimes.

The franc is a silver coin composed of 9 parts of silver and 1 part of copper, and weighs 5 grams. The Napoleon is a gold coin = 20 francs. A sou (a copper coin) = 3 centimes.

Ex. 1. Express 9 hectom. 9 dekam. 7 metres in *metres*, and 5 litres 3 decil. 7 centil. in *centilitres*.

(i) 9 hectom. 9 dekam. 7 metres = 997 metres.
(ii) 5 litres 3 decil. 7 centil. = 537 centilitres.

Ex. 2. Find the circumference of a wheel which will turn 1130 times in 2 kilom. 33 dekam. 91 decim.

2 kilom. 33 dekam. 91 decim. = 2339·1 metres ; and 2339·1 ÷ 1130 = 2·07.

∴ the circumf. reqd. = 2·07 metres = 2m. 7 centim. *Ans.*

Ex. 3. Make out the following bill :—

5 kilog. 27 dekag. of moist sugar at 1 fr. 50 c. per kilog., 12 kilog. 6gr. of lump sugar at 1 fr. 75 c. per kilog., 10 kilog. 15 dekag. of tea at 7fr. 5c. per kilog. and 12 kilog. 9hectog. of coffee at 4fr. 15c. per kilog.

5 kilog. 27 dekag at 1 fr. 50 c. per kilog. = (1·50 × 5·27) fr. = 7·905 fr.
12 kilog. 9 gr. at 1 fr. 75 c. per kilog. = (1·75 × 12·009) fr. = 21·01575 ..
10 kilog. 15 dekag at 7 fr. 5 c. per kilog. = (7·05 × 10·15) fr. = 71·5575...
12 kilog 9 hectog. at 4 fr. 15 c. per kilog. = (4·15 × 12·9) fr = 53·535...
154·01325...
= 154 fr. 1·325 c. *Ans.*

Ex. 4. A man walked a distance of $4\frac{1}{2}$ miles in 72 minutes, find his rate per hour of walking in French measure.

In 72 min. he walks $4\frac{1}{2}$ miles or $4\frac{1}{2} \times 1760$ yds.
∴ ... 1 hour ... $\frac{60}{72}$ of $4\frac{1}{2} \times 1760$ yds or 237600 in.

$$= \frac{237600}{39 \cdot 371} \text{ metres} = 6035 \text{ m. } \textit{nearly.}$$

Examples CLXXIV.

1. Express in terms of a *kilom.*, and also of a *centimetre* :—

(1) 17 kilom. 3 dekam. 3 millim. (2) 500 hectom. 37 millim.

2. Express in terms of a *sq. metre*, and also of a *sq. millimetre*:—

(1) 533 sq hectom. 3 sq. m. 2 sq. millim. (2) 725 sq. decim.

3. Express in terms of a *gram*. and also of a *kilogram* :—
(1) 1 hectog. 16 gr. 75 millig. (2) 53 dekag. 45 centig.

4. Express 763 sq. hectom. 5 sq. m. in terms of an *are*.

5. In 530·000465 kilolitres, how many *decilitres* ?

6. Express in terms of a *franc*, and also of a *centime* :—
(1) 24 fr. 14 c. (2) 480 fr. $5\frac{3}{4}$ c. (3) 5 fr. 8 c. (4) 555 dec. $3\frac{3}{4}$ c.

7. How many fields each containing 2 hectares, 47 ares can be made out of a farm of 313 hectar. 69 ares ?

8. A certain number of sawyers cut out 390 steres of wood ; on the average each sawyer cuts 32 steres 5 decisteres. How many sawyers were employed ?

9. Make out the following bill :—

27 m. 8 centim. of calico at 95 c. per m , 12 m. 12 centim. of lining at 70 c. per m., 3 dozen pairs of gloves at 2 fr. 75 c. per pair, and 19 m. 4 decim. of flannel at 2 fr 45 c. per m If a person give a ten-pound note to pay this bill, how much change in English money should he receive, £ 1 being = 25 fr. 25 c. ?

10. Divide 80 hectares of land between *A*, *B* and *C*, so that *A* shall have 2 hectar. 76 ares more than *B*, and *C* 11 hectar. 12 ares more than *B*.

11. If 6 horses in 4 days eat 144 kilog. 96 gr. of hay, how long will 675 kilog. 450 gr. serve 15 horses ?

12. From Paris to Lyons is 507 kilometres. The express trains take $10\frac{1}{2}$ hours for the journey, and the first class fare is 56·80 francs ; find the rate of the trains in miles per hour, and the fare per mile in English money, taking £1 = 25 francs.

13. A bankrupt's effects are worth 3535 fr. 98 c., his debts amount to 20037 fr 22 c. more than his effects ; what will a creditor to the amount of 350 fr. lose ?

14. A person's income is reduced from 56085 fr. 50 c. to 52720 fr. 37 c. by income-tax. (1) How much tax does he pay in the franc ? (2) How much tax does he pay in the £, supposing £1 = 25 francs ?

15. A needle-woman earns on Monday 3 fr. 40 c , on Tuesday 3 fr. 75 c., on Wednesday nothing, on Thursday 4 fr. 5 c., on Friday 14 c. less than what she earned on Monday, on Saturday 1 fr. 5 c. less than what she earned on Thursday ; find her average daily earnings during the week.

16. A person selling a kilog. of sugar at 1 fr. 80 c. gained $12\frac{1}{2}$ per cent. on the cost price ; find the cost price of a millier of the sugar.

17. At what rate per cent. will 4850 francs amount to 5820 francs in 3 years and 4 months ?

18. After the payment of an income-tax of 15 c. in the franc, a gentleman's income is 365318 fr. 10c. Express his income-tax as a fraction of his net income.

19. A person uses 53·48 kil. of thread in weaving 231·5 metres of linen, 1·15 metres broad; how many metres of linen ·95 metres broad can be woven with 37·75 kil. of the same thread?

20. How many Napoleons, five-franc pieces, francs, half-francs, and sous, are there in 4072 fr. 45 c., there being an equal number of each of the first two coins, and half that number of each of the last three coins?

21. A woman lays out at one shop 23 fr. 5 c. in calico; 34 fr. 45 c. in flannel; 3 fr. 50 c. in tape; 4 fr. 20 c. in thread, &c.; 65 c. in needles. At another shop 5 fr. in tea; 4 fr. 25 c. in sugar; 7 fr. 5 c. in cheese; 9 fr. 45 c. in candles; 6 fr. 3 c. in soap. (1) How much did she lay out at one shop more than at the other? (2) What change ought she to receive out of 5 Napoleons? What change ought she to receive in French, and also in English money, out of a £5 note? £1 being = 25 fr. 50 c.

22. *A* exchanges with *B* 200 sacks of wheat, each of 157 kilog. at 46 fr. 25 c. a sack, for 55 tons of iron at 21 fr. 50 c. the 100 kilog. What English money should pass between them to make it a fair bargain? £1 being = 25 fr. 12 c.

23. Divide 85 hectar. 50 ares between *A*, *B* and *C*, so that *A*'s share : *B*'s share :: 11 : 6, and that *C* shall have 9 hectar. more than the united shares of *A* and *B*.

24. A man made the following bill: 29 dozen pairs of gloves at 3 fr. 25 c. a pair, 116 pairs of stockings at 2 fr. 24 c. a pair, 84 pairs of stockings at 2 fr. 6 c. a pair, 4 pieces of calico, each containing 39½ m. at 95 c a m., 1 piece of Irish linen, 84 m. 6 centim. at 3 fr. 22 c. a m., 5 pieces of flannel, each containing 42 m. 27 centim. at 3 fr. a m., amount to 2634 fr. 3 c.; find the amount of his error.

25. In how many years will 450 fr. amount to 576 fr. at 3½ per cent. simple interest?

26. What will it cost to paint a room 7 m. 25 centim. long, 5½ m. broad, and 4 m. 75 centim. high at 4 fr. 5 c. a sq. m.? Find the cost in English money, a Napoleon being worth 15*s.* 9¾*d.*

27. A person invests 3000 fr. in the 5 per cents. at 87·73, what amount of stock will he hold?

28. A rectangular piece of ground is 32 m. 7 decim. long, and 19 m. 5 centim. broad. Find the cost of enclosing it with a path 1 m. 5 decim. broad at 3 fr. 5 c. a sq. m. (1) When the path is outside the piece of ground. (2) When the path is part of the ground.

29. A speculator bought 264 hectares 6 ares of land for 387074 fr. 25 c.; he sold $\frac{6}{9}$ of it at 1465 fr. the hectare; at what price per hectare must he sell the rest, in order to gain 9177 fr. 12 c. by his purchase?

30. If the driving wheel of a locomotive be 1 dekam. 25 decim. in circumference, and it makes 2½ revolutions in 1 second, how long will it take to travel 100 miles, if 1 mile = 1⅗ kilom.?

31. Two masons' wages, for 7 days' work of the 1st, and 5 days' work of the 2nd, amount to 45 fr. 90 c., and for 5 days' work of the 1st and 7 days' work of the 2nd, to 48 fr. 66 c. Find the daily wages of each.

32. Which is the best stock to invest 100000 fr, in the 5 per cents. at 121·90, or the 3 per cents. at 84? What is the difference of income in English money, if 24 fr. 96 c.=£1?

33. Make out the following bill :—

2 lbs. of sugar at 14 cents 9 mils a lb., 14½ lbs. of cheese at 17½ cents a lb., 7 lbs. 4 oz. of butter at $ ·245 a lb., 5¾ lbs. of tea at 39½ cents a lb, 40 eggs at 27 cents a dozen, 12 lbs. of currants at $ 15·12 cents a cwt. Find the value in English money, if $ 1=4*s.* 2*d.*

34. Suppose a person to invest 6000 fr. on the 1st of Jan. 1866 at 5 per cent. per annum; and also to invest the same amount on the 1st of Jan. of each of the next 3 years; what will his investments amount to at the end of the 4th year, supposing him not to touch any of the money, and to receive compound interest?

35. A person gives 5 Napoleons to pay a bill of £3. 12*s.* 6*d.*, when the exchange is £1=25 fr. 12½ c. (1) How much English money ought he to receive back? (2) How much French money?

36. If 3 kilog. of tea be worth 5 kilog. of cocoa, and 7 kilog. of cocoa be worth 11 kilog. of coffee, what will 5 kilog. of tea be worth, if 1 kilog. of coffee is worth 6 fr. 30 c.?

37. If 1 kilog. 7 dekag. of moist sugar be worth 99 dekag. of lump sugar, and 22 dekag. of lump sugar be worth 13 dekag. 5 gr. of coffee, and 2 kilog. of coffee be worth 9 hectog. 63 gr. of tea; what is the value of 100 kilog. of moist sugar, if a kilog. of tea be worth 9 fr. 50 c.?

38. *A* can plant with cabbages ⅛ of an hectare in 10 hrs. 15 min., *B* can plant the same space in 8 hrs. 45 min. What space can they together plant in 2½ hours? How many square yards?

39. It costs 217 fr. 25 c. to carpet a room 17·7 metres long, and 8·4 metres broad, what will it cost to carpet another room 13·1 metres long and 6·3 metres broad with the same material?

40. It takes 14375 sq. bricks to pave a path 184 m. long, and 4 m. 50 centim broad; find the length of the side of each brick.

41. If 38 kilog. 25 grs. of sugar cost 134 fr. 7½ c., how much sugar worth ⅚ as much per dekag. can be bought for 26 fr. 81½c.?

42. What number of dollars will amount to 1059·38 dollars in 5 years at 4½ per cent. simple interest?

43. A man buys a steam plough for 30807 fr. 84 c.; he ploughs 170 hectar. 4 ares of his own farm, and ⅔ of that quantity for a neighbour. He estimates the expenses at 10 per cent. on the cost of the steam-apparatus, and wages and fuel, &c., at 1210 fr.; what is the cost of ploughing an hectare?

44. If 20 men can dig a trench 160 m. long, 2 m. wide, and 1 m. 2 decim. deep in 8 days, what will be the depth of a similar trench 90 m. long, 1 m. 80 centim. broad, which 24 men can dig in 4½ days.

45. A person bought for 164 fr. 64 c. a certain number of boxes of oranges, each box contained 3 times as many oranges as there were boxes, and each orange cost twice as many centimes as there were boxes. Find the number of boxes, and also the number of oranges.

46. A Prussian buys and pays for the following articles of a Paris merchant, 15·5 metres of cloth at 14 fr. 75 c. the metre; 6·25 metres of velvet at 21 fr. 5 c. the metre; 19 metres of silk at 6 fr. 25 c. the metre; 28·50 metres of merino at 4 fr. the metre; half a dozen pairs of stockings at 25 fr. the dozen pairs; 5 pairs of gloves at 17 fr. the dozen pairs; in consequence of ready payment $\frac{1}{80}$ of the amount of the bill is deducted; he offers for payment 200 thalers; how many Napoleons ought he to receive from the merchant; 6 thalers, 20 silver groschen being = 25 francs? (1 Thaler = 30 silver groschens.)

II. DECIMAL COINAGE.

589. In the **Proposed Decimal System of Coinage,** the **pound** or **sovereign**, which is of the same value as the pound sterling at present, is taken as the unit of money, with sub-multiples *florins, cents, and mils,* which are respectively the tenth, hundredth and thousandth part of the pound, so that

10 Mils (m.) = 1 Cent. (c)
10 Cents = 1 Florin. (fl.)
10 Florins = 1 Pound. (£1.)

In this system, in a decimal of £1, the figure in the first place of decimals represents *florins*, in the second *cents*, and in the third *mils.*

Ex. 1. Reduce £15. 7c. 8m. to *mils*; and 25684 mils to £. *fl.* &c.

£15. 7c. 8m. = £15·078
= 15078 mils.

25684 mils = £25·684
= £25. 6fl. 8c. 4m.

Ex. 2. Reduce £4. 18*s.* 10¾*d.* to £. fl. &c, and £5. 6fl. 2c. 5m. to £. *s. d.*

4 | 3·
12 | 10·75
20 | 18·8958333...
£4·944791666...
= £4. 9fl. 4c. 4·7916m.

£5. 6fl. 2c. 5m. = £5·625
20
s. 12·5
12
d. 6·0
∴ the sum = £5. 12*s.* 6*d.*

Ex. 3. Find the value of £12. 3fl. 5m. + £23. 6m. − £37. 7fl. 8c. 9m. + £25. 46c. − £18. 9fl. + £7. 6fl. 9c. 5m. − £9. 9fl 9c. 6m.

£12. 3fl. 5m. = £12·305
£23 6m. = £23·006
£25. 46c. = £25·46
£7. 6fl. 9c. 5m. = £ 7·695
£68·466
£66·685
£ 1·781 = £1. 7fl. 8c. 1m. *Ans*

£37. 7fl. 8c. 9m. = £37·789
£18 9fl. = £18·9
£ 9. 9fl. 9c. 6m. = £ 9·996
£66·685

Ex. 4. Divide £13. 4 fl. 5c. 6m. × 15 by 48.

£13. 4fl. 5c. 6m. = £13·456
                15
£201·840

$$48\begin{cases} 8 \,|\, \underline{201·840} \\ 6 \,|\, \underline{25·230} \\ \quad \;\; £4·205 \end{cases}$$

£4·205 = £4. 2fl 5m. *Ans.*

Examples CLXXV.

1. Reduce :—

(1) £59. 1fl. 7c. to *mils* ; and 976358 mils to *£. fl. c. m.*

(2) £18. 6½c. to *mils* ; and £96. 2fl. 3c. 9m. to *mils*.

(3) £14. 4fl. 3½c. to *mils* ; and 254525 mils to *£. fl. c m.*

2. Express each of the following sums accurately in the *decimal coinage* ; that is, in *£. fl. c. m.* :—

(1) 4½*d.* ; 10*d.* ; 5½*d.* ; 5*s.* ; 6*s.* 4*d.* ; 2*s.* 1¼*d.* ; 3*s.* 0¾*d.*

(2) 4*s.* 3*d.* ; 15*s.* 8¼*d.* ; £12. 12*s.* 11¾*d.* ; £4. 8*s.* 5*d.*

(3) 15*s.* 6*d.* ; 12*s.* 11¾*d.* ; £5. 9*s.* 4¾*d.* ; £6. 17*s.* 11½*d.*

3. Express each of the following sums accurately in the ordinary way ; that is, in *£. s. d.* :—

(1) 9fl. 6·25m. ; 14fl. 8c. 7⅓m. ; 17fl. 8c. 3⅜m. ; 9fl. 6·34m.

(2) £3·621875 ; £7·82291$\dot{6}$; £29. 8fl. 7c. 6m. ; £25·944791$\dot{6}$.

4. Find the sum of £327. 9fl. 4c. 5m. ; £89. 4fl. 7c 8m. ; £5. 6fl. 5m. ; £479. 8c. 8m. ; £63. 75c. and £24. 8fl. 7m.

5. Add together £78. 75c. ; £14. 3¼fl. ; £9. 5fl. 8¾c. ; £35. 427m. ; £48. 7fl. 9c. 6m. and 56384m.

6. Subtract £123. 7fl. 8c. from £987. 6c. 5m.

7. Find the difference between £39. 9fl. 9m. and £54. 2fl. 3c. 4m. and between £54. 3fl. 7c. and £48. 9c. 6m.

8. Subtract £825. 7fl. 6c. 3m. from £1000.

9. Multiply £10. 9fl. 2½c. separately by 18 and 1008.

10. Multiply £34. 2fl. 8c. 9m. by 89.

11. Divide £4838. 5fl. 8c. 9m. by 63.

12. Divide £6852. 3fl. 8c. 7m. by 8760.

13. Divide £230. 9fl. 2c. 3m. by 77 and £342136. 8fl. by 7380.

14. How often is £5. 6fl. 7c. 8m. contained in £4479. 9fl. 4¾c. ?

15. Reduce 7fl. 3½c. to the decimal of £10 and of £500.

Examples worked out.

Ex. 1. *A*'s present age is to *B*'s as 9 : 7 ; and 34 years ago the proportion was 5 to 2. Find the present age of each.

Here, we have *A*'s present age to *B*'s as 9 : 7 ; and 9 is 4½ times (9 − 7). Similarly, *A*'s former age was to *B*'s as 5 : 2 ; and 5 is 1⅔

times $(5-2)$. Therefore, *A*'s present age is $4\frac{1}{2}$ times the difference of *A*'s and *B*'s ages; and his former age was $1\frac{2}{3}$ times the difference of *A*'s and *B*'s former ages.

But it should be borne in mind that the *difference* of the ages of two persons is *always the same*, though the *ratio* of the ages is *always varying*. Hence, we have

A's former age $=(1\frac{2}{3} \div 4\frac{1}{2})$ or $\frac{10}{27}$ of his present age;
but *A*'s former age $=$ *A*'s present age $-$ 34 years;
$\therefore$ *A*'s present age $-$ 34 years $=\frac{10}{27}$ of *A*'s present age;
$\therefore$ $(1-\frac{10}{27})$ of *A*'s present age $=34$ years;
or $\frac{17}{27}$ of *A*'s present age $=34$ years;
$\therefore$ *A*'s present age $=(34\times\frac{27}{17})$ years $=54$ years, } *Ans.*
and *B*'s present age $=\frac{7}{9}$ of 54 years $=42$ yerrs. }

Ex. 2. Five years ago, the ages of *A* and *B* were as $4:5$; and five years hence they will be as $13:15$; find their present ages.

In $(5+5)$ or 10 years, the ratio would be changed from $4:5$ to $13:15$. Now $5-4=1$, and $15-13=2$. Reducing the latter ratio to one in which also the difference between the terms $=1$, we get the ratio $\frac{13}{2}:\frac{15}{2}$.

Now, if *A*'s age five years before had been 4 years, it would have taken only $(\frac{13}{2}-4)$ or $\frac{5}{2}$ years to become $\frac{13}{2}$.

$\therefore$ $\frac{5}{2}:10::4:$ *A*'s age 5 years ago;
or *A*'s age 5 years ago $=10\times4\times\frac{2}{5}$ years $=16$ years.
$\therefore$ *A*'s age $=(16+5)$ or 21 years, } *Ans.*
and *B*'s age $=(\frac{5}{4}\times16+6)$ or 26 years. }

Ex. 3. Divide the number 237 into three parts such that 3 times the first may be equal to 5 times the second and to 8 times the third.

Since 5 times the 2nd $=3$ times the first, $\therefore$ the 2nd $=\frac{3}{5}$ of the 1st.

Similarly, the 3rd $=\frac{5}{8}$ of the 2nd $=\frac{5}{8}$ of $\frac{3}{5}$ of the 1st.

Hence the three parts are as 1, $\frac{3}{5}$ and $\frac{5}{8}$ of $\frac{3}{5}$, or as 40, 24 and 15.

Now $40+24+15=79$; $\therefore$ the 1st $=\frac{40}{79}$ of $237=120$, }
the 2nd $=\frac{24}{79}$ of $237=72$, } *Ans.*
and the 3rd $=\frac{15}{79}$ of $237=45$. }

Ex. 4. Divide *Rs.*210 among *A*, *B* and *C*, so that *Rs.*20 more than $\frac{2}{5}$ of *A*'s share, *Rs.*22 more than $\frac{3}{7}$ of *B*'s share, and *Rs.*2 more than $\frac{5}{6}$ of *C*'s share may be all equal.

$\frac{2}{5}$ of *A*'s share $+$ *Rs.*20 $=\frac{2}{5}$ (*A*'s share $+$ *Rs.*50);
$\frac{3}{7}$ of *B*'s share $+$ *Rs.*22 $=\frac{3}{7}$ (*B*'s share $+$ *Rs* $\frac{154}{3}$);
$\frac{5}{6}$ of *C*'s share $+$ *Rs.* 2 $=\frac{5}{6}$ (*C*'s share $+$ *Rs.*$\frac{12}{5}$).

Also (*A*'s share $+$ *Rs.*50) $+$ (*B*'s share $+$ *Rs.*$\frac{154}{3}$) $+$ (*C*'s share $+$ *Rs.*$\frac{12}{5}$)
$=$ *Rs.*$(210+50+\frac{154}{3}+\frac{12}{5})=$ *Rs.*$\frac{4706}{15}$.

Now, if *A*'s share $+$ *Rs.*50 $=1$, *B*'s share $+$ *Rs.*$\frac{154}{3}=\frac{14}{15}$,
and *C*'s share $+$ *Rs.*$\frac{12}{5}=\frac{12}{25}$.

Also $1+\frac{14}{15}+\frac{12}{25}=\frac{181}{75}$; and *Rs.*$\frac{4706}{15}\div\frac{181}{75}=$ *Rs.*130.

∴ *A's* share + *Rs*.50 = *Rs* 130, or *A's* share = *Rs*. (130 − 50) = *Rs*.80.
B's share + *Rs*.$\frac{154}{8}$ = *Rs*.130 × $\frac{14}{15}$, or *B's*... = *Rs*.($\frac{364}{8}$ − $\frac{154}{8}$) = *Rs*.70.
C's share + *Rs* $\frac{12}{8}$ = *Rs*.130 × $\frac{12}{35}$, or *C's* ... = *Rs*.($\frac{212}{8}$ − $\frac{12}{8}$) = *Rs*.60. *Ans.*

Ex 5. A person begins to speculate with a certain sum of money; in his first transaction he loses $\frac{1}{7}$th of the sum; in his second he gains 10 per cent. on his investment; in his third he loses $\frac{9}{11}$ths of the sum invested; in his fourth he gains $66\frac{2}{3}$ per cent. If he then has *Rs*.10000, with what sum did he start?

Suppose his capital in the beginning to be 1, then
after 1st transaction, he has $(1-\frac{1}{7})$ or $\frac{6}{7}$ of his capital.

... 2nd ... ... $\frac{110}{100}$ of $\frac{6}{7}$ or $\frac{66}{70}$ ... ...
... 3rd ... ... $\frac{66}{70}$ of $(1-\frac{9}{11})$ or $\frac{6}{35}$ ...
... 4th ... ... $\frac{166\frac{2}{3}}{100}$ of $\frac{6}{35}$ or $\frac{2}{7}$ ... ...

∴ by the question, $\frac{2}{7}$ of his capital = *Rs*.10000.
∴ his capital = *Rs*.10000 × $\frac{7}{2}$ = *Rs*.35000. *Ans.*

Ex. 6. How many francs are equivalent to £1, when gold purchased in London at 77*s*. $10\frac{1}{2}$*d*. per ounce standard, is sold in Paris at 4 per mile (*i.e.*, per 1000) premium on the fixed price? (An ounce Troy being = 31·1 grammes, and 1000 grammes of English standard gold being worth 3151 francs.)

Since 31·1 grammes = 1 oz.; ∴ 311 grammes = 10 oz.;
Hence 1 gramme = $\frac{10}{311}$ oz, and ∴ 1000 grammes = $\frac{10000}{311}$ oz.
Now 1000 grammes are bought in London for $77\frac{7}{8}$*s*. × $\frac{10000}{311}$;
and 1000 grammes are sold in Paris for 3151 frs. × 1·004;
∴ $77\frac{7}{8} \times \frac{10000}{311}$*s*. : 20*s*. :: 3151 × 1·004 frs. : reqd. no. of francs.
∴ the reqd. no. of francs = $\frac{3151 \times 1{\cdot}004 \times 20 \times 8 \times 311}{623 \times 1000}$
= 25·27 nearly. *Ans.*

Ex. 7. I bought 128 yards of cloth for £100, and am now obliged to sell it at a loss of as much money as I shall receive for a dozen yards. At what do I sell it per yard?

In the sum realised by selling 128 yds., my loss is the selling price of 12 yds.; but loss = cost price − selling price. Hence the cost price of the cloth = selling price of (128 + 12) or 140 yds.; *i.e.* 140 yds. are sold for £100, the cost price. Therefore the selling price per yard = £$\frac{100}{140}$ = £$\frac{5}{7}$ = 14*s*. $3\frac{3}{7}$*d*. *Ans.*

Ex. 8. A farmer bought a flock of sheep. He lost 5 of them and sold the remainder for *Rs*.4 a head more than the prime cost, and gained *Rs*.50 by the transaction. Had he sold them only at *Rs*.2 a head more than the prime cost he would have lost *Rs*.70. How many did he buy?

For each sheep sold the man gets *Rs*.(4 − 2) or *Rs*.2 less in the second case than in the first. And in this way for all the sheep sold,

he gets *Rs*.(50+70) or *Rs*.120 less in the second case than in the first. Hence, we have

Rs.2 : *Rs*.120:: 1 : no. of sheep sold.

∴ no. of sheep sold=60, and ∴ no. in the flock=60+5=65. *Ans.*

Miscellaneous Examples VIII.

1. A person by disposing of goods for *Rs*.1820 loses at the rate of 9 per cent.; what ought they to have been sold for to realise a profit of 7 per cent.?

2. If a person owes *Rs*.1000 on the 1st May, at what date ought he to pay *Rs*.750, so that he may retain the remaining *Rs*.250 till the 1st October?

3. I buy goods for *Rs* 600, and sell them immediately for *Rs* 680, giving three months' credit; what is gained per cent. reckoning interest at 8 p. c. per annum?

4. A gentleman sells a lac of rupees out of the 4 per cents, at 16 discount and invests the proceeds when exchange is at 2*s*. 1*d*. in 3 per cent. consols at 96. What income does he derive therefrom?

5. If the difference between the Simple and the Compound Interest on a sum of money for 2 years at 5 per cent. be £5. 18*s*. $9\frac{3}{4}$*d*.; what is the sum?

6. A banker, in discounting a bill due in 3 months at 4 per cent., charges 5*s*. $1\frac{1}{2}$*d*. more than the true discount; find the amount of the bill.

7. The debts of a bankrupt amount to *Rs*.21345. 4*a*. and his assets consist of property worth *Rs*.9167. 10*a*. 8*p*. and an undiscounted bill of *Rs*.5130 due 4 months hence, simple interest being reckoned at 4 per cent. How much in the rupee can he pay his creditors?

8. A person remits *Rs*.15480 to England at the rate of 1*s*. 6*d*. per rupee, and the money is invested in the purchase of 3 per cent. consols at $79\frac{1}{8}$. At what price should he sell out, in order to realise a gain of £24, after having paid $\frac{1}{8}$ per cent. commission on each of the last two transactions?

9. A company guarantees to pay $5\frac{1}{4}$ per cent. on shares of £100 each; another guarantees to pay $3\frac{1}{4}$ per cent. on shares of £10 each; the price of the former is £115 10*s*. and of the latter £7. 15*s*. Compare the rates of interest which the shares return to the purchasers.

10. What income shall I obtain in England, from *Rs*.78000 in Indian Government Bonds at $5\frac{1}{2}$ per cent., when my agents in Calcutta charge me 3 per cent. for drawing and remitting it, and the exchange on England is 1*s*. $10\frac{1}{2}$*d*. for the rupee?

11. A mixture of milk and water contains 32 seers and there is only 1 seer of water in it; how much water must be added to this mixture that in every 32 seers of the second mixture, there shall be $3\frac{7}{8}$ seers of milk?

12. When English money bears a premium of 5 per cent. in America, how much sterling should be given for 750 dollars each worth 4*s*. 6*d*. at par?

13. Two kinds of wheat are sold at the same price, in which 20 per cent. is gained on one kind, and 20 per cent. lost on the other. What percentage will be gained or lost if they be mixed equally, and then sold at the same price?

14. A merchant sells a mixture in the ratio of 1 : 3 of two different kinds of teas, at *Rs*.2. 4*a*. per lb. and gains $33\frac{1}{3}$ per cent. on the original cost. If he had mixed the teas in the ratio of 1 : 2 and sold the compound at *Rs*.2. 8*a*. per lb. he would have gained $33\frac{1}{3}$ per cent. on his selling price. Find the prime cost of each of the two kinds of teas.

15. A merchant owes £650. He gives a bill for £202 due 3 months hence, a second for £204 due 6 months hence and pays the remainder of the debt in ready money. Supposing the rate of interest to be 4 per cent. per annum, what ought he to pay in ready money?

16. A company has a capital of £5000000, of this $\frac{3}{5}$ is ordinary stock and the remainder is 5 per cent. preference: the working expenses and reserve absorb 48 per cent. of the gross receipts. Find the annual earnings, so that the company should just be able to pay the interest on the preference stock. What is the additional amount requisite for paying 1 per cent. on the ordinary stock?

17. If at compound interest the second year's interest is *Rs*.2310, and the third year's *Rs*.2425. 8*a*., what was the first year's interest?

18. A debt is paid on 23rd June by a bill dated at 6 months. Supposing the bill to be discounted on October 14th, the real discount would be *Rs* 35. Find the amount of the bill, the rate of discount being 5 per cent.

19. A gold chain is made of 3 parts gold to 1 copper, and the cost of manufacture is 10 per cent. on the value of the gold. A sovereign consists of 11 parts gold to 1 copper, and 120 oz. Troy of the mixture makes $467\frac{1}{2}$ sovereigns. What should be the price of a chain which weighs $2\frac{1}{2}$ oz., the value of the copper being neglected?

20. A man has an income of £200 a year; an income-tax is established of 7*d*. in the £, while a duty of $1\frac{1}{2}$*d*. per lb. is taken off sugar; what must be his yearly consumption of sugar that he may just save his income-tax?

21. The income-tax being 4*d*. in the £, a person has to pay

£69. 15*s.* 11*d.* less than when the tax was 7*d.* in the £, although his income has increased by £295. What was his income at first ?

22. Two passengers have together 150 seers of luggage and are charged for the excess above the weight allowed 8*a.* 4*p.* and *Re.*1. 0*a.* 8*p.* respectively. If the luggage had all belonged to one of them, he would have been charged *Rs.*2. 13*a.* Find how much luggage each passenger is allowed without charge.

23. A merchant having lost his cargo in the sea, which he had insured, the broker offered him a sum of money for his loss which the merchant refused as being 10 per cent. below the estimated value of his loss ; the broker then offered £379. 15*s.* more than at first, and the amount of the second offer was $5\frac{1}{2}$ per cent. in excess of the estimated value. What was that value, and what sum did the broker first offer ?

24. Calculate the profit made by a bookseller, assuming that he pays 11*s.* 4*d.* for a 16 shilling book, receives 25 copies as 24, and deducts 10 per cent. for commission.

25. An inclined roof rests upon two walls, one of which is $13\frac{1}{2}$ feet high and the other $7\frac{1}{2}$ ft. ; the distance between the two walls is 8 ft. ; what is the area of the roof, supposing its length to be 20 yards ?

26. The discount and interest on a certain sum for the same time are *Rs.*22 and *Rs.*24 respectively, find the sum.

27. If I buy the 3 per cents. at $78\frac{3}{8}$, and the $3\frac{1}{2}$ per cents. at $95\frac{9}{16}$, which is the better investment ? If I had invested £6962. 19*s.* $3\frac{3}{4}$*d.* in each, and the former rose and the latter fell $\frac{1}{16}$, how much should I lose or gain ?

28. A tradesman bought a quantity of goods and sold $\frac{3}{8}$ of them at a profit of 6 per cent. ; the price rising he got 10 per cent. profit on the remainder, and on the whole gained £114 ; what sum did he lay out ?

29. A man has *Rs.*41000 which he invests in the 3 per cent. stock at 87 and 5 per cent. stock at 104. What sums must he invest in the respective stocks to make $3\frac{1}{2}$ per cent. on the whole ?

30. I buy goods for *Rs.*5040 and incur 10 per cent. expenses ; what must I charge in order to make 10 per cent. profit on my capital after allowing 10 per cent. discount ?

31. Certain Railway shares pay an annual dividend of £3. 10*s.* A person having bought 12 shares, at such a price that they yielded $5\frac{5}{8}$ per cent. on his investment, sold them when the price had risen £5, and invested the proceeds in $3\frac{1}{4}$ per cent. stock at 85. Find the alteration in his income.

32. The capital of a Railway Company consists of £19,000000 4 per cent. debentures, £37,000000 5 per cent. preference stock, and £20,000000 ordinary stock. The receipts are £8 per mile per day ;

the length is 2500 miles ; the working expenses are 55 per cent. of the receipts. What dividend can it pay on the ordinary stock ?

33. A gentleman, deriving his income from an investment which pays 5 per cent., spends $\frac{3}{4}$ of his income and pays an income-tax of 8*d.* in the £ ; next year the investment pays $5\frac{1}{2}$ per cent., he spends 15 per cent. more, the tax is reduced to 6*d.* in the £, and he saves £7 more. What was his income ?

34. I bought 50 horses and sold 15 of them at a gain of 20 per cent., 25 at a gain of 16 per cent. and the rest at cost price. Had I sold all at a gain of 18 per cent., I should have gained *Rs.*560 more. Find the cost price of each horse.

35. Two kinds of tea are mixed together and the mixture sold at a gain of 15 per cent. If each kind were to be sold separately at the same price, 20 per cent. would be gained on the first kind, and 5 per cent. lost on the second. Find the proportion of the mixture.

36. A boy bought 200 eggs, and gained 25 per cent. by selling $\frac{3}{8}$ of them at 2 a penny, and the rest for 1*s.* $5\frac{1}{2}$*d.* more than what they cost him. At what rate did he buy them ?

37. Two boys buy oranges at 7 for 3*d.* and 5 for 2*d.* respectively. They buy equal numbers. Compare their rates of profit, if the former sells his oranges at 9 for 4*d.* and the latter at 7 for 3*d.*

38. A Railway bridge crosses a straight canal obliquely. The bridge on one side is 36 ft. lower down the canal than on the other ; and the breadth of the canal is 48 ft. Find the length of the bridge.

39. A tradesman lost 4 per cent. by selling an article for 15*s.* ; what should he have sold it for so as to gain 10 per cent. ?

40. A grocer buys 2 cwt. of tea ; the first cwt. he sells at 5 per cent. profit ; and the second which costs £1 more, at 12 per cent. profit. The difference in retail price being 4*d.* per lb., what is the cost price of each ?

41. The gross receipts of a Railway Company in a certain year are apportioned thus :—40 per cents. to pay the working expenses, 54 per cents. to give the shareholders a dividend at the rate of $3\frac{1}{2}$ per cent. on their shares, and the remainder *Rs.*283500 is reserved. Find the paid-up capital of the Company.

42. A man, buying goods by means of false scales, defrauds to the extent of 15 per cent. and 15 per cent. in selling ; find his whole gain per cent.

43. Eight years ago the ages of *A* and *B* were as 5 : 6 ; and eight years hence they will be as 9 : 10 ; find their present ages.

44. *A* owes *B* *Rs.*2725, and offers to pay him at a certain rate of discount instantly, instead of at the end of 2 years, when the debt will be due ; *B* can place out the money which he will receive at 5 per cent. interest, and by that means will gain by the transaction

*Rs.*25. Reckoning simple interest throughout, find at what rate the discount is calculated.

45. *A*'s present age is to *B*'s as 8 : 7 ; and 10 years ago the proportion was 11 : 9. Find the present age of each.

46. £1000 sterling is due from London to Portugal, when the exchange is 61⅛*d.* per milree. Whether is it better, for Portugal, to draw directly on London, or circuitously, at an expense of 1½ per cent., through Holland and France ;—exchange between Britain and Holland 11·90 florins per £ sterling, between Holland and France 10 florins for 21 francs, and between France and Portugal 480 rees for 3 francs ?

47. The original cost of a pipe of port is *Rs.*550, and it is sold to *A* at a certain loss per cent. ; then *A* sells it to *B* at the same losing rate ; but *B* sells it to *C*, at a profit of 12 per cent. for the original cost. What was the loss per cent. at which the wine was sold to *A* and *B* ?

48. If the Compound Interest of *Rs.*2500 for 2 years be *Rs.*204, what is the rate per cent. per annum ?

49. What fraction of $\sqrt[3]{(\cdot 0135)}$ is $\sqrt[3]{(\cdot 004)}$?

50. A narrow rectangular field *ABCD* has its length *AB* 160 yds. and breadth *BC* $31\frac{3}{7}$ yds. To what point *E* in the side *AB* must a straight line from *C* be drawn, so that *AECD* may contain an acre ?

51. I bought paper at the rate of 3*s.* 7½*d.* for 5 quires, and sold it so as to gain as much on the cost of 32 quires as 3 quires were sold for. At what rate did I sell it per quire ?

52. My age is 62, and my son's age 30 ; how long ago was my age 5 times that of my son ? and how many years hence (if we are both alive) will my age be a third of 5 times his age ?

53 Divide *Rs* 54339 into three sums, such that their amounts by Compound Interest at 5 per cent. per annum, for 20, 23 and 27 years respectively, shall be equal.

54. Given that 1 oz. Troy equals 31·1 grammes ; that 10 grammes of French standard gold are worth 31 francs ; and that the worth of a given weight of English standard gold is to that of the same weight of French standard as 3151 to 3100 :—To what number of Troy ounces of English standard gold is the franc equivalent, and what is the fixed number of francs equivalent to £1 ?—the English mint price for standard gold being £3. 17*s.* 10½*d.* per ounce.

55. How many francs are equivalent to £1, when gold purchased in London at 77*s.* 10½*d* per ounce standard is sold in Paris at 14½ per mille (*i.e.*, per 1000) premium on the fixed price ? And how many, when gold is at 1 per mille discount ?

56. A bankrupt owes £4594. 1*s.* ; he will pay a dividend of

3*s.* 6*d.* in the pound three months hence, a second of 3*s.* six months hence, and a third of 1*s.* nine months hence. What is the present value of his assets, money being worth 5 per cent. per annum ?

57. A person having to pay *Rs.*10850 two years hence, invests a certain sum in the 3 per cent. Government Securities (to accumulate interest till the debt is paid) and also an equal sum the next year. Supposing the price of the Government Securities to remain throughout at 73, what must be the sum invested on each occasion so that with its interest, there may be just sufficient to pay the debt at the proper time ?

58. A person borrows *Rs.*6180 in two separate sums, at the respective rates of $3\frac{1}{2}$ and 5 per cent. per annum ; and he repays the two loans at the end of 10 months, with interest amounting to *Rs.*225. Required the amount of each loan.

59. A person finds that if he invest a certain sum in Railway shares paying £6 dividend a share, the price of the share being £132, he will obtain £54 a year more than if he invested in the 3 per cents. at 93. How much has he to invest ?

60. A contractor sends in a tender of £5000 for a certain work ; a second sends in a tender of £4850, but stipulates to be paid £500 every 3 months ; find the difference of the tenders, supposing the work in both cases to be finished in 2 years, and money to be worth 4 per cent. Simple Interest.

61. (1) Given that the square of 15334 is 235131556 ; find that of 153347 without going through the operation of squaring.

(2) Given that the square root of 1038361 is 1019 ; find the square root of 103876864.

62. A landlord has an estate that brings him in £3000 a year, but this gross income is liable to deductions for rates and repairs to the extent of 12 per cent. He sells it at 26 years' purchase on the gross income, and invests the price in the 3 per cents. at $97\frac{1}{2}$. What difference is caused in his income ?

63. A man sold at 48 and 95 respectively £500 ordinary stock in the *A* Railway paying a dividend at the rate of $1\frac{3}{4}$ and £800 preference stock in the *B* Railway paying a dividend of 4 per cent. He then invested $\frac{1}{4}$ of the money in the Tramway Company where the £24 share paying interest at 6 per cent. was at £6 premium : £150 in the *C* Railway which paid no interest and the remainder in Bank shares at par ; what rate of interest must he receive from the Bank in order to increase his annual income by £12. 5*s.* ?

64. A corn merchant having bought 1300 quarters of wheat, sold one-fifth of it at a profit of 5 per cent., one-third at a profit of 8 per cent., and the remainder at a profit of 12 per cent. ; but had he sold all at a profit of 10 per cent., his gain would have been £16. 13*s.* 8*d.* more. What did the wheat cost him ?

65. The discount on *Rs.*825 for a certain length of time is *Rs* 75 ; what is the discount on the same sum (i) for twice that length of time, and (ii) for half that length of time ?

66. A person has *Rs.*24180 to invest, the $5\frac{1}{2}$ per cent. Government Loan being at 108, and the 6 per cent. Municipal Loan of *Rs.*1000 being at 1020 ; find how he must divide his capital between the Government and Municipal Loans, that he may obtain the same income from each.

67. Divide £444 among *A*, *B*, *C* and *D* in such a manner that £10 more than $\frac{3}{8}$ of *A*'s share, £20 less than $\frac{5}{6}$ of *B*'s share and £32. 10*s* more than $\frac{3}{4}$ of *C*'s share, and £65 more than $\frac{7}{12}$ of *D*'s share may all be equal.

68. What sum of money will amount to *Rs.*6996 9*a.* 7·2*p.* in 2 years, reckoning compound interest for the first year at 4 per cent and for the second at $3\frac{1}{2}$ per cent. per annum ?

69. A certain article of consumption is subject to a duty of 6*s.* per cwt. ; in consequence of a reduction in the duty the consumption increases one-half, but the revenue falls one-third. Find the duty per cwt. after the reduction.

70. Divide *Rs.*429 into four parts such that their simple interest for 4, 6, 7 and 10 months, and at 3, 4, 5 and 6 per cent. per annum respectively, shall be all equal.

71. *A* sells to *B* a horse, which had cost him *Rs.*300 at $12\frac{1}{2}$ per cent. profit ; *B* sold it to *C* at a profit of 5 per cent. on what it cost him. What would *A* have gained per cent., had he sold the horse to *C* for the money which *C* had paid *B* for it ?

72. A farmer gave for a horse a bill of £73 due in 1 month, and sold him at once for a bill of £87 at 4 months. Required the farmer's gain per cent., reckoning interest at $4\frac{1}{2}$ per cent.

73. A man has a certain amount of 5 per cent. stock. He sells out one-third of it at 104, and invests the proceeds in the 4 per cents. at 98. He sells out from the 4 per cents. when they have risen 2 per cent. and then purchases the same amount of 5 per cent. stock at 102 as he sold out originally. His gain being £202, find the amount of 5 per cent. stock originally held by him.

74. A tradesman selling goods for a certain price to be paid 6 months hence offers to give $\frac{1}{10}$th more of the same goods for the same price in ready money. What was the rate of discount ?

75. *A* and *B* each lends £250 for three years, *A* lends at $4\frac{3}{4}$ per cent. simple interest, and *B* at $4\frac{1}{2}$ per cent. per annum, compound interest. Find the difference in the amount of interest they receive.

76. If by selling cloth at *Rs.*14. 4*a.* for 5 yards my gain would be $6\frac{2}{3}$ per cent., what should I gain or lose per cent. by selling it at *Rs.*18. 12*a.* for 7 yards ?

77. A banker sells 400 English sovereigns at 25·35 fr., and buys Spanish piasters at 5·40 fr. Two months afterwards he sells again his piasters at 5·70 fr. and buys 400 sovereigns at 25·20 fr. What is his profit, and at what rate does he place his money?

78. A merchant fits out 3 ships in succession to run the American blockade; he reckons the total outlay on each ship after the first to be 25 per cent. more than on the one that preceded it. The first and third get into port, and he gains 160 per cent. on their cost, the second is taken. Find his loss or gain per cent. on the whole.

79. A man left £30000 stock in 3 per cent. Government Securities, to be divided among his three sons in the proportion of their ages, which were 15, 8 and 7 years respectively; afterwards when these securities stood at 10 discount the eldest son sold out, and invested the proceeds in 6 per cent. Bank shares at par. By how much did the annual income of the eldest then exceed that of the youngest son?

80. A man invests £864 in the following manner. One-half he invests in the $5\frac{1}{2}$ per cents. at 8 premium and the other half in Bank shares at 116 premium. After one year he sells out both his $5\frac{1}{2}$ per cents. and also his Bank shares, the former being now at 12 premium, and the latter 130 premium, and invests the whole of the proceeds in the $4\frac{1}{2}$ per cents. at $90\frac{5}{8}$. His annual income is now £1 less than it was before. What rate of interest did the shares pay?

81. Find the length of an edge of a cube of pure gold equal in value to the annual revenue of Great Britain (70 millions sterling), given that gold is 19·26 times as heavy as water, that a cubic foot of water weights 1000 oz. and that the value of fine gold is £4. 5*s.* per oz.

82. Prove the truth of the following extract from the *Times* Newspaper :—(See Ex. 54, above).

The quotation of gold at Paris is about 1 per mille discount (according to the last tariff), which at the English Mint price of £3. 17*s.* $10\frac{1}{2}$*d.* per oz. for standard gold, gives an exchange of $25·14\frac{1}{2}$; and, the exchange at Paris on London at short being $25·07\frac{1}{2}$, it follows that gold is about 0·28 per cent. dearer in Paris than in London.

MISCELLANEOUS EXERCISES.

1. The number of emigrants from the United Kingdom in ten years was 1,697,579;—649,742 being English, 158,226 Scotch, 31,983 foreigners. How many were Irish?

2. A library contains 3275 volumes, and each volume on the average contains 493 pages, and each page 39 lines. How many lines are there?

3. How many times has a batsman been out, if he has made 1073 runs and his average is 29?

4. What was the cost of each pig, if after buying 23 pigs I have £23. 7*s.* 2*d.* left out of £50?

5. A waggon loaded with 217 equal parcels weighs 2 tons 2 cwt. and the waggon itself weighs 18 cwt. 3 qrs.; find the weight of each parcel.

6. What amount will be left out of £50, after paying the following bills: £9. 17*s.* 4*d.*, £4. 12*s.* 6½*d.*, £5. 10*s.* 9¾*d.*, £27. 5*s.* 2¼*d.*?

7. How often does £789303. 6*s.* 6¾*d.* contain £1654. 14*s.* 5¾*d.*?

8. A sum of money was distributed among *A*, *B* and *C*. The shares of *A* and *B* together amounted to *Rs.*120, those of *A* and *C* to *Rs.*160, and those of *B* and *C* to *Rs.*184. Find the share of each.

9. I sold a horse for *Rs.*600, and thereby gained one-fifth of my outlay; what was my outlay?

10. How many ponies must a person buy at £8. 10*s.* each, so that after allowing 3*s.* 6*d.* for the food of each for a week, he may then gain £440. 11*s.* 3*d.* by selling each of the lot at £9. 4*s.* 7*d.*?

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11. The area of the British Colonies is 8,869096 square miles and the population is 292,680,168. How many people are there to the square mile? How much would a tax of 2¼*d.* per head on the whole population amount to in £. *s.* *d.*?

12. If the greater of two numbers be 19 times 508, and their difference be 15 times 112; find the sum and the product of the numbers.

13. If 7423971 be the dividend, 12130 the quotient, and 411 the remainder, what is the divisor?

14. Reduce £5672841. 16*s.* 9¾*d.* to *farthings*.

15. If £15942. 16*s.* 6*d.* were distributed equally among 216 persons, how much would each receive?

16. If the circumference of a coach-wheel measures 17 ft. $7\frac{1}{2}$ in., how often will it turn round in travelling a distance of 8 miles 264 feet ?

17. How often is 3 tons 27 lbs. 13 oz. contained in 228 tons 18 cwt. 3 qrs. 13 lbs. 12 oz. ?

18. Find the greatest number which will divide 13956 and 14565, and leave a remainder 7 in each case.

19. What is the price of a chair, if after buying 15 chairs my money has been reduced from *Rs.*250 to *Rs.*132. 13*a.* ?

20. A man whose weekly earnings are *Rs.*8 12*a.*, saves $\frac{1}{4}$ of that sum every fortnight ; when will he have saved *Rs.*224 ?

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21. If 2948 bricks are used per yard to build a tunnel which is 6285 yards long ; and 2175 bricks per yard to build another tunnel which is 6082 yards long ; state *in words* how many more are used for the one tunnel than for the other.

22. Multiply 562347892 by 297099011 in three lines.

23. The product of two numbers is 15580656, and one of them is 6552. Find the other.

24. Find the L. C. M. of 298717, 197457 and 207583.

25. Reduce 321005468 29 farthings to £. *s.* *d.*

26. A tax collector collected *Rs.*10. 1*a.*, *Rs.*210. 12*a.*, *Rs.*64. 0*a.* 2*p* and *Rs.*18. 11*a.* 8*p.*, but his pocket burst, and the money was scattered. He picked up *Rs.*303. 8*a.* 4*p.* Did he lose any money ? If so, how much ?

27. A farmer pays 30*s.* an acre rent on a farm of 215 acres. His working expenses amount to £589. 17*s.* $4\frac{3}{4}$*d.* His receipts are £1305. 6*s.* 8*d.* Find his net income.

28. How many hens have I bought at 1*s.* $8\frac{3}{4}$*d.* each, if I have £2. 16*s.* $9\frac{1}{4}$*d.* left out of a £5 note ?

29. How many minutes were there in the first six months of 1888 ? Find the G. C. M. of 23992091 and 8209897.

30. A man spends £84. 12*s.* 3*d.* in each of the first 5 months of the year. If he does not wish to spend more than £826. 8*s.* 9*d.* in the whole year, what must be his average monthly expenditure for the remainder of the year ?

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31. There are 57 boxes of rice each containing 809309 grains, and 76 other boxes each containing 719294 grains : write *in words* how many grains of rice there are altogether.

32. Find the G. C. M. of 230299, 3083035 and 1093729.

33. What is the least number that must be added to 56438971, that the result may be exactly divisible by 4064 ?

34. What is the least number that can be exactly divided by $\frac{1}{13}$, $2\frac{1}{3}$, 5, $6\frac{1}{3}$ and $\frac{1}{11}$?

35. Reduce 3208769841 square inches to *acres*.

36. I buy 60 gallons of wine at £1. 3*s*. 6*d*. a gallon and £1. 10*s*. is gained by selling it at £1. 2*s*. 6*d*. a gallon. How much water has been added ?

37. Simplify $\frac{1}{2}$ of $(\frac{1}{3}+\frac{1}{4}+\frac{1}{5})+7\times(\frac{1}{35}+\frac{1}{36})-\frac{1}{12}+\left(\frac{11\frac{1}{2}}{15\frac{1}{3}}-\frac{1}{4}\right)$.

38. The live stock on a farm consists of a certain number of horses worth 60 guineas each, an equal number of pigs worth £2. 10*s*. each, 3 times as many cows worth £18. 10*s*. each and 15 times as many sheep worth £1. 15*s*. each. The whole value of the live stock is £1030. 15*s*. How many are there of each kind ?

39. A man leaves *Rs*.50000 to his wife, and the remainder of his property to be divided equally among 4 children. It is found that each child has $\frac{1}{7}$ of the whole. How much did he leave altogether ?

40. A rupee is worth 2*s*. $0\frac{1}{2}$*d*., and a dollar 4*s*. $4\frac{1}{2}$*d*. ; find the least number of rupees which makes an exact number of dollars.

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41. If the distance of the Sun from the Earth be $92\frac{1}{2}$ million miles, and light travels from one to the other in 498 seconds, find the velocity of light in miles per hour.

42. Divide 198 ac. 3 ro. 16 po. 3 yds. 72 in. by 187.

43. Four bills, amounting to £27. 10*s*. $3\frac{3}{4}$*d*., £13. 4*s*. $7\frac{1}{2}$*d*., £43. 0*s*. 6*d*. and £5, are paid out of £100. What money is left ?

44. Twenty-five years ago a man was four times as old as his son, whose present age is 33. How old is the father now ?

45. Divide a lac of rupees between *A*, *B* and *C* in the proportion of 2, 3, 4, and the same amount between *D*, *E* and *F* in the proportion of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

46. How many payments of 14*s*. $11\frac{3}{4}$*d*. amount to £89. 2*s*. $6\frac{1}{4}$*d*. ; and how many plots each containing 2 ro. 16 po. 8 sq. yds. are there in 10 ac. 36 po. 15 sq. yds. ?

47. Find by Practice the cost of warming a building for 11 days 17 hrs. 28 min., if the cost is £4. 10*s*. per day.

48. A man runs $4\frac{7}{8}$ times round a course $930\frac{1}{4}$ yds. long. What fraction of 3 miles $301\frac{1}{2}$ yds. does he run ?

49. Simplify :—

$\frac{1864}{1408}$ of $\{\frac{7}{10}$ of £$3\frac{9}{14}+6\frac{3}{8}$ of £3. 0*s*. 9*d*. $-4\frac{8}{81}$ of £3. 2*s*.$\}$.

50. If I pay *Rs.*23. 13*a.* 6*p.* as income-tax on a rental of *Rs.*715. 5*a.*, what should I pay on a rental of *Rs.*5107. 8*a.* ?

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51. What prime numbers divide 222222 ; and what is the least number that must be added to it that the sum may be divisible by 47, 53 and 59 ?

52. Divide 2875962 by $5 \times 3 \times 11$ by short division, and find the true remainder.

53. Reduce 5762309875407 inches to *miles.*

54. Simplify $\dfrac{3\frac{7}{9} \times 1\frac{1}{17} + 4\frac{1}{12} - 3\frac{9}{16}}{5\frac{1}{9} - 7\frac{7}{8} + 28\frac{7}{20} + \frac{1}{3}} + \dfrac{3\frac{2}{3}}{4\frac{1}{7}} \times 3\frac{5}{6} \times 5\frac{6}{7} - 17\frac{1}{2}$.

55. Divide 2·3 by ·0015, and £$\cdot 0\dot{2}\dot{1}$ by ($1\cdot\dot{7}\dot{6}$*s.* − 3·2*d*).

56. What will be the cost of painting a room 20 ft. 3 in. long, 18 ft. 6 in. broad and 10 ft. $4\frac{1}{2}$ in. high, containing two windows, whose dimensions are $7\frac{1}{2}$ ft. by $4\frac{3}{4}$ ft. each, at the rate of 2*s.* 9*d.* per sq. yd. ?

57. 23 cwt. 3 qrs. 7 lbs. are bought at £2. 10*s.* 8*d.* per cwt. and 72 cwt. 2 qrs. 8 lbs. at £2. 7*s.* 10*d.* per cwt. Find by Practice the amount expended and give the average price per lb.

58. A bankrupt is indebted to *A*, *B*, *C* and *D*—*A*'s debt is twice *B*'s ; *B*'s three times *C*'s ; *C*'s half of *D*'s. How much should each receive of assets to the amount of *Rs.*45680 ?

59. If 2 men and 5 women can do a piece of work in 8 days of 9 hrs. each ; how long will it take 3 men and 6 women to do a piece of work twice as great, working 8 hours a day ; the work of a man being double that of a woman ?

60. Find the simple interest at 4 per cent. per annum on *Rs.*595. 9*a.* for 4 years and 17 weeks, reckoning 52 weeks to a year.

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61. What must I pay for a bill of exchange on London for £73. 15*s.* 6*d.*, the exchange being at the rate of 1*s.* $10\frac{7}{8}$*d.* for the rupee ?

62. Find the value of—

$\cdot 3\dot{8}\dot{7}$ of £8. 16*s.* 3*d.* + $6\frac{1}{2}$ of $\frac{10}{65}$ of 7*s.* $8\frac{1}{2}$*d.* + $\frac{7}{11}$ of 1*d.*

63. If the carriage of 30 mds. through 36 miles cost *Rs.*125 ; what weight ought to be carried 48 miles for *Rs.*66. 10*a* 8*p.* ?

64. Find the least number of weeks in which an exact number of half-guineas can be earned, the wages per week being 16*s.* 4*d.*

65. If a person receive $4\frac{1}{2}$ per cent. interest on his capital by investing it in the 4 per cent. Government stock ; what is the price of stock, and how much can be purchased for *Rs.*12000 ?

66. A person buys 64 animals—cows and horses—for *Rs.*6000. Each cow costs *Rs.*60, and each horse *Rs.*100. Find the number of horses purchased.

67. What would a banker gain by discounting on September 21st a bill of £318. 3*s.* dated July 31st at 4 months, at 5 per cent.?

68. A man sells a horse for *Rs.*246 and loses $26\frac{1}{3}$ per cent. on what the horse cost him; what was the original cost?

69. Four merchants *A*, *B*, *C* and *D* trade together; *A*'s stock of £400 was in trade 12 months, *B*'s stock of £450 for 9 months, *C*'s stock of £480 for 8 months and *D*'s of £405 for 6 months. The whole profit, being £1000, was in trade 12 months. How much ought each to receive?

70. A person in India wishes to invest *Rs.*24000 in the 3 per cent. consols at 90; the rate of exchange is 1*s.* 10*d.* for a rupee; brokerage in England is $\frac{2}{11}$ per cent.; how much stock would he realise, and what would be his yearly income?

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71. Show that the sum of the squares of six thousand and twenty-one and eight thousand and twenty-eight is equal to the square of ten thousand and thirty-five.

72. If when a number is divided successively by 11, 19, 23 (as in short division) the remainders are 8, 3, 14 respectively, find what the remainder would be if the same number were divided by 4807.

73. Simplify $\frac{1}{8}$ of $(\frac{1}{10}-\frac{1}{11})+\frac{\frac{1}{7}-\frac{1}{9}\div(\frac{4}{9}+\frac{4}{11})}{(\frac{1}{7}+\frac{1}{9})\div(\frac{4}{9}-\frac{4}{11})}\times\frac{\frac{1}{8}+\frac{1}{7}\div(\frac{1}{7}-\frac{1}{9})}{(\frac{1}{8}+\frac{1}{7})\div\frac{1}{7}-\frac{1}{9}}$.

74. 124 men dig a trench 110 yds. long, 3 ft. wide and 4 ft. deep, in 15 days of 11 hours each; another trench is dug by half the number of men in 7 days of 9 hours each, how many cubic feet of water is the latter capable of holding?

75. A cistern, without a lid, whose floor and walls are an inch and a half thick is 5 ft. 3 in. long, 3 ft. 7 in. wide and 2 ft. $5\frac{1}{2}$ in high in its external dimensions. Find its internal surface, and the cost of painting the same at 4*d.* per sq. foot.

76. In a certain firm *A* invests half as much again as *B*, and $\frac{2}{3}$ as much again as *C*, and *A*'s capital is *Rs.*12000 more than *C*'s. They gain *Rs.*1330; what is each man's share of the profits?

77. If 4 men earn as much in a day as 7 women, and one woman as much as 2 boys, and if 6 men, 10 women and 14 boys working together for 8 days earn £22, what will be the earnings of 8 men and 6 women working for 10 days?

78. A tank is 300 yds. long and 150 yds. broad: with what velocity per second must water flow into it through an aperture 2 ft. broad and $1\frac{1}{2}$ ft. deep, that the level may be raised 1 foot in 9 hours?

79. A person having invested a sum of money in the 3 per cents. receives annually therefrom £233 after deducting the income-tax of 7*d.* in the pound. Find the amount of stock, and also what it can be sold for when the $3\frac{1}{2}$ per cents. are at $109\frac{3}{8}$.

80. The length of the Eastern Bengal Railway being 110 miles and the capital employed in its construction £1500000, what must be the gross annual traffic receipts per mile in order that a dividend of 5 per cent. may be paid to the shareholders after allowing 45 per cent. of the gross receipts for current expenditure?

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81. *A* had some rupees, the number being composed of three 7's, preceded by 16, and followed by 216. *B* took some away, and the number which remained was expressed by the same digits in the same order with the exception of the digit 2. How many did *B* take?

82. If the remainders on dividing a certain number (by short division) by 5, 9, 7, 11 successively be 3, 5, 8, 4; find the remainder when the number is divided by 3465.

83. Simplify $2\cdot\dot{1}2345\dot{6} + \cdot12\dot{3}45\dot{5} + 1\cdot23\dot{4}5\dot{6} - 1\cdot1234\dot{5}\dot{6} - 1\cdot12345\dot{6}$.

84. Multiply 3·720789426 by ·0086341532 to five places.

85. Find by Practice the cost of 3 tons 17 cwt. 3 qrs. 23 lbs. 11 oz. of iron at *Rs.*9. 13*a.* 7*p.* per cwt.

86. If 21 men can in 12 days make 1260 thousand bricks working 12 hours a day, in how many days of 8 hours each, can 59 men make 177 lacs of bricks?

87. The cost of carpeting a room whose length is 22 ft. 8 in. with carpet at 3*s.* $4\frac{1}{2}$*d.* per sq. yd. is £7. 7*s.* 4*d.*; find the breadth of the room.

88. A merchant buys two pipes of wine, one for £112, the other for £120, and he also buys a third pipe; on mixing the three, he sells his wine at 50*s.* per dozen, gaining 25 per cent. on his outlay; what was the price of the third pipe? (The number of dozens in a pipe is 56.)

89. The wholesale price of books is 25 per cent. lower than the retail price, and 13 books are counted into the dozen. What percentage is obtained by the retailer?

90. I pay *Rs.*51000 to a bank for a bill of exchange payable in London. The rate of exchange is 1*s.* $10\frac{1}{2}$*d.* for the rupee and the bank charges me 2 per cent. on the amount payable in England. How much will my agent in London receive?

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91. When 582167 is divided by a certain number, the quotient is 762 and the remainder 761. What is the number?

92. A number diminished by $\frac{2}{3}$ of itself, when divided by 809, gives a quotient 327 and a remainder 456. What is the number?

93. What decimal multiplied by 125 will give the sum of $\frac{5}{8}$, $\frac{7}{16}$, $\frac{3}{4}$, ·09375 and 2·46?

94. The owner of $\frac{3}{8}$ of a ship sells $1\frac{1}{2} \div 4\frac{3}{8}$ of his share for £$21\frac{27}{49}$; what was the value of $\frac{3}{11}$ of $\frac{3}{8}$ of $2\frac{1}{1\frac{5}{7}}$ of the ship?

95. Find the value of ·375 of a guinea + $\cdot\dot{5}\dot{4}$ of 8*s*. 3*d*. + $\cdot 0\dot{2}\dot{7}$ of £2. 15*s*. and reduce the result to the fraction of a guinea and a half.

96. If the value of the rupee varies from 1*s*. 9*d*. to 1*s*. 9½*d*. and the franc from 9½*d*. to 10*d*.; find the maximum number of francs which it is always safe to give for *Rs*.500.

97. A person after paying 7*d*. in the £ for income-tax on his income, has £1632. 18*s*. 10*d*. remaining; what had he at first?

98 A farm contains 190 ac. 3 ro. 25 po. 13¾ sq. yds., of which 81 ac. 3 ro. 10 po. 27½ sq. yds. is pasture. What fraction of the whole farm is not pasture?

99. Extract the square root of $\cdot\dot{0}4761\dot{9} \div \cdot 06\dot{8}5714\dot{2}$, and the cube root of 206·425071.

100. A merchant buys goods for £568. 4*s*. and sells half of them at a gain of 1*d*. in the shilling on the cost price, one-third of them at a gain of 2*d*. in the shilling and the remainder at a gain of £15 15*s*. 8*d*. How much per cent. does he gain on the whole transaction?

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101. Find the G. C. M. and the L. C. M. of 157 days 7 hrs. 4 min. 7 sec, and 243 days 2 hrs. 11 min. 49 sec.

102. Multiply 32856 by 121711 in *three* lines only.

103. Find the value of $3\cdot\dot{6}$ of $\cdot\dot{9}5\dot{4}$ of $\cdot\dot{4}2857\dot{1}$ of 3*s*. 1*d*.

104. Divide 3·72808976 by 1·369840056, retaining six places.

105. *A* and *B* own a ship in shares which are in the ratio of 2 to 3. They dispose of parts of their shares to *C* so that *A*, *B* and *C* hold the ship in equal shares. What is the ratio of the payments that *C* must make to *A* and *B*?

106. Tea at 4*s*. 3½*d*. per lb. is mixed with tea at 3*s*. 7½*d*. per lb. so that the mixture contains 72 per cent. of the former. Find the weight of a chest of this mixture which is worth £6. 16*s*. 10*d*.

107. How many planks of teak, 12 ft. long and 71⅜ in. broad, will be required to floor a room 7½ yds. long and 5 yds. wide; a space 7½ ft. long and 5 ft. broad being left unplanked?

108. *A* barters sugar with *B* for rice which is worth 1⅞ annas a seer, but on weighing his sugar uses a false maund weight. *B* discovers this, and to make the exchange fair, raises the price of his rice to 2¼ annas a seer. Find the real weight of the false maund which *A* uses.

109. The diameter of a fore-wheel of a carriage is 1¾ ft. and that of the hind-wheel is 3 ft., how far will the carriage have travelled when the fore-wheel has made 100 more revolutions than the hind-wheel? (The circumference of a circle : diameter :: 3·1416 : 1.)

110. A person bought 10 Bank of Madras shares at *Rs.*1540 each and for $5\frac{1}{2}$ years got interest on his investment at the rate of 5 per cent. He then sold his shares at a loss of $22\frac{1}{2}$ per cent. How much did he make by the transaction and what rate per cent. per annum had he for his money?

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111. If when a number is divided continuously by 6, 7, 8, 9, 10, (as in short division) the remainders are 4, 2, 5, 1, 8 respectively, find the remainder when the number is divided by 30240.

112. A vulgar fraction has for its numerator 183, and its nearest approximate value in hundredths is ·43; what is the denominator?

113. *A* can copy a certain manuscript in 17 hours by writing at the rate of 3 lines per minute, *B* can copy the same in 24 hours. After 476 lines have been copied by *A*, in what time can *B* finish it?

114. If a cubic foot of marble weigh 2·716 times as much as a cub. foot of water, find the weight of a block of marble 9 ft. 6 in. long, 2 ft. 3 in. broad, and 2 ft. thick, supposing a cub. foot of water to weigh 1000 ounces.

115 If 48 men working 8 hrs. a day for one week can dig a trench 235 ft. long, 40 wide and 28 deep; in what time can 12 men working 10 hours a day form a Railway cutting 156060 cub. yards? (A week = 6 working days.)

116. A train starts from *A* at 12 o'clock and runs towards *C*, which is 100 miles distant, at the rate of 30 miles an hour; at the same time the mail cart starts for *C*, from *B*, which is half-way between *A* and *C*, and runs at 10 miles an hour; at what distance from *C* will it be overtaken by the train?

117. *A* and *B* enter into partnership. *A* supplies the whole of the capital amounting to *Rs.*45000 upon condition that the profits are to be equally divided, and that *B* pays *A* interest on half the capital at 10 per cent. per annum, but receives *Rs.*120 per mensem for carrying on the concern. Find their total yearly profits, when *B*'s share is equal to one half of *A*'s share.

118. What sum of money put out at Compound Interest will in 2 years amount to £1944·81, interest being at the rate of 10 per cent. per annum, and being paid *half-yearly*?

119 A gentleman receives 10 per cent. upon his investment in India. When the exchange is at 1*s.* $10\frac{1}{2}$*d.* he disposes of his investment at a premium of 60 per cent. and transfers his capital to 6 per cent. English Securities at par, which yield him an annual income of £1770. Find his original income in India per mensem.

120. A certain number of men and women subscribe a sum of money, the number of women being four times the number of men.

Each man subscribes as many annas as there are men altogether, and each woman as many pies as there are women altogether. The total amount subscribed being *Rs.*756 ; find the number of men and women.

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121. Find the least number which, when multiplied into 253125000, will make the product a perfect cube.

122. A farmer has 1134 sheep and 630 lambs. He forms them into flocks, keeping sheep and lambs separate, and having the same number of animals in each flock. If these flocks are as large as possible, how many animals are there in each ?

123. Calculate to four places $\frac{(11\cdot29+1\cdot306+\cdot0009) \text{ of } (4\cdot13+\cdot6)}{(5\cdot23+7\cdot98) \text{ of } (3\cdot1+6\cdot283)}$.

124. If one lb. Troy of gold be coined into $46\frac{29}{40}$ sovereigns and standard gold contains $10\frac{1}{2}$ parts of pure gold to $1\frac{1}{2}$ parts of copper ; find the weight of pure gold in one sovereign in grains. Also express the weight of copper in a sovereign as the decimal of one lb. Avoir.

125. If the work done by a man, a woman and a child be in the ratio of 3, 2, 1, and there be in a factory 24 men, 20 women, and 16 children, whose weekly wages amount to *Rs.*204 ; what will be the yearly wages of 27 men, 40 women and 15 children ?

126. A room is 19 ft. 5 in. long and 16 ft. 7 in. broad and the cost of painting the walls at 7*a.* 6*p.* per sq. yd. is *Rs.*43. 3*a.* Required the height of the room.

127. Divide a guinea between *A*, *B*, *C* and *D*, so that *B*'s share is $\frac{1}{3}$ more than *A*'s, *C*'s $\frac{1}{3}$ more than *B*'s and *D*'s $\frac{1}{3}$ more than *C*'s.

128. If the rate of exchange is 1*s.* $8\frac{9}{16}$*d.* per rupee, what must be paid in India for a bill for £850 on England ?

129. A person finds that if he invest a certain sum in Railway shares paying £6 per share when the £100 share is at 132, he will obtain £10. 16*s.* a year more for his money than if he invest in 3 per cent. consols at 93. What sum has he to invest ?

130. A certain sum put out at Compound interest amounts in two years to £270·4 and in three years to £281·216. Find the sum and the rate per cent.

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131. By what must 152207 by multiplied so that the product may consist of 8 digits, each digit being 1 ?

132. The population of India was by a former census found to be 190531440, of whom 139421250 were Hindoos. Find the ratio of the Hindoos to the whole population

133. A person settling his bills paid $\frac{1}{4}$ of his money to one

man, $\frac{3}{8}$ of the remainder to another, and $\frac{4}{7}$ of the rest to a third. If he had *Rs.*33 remaining, what had he at first?

134. Find the square root of $7 \cdot 4538 - 6 \cdot 8 + 8 \cdot 5 - 2 \cdot 03 \times 1 \cdot 17$.

135. Two equal sums were divided, the one among 141 men and the other among a certain number of women; each man received *Rs.*12. 8*a.* and each woman 12*a.* less; how many women were there?

136. Twelve years ago, *A* had *Rs.*13000 and *B* had *Rs.*9100. *A* has been more fortunate than *B*, and gained 3 per cent. where *B* gained 1 per cent.; *B* has now doubled his capital. How much has *A* got?

137. Bought a quantity of tea, and sold part of it, at the rate of *Re.*1. 0*a.* 6*p.* per lb., losing thereby 1 per cent. What was gained per cent. on the remainder which was sold at *Re.*1. 1*a.* 6*p.*?

138. What amount of stock must be purchased in the English 5 per cents. at $111\frac{1}{8}$ to produce the same yearly return as 3 lacs of rupees (a rupee = 1*s.* $10\frac{1}{2}$*d.*) invested in the Government $4\frac{1}{2}$ per cents. at $101\frac{1}{4}$?

139. A horse was sold at a loss for *Rs.*60. Had it been sold for *Rs.*81. the gain would have been $\frac{3}{4}$ of the former loss. Find the cost of the horse.

140 What is the smallest number of articles costing £5. 9*s.* $4\frac{1}{2}$*d* each, which can be purchased for an integral number of sovereigns?

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141. Multiply 6372 and 630072 by 567, and explain why the difference of the two products is divisible by 11, and by the squares of 7 and 10, and by the fourth power of 9.

(i) What change would occur in these factors if the two zeros instead of being in the centre of the digits 6, 3 and 7, 2 occupied some other position still being together?

142. Reduce the expression $\left(\frac{3\frac{1}{3}}{7} + \frac{2}{10\frac{1}{2}} - \frac{5}{18} \text{ of } \frac{4}{7}\right) + \frac{4}{7}$ to its simplest form; and find the value of £$\frac{2}{3}$ + $\frac{1}{3}$ of a guinea − $\frac{17}{18}$ of 2*s.* 6*d.*

143. Divide 1215013·8 by 2·023 and ·000072072 by ·000012. Find the fraction corresponding to $1 \cdot 0\dot{1}$, and divide 27·36 by $3 \cdot \dot{1}0\dot{9}$.

144. What is the least number of dollars at 4*s.* 2*d.* each, which is equal to an exact number of sovereigns?

145. If the income-tax be 6 pies in the rupee for the first half of the year and 3 per cent. in the second, what is the gross income of a gentleman whose net annual receipts amount to *Rs.*1454. 1*a.*?

146. *A* can do $\frac{1}{4}$ of a piece of work in $\frac{3}{8}$ of the time that it would take *B* to do $\frac{5}{6}$ of it, and *B* can do $\frac{7}{8}$ of the same work in $\frac{1}{2}$ the time *C* would occupy in doing another piece of work half as large again as the first. If *C* can finish the first named piece of work in 6 hours, how long would *A* and *B* together be in doing it?

147. A debt of *Rs.*700 is discharged by a payment of *Rs.*180 in cash, and a bill for *Rs.*533 due 6 months hence. At what rate is discount calculated ?

148. Exchange on London is at the rate of 1*s.* 7*d.* on demand, and 1*s.* $7\frac{1}{8}$*d.* at 60 days' sight. What is the rate per annum for money in the latter case ?

149. A man buys wine at 4*s.* a gallon ; he mixes it with water, and by selling the mixture at 3*s.* a gallon gains 20 per cent. on his outlay. How much water did each gallon of the mixture contain ?

150. What sum must a person invest in the 3 per cents. at 90, in order that by selling out £1000 stock when they have risen to $93\frac{7}{8}$, and the remainder when they have fallen to $84\frac{3}{8}$, and investing the whole proceeds in the 4 per cents. at par, he may increase his annual income by £9. 5*s.* ?

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151. The yearly expense of a village school was *Rs.*300. This is paid partly by a rent of $29\frac{22}{28}$ bighas of land, at *Rs.*3. 8*a.* per bigha, partly by a tax on the village of *Rs.*100, partly by Government allowance of *Re.*1. 4*a.* per scholar, and the rest was made up by the scholars, of whom there were 45 ; what did each pay ?

152. How many times is $\frac{1}{4}$ of $13\frac{1}{3}$ of 2*s.* $2\frac{1}{4}$*d.* contained in $2\frac{1}{4}$ of 3*s.* 4*d.* + $4\frac{4}{11}$ of 1*s.* $1\frac{3}{4}$*d.* + $\left(\frac{27\frac{5}{8} \text{ of } 28\frac{1}{4}}{20\frac{3}{8} \text{ of } 75\frac{1}{8}} + \frac{3\frac{6}{7} \text{ of } 1\frac{1}{3}}{7\frac{5}{7} \text{ of } 2\frac{2}{3}}\right)$ of 16*s.* 8*d.* ?

153. Find by Practice the cost of replacing a cistern, to weigh 8 cwt. 2 qrs. 14 lbs. at the rate of £2. 0*s.* 6*d.* per cwt., if the plumber allows £1. 11*s.* 6*d.* per cwt. for the lead of the old one which weighs 6 cwt. 1 qr. 10 lbs.

154. If one pound of standard gold of 22 carats (*i. e.* parts in 24) fine be worth £46. 14*s.* 6*d.*, find the value of a gold-mohur of weight 7 dwts. 23 grs. of fineness 993 in 1000.

155. Find the least number of ounces of pure silver worth *Rs.*2. 14*a.* $6\frac{6}{11}$*p.* per oz. that, with the proper proportion of alloy, can be coined into an exact number of rupees

156. A work can be completed in 36 days by 30 men working 6 hours a day ; in what time would 18 men and 60 women working 9 hours a day complete it ; supposing that 3 men can do as much as 5 women ; and that in the longer days a man does only $\frac{4}{5}$ per hour of what he does per hour in the shorter days ?

157. A contractor agrees to supply $10\frac{1}{2}$ lacs of bricks for a particular work. His bricks cost him $3\frac{1}{2}$ rupees per 1000 to make, and of these $12\frac{1}{2}$ per cent. are rejected. How many bricks must he make in order to fulfil his contract and what price per 1000 must he put on those supplied in order to gain 25 per cent. on his outlay ?

158. A merchant sells 60 mds. of rice at a profit of 8 per cent. and 94 mds. at a profit of 10 per cent. ; if he had sold the whole at a profit of 9 per cent. he would have received *17a.* less than he actually did ; how much per maund did he pay for the rice ?

159. If a cloth 4 yds. long and 15 in. wide, cost *Rs.*3. *2a.* ; how much should you give for a cloth 19 yds. long, 12 in. wide, and every square inch of which is worth $\frac{5}{8}$ths of the value of a square foot of the former ?

160. In a field of cabbages the distance between the rows of cabbages is 2 feet, the distance between the cabbages in a row is 9 inches ; how many cabbages are there in an acre ?

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161. A person goes to a bookseller's shop with a certain sum of money, and after buying 20 books at *Rs.*2. *4a.* each, finds that $\frac{5}{8}$ of his money remains. How much had he when he entered the shop ?

162. A person with a monthly income of *Rs.*264 spends as much in 4 months as he earns in three. After 12 years he divides his savings among his three children in such a manner that the eldest has twice as much as the second, and thrice as much as the youngest. How much did each receive ?

163. Simplify :—

(i) $\dfrac{2\times\sqrt{(1+\frac{1}{8})}+\sqrt{(1-\frac{1}{8})}}{5\times\sqrt{(1+\frac{1}{3})}\times\sqrt{(1-\frac{1}{8})}}$ (ii) $\sqrt{\left(\dfrac{\cdot 00195\dot{3}\times\cdot 00027}{\cdot 0001\dot{9}}\right)}$.

164. How many rings, each weighing 4 dwts. 18 grs. can a goldsmith make from a mixture of 1 lb. 10 oz. 1 dwt. 18 grs. of pure gold with 7 oz. 7 dwts. 6 grs. of alloy ?

165. Find by Practice, the time of building a wall 27 yds. long by 6 ft. high, of which one square yard is built in 3 hrs. 18 min. 45 sec.

166. In the Centigrade thermometer the freezing point is zero and the boiling point is 100° ; in Fahrenheit's the freezing point is 32°, and the boiling point is 212° ; what degree Centigrade corresponds to 68° Fahrenheit ?

167. If the rate of wages vary as the price of rice, and if 57 men working for 35 days receive *Rs.*405. *3a.* *9p.* when rice is sold at the rate of 136 measures for *Rs.*39 ; find the price of rice per measure when 70 men working for 19 days receive *Rs.*353. *4a.* *6p.*

168. Tea shares, original value *Rs.*1000, are selling at *Rs.*1250. They pay a dividend of 5 per cent. on the original value ; what income would *Rs.*57975 invested in them give ?

169. A ship's hold is 99 ft. long, 40 ft. broad and 5 ft. deep ; how many bales can be stowed in it each 3 ft. 6in. long, 2 ft. 8 in. broad and 2 ft. 6 in. deep, leaving a gangway of 4 ft. broad ?

170. A person has 200 shares in a Railway Company for which

he paid *Rs.*1000 per share. When the shares are paying 2 per cent. he sells them all at *Rs.*460 per share, and invests the proceeds in the Government 3 per cents. at 92. Find the alteration in his income.

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171. A steamship whose speed averages 14 miles an hour reaches a certain port in 12 days ; how many days afterwards will a sailing vessel arrive, which starts at the same time and sails on an average 8 miles an hour ?

172. At an examination $\frac{1}{8}$th of a class gains $\frac{7}{8}$ths of the maximum number of marks, $\frac{1}{10}$th gains $\frac{3}{4}$ths, $\frac{2}{5}$ths gains $\frac{1}{2}$, $\frac{1}{4}$th gains $\frac{1}{4}$ and the rest $\frac{1}{8}$. The average number of marks gained by the whole class is 166 ; what is the maximum ?

173. A bag contains 160 coins consisting of half-crowns, shillings, six pences and four-penny pieces, and the values of the sums of money represented by each denomination of coin are the same ; how many of each are there ?

174. *A* and *B* engage in trade, their capital being as 3 : 2. At the end of 3 months *A* takes out a sum equivalent to $\frac{1}{8}$ of *B*'s capital, and at the end of another 3 months *B* puts in a sum equivalent to what *A* took out. If *A*'s profits are at the end of the year £110 more than *B*'s, find the amount of the profits of each.

175. The hands of a clock which gains uniformly at the rate of 15 sec. a day were set at sunset on the evening of the first of the month at 6 o'clock. The true time of sunrise on the 3rd, was known to be a quarter to six but the clock indicated a quarter past 6. Find the error made in setting the clock on the 1st.

176. A person pays an income-tax of 4*d.* in the £ during the first half of the year and of 3*d.* in the £ during the second half, and finds that owing to an increase in his income he pays the same amount of tax for the second as for the first half of the year. If his gross income for the year is £700, find his net income.

177. The length and breadth of an enclosure which is in the form of a parallelogram are respectively equal to 47 yds. 2 ft 4 in. and 22 yds. 2 ft. 11 in. ; what should be the breadth of another equiangular enclosure (which is also a parallelogram) if its length is 63 yds. 1 ft. 5 in., and its area is $\frac{5}{8}$ths of the former ?

178. What must be the price of a £50 Railway share which pays a dividend of $2\frac{1}{2}$ per cent., if the purchaser is to make 4 per cent. upon his outlay ?

179. A merchant sells tea to a tradesman at a profit of 60 per cent.; but the tradesman becoming a bankrupt pays only 2*s.* 6*d.* in the £. How much per cent. does the merchant gain or lose by his sale ?

180. A merchant made a mixture of wine at 28*s.* a gallon, with brandy at 42*s.* a gallon, and he found that by selling the mixture

at 35*s.* a gallon, he gained 15 per cent. on the price of the wine, and 20 per cent. on the price of the brandy. In what ratio were the wine and brandy mixed together?

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181. Find the product of ·18988 and 8·08, divide it by ·0235 and subtract the square of 8·08 from the quotient.

182. A merchant buys in Calcutta 210 bags of rice at *Rs.* 10. 12*a.* per bag of 164 pounds. He sends them by rail 320 miles at $6\frac{3}{8}$ pies per ton per mile, but during the journey $7\frac{3}{4}$ pounds are stolen from each bag. Find at how many measures per rupee he must sell the remainder in order to clear *Rs.* 95. 15*a.* by the transaction. (One measure = $3\frac{1}{8}$ pounds.)

183. A building worth *Rs.*9000 was burnt, of which $\frac{1}{8}$ belonged to *A*, $\frac{1}{4}$ to *B* and the rest to *C*; what loss will each sustain, supposing that *Rs.*5400 of the value of the house were insured?

184. If 10 compositors who can set 3 letters in 5 seconds finish 27 pages in an hour and a half, how many compositors who can set 5 letters in 6 seconds, will complete 50 pages in an hour?

185. The cost price of a book is 12*s.* 6*d.*: the cost of the sale 8 per cent. on this, the profit 22 per cent.; find the retail price of the book.

186. There is a piece of work which 8 men working in pairs would accomplish in 20 days, and working singly in 30 days. One man is engaged on the work for 25 days; at the end of which time 7 men arrive to his assistance, and continue at the work for 4 days, when they are strengthened by the arrival of 7 men more. In what time would the work be finished, supposing all the men to continue at work, and operate in pairs whenever practicable?

187. A cask of $144\frac{3}{8}$ gallons is bought for £50 and kept 10 years, during which time $\frac{7}{18}$ of a gallon evaporated yearly; at what rate per gallon must the contents be sold so as to clear 20 per cent. on the amount of the original outlay at 4 per cent. per annum, Simple Interest?

188. Supposing a gallon of water to contain $277\frac{1}{4}$ cubic inches; find what number of gallons of water would cover a square mile to the depth of 5 inches.

189. The diameter of the fore-wheel of a carriage is $\frac{5}{8}$ of that of the hind-wheel and the former makes 528 revolutions in passing over $\frac{3}{4}$ of a mile. How many revolutions does the hind-wheel make in passing over a mile? and what is the diameter of each wheel?

190. A person borrows £500 at 5 per cent. per annum and subsequently £400 at $3\frac{1}{2}$ per cent.; if the amount of both sums 6 months after the latter was borrowed is £957, find the time for which interest is paid on the former sum.

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191. Find the value of

$$\frac{2\cdot3475 \text{ of } 1 \text{ ton} + 4\cdot6875 \text{ of } 1 \text{ cwt.} + \cdot5 \text{ of } 3 \text{ qrs.} - 1\cdot4 \text{ lbs.}}{1\cdot05 \text{ of } 1 \text{ ton.}}$$

192. Simplify :—

(i) $\frac{7}{15} \cdot \frac{\sqrt{(2-\frac{3}{5})} \times \sqrt{(3+\frac{4}{7})}}{3\sqrt{(2-\frac{3}{5})} + \sqrt{(3+\frac{4}{7})}}$. (ii) $\frac{10\frac{2}{11}}{3 \times 3\cdot\dot{3}\dot{9}} - \frac{\sqrt{(75\frac{1}{9})} - 6\frac{7}{8} \text{ of } \cdot\dot{7}\dot{2}}{5\frac{8}{9} - \sqrt{(2\frac{34}{81})}}$.

193. From a vessel containing 50 seers of milk, 10 seers are taken away; the vessel is then filled up with water, and 10 seers of the mixture drawn off; how many seers of milk are left?

194. Find the square root of $\cdot\dot{0}1234567\dot{9}$; and also of 002 to 4 places of decimals.

195. If a pound of pure silver be worth 62 shillings, the shilling containing 222 parts of pure silver in 240; what will be the value in shillings of a rupee weighing 180 grains, the rupee containing 925 parts of pure silver in 1000?

196. *A*, *B* and *C* can together do a piece of work in 12 days; *A* can do it in 28 days and *B* in 26 days. All three work together for 7 days, when *A* leaves off work. In how many days will *B* and *C* complete the work?

197. A rectangular parish, 6 fur. long and 4 fur. broad, is enclosed; a belt of plantation, 200 ft. wide is carried the whole way round; a main road, 60 ft. wide, runs across the land in the direction of its length and a cross road, 41 ft wide in the direction of its breadth: how many acres of field are there?

198. If the cost of making bread be one rupee per bushel of wheat, what is the price of wheat, when the two-anna loaf is twice as large as it is when wheat is *Rs.* 5 a bushel?

199. A publisher wishes to get 14*s.* for each copy of a work; what price should he put upon it so as to be able to allow the trade 30 per cent. discount?

200. A merchant receives a bill on London at 3 months of *Rs.* 3000, which he keeps till maturity and then exchanges at the rate of 1*s.* $10\frac{1}{2}$*d.* per rupee; when he finds he receives as much as he would have done had he discounted the bill when drawn at 4 per cent.; what was the rate of exchange when the bill was drawn?

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201. Add together $3\frac{3}{8}$ of $2\frac{1}{5}$ of $7\frac{11}{20}$ of £1, $9\frac{3}{7}$ of $3\frac{8}{9}$ of 1*s*, and $8\frac{1}{4}$ of $4\frac{1}{5}$ of 1*d.*, and divide the sum by $\frac{11}{12}$ of $\frac{5}{14}$ of $3\frac{1}{2}$*d.* Is the quotient an abstract or a concrete number?

202. Simplify :—

$$\frac{\frac{2}{15}+\frac{1}{3}}{2\cdot\dot{6}} \text{ of } \cdot5\dot{3} \div \left\{1\frac{1}{17} \text{ of } \left(1 - \frac{4^2}{9^2}\right) + \frac{8}{11} \text{ of } \cdot1\dot{6} \text{ of } \left(\frac{1}{2}+\frac{5}{12}\right)\right\}.$$

203. If 8 men or 12 boys can do a piece of work in 20 days how long will it take 3 men and 5 boys to complete a piece of work twice as great?

204. A grain of gold beats out into a leaf of 54 sq. in.; and a cubic foot of gold weighs 1211 lbs. Avoir. How many leaves together are as thick as a sheet of paper when 175 sheets of paper placed together are an inch high?

205. Two straight rods, each 1 foot in length, divided into 11 and 12 equal parts respectively, are placed side by side with their ends together. What fraction of an inch would the distance of the third division of the first be from the third division of the second?

206. In 1861 three towns had populations of 17650, 19600, and 18760 respectively. In 1871 the population of the first had decreased 18 per cent., that of the second had increased 21 per cent, while the population of the third had increased by 4690; find the average population of the three towns in the year 1871.

207. On what sum of money will the Compound Interest for 2 years be the same as the Simple Interest on £943 for 10 years, reckoning interest at the rate of 5 per cent. per annum?

208. If the true discount on a bill of £14641 be £4641 at 10 per cent. Compound Interest, how many years has the bill to run?

209. Five thousand copies are issued of a book the price of which is *Rs.* 3 per copy; the cost of printing is 8*a.* per copy, binding *Rs.* 2 per dozen, and of carriage, advertising, &c. *Re.* 1 per dozen; the publisher disposes of them to the retail bookseller charging 25 copies as 24, and 30 per cent. less than the selling price and upon the whole receipts takes 10 per cent. commission for himself; what are the gains respectively of author, publisher, and bookseller on this edition?

210. For two-thirds of the distance up a ghaut the rise is 1 foot in 24 (measured along the road) and for the remaining third the rise is 1 in 16. The top of the ghaut is 1400 feet above the bottom; what is its length?

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211. How many acres are contained in three countries, of which the first comprises 723100 square miles, the second 12342, and the third 89704 square miles?

212. The gallon contains 277·27 cubic inches, and a cubic foot of water weighs 62·42 lbs. Find the weight of a pint of water to two places of decimals.

213. Divide £954. 9*s.* between *A*, *B* and *C*, so that *A*'s share may be to *B*'s share :: 3 : 5, and *B*'s share : *C*'s share :: 10 : 11.

214. *A*, *B*, *C* and *D* working together can perform a piece of work in 8 days. *A* and *B* or *B* and *D* together take twice as long

as *A*, *B*, *C* and *D* together to perform the same work. *A* works during the whole of the day, *B* during three-fourths, *C* during a half and *D* during $\frac{1}{4}$th of the day. In how many days will the work be finished?

215. An annual tax of *Rs.*2255 is laid upon a district containing four villages—*A*, *B*, *C*, *D*—and the rate to be paid by each of the villages *A*, *B* and *C* is to the rate to be paid by *D*, as 3 to 2; what are the annual payments due from the villages?

216. If 10 men or 15 boys can reap 20 bighas of corn in 6 days working 14 hours a day, how many boys must be employed to assist 3 men to reap 6 bighas in $1\frac{2}{3}$ days of 8 hours a day?

217. A shop-keeper buys $\frac{1}{2}$ cwt. of tea at 4*s.* 2*d.* per lb, and mixes it with tea which costs him 2*s.* 11*d.* per lb. How much of the latter must he add to the former that he may sell the mixture at 3*s.* 8*d.* per lb. and gain 20 per cent. on his outlay?

218. Find the cost price of an article, which, if sold at 7 per cent. profit realises 5*s.* more than if it were sold at 17 per cent. loss.

219. *A* owes *B* £500, in liquidation of which debt he gives him a bill of £300 due 10 years hence, another bill due 4 years hence, and £133. 6*s.* 8*d.* in cash. What is the value of the latter bill, interest being at the rate of 5 per cent. per annum and allowing true discount?

220. A man has £10000 of 3 per cent. stock which he sold out at 72. He placed one half into the 4 per cents. at 80 and the other half into the 5 per cents. at 90. The former having fallen to 76, he transferred his stock from them to the $5\frac{1}{2}$ per cents. at 95. State the difference between his present and his original income.

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221. Two men *A* and *B* start together, and when *A* has gone a mile, *B* has gone $\frac{9}{200}$ of $1\frac{2}{5}$ of $\frac{1\frac{1}{5}}{5\frac{2}{5}}$ of $\frac{\frac{2}{3}+\frac{1}{2}}{\frac{3}{5}-\frac{1}{5}}$ of $71\frac{8}{7}$ of

$$\frac{\frac{\frac{2}{3}}{1-\frac{1}{25}}+\frac{1}{3}+\frac{1}{7}}{1-\frac{1}{7}\text{ of }\left(\frac{\frac{2}{3}}{1-\frac{2}{5}}+\frac{1}{3}\right)}$$ of a mile; which is in advance of the other?

222. Express the difference between $\cdot\dot{3}7\dot{8}$ of 13*s.* $10\frac{1}{2}$*d.* and $\cdot 3\dot{7}\dot{8}$ of 16*s.* 6*d.* as the fraction of

$$\cdot 42\dot{6} \text{ of } \frac{3\cdot\dot{3}}{\cdot 08} \text{ of } \frac{\cdot 3}{\cdot\dot{7}3\dot{5}} \text{ of } \frac{\cdot 147\times 4\cdot\dot{4}}{11\cdot 1} \text{ of } £1.\ 17s.\ 6d.$$

223. The Hindoo year consists of 365 days 6 hours $12\frac{1}{2}$ minutes, the Mahomedan of 354 days 8 hours 48 minutes. After what length of time would the accumulated difference between them amount to the tropical year of 365 days 5 hours 48 minutes 49 7 seconds?

224. If the rates of running of A, B are as 14 : 11, and if A give B a start of 36 yards, how far must the winning post be for A, to get in by the same distance?

225. What is the cost of making a ditch, 64 yds. 1 ft. 4 in. long, 2 yds. 2 ft. 2 in. wide and 3 yds. 0 ft. 6 in. deep at $1\frac{7}{8}d.$ per cubic foot? How long would 4 men, 3 women and 2 boys together take to make it, working 8 hrs. a day, if 2 men equal 3 women and 4 women equal 3 boys, and a boy earns $1\frac{1}{2}d.$ per hour?

226. A can do $\frac{1}{2}$ of a piece of work in 24 days, and B can do as much work in 3 days as A can do in 4 days; they work together for 4 days. A then leaves and C joins B and they work together for 6 days; then A returns, and the three finish the work in 11 days; how long would it have taken C to finish the whole piece of work?

227. Two watches are both set right at noon on the 15th of June, 1859; one gains $1\frac{1}{2}'$ in a day, the other loses $1'$; when will they be together again, and what o'clock will it be by each of them?

228. Two ships sail from the same port; one of them sails west, 50 miles; and the other sails north, 48 miles. Find the distance between them in miles to 4 places of decimals.

229. If the daily wages of a labourer rise from four and three quarters to six annas, what percentage of the increase in the price of food and other commodities will cause his position to be unaltered?

230. A person invests £6200 in the three per cents. at $89\frac{1}{8}$, and pays income-tax of 10*d.* in the £; on the stock rising to 92, he sells out and invests the proceeds in £50 Railway shares at par which yield an annual dividend of 3 per cent., clear of income-tax. Find the alteration in the income.

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231. Add together ·062435 of £100 + 7·4375 of 10*s.* + $1\cdot35\dot{6}$ of 7*s.* 6*d.* + 2·784 of $2\frac{1}{2}d.$, and reduce the result to the fraction of £29. 10*s.* $7\frac{1}{2}d.$

232. The area of a rectangular enclosure being 33 sq. poles 1 yd. 6 ft. 108 in., and the length 9 poles 1 ft. 6 in.; what is the width?

233. If the rate of interest for money is 6 per cent., what should be the rate of exchange for bills payable at sight in England when the rate for those payable 6 months after sight is 1*s.* 11*d.* per rupee?

234. How much per cent. must be added to the cost price of goods that a profit of 20 per cent. may be made after throwing off a discount of 10 per cent. from the labelled price?

235. Given that gold is worth £3. 17*s.* 10*d.* per oz. and silver 4*s.* 10*d.* per oz., and that the weights of equal volumes of gold and silver are as 19 : 11; and the length in inches (to 3 places of decimals) of an edge of a cube of silver equal in value to a cubic inch of gold.

236. If the volume of a sphere $= \frac{4}{3} \times 3 \cdot 1416 \times$ the cube of the radius, find how many spherical balls each $\frac{1}{4}$ inch in diameter can be made out of a cubic inch of clay, and how much clay will remain?

237. The top of a tank is a rectangle, whose sides are 9 feet and 15 feet; it is of the same horizontal section throughout its depth. What must be its depth in order, that it may contain 12960 gallons of water, one gallon containing 277·274 cubic inches?

238. A person buys saddlery in London for £31 and pays £3. 5*s.* for freight and insurance to Calcutta. On the arrival of the goods he pays 7 per cent. duty on the declared value, which was the London price turned into rupees at the current rate of exchange. If he had bought them in Calcutta he would have paid 40 per cent. above the declared value. How much did he gain by buying the goods in London, the rate of exchange being *Rs.*10. 13*a.* for a sovereign?

239. A dealer buys 10 horses at *Rs.*400 each, 8 horses at *Rs.*500 each and 4 horses at *Rs.*600 each. He keeps the horses for 6 months, during which each costs *Rs.*15 per month, and then sells them clearing $12\frac{1}{2}$ per cent. on his original outlay after paying all his expenses. Find the selling price.

240. A man who can walk down a ghaut at the rate of $4\frac{1}{2}$ miles and up at the rate of $3\frac{1}{4}$ miles an hour, descends, and returns to his starting point after walking for 2 hours 4 minutes. How far did he walk?

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241. Multiply 3 mi. 5 fur. 17 po. 5 yds. 1 ft. 10 in. by 7; and 1574 hhds. 62 gals. 3 qts. 0 pt. 2 gills by 27. Reduce 23 ac. 1 ro. 27 po. 24 sq. yds. 6 sq. ft. 103 sq in. to *square inches.*

242. The length of a room is treble its breadth. The cost of flooring, at *Rs.*3. 12*a.* per sq. yd., is *Rs.*281. 4*a.*, and that of painting the four walls, at 3*a.* per sq. foot, is also *Rs.*281. 4*a.* What is the height of the room?

243. *A*'s rate of working is to *B*'s as 4 to 3, and *B*'s is to *C*'s as 2 to 1. How long will it take *C* to do what *A* would do in 6 days?

244. An Indian officer, whose annual pay was estimated in rupees, lost £41. 12*s.* 6*d.* in one year by a fall in the value of the rupee from 1*s.* $11\frac{1}{4}$*d.* to 1*s.* $10\frac{1}{4}$*d.*; what was his salary estimated in rupees?

245. A warehouse consists of seven floors; the rent of each floor is ·875 times that of the floor below; the rent of the middle floor is £120. 1*s.*; compare the rents of the highest and lowest floors, and find that of the lowest.

246. If a certain amount of work is done by 9 men, 12 women and 13 boys in 11 days, how long will the same work take if 18 men,

3 women and 5 boys are set to do it : assuming that the ratio of a man's work to a woman's is as 5 to 3, and a woman's work to a boy's as 4 to 3?

247. The travelling expenses of 7 tourists for 5 weeks amounted to Rs.752. 8*a.* ; a second party of 18 made the same tour in 6 weeks, their average weekly expenditure per man being $\frac{4}{5}$ of that of the first party. What were the total expenses of the second party?

248. An armourer undertakes to supply 2000 swords at 17*s.* 3*d.* each. He estimates that if 5 per cent. fail to stand the required test and are worthless, the profit will be 15 per cent. on his whole outlay. At the trial, 35 per cent. of the swords prove worthless. How much does the armourer lose by the contract?

249. In a hundred yards race *A* can beat *B* by 4 yards, in a quarter of a mile race *C* can beat *A* by 11 yards ; by how much can *C* beat *B* in a mile race, supposing that the average speeds of each man when running a hundred yards, a quarter of a mile, and a mile, are proportional to 9 : 8 : 7?

250 A field containing 26 ac. 3 ro. 10 po. is let in equal allotments to 66 agricultural labourers at a rental of 3*d.* a pole per annum, a reduction of 15 per cent. being offered to those tenants who shall pay their rent on the day that it becomes due. When all the year's rent have been paid, the landlord finds that he has received the sum of £49. 4*s.* 9*d.* How many labourers paid to the day?

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251. Find the sum, difference, product, and quotient of two dozen dozen and half-a-dozen dozen.

252. If 8 lbs. of coffee cost as much as 5 lbs. of tea, and 7 lbs. of coffee as much as 40 lbs. of sugar, what is the price of each when 1 lb. of tea, of coffee, and of sugar, together cost *Rs.*2. 5*a.*?

253. Two cog-wheels, one with 15 teeth, the other with 28 teeth, work together. If the former turns round 16 times in $7\frac{1}{2}$ seconds, how many times will the latter turn round in 21 seconds?

254. *A*, *B* and *C* play at cricket. *A*'s runs are to *B*'s and *B*'s runs are to *C*'s, as 3 to 2. They get altogether 342 runs. How many does each get?

255. If 3 kilometres are as much under 2 miles as 5 kilometres are over 3 miles, what is the length of a kilometre?

256. The income-tax is reduced from $10\frac{1}{2}d.$ to 5*d.* in the pound, but a man's gross receipts are at the same time reduced by 10 per cent. owing to the stoppage of a mine. Find by what percentage his net income is altered.

257. Two settlers in New Zealand own adjoining farms of 3000 and 5000 acres respectively. They unite their farms, taking at the same time an additional partner, who pays them £8000, on the

understanding that a third share of the land shall in future belong to each. How is the £8000 to be divided between the original owners?

258. The debts of a bankrupt amount to £1067. 5*s*. 3*d*., and his assets consist of property worth £458. 7*s*. 8*d*., and an undiscounted bill of £256. 10*s*. due 4 months hence, simple interest being reckoned at 4 per cent. How much in the pound will he pay?

259. A journey of 560 miles was made by rail, steamer and coach. The distance by coach was one-fourth, and the distance by sea three-fourths of that by rail. The fare per mile by coach was double, and by sea four-fifths of that by rail. What was the expense of the whole journey, railway fare being $1\cdot\dot{5}7142\dot{8}$*d*. per mile?

260. One clerk has $24\cdot\dot{4}2857\dot{1}$ and a second clerk has $38\frac{4}{7}$ sheets to engross; they call in a third clerk and agree to divide the work equally among three, and to pay the third clerk at the rate of $\cdot 2430\dot{5}$ shilling per sheet; how much will he receive from each of them?

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261. Arrange the figures 194678 in the six different ways in which 194 are the *first* three figures in different order, and also in the six different ways in which 678 are the *last* three figures in different order; and add the twelve arrangements together.

262. Multiply the difference between 12 cwt. 3 qrs. 17 lbs. 10 oz. and 5 cwt. 2 qrs. 23 lbs. 11 oz. by 528. Divide the sum of the same two quantities by 324.

263. Gold is 19 times as heavy as water, and copper 9 times. In what proportion should these metals be mixed that the mixture may be 15 times as heavy as water?

264. Exchange *at six months' sight* is at 1*s*. 10*d*. per rupee. By depositing in a bank for 6 months, certain interest can be had at the rate of 3 per cent. per annum; at the end of the 6 months, exchange *at sight* is 1*s*. $9\frac{1}{2}$*d*. per rupee. What is the gain or loss per cent. on remitting from India either (i) *at six months' sight*; or (ii) depositing at interest for 6 months, and then remitting *at sight*?

265. Calculate the value of $\sqrt{\left\{\dfrac{(\cdot\dot{7}\dot{8})\times(\cdot 00\dot{0}\dot{4})}{(\cdot 0\dot{1}\dot{3})\times(\cdot\dot{0}1\dot{5})}\right\}}$.

266. *C* does half as much in a day as *A* and *B* can do together, and *B* does half as much again as *A*. If all three working together can mow 20 acres of grass in 16 days, how long would each working by himself, take to mow 5 acres?

267. If it cost *Rs*.497. 4*a*. to decorate a wall space measuring 69 ft. 4 in. by 6 ft. 9 in., what will it cost for one measuring $22\frac{1}{2}$ yds. by $3\frac{1}{3}$ yds., the style of decoration used in the second case being half as expensive again as in the first case?

268. If the cost of provisioning a gunboat carrying 84 men be £598. 10*s.* when the ship is at sea for 95 days, what will it cost to provision for 33 days a ship carrying a crew of 110 men ?

269. If the true discount on *Rs.*1000 at $3\frac{1}{3}$ per cent. Simple Interest, be *Rs.*166. 10*a.* 8*p.*, when is the sum due ?

270. Arrange in order of magnitude :— $\sqrt{(50)}$, $\sqrt[3]{(344)}$, $\sqrt[4]{(2402)}$.

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271. Reduce $\dfrac{1+\dfrac{2}{3\frac{1}{4}}}{1-\dfrac{2}{3\frac{1}{4}}} + \left(1+\dfrac{4}{9-\dfrac{3}{1-\frac{3}{8}}}\right)$ to a decimal.

272. Find by Practice the cost of a fence 3 fur. 11 po. $3\frac{1}{4}$ yds. long, at £183. 6*s.* 8*d.* per mile.

273. What is the whole cost of 5 pairs of gloves at 2*s.* $11\frac{3}{4}$*d.* per pair, 24 yds. of muslin at 1*s.* $9\frac{1}{2}$*d.* per yard, $17\frac{1}{2}$ yds. of ribbon at $8\frac{1}{4}$*d.* per yard, and 35 yds. of flannel at 1*s.* $7\frac{3}{4}$*d.* per yard ?

274. A piece of work is done by three men *A*, *B* and *C* in 5 days in the following manner : *A* works the whole time, *B* only on the first and second days, *C* only on the third, fourth and fifth days. The work might also have been done by *B* and *C* working together for 6 days without the assistance of *A*. If *B* and *C* working together for 2 days can do as much work as *A* can do alone in 3 days, find how long it would take *A*, *B* and *C* each to do the work separately.

275. In an examination *A* obtains 10 per cent. less than the minimum number of marks required for passing, *B* obtains $11\frac{1}{9}$ per cent. less than *A*, and *C* $41\frac{8}{17}$ per cent. less than the number of marks obtained by *A* and *B* together. Does *C* pass or fail ?

276. A rectangular swimming bath is 60 ft. long and 40 ft. broad ; it can be filled by a supply pipe in 5 days, and if 6000 cub. ft. of water be thrown in, the rest can be filled in 3 days 18 hours. Find the depth of the bath.

277. What will £3255. 4*s.* 2*d.* amount to in a year and a half, if put out at Compound Interest at 8 per cent. per annum, the interest being added at the end of each half year ?

278. A person rows a distance of $1\frac{1}{2}$ miles down a stream in 20 min., but without the aid of the stream it would have taken him half an hour ; what is the rate of the stream per hour ? and how long would it take him to return against it ?

279. Three sums of money are in the proportion of 2 : 3 : 5 ; and when each has been reduced by *Rs.*25, the remainders are in the proportion of 1 : 2 : 4. Fiud the sums of money.

280. A sum of £3750 was sold out of the 3 per cents. at 95 and put out at Compound Interest for two years at 4 per cent.; the amount being then invested in the $3\frac{1}{2}$ per cents. at 104. Find the alteration in income.

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281. Show that the sum of the squares of three thousand and nine and four thousand and twelve is equal to the square of five thousand and fifteen. Write this sum in figures.

282. In a book on Arithmetic an example was printed thus :—

"Add together $\frac{1}{14\frac{2}{3}}$, $\frac{1}{19\frac{1}{4}}$, $\frac{1}{\quad}$, $\frac{1}{13\frac{3}{4}}$," the denominator of one fraction being accidentally omitted. The answer given at the end of the book was $\frac{1}{2}\frac{1}{8}$. Required the missing denominator.

283. A 7 ft. strip along two adjacent sides of a rectangular garden forms a border for flowers. If the dimensions of the garden be 42 ft. × 34 ft., what is the exact proportion of the strip to the whole area?

284. If a family, by using 6 gas-burners 5 hours a day, pay *Rs.*12. 8*a.* per quarter, when gas is at *Rs.*2. 8*a.* per 1000 cub. ft., what will a family, using 8 burners 3 hours a day, pay per quarter, when gas is at *Re.*1. 14*a.* per 1000 cub. ft.?

285. A grocer buys 15 lbs. of tea; he sells 8 lbs. at 2*s.* $7\frac{1}{2}$*d.* per lb., and the rest at 2*s.* 9*d.* per lb.; and finds that he has made 15 per cent. profit. What rate per cent. profit would he have made if he had sold it all at 2*s.* $9\frac{1}{4}$*d.* per lb.?

286. Calculate the value of each of the following to 3 places of decimals :—

(i) $\frac{\sqrt{9}-\sqrt{\cdot 9}}{\sqrt{4}+\sqrt{\cdot 4}}$. (ii) $\frac{\sqrt{9}+\sqrt{\cdot\dot{9}}}{\sqrt{4}-\sqrt{\cdot\dot{4}}}$. (iii) $\frac{1}{\sqrt{(12)}+2}+\frac{1}{\sqrt{(12)}-2}$.

287. A rectangular field which is twice as long as it is wide costs $1\cdot01\dot{3}\dot{8}$*s.* per square yard to turf. If the whole cost is £191. 17*s.* $0\frac{3}{4}$*d.*, find the lengths of the sides of the field.

288. A person buys a piece of land at £25 an acre, and by selling it in allotments finds that the value is increased by one-half, so that, after reserving 20 acres for himself, he clears £200 on his purchase money by the sale of the remainder. How many acres were there?

289. Gold is sold at the Mint at £3. 17*s.* 9*d.* per oz. and is mixed with alloy, worth 5*s.* 2*d.* per oz. in the ratio of 11 : 1. If sovereigns be coined of this mixture, each weighing 5 dwts. 3·47 grs., what is the Mint profit per 100 sovereigns?

290. 922 ft. 77 in. of colouring have to be done on the walls

and ceiling of a room whose form is that of a cube. Find its dimensions.

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291. In a long division sum the dividend is 529565 and the successive remainders are 246, 222 and 542. Find the divisor and the quotient.

292. What two numbers of 4 digits each can have 119 as their G. C. M. and 13923 as their L. C. M.?

293. A sum of money is to be divided amongst 11 men and 18 boys, and 5 men are to receive as much as 9 boys. When 3 men and 3 boys have received their shares, what fraction of the whole sum will remain?

294. An engine while driving machinery burns coal at the rate of 1 ton 12 cwt. 5 qrs. in 8 hrs. 40 min. When the machinery is not in motion, the consumption of coal is only $\frac{4}{11}$ of this rate. How much coal will the engine burn in 1584 hours, during $\frac{1}{8}$ of which time the machinery is at rest?

295. Find correct to 7 places of decimals the value of

$$\frac{1}{9}+\frac{1}{3.9^3}+\frac{1}{5.9^5}+\frac{1}{7.9^7}+\&c\ldots$$

296. *A* can do a piece of work in 25 days, *B* can do it in 20 days and *C* in 24 days. The three work together for 2 days, and then *A* and *B* leave; but *C* continues, and after $8\frac{2}{5}$ days, is rejoined by *A*, who brings *D* along with him, and these three finish the remainder of the work in 3 days more. In what time would *D* alone have done the whole work?

297. What sum will amount to £1591. 13*s*. 2·16*d*. in 3 years at Compound Interest; the rate of interest for the first, second and third years being 3, 2 and 1 per cent. respectively?

298. What must be the rate of interest in order that the discount on £387. 7*s*. $7\frac{1}{2}$*d*. payable at the end of 3 years may be £41. 10*s*. $1\frac{1}{2}$*d*.?

299. If after adding 14 gals. of water to a pipe of wine, and selling it at 13*s*. 6*d*. a gallon, I gain 25 per cent.; find the prime cost of the wine.

300. A boat is rowed down a river at the rate of a mile in 6 min., and up the river at the rate of a mile in 10 min., the crew working equally hard; find the velocity of the current.

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301. The officers of a regiment are ·042 of its strength, but after 50 privates have been added the officers are ·04 of the whole. What is the number of officers?

302. A pound of powder costs 3*s.* and the charge of a gun is $2\frac{1}{4}$ drams; how many shots will 6*s.* 9*d.* worth of powder furnish; and what will be the cost of powder for 2560 shots?

303. A regiment of 1000 men are to have new coats; each coat is to contain $2\frac{1}{2}$ yds. of cloth, $1\frac{1}{4}$ yds. wide, and to be lined with shalloon $\frac{7}{8}$ in. wide; how many yards of shalloon will be required?

304. If the driving wheel of a railway engine be 16 ft. in circumference, and the velocity of the engine 25 miles an hour; how many revolutions does the wheel make in 5 minutes?

305. A cistern is supplied by 4 taps which would fill it (if empty) in 3, 4, 5 and 6 hours respectively, and discharged by two others which would empty it (if full) in $2\frac{1}{2}$ and $3\frac{3}{4}$ hours respectively. Suppose the cistern empty, and all the taps open, how long would it be in filling?

306. A sailing vessel reaches Madras from Calcutta in 6 days; a steamer whose speed is to that of the sailing vessel as 3 : 2, starts at the same time, but meets wtih detentions that average 6 hours daily. Which will reach Madras first, and by how much?

307. It it is high water at noon on a certain day, find after how many days it will again be high water at noon, supposing the time of high water to be three-quarters of an hour later every day.

308. A fixed rent of £780 per annum is converted into a corn-rent of one half wheat at 48*s.* per quarter, and the other half barley at 30*s.* per quarter. What will be the rent when wheat has advanced to 56*s.* and barley to 32*s.* per quarter?

309. At what time between 6 and 7 will the two hands of a watch be exactly 23 minute spaces apart?

310. A person invests a certain sum in the 3 per cents. when they are at $96\frac{3}{4}$; had he waited till they had fallen to $96\frac{1}{8}$, he would have obtained £16 more of stock. How much money did he invest, $\frac{1}{8}$ per cent. being charged as brokerage in both cases?

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311. In a certain sum the dividend is 31884740, the quotient 40930; find the divisor and remainder.

312. A manufactory turns out 50 tons of iron goods weekly, using up for that purpose 51 tons of iron at £6. 15*s.* per ton, 100 tons of coal at 11*s.* 6*d.* per ton, and £45 worth of other materials; rent, rates and taxes amount to £219 annually; wages and incidental expenses to £75 per week. At what price per cwt. must the iron goods be sold in order that the works may gain 8 per cent. per annum on a capital of £35000?

313. Find by Duodecimals how many cubic feet are contained in a beam 25 ft. 5 in. long, 1 ft. 7 in. broad and 1 ft. 2in. thick. If an

inch in thickness be taken off from each of the four sides, the length of the beam remaining the same, find how much wood has been removed.

314. The average of 25 returns is 43. The first 9 average 52, the next 12 equal 37. Find the average of the last 4.

315. If the par of exchange be two English shillings for the Indian rupee, but if an Indian bill of exchange for *Rs.*540 12*a.* be negotiated in London for £51. 10*s.*; how much per cent. below par is the rate of exchange?

316. *A*, *B* and *C* do $\frac{4}{5}$ths of a piece of work together in 24 days, *A* does the same amount of work as *B* does in the same time; had *A* or *B* been absent, then the two others would have accomplished $\frac{3}{8}$ths of the work in 28 days. In what time can each separately do the work?

317. A bill for £202. 16*s.* was drawn on March 2nd, at 7 months: what will it be worth on May 12th, discount being calculated at $3\frac{1}{2}$ per cent.?

318. A merchant bought a fifty gallon cask of wine for *Rs.*741. Supposing it to have lost $3\frac{1}{2}$ gallons, at what price per dozen bottles (nine bottles holding a gallon) should he sell it in order to gain 15 per cent. upon the whole original cost?

319. When the East India 4 per cent. stock is at 12 discount and the 5 per cent. ditto at $5\frac{3}{8}$ premium, find the difference in the rates of interest obtained by investing in these stocks respectively. A person has a certain sum to invest, and finds that the latter stock will give him an annual income of £3. 7*s.* 6*d.* more than the former; what is the sum?

320. A watch is 5 min. fast at noon on the first of the month; on the 11th at true noon it shows 12 hrs. 55 min. Find its rate of gaining; and the true time to the nearest second, when the watch shows 5 min. past noon on the 16th.

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321. Find the value in acres, &c. of

$$\cdot 75 \text{ of } \frac{1\frac{8}{9}}{1\frac{4}{5}} \text{ of } \frac{8\cdot 5}{2\cdot 25} \text{ of } \frac{1\cdot\dot{3}}{\cdot 0\dot{2}\dot{6}} \text{ of } \frac{\pounds 3.\ 5s.}{14s.\ 3d.} \text{ of } \frac{1 \text{ yd. } 1 \text{ ft. } 6 \text{ in.}}{6 \text{ yds. } 2 \text{ ft.}} \text{ of } \frac{136 \text{ gals. } 2 \text{ qts.}}{178 \text{ gals. } 3 \text{ qts.}}$$

$$\text{of } \frac{1 \text{ lb. } 6 \text{ oz } 17 \text{ dwts. } 2 \text{ grs.}}{2 \text{ lbs. } 2 \text{ oz. (Avoir.)}} \text{ of } \frac{77 \text{da. } 4 \text{hrs. } 30 \text{ min.}}{6 \text{da. } 12 \text{ hrs.}} \text{ of } 517 \text{ sq. ft. } 72 \text{ sq. in.}$$

322. A drawing room, 36 ft. 10 in. long and 23 ft. 2 in. wide, is surrounded with a cornice $3\frac{1}{2}$ in. wide, the gilding on which costs £4. 11*s.* $10\frac{1}{2}$*d.*; how much was that per square foot?

323. Gun metal is composed of 9 parts of copper to one of tin, and bell metal is equal to 80 parts of copper to 10·1 of tin,

5·6 of zinc and 4·3 of lead. What quantity of zinc, lead and tin must be added to 24 tons of gun-metal to convert it into bell-metal? Give the answer in tons.

324. A man having *Rs.*50000 to invest, expends a proportion of $\frac{1}{2}$ pie in the rupee, in purchase of building land at *Rs.*63. 4*a* 9*p*. per acre, and a proportion of 7*a*. 2*p*. in the rupee in iron, at *Rs.*10. 2*a*. 5*p*. per cwt. How much of each did he obtain; and what will be the interest for $2\frac{1}{2}$ years, on the balance of his capital at $4\frac{1}{2}$ per cent.?

325. A cistern can be filled by two taps in 20 and 24 minutes respectively, when in good order, but after they have both been running together 15 minutes, it is found that the cistern is only $\frac{7}{8}$ full. Determine the rate of leakage.

326. A clerk's salary before the imposition of the income tax was £200 a year. How much must it be now with an income tax of 16*d.* in the pound, that he may still possess £200 a year net income?

327. Supposing the price of the 3 per cent. consols to be $97\frac{3}{8}$ brokerage $\frac{1}{8}$, what will be the Simple Interest of £260 for 2 years?

328. A man bought a bigha of land for *Rs.*140 and intending to sell it, fixed such a price, that by selling it at $12\frac{1}{2}$ per cent. under the intended price he would still have a gain of $12\frac{1}{2}$ per cent. on the prime cost. At what price he intended to sell the land?

329. A rectangular pile of wood is 12 yds. high and 10 broad, find the number of oblong pieces 18 ft. long 8 in. broad and 4 in. deep contained in it, supposing the cost of covering the pile with matting at 4*d.* per square foot to be £87.

330. What time must elapse between the time of placing *Rs.*250 in the Government Saving's Bank, and taking out the amount just as it goes over *Rs.*300, supposing Compound Interest at 5 per cent. per annum?

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331. Find the sum of *all* the numbers that can be formed by the digits 2, 3, 4, 5 taken all together.

332. A body of men in cloumn of 625 ranks of 64 abreast, was drawn up in a solid square. How many would there be in each face?

333. If 560 flag-stones, each $1\frac{1}{4}$ ft. square, will pave a court-yard, how many will be required for a yard twice the size, each flag-stone being 14 in. by 9 in.?

334. A man must get $\frac{1}{2}$ marks to pass an examination; he answers $\frac{1}{2}$ the questions, but to $\frac{5}{8}$ of his answers gets on an average only $\frac{3}{4}$ of the marks and thus gets 25 marks too few to pass. How many does he get altogether?

335. If 5 men build a tower 30 ft. high in 3 days, in how long a time can 4 men build a similar tower 64 ft. high? If each foot of

the tower above the 30th takes on an average twice as long to build as each foot below it, how long would they take?

336. Determine the value of each of the following expressions to 4 places of decimals :—

(i) $\frac{2+\sqrt{3}}{1+\sqrt{3}}$. (ii) $\frac{15+\sqrt{10}}{15-\sqrt{10}}+\frac{30-\sqrt{10}}{30+\sqrt{10}}$. (iii) $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$.

337. If the 3 per cent. stock is at £87 and the $3\frac{1}{4}$ per cent. at £92, which is the most advantageous investment? And if I have £3000 stock in the least advantageous, and I wish to change it into the other, how many £100 shares of stock shall I obtain? Brokerage of £$\frac{1}{8}$ per cent. being charged on all transactions.

338. A reduction of 40 per cent. in the price of a half-crown tea enables me to buy for my money 6 lbs. more than I could previously buy for a sovereign. What amount have I?

339. A person borrows *Rs.*1000, and at the end of each year pays *Rs.*250 to reduce the principal and to pay interest at 4 per cent. on the sum which has been standing against him through the year. How much will remain of the debt at the end of 3 years?

340. By paying an income-tax of 9*d.* in the pound, a man's income is reduced to £1212. 15*s.* Find (i) his original income, and (ii) the sum of money which must be invested in the $4\frac{1}{2}$ per cent. stock at $110\frac{5}{8}$ to produce that income, a brokerage of $\frac{1}{8}$ per cent. being charged on the purchase of the stock.

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341. A farmer bought 4 horses and 7 cows for *Rs.*2380, the prices of a horse and a cow being as 5 : 2; how much did he give for each?

342. Two rooms contain equal quantities of air. The area of the floor of one of them is 340 square feet and its height is 12 feet. Find the area of the floor of the second, whose height is 17 feet.

343. An article is first sold at a profit of 10 per cent.; the purchaser then again sells it for *Rs.*2. 4*a.* 8*p.*, and his gain is 15 per cent. of the price at which he sells it. How much did the article originally cost?

344. The area of each of the 64 squares of a chess-board is 4·2025 inches, and the outer rim of the board is ·3 of an inch wide; find the length of a side of the board.

345. If 7 men and 2 women earn *Rs.*510 in 8 weeks, and 4 men and 2 women earn *Rs.*465 in 12 weeks, what are the weekly wages of a man and a woman?

346. A gentleman divided £4. 18*s.* among 150 school-children, giving the girls a shilling each, and the boys six-pence; how many boys and girls were there?

347. A man after a tour in Switzerland found that he had spent every day half as many shillings as the total number of days he had

been from home. His tour costs £57. 12*s*. How many days did it occupy?

348. On heating a piece of metal its volume is increased 2·4 per cent.; by what percentage of the new volume does the metal decrease on cooling again to its original temperature?

349. *A* bequeaths *B* a certain sum of money, which after paying a legacy duty of 10 per cent. yields an income of £810 when placed at 3 per cent. interest. Find the amount bequeathed.

350. If £31250 put out at Compound Interest amount in 3 years to £43904, what is the rate per cent.?

351. The total area of three estates is 1768 acres. If the areas of the two smaller estates be respectively three-fifths and two thirds that of the largest, find the acreage of each.

352. A debtor owing £11500 paid a composition of 6*s*. 3*d*. in the pound. How much would a creditor to whom ·2 of the whole owing receive?

353. *A* and *B* ride a race of 31 miles on bicycles. The driving wheel of *A*'s machine makes 3,410 revolutions per hour and has a circumference of 168 inches; that of *B* makes 3,520 revolutions per hour, and has a circumference of 162 inches; which will win, and by how much?

354. A man who has a certain capital calculates that if he invest it in the 3¼ per cent. stock at 91 his income will be £25 more than if he invest it in the 3 per cent. stock at 88. What is his capital?

355. *A*'s present age is to *B*'s present age as 8 : 7; 27 years ago their ages were as 5 : 4. Find *B*'s age.

356. If *A*'s income be 150 per cent. more than *B*'s, how much per cent. is *B*'s income less than *A*'s?

357. If the price of the 4 per cents. just before the payment of a half-yearly dividend be 93, what ought to have been the price three months previously, supposing no change in the value of money to have taken place during that interval?

358. A cubical box contains 9261 cubic inches; find the expense of gilding it at 3 farthings a square inch, and reduce the result to the decimal of 10 guineas.

359. In an examination in which the full marks were 6000, *A* got 12 per cent. more than *B*, *B* 16 per cent. more than *C* and *C* 20 per cent. more than *D*; if *A* got 4872, find what percentage of the full marks were obtained by *D*.

360. It is desired to put a cubical case, whose content is 4019·679 cub. feet, through a square hatchway whose area is 37791·36 sq. inches; shew whether this can be done.

361. Divide $15\times48\times49\times50+16\times48\times49\times51+17\times48\times50\times51+18\times49\times50\times51+4\times48\times49\times50\times51$ by $48\times49\times50+48\times49\times51+48\times50\times51+49\times50\times51$.

362. Express $\sqrt{\left(\dfrac{\cdot678\times9\cdot01}{\cdot0234}\right)}$ correctly to the nearest integer.

363. A man rode a bicycle from A to B, a distance of 54 miles at an average rate of 8 miles an hour; another man started from A on horseback half an hour after the bicyclist and arrived at B 15 min. before him. Find the ratio of their speeds.

364. Add together $\cdot2\dot{6}+\sqrt{(\cdot1225)}+\sqrt[3]{(\cdot015625)}$, and express the sum as the vulgar fraction of $\cdot3\dot{6}$.

365. Find the value of $\sqrt{(\cdot00139876)}-\sqrt[3]{(\cdot000030664297)}$.

366. A woman bought eggs at the rate of 8 for 5*d.*, and sold them so as to gain 1½*d.* on a dozen; (1) what did she charge for each? (2) What did she gain per cent.?

367. A merchant mixes 54 gallons of wine at *Rs.*24 per gallon, with 36 gallons at *Rs.*14 per gallon. How much water must be added to the mixture in order that by selling it at *Rs.*20 per gallon he may gain 10 per cent.?

368. If 12 labourers working for 10 hours a day can accomplish their task in 9 days, but after 5 days two are taken ill, find how much longer per day they must work in order to finish it at the proper time; (i) when no fresh hands are employed, (ii) when two boys are added, a boy's work being ⅓ that of a man's.

369. A shilling weighs 3 dwts. 15 grs., of which three parts out of forty are alloy and the rest pure silver: if the value of silver rises 8 per cent., what must be the reduction in the weight of pure silver in a shilling?

370. If the 3 per cent. consols be at 90⅛, what sum must I invest in order to secure from them a yearly income of £466, after paying an income-tax of 7*d.* in the £, brokerage being ⅛ per cent.?

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371. How many ninths of a shilling are there in 9⅓ shillings? How many ninetieths? And what is that number whose ninth part is nine dozen and nine?

372. Simplify :— $\left(\dfrac{2\frac{3}{4}+3\frac{2}{3}}{4\frac{1}{3}+5\frac{1}{4}}+\dfrac{3\frac{2}{3}}{10\frac{1}{2}}\right)\times\left(\dfrac{2\frac{4}{11}}{2\frac{3}{8}}+\dfrac{2\frac{7}{11}}{8\frac{7}{10}}\right)-\dfrac{\cdot281}{1\cdot405}$.

373. M starts from C and travels towards D at the rate of 6 miles per hour; two hours afterwards N starts from C, and going 10 miles per hour reaches D 4 hours before M. Find the distance from C to D.

374. *A*, *B* and *C* are partners. *A* receives $\frac{2}{5}$ths of the profits *B* and *C* dividing the remainder equally. *A*'s income is increased by *Rs*.220 when the rate of profit rises from 8 to 10 per cent. Find the capital of *B* and *C*.

375. A sum of money amounts in 3 years at 5 per cent. Compound Interest to *Rs*.9261 ; what would be its amount in 5 years ?

376. How often between 11 and 12 are the hands of a clock an integral number of minute-spaces apart ?

377. *A* and *B* walk a race of 25 miles ; *A* gives *B* 45 minutes start ; *A* walks uniformly a mile in 11 minutes and catches *B* at the 20th milestone ; find *B*'s rate and by how much he lost in time and space.

378. *A*, *B* and *C* are employed on a piece of work. After 15 days *A* is discharged, one-third of the work being done. *B* and *C* continue at the work, and after 20 days more *B* is discharged, one-third more of the work being done. *C* finishes the work in 30 days. In what time would the work have been done, if *A* and *B* had continued to work ?

379. The gold coinage of one nation contains 1 part of silver to 11 parts of gold without any alloy ; that of another nation, 1 part of alloy to 23 parts of gold. It is found that $88\frac{1}{3}$ of the first weigh as much as 46 of the second. The intrinsic value of silver is one-sixteenth that of gold. Determine the part of exchange.

380. A man sold a horse for *Rs*.250 and thereby lost $\frac{1}{6}$ of his purchase money. What part of his purchase money would he have gained had he sold the horse for *Rs*.400 ?

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381. Find the greatest and least number of 6 digits which are exactly divisible by 789.

382. Divide $\frac{1}{3}[3+\frac{1}{3}\{3+\frac{1}{3}(3+1\frac{1}{3})\}]$ by $\cdot125$.

383. A company of Sepoys proceed in 5 equal rows, and after some time arrange themselves in 7 equal rows. Find the least number above 1000 which the company could contain.

384. A clock loses $8\cdot5$ sec. an hour when the fire is alight, and gains $5\cdot1$ sec. an hour when the fire is out, but on the whole it neither loses nor gains ; how long in the 24 hours is the fire burning ?

385. An agent has to receive a rent paid in corn from a tenant, and to deliver it to the landlord. At both payments he uses, so as to benefit himself, a false balance, such that 9 lbs. in one scale balances 10 lbs. in the other. Corn being worth 49*s*. a quarter, the value of his plunder is £46. 11*s*. What is the corn rent ?

386. *A* and *B* fire at targets, and have 55 cartridges each ; *A* fires twice in 3 minutes, and *B* three times in 5 minutes : how many times will *B* have to fire after *A* has finished ?

387. A hollow cubical box, made of material which is $1\cdot\dot{3}$ in. in thickness, has an interior capacity of 50653 cub. ft.; determine the length of the outside edge of the box.

388. The expense of painting the wainscot of a room is *Rs.*167. 10*a.*, at *Re.* 1. 2*a.* per sq. foot, the wainscot being 2 ft. deep. What will be the expense of carpeting the rooms with carpet 27 inches wide at *Rs.*2. 4*a.* per yard, the breadth of the room being to its length as 62 is to 87?

389. *A* and *B* run a race; their speeds are as 17 to 18. *A* runs $2\frac{1}{2}$ miles in 16 min. 48 sec.; *B* having run $2\frac{1}{2}$ miles, takes 34 minutes in finishing the course; determine the length of the course.

390. What are the times between 3 and 4 when the hands of watch are equally distant from Fig. III?

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391. Find all the numbers of 5 digits divisible by 9, which have unity for their 1st and 5th digits and 2 for their middle digit. Enunciate and prove the principle upon which you proceed.

392. How many revolutions will be made by a wheel, which revolves at the rate of 151 revolutions in 3 min., while another wheel revolving 241 times in 17 min. makes 723 revolutions?

393. Simplify :—

(i) $10\frac{1}{2}\left\{\frac{2\frac{1}{2}}{7}+\frac{3\frac{1}{3}}{9}\right\}-\frac{4\frac{1}{7}-2\frac{1}{4}}{6\frac{1}{3}+2\frac{1}{7}}\times 4\frac{80}{83}$. (ii) $\left\{\sqrt[3]{\frac{\frac{1}{7}+\frac{1}{6}}{\frac{1}{6}-\frac{1}{7}}+2\left(\frac{\frac{1}{8}+\frac{1}{6}}{\frac{1}{6}-\frac{1}{8}}\right)}\right\}^{2}$.

394. In the certre of a room 21 ft. square, there is a square Turkey carpet; the rest of the floor is covered with oil-cloth. The carpet and oil-cloth cost respectively *Rs.*8. 4*a.* and *Rs.*4. 4*a.* per sq. yd.; and the whole cost of both is *Rs.*352. 4*a.* Find the width of the oil-cloth border.

395. The masters of a school are $\cdot041\dot{6}$ of its whole number, but after 40 new boys have been added the masters become $\cdot0375$ of the whole. How many boys and masters were there before the new boys came?

396. A traveller meets two Arabs in the desert, one of whom has 8 loaves and the other 5, and the loaves are shared equally by the three. In what proportion should the Arabs be paid?

397. *M* invests one-third of his property in Bank stock, one-sixth in Consols, and the remainder in Railway shares. When he sells out he makes a profit of 5 per cent., 3 per cent., and 2 per cent. respectively on the investments, and realises £6190. Required the amount of his property originally.

398. The average of 8 results is 20; that of the first two is $15\frac{1}{2}$, and of the next three is $21\frac{1}{3}$; the sixth is less than the seventh by 4, and less by 7 than the eighth. Find the last.

399. The telegraph posts on Railways are generally erected at intervals of 60 yards. Shew that if a traveller count the number of the posts, which pass his eye in two minutes, that number will nearly express in miles per hour the speed of the train.

400. The breadth of a room is two-thirds of its length and three-halves of its height, and the contents are 5832 cubic feet. Find the dimensions of the room.

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401. Decompose 831600 into its prime factors, and find the least multiplier of it, which will make the product a perfect cube.

402. A man having lived at the rate of *Rs.*5000 a year for 6 years, finds himself in debt, and reduces his expenditure to *Rs.*4500. He is out of debt in 4 years. What is his income ?

403. A boat propelled by 8 oars which take 30 strokes per minute travels at the rate of $9\frac{3}{4}$ miles per hour ; find the rate of a boat propelled by 6 oars which take 28 strokes per minute, the work done by each oar during one stroke in the latter case being a quarter as much again as in the former case.

404. The value of a certain length of a material, *A*, is $\frac{9}{11}$ of the value of $\frac{1}{5}$ as much again of another, *B* ; and the weight of 17 yds. 2 ft. of *A* is $\frac{68}{51}$ of the weight of 15 yds. 1 ft. 9 in. of *B*. If the value of 3 cwt. 27 lbs. of *A* be *Rs.*351, what is the value of 1 cwt. 2 qrs. 12 lbs. of *B* ?

405. *A* and *B* barter : *A* has 27 tons of coal worth £1. 2*s.* 6*d.* a ton, but insists on having £1. 5*s.* a ton ; *B* has hops worth £2. 14*s.* a pocket, which he raises in price in proportion to *A*'s demand. *A* receives 6 pockets of hops : what cash does he get besides ?

406. A rectangular field whose diagonal measures 825 feet, has one of its sides ·75 the length of the other. Find the length of each side in yards, and the area in acres.

407. The sum which will pay *A*'s wages for $61\frac{1}{4}$ days will pay *B*'s wages for $81\frac{2}{3}$ days. For how many days will it pay the wages of *A* and *B* together ?

408. Four persons possess respectively 14 mangoes, 24 oranges, 34 plantains and 44 peaches. Each of them gives one of his kind to each of the others, and it is then found that they are all equally rich. If the price of each plantain be 4 pies, find the price of each of the others.

409. *A* can excavate 14·2884 cubic yards per day ; how many can *B* do per day, if *A* could do *B*'s daily quantity in $\frac{11}{12}$ of the time that *B* would take to do *A*'s daily quantity ?

410. If money invested in the 3 per cent. consols give exactly

3 per cent. after the payment of 1*s*. in the £ income-tax, find the price of the consols, allowing $\frac{1}{8}$ per cent. to a broker for purchase.

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411. A multiplication sum having been worked is partially rubbed out, the figures remaining :—the multiplicand 7699, the first two (of four) figures in the multiplier 42, and the last two figures 47 in the product. Restore the complete work.

412. Three men, whose strides are 2 ft. 9 in., 3 ft. and $3\frac{1}{2}$ ft. respectively, walk 2 miles. Find how often they will step together.

413. Of the books in a library, $\frac{1}{3}$ are literature, $\frac{1}{4}$ mathematics, $\frac{1}{5}$ history, $\frac{1}{6}$ philosophy, and the rest novels. What is the least number of novels that the library can have ?

414. A coal merchant had 150 tons of coal, of which he sold 50 tons at *Rs*.27 per ton, and found that he was only gaining $7\frac{1}{2}$ per cent. At what rate must he sell the remainder, so that he may gain 10 per cent. on the whole ?

415. A speculator sells at a profit of 50 per cent. ; but his purchaser fails, and only pays 10*s*. in the £. How much per cent. does the speculator gain or lose by his venture ?

416. The wheel of an engine 8 ft. 2 in. circumference slips back along the rail 1 ft. during each revolution, and goes round 12 times in 1″ ; how long is it going 30 miles ?

417. If 4 men dig a piece of ground 350 yds. long and 144 yds. broad, in $2\frac{1}{3}$ days, how long will it take 5 boys to dig one 600 yds. long and 168 yds. broad, the resistance of the soil in the two cases being as 4 : 5, and 4 men having the strength of 7 boys ?

418. The cost of painting a room 9 ft. 6 in. high, 15 ft. 3 in. long and 10 ft. broad is *Rs*.47. 8*a*. ; what must be the height of another room whose length and breadth are respectively 18 ft. 2 in. and 11 ft. 7 in., if the painting of the walls at the same cost per sq. yard amount to *Rs*.74. 6*a*. ?

419. If the discount on a sum due at the end of $2\frac{1}{2}$ years be $\frac{89}{87}$ of simple interest, at what rate is the interest calculated, and if the discount and interest together amount to £46. 5*s*. $5\frac{1}{2}$*d*., what is the sum of money ?

420. I bought goods at 23*s*. 9*d*. with 4 months' credit, and sold them forthwith at 25*s*. 6*d*. with such allowance of credit, as made my gain $6\frac{3}{8}$ per cent. How long credit did I give, reckoning interest at 4 per cent. per annum ?

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421. 650 horses are conveyed in transports to the seat of war at a cost for food of *Rs*.15420. A storm occurs just after $\frac{1}{4}$th of the voyage is completed, in which 10 horses are killed. If the expense

of the food of each horse be 8*a.* per day, what was the length o the voyage?

422. Brussels carpet is 2 ft. wide, costs *Rs.*3. 12*a.* per yard, and will last 5 years; Kidderminster carpet is $2\frac{1}{2}$ ft. wide, costs *Rs.*2. 8*a.* per yard, and will last 3 years; find the ratio of their cost, not reckoning interest on the outlay.

423. Find the difference between $1\cdot6$ of $3\cdot4$ of £$1\cdot125$ and $\frac{1}{4}$ of $3\cdot\dot{6}$ of £$9\cdot1125$, and find the value of

$$\frac{6\cdot27\times0\cdot5}{(\frac{1}{3}\text{ of }\frac{3}{4})\times8\ 36}+\frac{(\frac{1}{4}\text{ of }\frac{1}{10})\times(\frac{3}{4}\text{ of }21\frac{1}{3})}{(\frac{2}{3}\text{ of }\frac{5}{6}+1.\dot{4}}$$

424. *A*, *B*, *C* rent between them a field of grass for 12 weeks for Rs.128. 5*a.* 4*p.*, and put into it 3, 4, 5, score sheep respectively. After 2 weeks *A* added 2 score, and at the end of 9 weeks removed all; *B* withdrew 1 score after 3 weeks, and added 2 score at the end of the 5th week. What ought each to pay? At what rate per acre per annum is this rent, supposing the field to be rectangular; the shorter side 110 yards, and the longer side 440 yards?

425. Three merchants *A*, *B*, *C* engage in a speculation (of which *A*'s share is $\cdot\dot{1}4285\dot{7}$, *B*'s $\cdot\dot{4}2857\dot{1}$) and agree to deposit $\frac{1}{7}$th of their respective contributions to the required capital at once, $\cdot\dot{2}3076\dot{9}$ at the end of a month, and the remainder at the expiration of another month: after the second instalment *B* dies, and *A* and *C* make up the remainder, *C* paying three times as much as *A*; supposing $1\cdot\dot{8}5714\dot{2}$ of the capital to be *Rs.* 16900, what were *A*'s and *C*'s last deposits?

426. If, while the common rate of interest is $3\frac{1}{2}$ per cent. I can get my tradesman's bills discounted as if it were 5 per cent., what is the advantage to me, in an expenditure of £450 of ready money payments, rather than leaving my bills unpaid for 12 months?

427. *A* is at Calcutta and *B* at Ichapore ($21\frac{1}{4}$ miles apart). *A* leaves for Ichapore at 9 A. M., and *B* for Calcutta at 9-30 A. M. They meet at 12-8 (noon), and find that *B* has walked $\frac{1}{4}$ mile an hour more than *A*. How many miles has each travelled?

428. The area of a verandah 3 ft. broad around a room 26 ft. long, is 300 sq. ft. Find the breadth of the room.

429. Inside a rectangular vessel 9 ft. by 7 ft. is another rectangular vessel, whose sides are 6 inches apart from the sides of the larger vessel. The smaller vessel is filled with a certain fluid which costs *Rs.*900 at the rate of $\frac{1}{4}$ of an anna per cub. inch. Find the depth of the vessel.

430. There are three pendulums. The first makes 35 beats in 36 seconds, the second 36 beats in 37 seconds, and the third

37 beats in 38 seconds. Supposing they commence together, find how many times they will beat simultaneously in 24 hours.

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431. In passing through a certain district, a person went 5 miles due north, then 6 miles due east, then 10 miles due south, and lastly straight to the place whence he started. Find the area of that part of the district round which he travelled.

432. There is an oblong room, whose $B : L :: 3 : 5$, $L : H :: 3 : 2$, and H : the thickness of its walls :: $3 : \frac{3}{2}$. How many stones of 13 cub. ft. each would be required for the construction of its walls, given that there are two doorways each $4\frac{1}{2}$ ft. × 6 ft. and two windows each $3\frac{1}{2}$ ft. × 4 ft. and that the area of the room is 135 sq. ft. ?

433. *A* can do as much work in 17 days as *B*, *C* and *D* together in 4 days ; *B* can do as much in 7 days as *A*, *C* and *D* together in 3 days ; and *C* can do as much in 13 days as *B* and *D* together in 7 days. How long would it take *A*, *B* and *C* together to do what *D* could alone perform in 13 days ?

434. The earth from a rectangular trench 8 ft. deep, one of whose sides is ·75 of the other, and whose diagonal measures 25 yards, is required to fill up a square pit whose depth is 6 ft. Find the length of a side of the square and how many coolies it would take to fill up the pit in 16 days of 10 hours each, supposing that each cooly can carry $\frac{1}{2}$ cubic foot of earth at a time and make 9 trips per hour.

435. A train starts from *P* to go to *Q* ; after it has travelled 1 hr. 12 min. it meets with an accident which delays it 48 min., and diminishes its rate by 5 miles ; hence it arrives 2 hours behind time. Had the accident happened 20 miles further on, it would have been only 1 hr. 48 min. late. Find the original rate of the train and the distance from *P* to *Q*.

436. A cistern can be filled by a pipe in 6 hours and emptied by another in 4 hours. They are alternately opened and closed for $1\frac{1}{2}$ hours. Find the time in which the cistern when full can be emptied.

437. If 21 horses and 217 sheep can be kept 10 days for the same sum as it would cost to keep 9 horses and 60 sheep for 27 days, find how many sheep eat as much as 3 horses.

438. A man buys 27 sheep for *Rs.*90 and sells 12 of them at a loss of 3 per cent. ; at how much price must he sell the remainder per head, in order that he may gain $2\frac{1}{2}$ per cent. on the whole purchase ?

439. Two men undertake to do a piece of work for *Rs.*7. 8*a.* One could do it alone in 5 days, the other in 6 days. With the assistance of a boy they finish it in 2 days. How should the money be divided ?

440. A book sent from England costs me (including 1*s.* 6*d.*

postage) 16*s*. 1*d*. But my bookseller allows me two pence in the shilling discount on the published price. What is the published price ?

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441. Find the diagonal of a square whose side is one-fifth of a mile.

442. A square area is bounded by 36 yds. of wire fencing. If the enclosed area be increased by 40 sq. yds., still retaining the form of a square, how many additional yards of fencing will be required ?

443. A broker charges one-eighth per cent. commission on the money invested. He is handed over *Rs.*14800 to invest in a certain stock at $92\frac{5}{8}$ and succeeds in obtaining the stock at $92\frac{1}{2}$, keeping the balance to himself ; what is his total profit ?

444. A person, who pays 5*d*. in the £ income-tax, finds that a rise of interest, from 6 to $6\frac{1}{2}$ per cent., increases his income by £23. 10*s*. What is his capital ?

445. A man who has sold tea at *Rs.*2. 8*a*. per seer making a profit of 25 per cent., lowers his price so as to gain only 2*a*. per seer ; in what ratio must his monthly sale increase that he may make twice as much as before ?

446. The cost of converting wheat into bread is 20 per cent. on the cost of the wheat. If the price of wheat fall 20 per cent., and that of converting it into bread rises 20 per cent., for what should a loaf as large as an old 10*d*. loaf be sold, so as to gain 20 per cent. ?

447. There are three cubical boxes ; the edge of the first is 12 in., that of the second 20 in., and that of the third 30 in. Find the length of the edge of a cubical box which shall contain as much as all three.

448. During a distillation the fluid that comes over in three consecutive hours contains 53, 50 and 48 per cent. of alcohol respectively. The ratio at which it comes over during these 3 hours are in the ratio of 1, 2 and 3. What is the percentage of alcohol in the whole mixture ?

449. Two monkeys, having stolen a pile of walnuts and filberts from a garden, are on the point of beginning their feast, when they see the injured owner of the nuts approaching with a stick. At once they see that he will take $2\frac{1}{2}$ minutes to reach them. There are twice as many filberts as walnuts, and one monkey finishes the latter at the rate of 15 per minute in $\frac{4}{5}$ of the time and runs away ; the other manages to eat the filberts *just* in time. If the first monkey had stopped to help the other till all were finished, find when they would have got away, (i) if they eat filberts at equal rates, (ii) if the first monkey eats filberts at the same rate as he eats walnuts.

450. A goldsmith mixes together silver, copper, zinc, gold, and brass, and makes an idol weighing 30 lbs., which he sells for *Rs.*720.

including a profit of 15 per cent. and 5 per cent. for interest on his outlay. For every one pound of gold he has 3 of silver, 12 of copper, 6 of zinc and 8 of brass ; supposing that a lb. of brass costs *Rs.*2, a lb. of silver 11 times as much as brass, a lb. of copper $16\frac{2}{3}$ per cent. less than that of brass, and a lb. of zinc $16\frac{2}{3}$ per cent. less than that of copper ; what was the cost price of the gold in the mixture ?

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451. Write in figures twelve thousand twelve hundred and twelve.

452. One-fifth of the difference of two numbers = 58, and one-fifth of their sum is less than their difference by 92. Find the numbers.

453. Reduce 3 roods 7 po. 28 sq. yds. 2 ft. 36 in. to the decimal of 1 ac. 31 po. 27 sq. yds. 2 ft. 36 in.

454. Reduce $(\cdot 5\dot{7}\dot{5} + 16 \times \cdot\dot{1}\dot{5})$ of 4 viss to the decimal of $1\frac{19}{56}$ cwt. a viss being equal to 3 lbs. 2 oz. Avoir.

455 The materials of an old building were sold for *Rs.*1500 on condition that they should be removed within 30 days under a penalty of *Rs.*10 per day for every day beyond 30 days. The purchaser employed 40 men at $3\frac{1}{2}$ annas per day to do the work, and after selling the materials for *Rs.*2365, he cleared *Rs.*190 by his bargain. Find the number of days the men were at work.

456. In a rectangular area 100 yds. long and 50 yds. broad, there are two paths crossing one another, each parallel to one side of the rectangle, and each 4 yds. broad. Find the cost of paving the area with stone at 12*a.* per sq. yd., and of covering the paths with gravel at 6*a.* per sq. yd.

457. A bankrupt has goods worth *Rs.*9750 ; and had they realised their full value, his creditors would have received 13*a.* in the rupee ; but $\frac{3}{8}$ths were sold at 17·5 per cent. and the remainder at 23·75 per cent., below their value. What sum did the goods fetch, and what dividend was paid ?

458. Two clocks begin to strike 8 together. The one strikes in 14 seconds and the second strikes in $10\frac{1}{2}$. Find the interval between their fifth strokes.

459. If 3 men, 2 women and 6 children, or 4 men and 7 children can dig 125 cub. ft. of earth in 6 days ; how many cub. ft. would 4 men, 4 women and 4 children dig in 20 days, 2 women being supposed to do as much as 3 children ?

460. Two trains set out from *A* and travel towards *B*, which is 400 miles distant from *A*. The one leaves *A* at 2 A.M. and travels at 23 miles an hour. The other leaves *A* at 2-30 A.M. and overtakes the former 75 miles from *B*. Find the rate of the latter train.

PROBLEMS.

1. *A* pays £9. 3*s.* 4*d.* more rates than *B*, their incomes being equal; living in different towns they are rated at 2*s.* and 1*s.* 4*d.* in the pound respectively: what is their income?

2. In running a 3 mile race on a course $\frac{1}{8}$ of a mile round, *A* overlaps *B* at the middle of the 7th round. By what distance will *A* win at the same rate of running?

3. A person bought a French watch, bearing a duty of 25 per cent., and sold it at a loss of 5 per cent.; had he sold it for *Rs.*30 more, he would have cleared 1 per cent. on his bargain. What had the French maker for the watch?

4. A four-wheeled carriage travels round on a circular railway. The circumferences of the two wheels of the carriage and of the two circles of rails, are proportional to 6, 7, 7000, 7014. Find the number of revolutions made by each of the four wheels in a complete circuit.

5. *A* and *B* run a race. *A* starts at the rate of 400 yards a minute, and in every successive minute diminishes his pace by a yard a minute. *B* increases his pace by the same, and overtakes *A* in 4 minutes. What was *B*'s pace at starting?

6. Three tramps meet together for a meal: the first has 5 loaves, the second 3, and the third, who has his share of the bread, pays the other two 5*a.* 4*p.*; how ought they to divide the money?

7. A person puts £1197 out at 30 per cent. per annum, interest, and spends at the end of the year £300 *more* than the annual interest on £1197, and thus at the end of a certain time has nothing left. If he had spent £300 *less* than the annual interest, how much would he have had at the end of the same time?

8. A person borrows two equal sums at the same time at 5 and 4 per cent. respectively, and finds that if he repays the former sum with interest on a certain date 6 months before the latter, he will have to pay in each case the same amount, *viz.*, £1100. Find the amount borrowed and the time for which interest is paid.

9. The Fort Barracks are lighted with gas from 100 burners. Find the cost of lighting them per night of 10 hours at the rate of *Rs.*5$\frac{1}{4}$ for 1000 cub. ft. of gas, assuming that for the first 3 hours each burner consumes 1 cubic inch per second, and during the remainder of the night the light is so reduced that the consumption of gas by each burner is only $\frac{3}{4}$ths of that quantity per second.

10. The distance by Railway from Madras to Salem is 206$\frac{3}{4}$ miles. A Passenger Train travelling 20 miles an hour leaves Madras at 7 A. M.; and a Special Train at 10 A. M. the same day. At what rate must the latter travel, so as just to overtake the former at Jollarpett Junction (132 miles from Madras); and at what hour must

a Goods' Train leave Salem for Madras travelling 15 miles an hour, so as to reach Jollarpett at the same time as the other Trains ?

11. A person having to pay *Rs.*10572 two years hence, invests in the 4 per cent. Transfer Loan to accumulate interest till the debt shall be paid, and also an equal sum the next year. Supposing the investment to be made when paper is at $86\frac{1}{4}$, and the price to remain the same, what sum must be invested on each occasion that these be just sufficient to pay the debt at the given time ?

12. *A* owes *Rs.*356 and *Rs.*743 to be paid in 10 months and $2\frac{1}{2}$ years respectively. At what time should *A* pay the whole in a lump-sum to clear off the debt, so that neither may lose by the transaction ?

13. *A* has stock in the 3 per cent. consols which produces him £300 per annum. He sells out one-half at 92 and invests the proceeds in the South Devon Railway where a £50 share is worth £23. What dividend per cent. per annum ought the South Devon Railway to pay so that he may increase his income £50 per annum by the operation ?

14. A person spending annually *Rs.*2400 saves *Rs.*27. 8*a.* of it quarterly by ready payments. If by this means he increases his annual saving $20\frac{5}{8}$ per cent., what is his annual income ?

15. A person going from Pondicherry to Ootacamond travels 90 miles by steamer, 330 miles by rail and 30 miles by horse transit. The journey occupies 30 hours 50 min. and the rate of the train is 3 times that of the horse transit and $1\frac{1}{2}$ times that of the steamer. Find the rate of the train.

16. A tradesman finds that if he asks for his goods 15 per cent. above the wholesale price, he can sell his whole stock in 4 months, whereas if he asks 20 per cent. he requires 6 months to sell the same amount. Which will he find the more profitable system at the year's end ?

17. A cistern has two pipes, *A* and *B*, which singly could fill it in 9 hours and 10 hours, respectively. It has also two taps *C* and *D*, which singly could empty it in 12 hours and 8 hours, respectively. Suppose that when the cistern stands half full of water, *A* and *D* are turned on for 3 hours ; that then *B* is also turned on for the next two hours ; and that then *A* and *D* are turned off, and *C* is turned on for the next 8 hours ; after which all are shut, and the cistern is found to contain 95 gallons more than its half content :— Find the content of the cistern. Find also how much per hour the cistern would lose or gain, if all the pipes were set open at once.

18. Of the whole cost of constructing a Railway, $\frac{5}{7}$ is held in shares, and the remainder, £400000, was borrowed on mortgage at 5 per cent. Find what amount of gross annual receipts,—of which 40 per cent. will be required for the working expenses of the line, and 8 per cent. for a reserved fund,—which will yield to the shareholders a dividend of $4\frac{1}{2}$ per cent. on their investments.

19. An investment was made by a certain person in the 4 per cents., when they were selling at 20 discount; twelve months afterwards, when they were selling at 16 discount he sold out; what interest did he get upon his investment?

20. A person borrows £1261 at 5 per cent. compound interest which he wishes to pay off in 3 equal yearly instalments commencing at the end of the first year; what ought he to pay yearly to effect this?

21. A bath can be filled by the cold water pipe in 9 minutes, and by the hot water pipe in $11\frac{1}{4}$ minutes. A person leaves the bath-room after turning on both pipes simultaneously, and returns at the moment when the bath should be full. Finding however that the waste-pipe has been open, he now closes it. In $3\frac{3}{4}$ minutes more the bath is full. In what time would the waste-pipe empty it?

22. A square field contains 22 acres 2 roods: how long will it take a man to run round the boundary, running at the rate of 12 miles an hour? If the field be increased by 9 acres, so as to form a rectangle whose shorter side is the former side of the square, at what rate does a man run who runs round it in 1 min. 39 sec. longer than was occupied in running the square field?

23. A cubical block of metal of 7·84 inches side weighs ·25 lbs. per cub. inch. A hole of square sectional area is to be cut completely through the metal perpendicular to a face of the cube, in order that the weight of the metal left may be 100 lbs. Find to three places of decimals the side of the square section.

24. A rectangular cistern $10\frac{1}{8}$ ft. in length, $6\frac{1}{4}$ ft. in breadth and $3\frac{1}{8}$ ft. in depth, contains $140\frac{5}{8}$ cubic. ft. of water. What is the least number of bricks, each 9 in. long, by $4\frac{1}{2}$ in. wide, by 3 in. thick, that must be thrown into the cistern to make the water rise to the top, a brick being found to absorb water to the extent of one-fifth of its volume?

25. A man bought a house, which cost him 4 per cent. upon the purchase money to put into repair; it then stood empty for a year, during which time he reckoned he was losing 5 per cent. upon his total outaly. He then sold it again for £1192, by which means he gained 10 per cent. upon the original purchase money. What did he give for the house?

26. Divide £3010 into three sums, so that if the first be put out at simple interest for 3 years at 4 per cent., the second for 5 years at 3 per cent., and the third for 2 years at $2\frac{1}{2}$ per cent., the amount of the second shall be double that of the first, and the amount of the third treble that of the second.

27. How many persons can be accommodated in a concert room which is 117 ft. long and 90 ft. wide; allowing a space of 2 ft. 3 in. by 1 ft. 6 in. for each sitting, and a gangway 3 ft. wide the whole length of the room?

28. At what distance from London will a train which leaves London for Rugby at 2-45 P. M., and goes at the rate of 41 miles an hour meet a train which leaves Rugby for London at 1-45 P. M., and goes at the rate of 25 miles an hour, the distance between London and Rugby being 80 miles?

29. If the manufacturer makes a profit of 20 per cent., the wholesale dealer a profit of 25 per cent., and the shopkeeper a profit of 40 per cent., what was the cost of the manufacture of an article bought at a shop for 17*s*. 6*d*.?

30. The external length, breadth and depth of a rectangular tin vessel are 14, 10 and 9 inches respectively, and the thickness of the tin ½ an inch: when the vessel is empty it weighs 1500 oz., and when filled with water 2040·6 oz. Find the weight of a cub. foot of water.

31. A boat's crew row over a course of a mile and a quarter against a stream which flows at the rate of 2 miles an hour in 10 minutes. The usual rate of the stream is half-a-mile an hour. Find the time which the boat would take in the usual state of the river.

32. A soldier has 5 hours' leave of absence; how far may he ride on a coach which travels 10 miles an hour, so as to return to the camp in time, walking at the rate of 5 miles an hour?

33. An express train starts from a station *A* at one o'clock for a station *B* at 30 miles an hour, and in 15 minutes is followed by an ordinary train at 20 miles an hour. A train from *B* to *A* at 25 miles an hour, after travelling one hour, meets the express, and in 20 minutes more meets the ordinary train. At what time did the train leave *B*?

34. Two men, *A* and *B*, start from Cambridge, at 4 and 5 o'clock A. M. respectively, to walk to London, a distance of 50 miles; *B* passes *A* at the 20th milestone, and reaches London at 5 P. M. When will *A* arrive there?

35. Two ships are built. Twice as many ship-carpenters are employed about the first as about the second; the first is built in 9 months, the second in 8 months; the wages of each man of the first set are 7*d*. per hour, and they work 12 hours a day; the wages of each of the second set are 6*d*. per hour, and they work 10½ hours a day. The cost of the first in carpenter's wages was £6000; what was that of the second?

36. If when 25 per cent. is lost in grinding wheat, a country has to import ten million quarters, but can maintain itself on its own produce if only 5 per cent. be lost; find the quantity of wheat grown in the country.

37. Two trains start at the same time, the one from Calcutta to Delhi, the other from Delhi to Calcutta. If they arrive in Delhi and Calcutta respectively 1 hour and 4 hours after they passed each other, show that one travels twice as fast as the other.

38. A franc being worth $9\frac{3}{4}$*d*., find the sum of money which can be paid by an exact number of either shillings or francs, the number of francs exceeding the number of shillings by 27.

39. A poor fellow ignorant of Arithmetic, had *Rs*.3. 4*a*. in his purse; he talked of spending $\frac{1}{2}$ of it on cakes, $\frac{1}{3}$ of it on fruit, and $\frac{1}{4}$ of it on a knife. Show (i) his mistake, and (ii) how he might have spent his money in proportion to the fractions he intended.

40. A person sells 40 horses at a gain of 10 per cent., and 50 horses at a gain of 20 per cent. Had he sold all of them at a uniform profit of 15 per cent. he would have got *Rs*.40 less. Find the cost price of each horse.

41. A ship's crew sailed with provisions for 30 days; after being at sea 20 days they encountered a storm in which they lost 6 men, and 2 days after the storm they took on board 18 men who had been wrecked and were without provisions; they then found that to make their supplies last so long as was intended each man's daily allowance must be reduced to $\frac{5}{8}$ of what had been before; how many men were on board when the ship set sail?

42. A man has his money in a bank, which pays him 5 per cent. interest. He buys a £100 share in a company, of which £20 is to be paid up at once. At the end of the first year, a dividend is declared at the rate of 10 per cent. on the paid-up capital. At the end of the second year, a further call of £20 per share is made, and no dividend is declared. At the end of the third year, a dividend is declared at 12 per cent. on the paid-up capital. How much will he have gained or lost by his speculation at the end of the three years?

43. The transverse section of a river channel is a semi-circle whose diameter is half-a-mile, how much water passes through it in 7 hours, supposing the average velocity of the stream to be 2 miles an hour? [Area of a circle $= 3\frac{1}{7} \times (\text{radius})^2$.]

44. If the wholesale dealer sell to a retailer at 10 per cent. profit; and the retailer sell to the consumer at 50 per cent. profit, what proportion of the price paid by the consumer is profit?

45. If when corn is 15*s*. 9*d*. a quarter, and hay $5\frac{1}{2}$*d*. per stone, 7 horses can be kept 8 days for £4. 1*s*. 3*d*.; how many weeks can 16 horses be kept for £95, when corn is 2*s*. a bushel, and hay 70*s*. a ton, supposing that 126 lbs. of hay are consumed with 1 bushel of corn?

46. In a certain manufactory, 158 men of ordinary ability, and working the same number of hours each day, execute a certain piece of work; but if the abilities of 2 of them had been, respectively, $\frac{1}{7}$ and $\frac{1}{9}$ less than ordinary, and the abilities of 2 others $\frac{2}{7}$ and $\frac{3}{8}$ more, the work could have been finished $\frac{22}{63}$ of an hour sooner. Haw many hours a day did the men work?

47. If gold be beaten out so thin that an oz. Avoir. will form

a leaf of 20 sq. yds., how many of these leaves will make an inch thick, the weight of a cubic foot of gold being 10 cwt. 95 lbs.?

48. Two houses are built; the first is twice as long in building as the second; half as many men again are employed in building the first; their wages per hour are one-third higher, and they work 10 hours a day and 6 days a week, whilst the others work only 8 hours a day and 5 days a week; the cost of the second in workman's wages was *Rs.*1000. What was that of the first?

49. A man fastens his cow by a rope to a stake in the hedge of an oblong grass-field. What must be the length of the rope if the cow be allowed 1 acre to graze upon? Given that the area of a circle $=3\frac{1}{7}\times(\text{radius})^2$.

50. A sold a horse to *B*, who sold it to *C* at a profit of 5 per cent, who sold it for *Rs.*1071, gaining thereby 20 per cent. What did *B* give for the horse? What gain per cent. was the last price on the first price?

51. Suppose that 15 men would be necessary to excavate 966 cub. yds. in 8 days of $10\frac{1}{2}$ hours each:—How many men did a contractor engage for 12 days of $7\frac{1}{2}$ hrs, to excavate 575 cub. yds, if he found it necessary to engage 4 additional men during the last 4 days, in order to complete the work in 12 days?

52. The ready-money price of a dress at a tradesman's, who allows mercantile discount in the ordinary way for ready money, is *Rs.*85. 8*a.* and the credit price is *Rs.*90. What ought the credit price to be, in order that, while charging the same ready-money price, he may allow twice the rate of discount?

53. I sell a horse for *Rs.*306, and lose, 15 per cent. on what I gave for him, whereas I ought to have gained 25 per cent.; how much did I sell him under the price I had fixed on?

54. A baker fixes the price of a quartern loaf, when the cost of converting wheat into bread is 5 per cent. on the cost of the wheat, with a view to gaining 20 per cent. on the cost of his bread. Shew that if wheat falls 16 per cent. and the cost of converting it into bread rises 20 per cent., he ought to sell seven loaves for the price at which he before sold six; and that if he does not alter his prices he will gain on his outlay 40 per cent. instead of 20 per cent.

55. A bar of metal, one square inch in section, can just suspend a weight of one ton; also a cubic inch of the metal weighs 4 oz. Find the length of the longest wire of uniform section into which a cubic foot of the metal can be drawn, so as not to break when suspended at one end.

56. *A*, *B* and *C* enter into partnership; *A* and *B* place their capital in the hands of *C*, who is to have the sole management, but who does not invest any money in the business. It is agreed that when the net annual value of the profits is not less than 5 per cent.

on the capital, *A* and *B* shall have 4 per cent. on their money, and of the rest *C* shall receive one-half, and the other half shall be divided between *A* and *B* in proportion to their investments. If *A* embark in the concern half as much again as *B*, and *C* receive *Rs.*2500 when the profits are 5 per cent., find the amount invested by *A* and *B*, and the share of each partner when the net profits are *Rs.*35000.

57. If 40 men, working 24 days of 8 hrs. can build a wall $5\frac{1}{3}$ ft. high, 3·4 feet thick and 36 yds. long, how many days of 10 hrs. will 480 boys require, to raise a wall $3\frac{1}{2}$ times the size? It being known that whilst 12 boys can build a cubic yard, 5 men can build $\frac{1}{8}$ of $\frac{2\frac{1}{2}}{\cdot 06}$ of a pillar 12 feet high, with a square base whose breadth is $1\frac{1}{2}$ feet.

58. *A* and *B* who are at the opposite extremities of the diameter of a circular area 135 miles in circumference, start to go round it at the same time in the same direction, *A* at the rate of 11 miles in 2 hours and *B* at the rate of 17 miles in 3 hours. How many rounds will each take before the one will overtake the other, and how long will the chase continue?

59. In a mixture of wine and water, the ratio of the wine to the water is 2 : 3. A gallon more wine being poured into the mixture, there is now as much wine as water; of how many gallons did the original mixture consist?

60. On the Eastern Counties Railway an ordinary train takes 50 minutes from Ely to Cambridge, an express train a quarter of an hour less. Supposing an express to leave London at 10·57 A. M. and arrive in Cambridge just as an ordinary train is leaving which arrives in London at 4-20 P.M., find the time respectively taken by these trains.

61. In a theatre the number of seats in the stalls is $\frac{2}{3}$ of the number of seats in the boxes, and are together twice as numerous as the seats in the pit. The number in the pit=4 times the number of seats in the gallery; and taken together afford as many places as the boxes. The number in the gallery is 100, find how many persons the house will accommodate, and state the number of seats in each portion.

62. *A*, *B* and *C* are sent to empty a cistern by means of two pumps of the same bore. *A* and *B* go to work first, making 37 and 40 strokes respectively per minute; but after 5 minutes they make each 5 strokes less a minute; but after 10 minutes more *A* gives way to *C*, who works at the rate of 30 strokes per minute. The cistern is emptied in 22 minutes altogether, and the men are paid 12*s.* 7*d.* for their labour. What should each receive?

63. A ship having a crew of 26 persons carries provisions for 21 days; after having been at sea for 11 days they pick up a party from a wreck, and it is then found that the provisions will be exhausted in 5 days; how many persons were taken from the wreck?

64. In a company paying 10 per cent. on all its capital, a person buys a £100 share at 92 premium ; and afterwards he takes up a new share allotted to him at par ; and he finds that he makes 6 per cent. on his money ; what was the amount of the new share ?

65. *Y* sells *Z* a horse ; *Z* gives *Y* a bill at 3 months for *Rs.*555. At the expiration of 1 month *Y* gets the bill discounted at 6 per cent., what ready money should he receive ?

66. How many pipes each $1\frac{1}{2}$ ft. in length will be required for draining a square field containing 11 ac. 4 po with parallel drains ; $5\frac{1}{2}$ yds. being the distance between each two drains and also the distance between the extreme drains and the fences ?

67. What is the distance between the hour and the minute-hands of a watch at 17 minutes past 4 ?

68. A bath is supplied with water from two pipes, one of which can fill it in $12\frac{1}{2}$ minutes, the other in 15 minutes ; there is also a discharging pipe which would empty it, when filled, in 10 minutes. The first pipe is open alone for 4 minutes and then the first and second open together for one minute ; if now the third pipe be opened as well, how long will it take to fill the bath ?

69. A certain reef of quartz when crushed yields ·0011 per cent of gold. If the working expenses amount to 62·5 per cent. of the gross receipts, and the profit on each 100 tons crushed amounts to £52. 10*s*.; find the number of grains in sovereigns.

70. Two watches *A* and *B* whose rates are uniform, at noon yesterday indicated 11·55′ and 12-2′ respectively. *A* indicated the correct time at 9 P. M. yesterday, and *B* at 6 A. M. this morning. When did *A* and *B* last agree, and what time did they then indicate? When will they agree next ?

71. A merchant sells 49 quarters of wheat at a profit of 7 per cent. and a certain number of quarters at a profit of 11 per cent. The price of a quarter of wheat being £3. 12*s*. 6*d*., he would have lost £2. 10*s*. 9*d*. if he had sold the whole at a profit of 9 per cent. Find the total number of quarters of wheat sold by him.

72. A cistern the cubic contents of which are 360 cubic feet, has two pipes which can empty it in 3 and 4 hours respectively. It has also a third pipe with an orifice of 1 sq. ft. through which water flows into the cistern at the rate of 1 yard per minute. If all the three pipes be opened together when the cistern is full, in what time will it be emptied ?

73. *A* and *B* start at the same time on a journey. *A* walks at the rate of 4 miles an hour and *B* of 3 miles an hour. When *A* has gone half way, *B* gets a ride, and goes at twice the rate of *A*, until he has ridden a distance equal to $\frac{2}{15}$ of the whole journey beyond the spot at which he passes *A*. *B* then walks the remainder

of the journey, *A* having walked it all. Will *A* or *B* arrive first? And what fraction of the whole journey will the other still have to travel?

74. *A* and *B* exchange goods; *A* gives 13 cwt. of hops, the retail price of which is 56*s*., but in barter he rates them at £3. *B* gives 10 barrels of beer, the retail price of which is 1*s*. a gallon, but the value of which he raises in proportion to the increased price of the hops. How much must *B* give in money?

75. If gold be at a premium of 20 per cent., and a person buy goods marked 135 dollars, and offers gold to the amount of 135 dollars, what change ought he to receive in notes, 5 per cent. being abated for ready payment?

76. A grocer buys twice as much black tea as green, giving 2*s*. a pound more for the green than the black; he retails it when mixed at 5*s*. a pound, and makes 25 per cent. on his outlay. What did he give for each sort of tea?

77. Ash saplings after five years' growth are worth 1*s*. 3*d*., and increase in value 1*s*. 3*d*. each year afterwards. Each is allowed 40 sq. yds. of ground, and they are cut after 20 years' growth; what will then be the value of an acre?

78. A man and a boy are to work on alternate days at a piece of work, which would have occupied the boy alone 13 days. If the boy take the first day, the work will be finished half a day later than if the man commences. Find how long they would take to do it working together.

79. A man has three vessels, *P*, *Q*, *R*, holding 1, 2, 4, gallons respectively; *P* is empty, *Q* is full of water, *R* is full of wine. He fills *P* from *Q*, replenishes *Q* from *R*, and empties *P* into *R*. When he has performed this operation twice, what will be the proportion of the wine in *Q* to the water in *R*?

80. Two boats start to row a race at 3 o'clock. The race is over at $6\frac{2}{3}$ minutes past 3, the losing boat being 40 yards behind at the finish. At 4 minutes past 3, this boat being 700 yards from the winning post. Find the speed of each boat in miles per hour.

81. An up-train 88 yds. long, travelling at the rate of 35 miles an hour, meets a down train 88 yds. long, at 10 o'clock and passes it in 6 seconds. At 15 m. 3 sec. past 10 the up-train meets a second down-train, 132 yds. long, and passes it also in 6 seconds. At what time will the second train run into the first?

82. A tree grows each year one inch less than it did the previous year, and it grew a yard during its first year; the value of the tree at any time is equal to the number of pence in the cube of the number of yards in its height: find what the tree is worth when it has done growing.

83. A train, going at the rate of 60 miles an hour overtakes another train, 160 yds. long, going 45 miles an hour in the same direction on a parallel line, and passes it in half a minute. In what time would the first train pass another, of its own length, going in the opposite direction at the rate of 30 miles an hour?

84. The apparent intensity of a light varies inversely as the square of its distance. Find the point (i) between two lamps 50 yards apart, (ii) in a straight line beyond one of them, at which one appears twice as bright as the other.

85. If three concentric rings be drawn in one plane and the diameter of the external ring be 139 feet, find the radii of the others to three places of decimals so that the whole area may be divided into three parts by the rings in the ratio 4, 5, 11. [The area of a circle $= 3{\cdot}14159 \times (\text{radius})^2$].

86. Four coins of equal weight are made of gold and silver in the ratio 1 : 2, 2 : 3, 3 : 4, 4 : 5 respectively. They are melted together and recast. In what ratio will the gold and silver be in the new coins?

87. From a cask of wine $\frac{1}{4}$ is drawn off and the cask is filled up with water; $\frac{1}{4}$ of the mixture is then drawn off and the cask is again filled with water. after this process has been repeated 4 times altogether, what fraction of the original quantity of wine will be left in the cask?

88. Two passengers are charged for excess of luggage *Re.*1. 12*a.* 8*p.* and *Rs.*4. 3*a.* 4*p.* respectively. Had the luggage all belonged to one person he would have been charged *Rs.*9. 8*a.* for excess. How much is allowed free, the charge for excess being 4*p.* per seer?

89. A person shooting at a target at a distance of 500 yards, hears the bullet strike the target 4 seconds after he fired. A spectator equally distant from the target and the shooting point, hears the shot strike $2\frac{1}{2}$ seconds after he heard the report of the gun. Find the velocity of sound.

90. A merchant buys China tea at 3*s.* 6*d.* per lb. To improve the flavour, he adds 2 oz. of Assam to every lb. of China tea, and finds that the mixture costs him 4*s.* a lb. How much per lb. did he give for Assam?

91. If a certain number of workmen can do a piece of work in 25 days, in what time will $1\frac{2}{3}$ of that number of men do a piece of work twice as great, supposing that 2 of the first set can do as much work in an hour as 3 of the second set can in $1\frac{1}{2}$ hours, and that the second set work half as long a day as the first set?

92. A train running at the rate of 40 miles an hour meets a person walking along the line in the opposite direction at the rate of 4 miles an hour, and passes him in $5\frac{5}{8}$ seconds. Find the

length of the train. Had the person been going in the same direction as the train, in what time would it have passed him?

93. *Rs.*180 is to be divided among 60 men, women and children. The sums of the men's shares, women's shares and children's shares are in the proportion of 5 : 4 : 3, but their individual shares are as 3 : 2 : 1 respectively. Find the number of men, of women, and of children.

94. A spirit merchant buys two sorts of spirit in equal quantities, one at 1*s.* per gallon more than the other; he mixes them and sells the mixture at 16*s.* 6*d.* per gallon, gaining 10 per cent. on his outlay. What was the price paid per gallon by the merchant?

95. A man spends *Rs.*1200 in buying horses and cows, at the respective rates of *Rs.*54 and *Rs.*48 each. He sells them at a uniform rate of *Rs.*51 each, and gains 2 per cent. How many horses did he buy?

96. A crew, who can row at 4 miles an hour in still water, rows down a certain distance and up again in 1 hour 36 min. Find the distance in miles, if the velocity of the current be $1\frac{1}{2}$ miles.

97. Two trains, running on parallel rails in opposite directions at 40 and 30 miles per hour respectively, are observed to pass one another in 6 seconds. When they are running in the same direction, a man in the faster train finds that he passes the slower train in 24 seconds. What is the length of each train?

98. On a piece of work 3 men and 5 boys are employed, who do half of it in 6 days. After this one more man and one more boy are put on, and $\frac{1}{3}$ more is done in 3 days. How many more men must be put on that the whole may be completed in 1 day more?

99. A man has £1583. 17*s.* 11*d.* in the 3 per cent. stock and £982. 12*s.* 6*d.* in the $3\frac{1}{2}$ per cent. stock; he transfers a certain sum from the former to the latter when the stocks are at 91 and 98 respectively and thus makes the income derived from each the same. How much has he finally in the 3 per cent. stock?

100. A person has a certain sum to invest; he divides it equally between the $3\frac{1}{2}$ per cents. at 84 and the 4 per cents. at 88 and finds that the latter stock gives him an annual income of £3. 7*s.* 6*d.* more than the former; what is the sum?

101. Two clocks begin striking the hour of noon together on a certain day, the interval between every two strokes being 1″ and 2″ respectively. They gain 1″ and 2″ respectively in every 24 hours. Shew after what length of time they will end striking the hour of noon together, both shewing the correct time at the fall of the last stroke.

102. The height of a room is $\frac{1}{4}$ of the sum of its length and breadth, and the cost of papering its walls at 6*p.* per square foot is *Rs.*31. 4*a.* Find its height.

103. A mixture consists of milk and water mixed in the ratio of 8 : 1 ; 5 seers of water were then added, and it was found that the quantity of water is $\frac{9}{11}$ of the quantity of milk. Find the quantity of milk in the mixture.

104. In a certain meadow there is a crop of 525 stone of grass, which grows uniformly. If 11 oxen turned in would consume all the grass in 48 days, but 6 oxen would require 98 days, what weight of grass would each ox eat in a day ?

105. Four apples are worth as much as 11 plantains, 8 mangoes as much as 5 apples, 2 oranges as much as 7 plantains, and 9 plantains sell for a 2-anna piece. I wish to buy an equal number of each of the four fruits, and to spend an exact number of 2 anna pieces ; find the least sum I require.

106. Three boys begin to fill a cistern : one of them brings a pint at the end of every 3 minutes, another a quart every 5 minutes, and the third a gallon every 7 minutes. If the cistern holds 53 gallons, in what time will it be filled ?

107. Two trains on the same railway are running past each other in opposite directions, one at 40 and the other at 30 miles per hour. Each has an engine and tender ; the first has 12 carriages, and the second 17. If the length of an engine and tender be 40 ft. and the length of a carriage 32 ft., and the coupling spaces be each 5 ft., how much time will elapse from the moment that the engines meet till the last carriages of each train have passed each other ?

108. An empty cistern had two supplying pipes *A* and *B*, and two taps *C* and *D*. *A* would fill the cistern in $42\frac{1}{2}$ min. and *B* in 46 min. ; and *D* can carry off per minute half as much again as *C*. After *A* and *B*, running together, have supplied a certain quantity, *C* is allowed to run with them, and takes 51 min. to empty the cistern ; but had *D* been turned on along with *C*, the two would have taken only $5\frac{8}{4}$ min. to empty it. In what time would the cistern have been emptied if *D* had been turned on instead of *C*? And how much of the cistern was filled when *C* was set upon ?

109. A tradesman imported a quantity of foreign goods for which he had to pay a duty of 15 per cent., but the demand for them having gone off he is obliged to sell at a loss of $5\frac{5}{7}$ per cent. ; a month earlier he could have sold them for *Rs.*1242 more, and then would have cleared $2\frac{6}{7}$ per cent. on the transaction ; what price did he pay for the goods ?

110. The paper duty was $1\frac{1}{2}$*d.* a lb. ; the weight of a newspaper $\frac{1}{4}$ lb. ; the paper manufacturer made 20 per cent. profit on his outlay, and the proprietor of the newspaper 10 per cent. on his sale. What reduction should be made in the price of the newspaper, now that the duty on paper is abolished, allowing to each tradesman the same rate of profit ?

111. In buying rice on three occasions I found the prices per bag were as 1 : 2 : 3 respectively. The price of 30 bags, 10 bought on each day, was *Rs.*480. Find the price per bag on each day.

112. A field of 7 acres is sown with wheat, barley, and maize, the areas of the crops being respectively as $2\frac{1}{4} : 3\frac{1}{2} : 4\frac{1}{3}$. If the values of an acre of each be also respectively in the same ratios, and an acre of wheat be worth £7, what is the worth of all the crops in the field?

113. A clock and a watch together cost *Rs.*24. If the value of the watch were to rise 30 per cent. and that of the clock 40 per cent., both would together cost *Rs.* 32. What is the price of each?

114. A person went to a tavern with a certain sum of money; there he borrowed as much as he had about him and spent 1*a.* out of the whole; with the remainder he went to a second tavern where he borrowed as much as he had left and also spent 1*a.*, and he then went to a third tavern borrowing and spending as before, after which he had nothing left. How much had he at first?

115. An oz. of gold is worth £3. 18*s.*, and an oz. of silver is worth 7*s.*: and a mass of gold and silver weighing 170 oz. is worth £166. What is the worth of another mass, in which the numbers respectively of gold and silver in the first mass are interchanged?

116. A cord 95 ft long, when fully stretched, reaches from the top of a pole 57 ft. high, standing vertically, to a point in the ground. A shorter pole is placed vertically between the foot of the longer pole and this point at a distance of 48 ft. from the point, and it is found that the cord just reaches its top. Find the height of the longer pole over the shorter pole, it being given that the length of the cord between them is $\frac{7}{19}$ of the length of the entire cord.

117. A person bought 4 railway tickets to go 60 miles. Two were for the 1st class, one for the 2nd, and the fourth a half first class ticket for a child. The cost of a second class ticket was $\frac{2}{3}$ of that of a first class, and the whole sum was *Rs.*15. 13*a.* 4*p.* Find the price of each ticket, and the rate per mile for the first class.

118. *A* and *B* set out from the same place in the same direction. *A* travels uniformly 18 miles per day, and after 9 days turns and goes back as far as *B* has travelled during those 9 days; he then turns again, and pursuing his journey overtakes *B* at the end of $22\frac{1}{2}$ days after the time they first set out. Find the rate of *B* in miles per day.

119. A person bought goods on the continent; the cost of freight and insurance was 15 per cent., and that of duty 10 per cent. on the original outlay; he was obliged to sell them at a loss of 15 per cent.; but if he had made *Rs.*80 more of them he would have gained 1 per cent. What was the original outlay?

120. It is agreed that the rent of a farm shall consist of a fixed sum together with the value of a certain number of maunds of wheat; when wheat is *Rs.*3. 8*a.* per maund, the rent is *Rs.*312. 8*a.*, when

wheat is *Rs.*3. 12*a.* per maund, the rent is *Rs.*325 ; what will the rent be when wheat is *Rs.*5 a maund ?

121. Riding a journey of 27 miles into town, I meet the coach which left town at the same moment that I started from home (7 o'clock), at the 18th milestone from town. Supposing that it travels 10 miles an hour, determine the hour when we meet, and the time when, (proceeding at the same rate as before) I shall reach Calcutta.

122. *A*, *B* and *C* are partners ; *A* receives $\frac{1}{2}$ profits, and *B* twice as much as *C* ; find the capital of *C*, *A*'s income being diminished *Rs.* 400 by a fall of $\frac{1}{2}$ per cent. in the rate of profit.

123. A person increased his capital annually by one-third part, and at the end of 4 years, one year's interest thereon at $4\frac{1}{2}$ per cent. amounted to *Rs.* 2700. What capital did he start with ?

124. Divide *Rs.* 2025 among *A*, *B*, *C*, *D*, *E*, so that *A*'s share : *B*'s share : : 1 : 2 ; *C*'s share : *B*'s share : : 5 : 4 ; *D*'s share : *C*'s share : : 6 : 5 ; and *E*'s share : *D*'s share :: 4 : 3.

125. The length of a room is 20 ft. ; the cost of carpeting the floor at 6*s.* 6*d.* per sq. yd. is £11. 11*s.* $1\frac{1}{2}$*d.* ; and the cost of papering the walls (inside) at 10*d.* per sq. foot is £36 ; the area of the walls (outside) the room is 1056 sq. ft. ; find the thickness of the walls.

126. To what uniform depth must a piece of ground 414 yds. long, 37 yds. wide be excavated, that the earth taken out may form an embankment of 25530 cub. yds., supposing the earth to be increased *one-ninth* in volume by removal ?

127. A train which travels at the uniform rate of 30·8 ft. a second leaves Madras at 7 A. M. ; at what distance from Madras will it meet a train which leaves Arconum for Madras at 7-20 A. M. and travels one-third faster than it does, the distance from Madras to Arconum being 42 miles ?

128. When the income-tax was 7*d.* in the pound, a person had to pay £63 more than when it was 4*d.* in the pound, although his income had diminished in the interim by £225. What was his income at first ?

129. It costs *Rs.*90 to mat the floor of a room whose dimensions are as 6 : 5 at 12*a.* per sq. ft. ; and it takes *Rs.* 155 more to paper its walls *outside* at 10*a.* per sq. foot. If the height of the room be 8 ft, and there are four doorways in it, the area of each of which is 4 sq. ft., find the thickness of the wall.

130. The price of a mixture of two metals, weighing 14 oz. is *Rs.*34. If the proportions of the two metals in the mixture be interchanged, its price is *Rs.*36. Supposing the difference between the prices of 1 oz. of each metal to be *Re.* 1, find the price per cwt. of each of the metals.

131. If a publisher, in selling a book for cash, rates it at 25 per cent. below publishing price, and then charges for 13 copies as 12, how long credit could he allow, so that, on the principle of true discount at 4 per cent. per annum, the sum to be received for a book should be just 29 per cent. below publishing price?

132. A boatman rowing against the tide passes a body floating with the tide, and in 9 minutes afterwards is a mile distant from it; in 35 minutes more he rows $2\frac{1}{2}$ miles, and then returns. At what rate per hour does he return, supposing the tide to flow uniformly in one direction?

133. A dealer buys 18 cwt. 3 qrs. at 1*s.* 3*d.* per lb, which, to obtain a fair profit, he should retail at $8\frac{1}{3}$ per cent. above cost price. But, while he professes to sell at the rate of 3 lbs. for 3*s.* 10*d.*, he serves his customers, to his own advantage, with a false balance, in which 10 lbs. weighs $10\frac{1}{2}$ lbs., and at the same time he uses a false lb. of 6860 grains. How much does he make beyond the fair profit?

134. Divide 33 cwt. 2 qrs. 22 lbs. into three such parts that 6 times the first, 9 times the second, and 10 times the third may be equal amounts.

135. The interval between the firing of two guns, at a railway station was 6 minutes, and a passenger in a train, approaching the station at a uniform rate, heard the second report 5 min. 51 sec. after hearing the first. Now, suppose that the sound of the train's approach to have become audible at the station when the train was 2 miles off, how soon after that did the train pass the station,—sound travelling 1125 feet per second?

136. A boatman rows 5 miles with the tide in the time he would take to row 3 miles against it; but if the hourly velocity of the current were $\frac{1}{2}$ mile more, he would move twice as rapidly with the tide as against it. What is his power of rowing in still water?

137. The only three creditors of an insolvent, whose assets amounting to £200 can only pay 5*d.* in the £, agree among themselves to take dividends in the proportion of the number of £. *s.* and *d.*, respectively contained in the amounts due to them. The dividends thus taken are in the proportion of 12 : 7 : 6. What are the amounts of their debts?

138. The sum of £2100 is due in 4 years, but it is paid by instalments as follows:—£275 at the end of 2 years, £460 at the end of the 3rd year, £500 at the end of the 4th year, and £600 at the end of the 5th year. What amount should be paid at the end of the 6th year, in order to clear off the balance, simple interest being reckoned at 5 per cent. per annum.

139. A metal is composed of 11 parts gold and 1 copper, another of 5 parts gold and 8 silver, and a third of 12 parts silver and 1 copper. These are mixed together in the ratios of 8 : 9 : 10, and

the compound weighs 1 cwt. 5 lbs.; how much gold, silver and copper respectively are contained in the mixture?

140. A barrack has to be built to hold 50 beds which are to be arranged in single file, with their lengths parallel to the shorter side of the room, each bed being 6 ft. 2 in. long and 3 ft. 1 in. broad. A space of 2 ft. 3 in. is to be left between each bed, and a passage 3 ft. 6 in. wide is left on all sides between the walls and the bed. Given that each individual requires 1000 cub. ft. of air per night, and that each bed with its occupant takes up 50 cub. ft. of space, required the dimensions of the room.

141. Two carriages are in motion in parallel lines and in the same direction along a straight road, the distance between their fore-wheels is 600 yds. The circumferences of the fore-wheels are to each other as 7 : 6, that of the first being 3 yds. 1 ft.; the fore-wheels of the first and second carriages turn round respectively 14 and 15 times every minute; find (i) when they were alongside of each other; (ii) when they were 100 yds. apart.

142. The area of a rectangular space is an acre, and its length and breadth are in the ratio of 5 : 2; around this space on the inside is a path whose breadth at any point is $\frac{1}{20}$ of the distance to the opposite path; find how many bricks will pave the path, allowing 45 bricks for a square yard.

143. A wine merchant buys 3 kinds of wine and mixes them in this proportion: 1 pipe of the first kind, the price of which is Rs.800 a pipe, 3 pipes of the second, the price of which is Rs.900 a pipe, 2 of the third kind. He keeps this mixture for 12 months and then sells it at Rs.1045 a pipe, clearing 10 per cent., after allowing 4 per cent. for interest of capital. What was the original price of the third kind of wine?

144. The Governors of Queen Anne's Bounty advance £845 on mortgage of a living on the following conditions: the principal to be repaid in 30 years by equal annual instalments and interest at the rate of $3\frac{1}{2}$ per cent. to be charged on the part unpaid. If the sum due in any particular year be £43. 18*s*. $9\frac{3}{5}$*d*., find how many previous annual payments have been made.

145. If 9 oxen are kept for the same money as 7 horses (for any given time), and a team of oxen are one-fifth as long again in ploughing 97 acres as the same number of horses are in ploughing 90 acres and a field costs as much whether ploughed by oxen or by horses *viz*., £7. 5*s*. 6*d*.; the same men being required in both cases, and paid by the time, what is due to them?

146. A shed rests upon two walls, one 10 ft., the other 8·7 ft. high, and covers a space of ground 8·4 ft. in breadth by 15 ft. in length. Find the price of roofing it at *Rs*.9 a square yard.

147. A bill of £61 due 4 months hence would with interest

amount to £63. 0*s.* 8*d.* at the end of the year, find its present value at the same rate of interest. If at a shop where a year's credit is allowed, I by paying ready money obtain 13 articles to the dozen, at what rate per cent. is the discount allowed?

148. *A* with a capital of *Rs.*60000 began business on the 1st day of January, and wishing to extend his trade, he took in *B* as partner, with a capital of *Rs.*50000 on the 15th March following; and on the 27th May they admitted *C* as a third partner, who brought *Rs.*70000 into the concern. On taking stock at the end of the year, they find the profits of the firm to be *Rs.*24850 : how must this sum be divided amongst the partners?

149 From a place *A*, a messenger goes to a place *B*, distant 21 miles from *A*, and immediately returns, going at the rate of 4 miles an hour; and simultaneously with the messenger's departure from *A*, another messenger starts from *B* at the rate of 3 miles an hour, goes to *A*, and immediately returns : find the distance between the two points at which they cross each other.

150. A company is formed in which the liability of each partner is limited to the amount of his shares. There are 500 shares of £10 each; after 3 calls have been made of £2 on a share, it is found that the concern is a failure, and its affairs are wound up. At this period its assets amount to £10217. 0*s.* 0$\frac{3}{4}$*d.* and its liabilities to £15763. 17*s.* 6*d.* How much will the company be able to pay in the pound after all the remaining calls are paid up?

151. If 36 oxen in four weeks eat up the grass on a field of 12 acres and what grows upon it during the time; and 21 oxen eat up the same in 9 weeks; how many oxen will it maintain for 18 weeks, supposing the grass to grow uniformly during the time?

152. The premium of gold at Paris is 7$\frac{1}{2}$ per mile, which, at the English mint price of £3. 17*s.* 10$\frac{1}{2}$*d* per oz for standard gold gives exchange 25·35$\frac{3}{4}$; and the exchange at Paris on London at short, being 25·33$\frac{1}{2}$, it follows that gold is about 0·09 per cent. dearer in Paris than in London.

153. Incomes below £150 a year being subject to 5*d.* in the pound income-tax, and incomes above £150 to 7*d.* in the pound; find what income above £150 a man must have, that he may be just 7$\frac{3}{4}$*d.* a year poorer than a man who has £149. 10*s.* a year.

154. *A* has £90000 stock in the 3 per cent. South Sea Annuities, and is offered by Government the choice of being paid off at par at the end of the year, or of receiving £110 of a new 2$\frac{1}{2}$ per cent. stock for each £100. He chooses the former alternative; and, on being paid off, is able to invest his money in the 3 per cent. consols at 92. Find the amount of his stock in consols and the excess of his income above what it would have been if he had agreed to the proposed conversion.

155. If 28 men can excavate 750 cub. yds. in 4 days working $6\frac{3}{4}$ hrs. a day ; what uniform length of day will 24 men require, to excavate 615 cub. yds. in $3\frac{1}{2}$ days, supposing that any 5 of the latter party can do as much in 4 hours as any 6 of the former can do in $3\frac{1}{3}$ hours, and that 2 men will be withdrawn from the latter party after $2\frac{1}{2}$ days' work ?

156. Eight billiard balls revolve with equal velocities in eight concentric circular grooves. They start from a position in which they are all in the same radius of the outermost circle. The innermost ball occupies 5 sec. in traversing its groove once. After what time will they all be again in the position in which they started, the radius of the circular grooves being proportional to the nos. 1, 3, 5, 7, 9, 11, 13, 15. When first will the 1st, 3rd, 5th, 7th, be in the same position in which they started ?

157. A vessel is full and contains 56 gallons of water. Two pipes are opened, through one of which the water runs off continuously at the rate of 100 gallons per hour, whilst the vessel is irregularly, supplied by the other which is first open for 2 min., when the water flows in at the rate of 36 gallons per hour, and then closed for 3 min. and so on. Find when the vessel will be emptied to parts of a second.

158. A packet sails from Dover to Calais, and starts at the rate of $1\cdot5$ miles in $1\cdot\dot{6}$ of an hour ; but, after sailing $(\cdot\dot{1}\dot{8} + \cdot\dot{2}\dot{7})$ hours at this rate, the wind drops, and, for the next $(\cdot35 + \cdot2)$ hours she makes only $\cdot\dot{5}$ of the way she did ; however, the wind again freshening, her rate of sailing per hour is in consequence increased by $\cdot\dot{5}7142\dot{8}$ of the difference between her former ones ; supposing her rate thence-forward to continue uniform, and the distance between the two ports to be $22\cdot03\dot{5}7142\dot{8}$ miles, find what time was occupied in the passage.

159. *A* and *B* start at the same time walking to and fro a course of 1 mile ; *A* walks a mile in 13 minutes, *B* in 18 minutes ; find the series of points where *A* crosses *B* and the series of points where *A* overtakes *B*, distinguishing those where *A* is walking towards the starting point from those where he is walking from it.

160. If a man can beat a boy $\frac{3}{8}$ of a mile in a race of 4 miles, how much start must the boy have in order to reach the tenth milestone at the same time as the man ? And supposing the man's rate is 5 miles an hour, how long would it take the boy to go $8\frac{1}{2}$ miles ?

161. A baker's outlay for flour is 70 per cent. of his gross receipts, and other trade expenses are 20 per cent. The price of flour falls 50 per cent. and the other trade expenses are thereby reduced 25 per cent. What reduction should he make in the price of a five-anna loaf allowing him still to realise the same amount of profit from it ?

162. If I am allowed $1\frac{1}{4}$ per cent. discount on an amount

charged to me for goods and give my acceptance at 5 months for the net sum ; and if by selling the goods forthwith for a bill of £162. 12*s.* 2*d.* payable in 7 months, my present gain is $\cdot 11\frac{1}{9}$ per cent. ; what is the amount originally charged to me, interest being reckoned at 5 per cent. per annum ?

163. Three persons invest sums of money in a business in the ratio of 3 : 2 : 1, but agree to divide the profits in the ratio of 4 : 2 : 1 ; the profits of the concern are 10 per cent. on the whole money invested ; what interest will each partner make of his money ?

164. Two hands (as in a clock-face) revolve round a common centre 20 and 30 times respectively in 24 hours, starting together from a given position in the same direction. How often during 24 hours will they be in a similar condition after complete revolutions ? Also after what times will they be $\frac{3}{4}$ths of a revolution apart ?

165. *A* and *B* have invested *Rs.*1500 and *Rs.*2450 respectively in the same business for a year ; it is expected that the rate of profit will be doubled the following year, and *A* calculates that if he doubles his capital the profits of the business for the two years will amouut to *Rs.*1039. 8*a.* ; what does he expect as his share of this ?

166. Two circular gold plates, each an inch thick, the diameters of which are 6 in. and 8 in. respectively, and melted and formed into a single circular plate, also an inch thick ; find its diameter. Having given that the area of a circle equals $\frac{22}{7} \times (\text{radius})^2$.

167. A boy runs away from home towards Dover at 8 A. M., taking the longer of two roads (one of which is 25, the other 28 miles) and walking $3\frac{1}{2}$ miles per hour. At 10 A. M , he is missed, and immediately his father pursues on horseback, riding 8 miles per hour, but taking the wrong road. On reaching Dover he spends half an hour in baiting his horse and making enquiries, and then starts back by the other road ; find where he meets the truant. He takes him up behind, and the horse so laden goes $6\frac{1}{18}$ miles per hour ; at what o'clock will they reach home ?

168. *A*, *B* and *C* row from Cambridge to Ely, and it is observed that the rate of the current is $\cdot\dot{6}$ of the difference between *A*'s and *C*s rates of rowing ; now supposing *C*'s rate of rowing per hour to be $\cdot\dot{9}\dot{0}$ of *B*'s, $\cdot 08\dot{3}$ of *A*'s to be $\cdot\dot{0}38461\dot{5}$ of 13 miles and ·0625 of *B*'s to be $\cdot 057291\dot{6}$ of *A*'s, supposing also that *B* left Cambridge $\cdot\dot{6}$ hour after *A*, and *C* ·6 after *B*, find how far *B* and *C* were asunder, when *A* was 7 miles from Ely, ·00625 of the distance between Cambridge and Ely being ·125 of a mile.

169. *A* and *B* are bound to perform certain works in 3 and 4 hours respectively. Compare their quantities of work and rates of working, (i) if upon exchanging works they could perform them in

2 and 6 hours respectively, (ii) if supposing them to do the works in common they could finish them in 4 hours.

170. Part of the journey from *A* to *B* is performed by coach which charges carriage in proportion to bulk, and the rest by rail which charges according to weight; a parcel of 15 lbs. and whose length, breadth and depth are 9 in., 7 in. and 5 in. respectively cost 10*s*. $1\frac{1}{2}$*d*., while another of 30 lbs. and whose dimensions are 10 in., 8 in., and 6 in., costs 19*s*.; find the carriage of a parcel of 47 lbs. equal in bulk to the sum of the other two.

171. A boat whose speed was $9\frac{3}{4}$ miles an hour sailed from *A* to *B* a distance of 65 miles; and a second boat, which left *A* $2\frac{1}{2}$ hours after the first, arrived at *B* 5 min. before the first. Compare the rates of sailing.

172. A person sets out to walk from *A* to *B* at the rate of 4 miles an hour. After he has walked $1\frac{2}{3}$ miles he is overtaken by the coach which started a quarter of an hour after him. At a distance of 13 miles from *A* he meets the coach returning from *B* where it has stayed for half an hour. Find the distance from *A* to *B*.

173. A landlord has an estate which after deducting 10 per cent. for repairs and rates brings him a net income of £2250. If he were to sell the estate at 21 years' purchase on the gross income and invest the money obtained in the 3 per cents. at 92, what would be his income after deducting 10*d*. in the £ for income-tax?

174. *A* purchases equal amounts of stock in the 3 and 5 per cents.. The half-yearly interest on the latter is payable 3 months after that on the former. If *A* upon receiving the interest on the first stock, lend it to *B* for 3 months, on condition of being paid back a sum equal to the interest on the second stock, at what rate per cent. per annum, does *A* lend the money?

175. The volume of a certain figure increases in the same proportion as either one of its three axes increases, the other two remaining the same. Given that the weight of such a figure of tin whose axes are 8 in., 6 in., 3 in., is 318·4797 oz.; find the weight of another such figure of lead whose axes are $1\frac{1}{3}$ ft., 9 in., 2 in, supposing that tin and lead are respectively 7·299 and 11·352 times as heavy as water.

176. A man invests *Rs.*5120 in a stock (*A*), he sells out when the stock has risen 50 per cent. and invests the proceeds in a stock (*B*). He sells out of (*B*) when its value has fallen 25 per cent., and invests successively in stocks (*C*), (*D*), (*E*), (*F*). He gains by (*C*), (*E*) as he gained by (*A*), and loses by (*D*), (*F*) as he lost by (*B*). Find his ultimate gain.

177. A homœopathist takes a fluid ounce of medicine, and mixes it with 9 times its quantity of water; he then takes a fluid ounce of the mixture and mixes it with 9 ounces of pure water. How many times must he repeat this operation that the strength of the

medicine may be the same as if he had put the fluid ounce originally in the ocean, supposing that the ocean contains 10^{18} fluid ounces?

178. A cistern, holding 1200 gallons, is filled by 3 pipes *A*, *B* and *C* in 24 minutes. The pipe *A* requires 30 min. more than *C* to fill the cistern, and 10 gals. less run through *C* per minute than through *A* and *B* together. What time would each pipe take to fill the cistern by itself?

179. The shorter of two roads between *A* and *B* is 15 miles and goes over a hill the summit of which is 3 miles from *B*, and the longer is 36 miles on level ground. A thief runs away from *A* by the shorter road at the rate of 3 miles per hour up-hill and 4 miles per hour down-hill. Three-fourths of an hour afterwards, a Constable, whose speed up-hill is 4 miles per hour and down-hill 5 miles, starts in pursuit of the thief but takes the wrong road. The thief, an hour after he turns into the longer road at *B*, sees the Constable $1\frac{1}{5}$ miles ahead of him, and at once turns back and retraces his steps increasing his speed by $1\frac{1}{2}$ miles per hour on the level ground, his speed up and down the hill being the same as before. If the speed of the Constable be to the original speed of the thief as 4 : 3 on the level ground, find where the Constable will overtake the thief.

180. A contractor employs a fixed number of men to complete a work. He may employ either of two kinds of workmen : the first at 26*s*. 6*d*. per week each, the second at 18*s*. 6*d*. per week each; the work of one of the former is to that of one of the latter as 5 to 4. If he finishes it as quickly as possible, he spends £270 more than he would have done if he had finished it as cheaply as possible, but takes 4 weeks less time. What would it have cost if he had employed equal numbers of the two kinds of workmen?

181. Ash saplings after 5 years' growth are worth 1*s*. 3*d*., and increase in value 1*s*. 3*d*. each year afterwards. For their growth each requires twice as many sq. yds. as the number of years it is intended to grow before cutting. A plantation is arranged so that each year the same number may be ready for cutting. Find the greatest annual income which can be obtained per acre allowing 20 per cent. for expenses.

182. On a Railway line, telegraph posts are fixed at a distance of 80 yards from one another. A watchman from the 100th telegraph post, expecting a train, observes it at the 122nd post, and at the same time sees a lamb at the next post to where he is, proceeding towards the train at the rate of 3 miles an hour. He at once runs and overtakes the lamb at the 102nd post, and without loss of time driving the lamb off the line, returns to his place, walking at half his former speed. Just as he reaches his place, the train comes up to the 102nd post. What is the speed of the train?

183. The 2 P.M. goods train from Madras, travelling at the rate of 18 miles an hour, meets with an accident in the way and

reaches Arconum (48 miles from Madras) $\frac{5}{8}$ of an hour late. The passenger train from Erode, which ought to have left Arconum at 5 P.M., is consequently delayed, and leaves Arconum for Madras 6 minutes after the arrival of the 2 P.M. train and meets the 6 P.M. mail train from Madras midway between Madras and Arconum. If the rates of the passenger and the mail train be as 2 : 3 find where the mail train will overtake the 2 P.M. goods train, leaving stoppages out of consideration.

184. What is the least number of years for which interest must be reckoned on £145. 7*s.* 6*d.* at 4 per cent., so that the interest may be an exact number of £s.?

185. On a certain day mackerel were being sold at a certain price per dozen, on the next day as many fish could be bought for 1*s.* as dozens for £1 the day before. The whole price of 20 fish, 10 bought one day, and 10 the other, was 2*s.* 2*d.* Find the price of a fish on each day.

186. *A* is to give *B* 3 marbles every time *B* wins the game, and to receive 4 every time *B* loses. They begin with 15, 18 respectively, and leave off with 5. Shew that this may have happened after 5, 12, 19, &c. games.

187. *A* and *C* can do a piece of work in 20 days, *B* and *D* can do the same in 25 days, *A* and *D* in 30 days. *A* works 13 days, *B* 10 days, *C* 8 days, *D* 15 days and *E* 2 days: and thus complete the work. If *E* is twice as good a workman as *D*, find the time in which the 5 men together can do another eight times as great as the former.

188. *A* travels from *P* to *Q*, and *B* from *Q* to *P*. The point where they meet is nearer *Q* than *P* by 160 yds. If *A*, after reaching *Q*, turns back and goes towards *P*, and *B*, after reaching *P*, turns back and goes towards *Q*, the point where they next meet would be 640 yds. from *P*. Find the distance from *P* to *Q*, and compare the rates of motion of *A* and *B*.

189. *A* works 7 days and takes rest for 3 days; again works 7 days, takes rest for 3 days, and so on. If he earns 3*s.* for every working day, (i) what will he earn in 47 days? (ii) when will he earn £3. 3*s.* 3*d.*?

190. *A* and *B* embark in a trade for 5 years; *B* is to have $\frac{7}{16}$ of the net annual profits for the first half of the time, and half of them for the remainder. After $3\frac{1}{2}$ years, the annual profits, by a lowering of the tariff, were increased in the proportion of 6 : 5, and at the same time, became liable to a reduction of 7*d.* in the £ by the laying on of the income-tax. At the termination of the partnership, *B*'s share of the total net profits amounted to *Rs.*9870; find the annual profits before the duties were reduced.

191. I have two godowns which I want to fill with cases of tea all of equal size. Exclusive of passage room, the larger godown

is 28 ft. 9 in. long, 17 ft. wide, and 9 ft. 9 in. high. The other is 21 ft. 1 in. long, 9 ft. 11 in. wide, and 8 ft. 8 in. high. Now I want to have my cases made of the largest possible dimensions each way, so as not to lose unnecessary space. What must be their length, breadth and depth? And how many cases must I make?

192. A certain piece of work can be done by *A* and *B* in 25 days, *C* and *E* can do the same in 35 days, and *A* and *E* 7 times as much in 350 days. They all work at it, and with the assistance of a fifth person *D*, complete the work,—*A* working for 12 days, *B* 13 days, *C* 15 days, *D* 5 days and *E* 14 days. If the energies of *B* and *D* be in the ratio of 1 to 4, find in what time, *A*, *B*, *C* and *D* together can do another work, 9 times as great as the former.

193. Divide *Rs*.420 among 5 persons, so that *A* may have $\frac{1}{6}$th of what all the others have together; *B* $\frac{1}{5}$th of what the other four have together; *C* $\frac{1}{4}$th of what the other four have together; *D* $\frac{1}{3}$rd of what the other four have together; and *E* the rest.

194. *A* and *B* set out at the same time from the foot of a hill to go to the summit. Four hours and 45 min. after starting, *A* meets *C* who is going down the hill at a point *R*; but had *A* waited $\frac{1}{2}$ an hour at *P* till *B* arrived there and then proceeded as before, he would have met *C* 3 hours after leaving *B*, and reached the summit 2 hrs. 15 min. after meeting *C*, and 2 hrs. 25 min. before *B*. If *B* is travelling 6 miles an hour, find the rates of *A* and *C*.

195. The men employed by a certain farmer work 12 hrs., the women 9 hrs. and the boys 8 hrs. each day: for labouring the same number of hours, each man receives a half more than each woman, and each woman a third more than each boy: the entire sum paid to all the women each day is double of the sum paid to all the boys; and for every 5*s*. earned by all the women each day, 12*s*. are earned by all the men. Find the number of each class employed, the entire number being 59.

196. A tank, 50 yds. square at the top, has flights of uniform steps on all sides leading to the bed which is 40 yds. square. If the breadth and the depth of the steps be each one yard, find the volume of water in the tank when two steps are completely visible on all sides. If the water be let out by means of a sluice the area of whose section is 10 sq. ft. and the velocity of water 2 miles an hour, find in what time the tank will be emptied.

197. Two sums of money which are in the ratio of 7 : 5 were borrowed on the same day at the respective rates of 4 and 5 per cent. per annum; if the latter were paid 6 months before the former, the difference in the amounts of both sums would be *Rs* 455. If the interest on the former sum exceeded that on the latter by *Rs*.55, how long did the latter loan continue?

198. A person whose rates of walking up-hill, on level ground and down-hill are in the ratios 2 : 3 : 4, walks a certain distance,

the parts of which up hill, level, and down-hill are as 3 : 4 : 5. If he is able to walk the whole distance in 49 hours, in what time would he have walked it, if the road had been (i) level throughout ; (ii) half up-hill and half down-hill.

199. In the making of pins 3 men who file the points can keep at work exactly 5 men who put on the heads, no man in either of these two sets can do the work of the other set. Suppose one of the first set to stay away for a week, by what fraction are the earnings of each of the remaining men diminished, supposing them to work by the piece, and divide their earnings equally ?

200. In a Railway train the total lengths of all the first-class carriages, all the second-class carriages, and all the third-class carriages, are the same ; the length of a first-class carriage being 12 ft., of a second-class carriage 18 ft., and of a third-class carriage 24 ft. Each first-class carriage contains the same number of first-class passengers, each second-class carriage the same number of second-class passengers, and each third-class carriage the same number of third-class passengers ; the number of passengers in a first, second, and third class carriage being different. The total fares received from the first-class, second-class, and third-class passengers are the same. If *Rs.*5, *Rs.*3, and *Rs.*2 be the fares of a 1st, 2nd, and 3rd class passenger respectively, find the least amount of the total fares.

201. A vessel is sent to the Arctic Ocean to catch whales, and the captain who is entitled to $\frac{1}{18}$ of the entire produce of oil receives £735 at the end of a 4 years' voyage. Supposing the oil to be worth 60 guineas per ton (each whale on an average yielding 6 tons), and the crew to have been $\frac{3}{8}$ of the entire period on the look out, what was the average number of days between the capture of each whale ?

202. If 56 Indian workmen, each earning 6*a.* per day, can do the same piece of work in 25 days that takes 20 English workmen, each earning 3*s.* 6*d* per day, 15 days to complete ; taking the value of the shilling at 12*a.*, determine which class of workmen it is more profitable to employ. If a piece of work done by Indian workmen cost *Rs.*3000, what would be the cost of the same work done by English workmen ?

203. There is a serpent's hole at the bottom of a post and on the top of the post is perched a peacock. The post is 9 cubits high. The peacock observing the serpent gliding to its hole when at a distance from it equal to 3 times the height of the post, flies down obliquely and seizes it when both have gone over an equal distance. At what distance from the serpent's hole did they meet ?

204. An ordinary train leaves one terminus at 6 A.M. and reaches the other terminus at 8 P.M , losing as much time in stoppages as it would take to travel 40 miles without stopping. An express train follows the ordinary 2 hours later and reaches the second terminus 3 hours earlier, losing only half as much time in stoppages as the

ordinary train. If the distance between the termini be 240 miles, find the rates of the trains.

205. The time which an express train takes to travel 480 miles is to the time taken by an ordinary train as 11 : 20. The ordinary train takes as much time in stoppages as it would take to travel 70 miles without stopping ; while the express train loses only $\frac{11}{28}$ of the time lost by the ordinary train in stoppages and it also travels 13 miles an hour quicker. Find the rate of each train.

Additional Examples.

1. Find the least number which when divided by the L. C. M of 3, 5, 7, 9 and 11 will leave a remainder 1. [*Ans.* 3466.

2. Find the least number which will be exactly divisible by the L. C. M. of 7, 9, 11 and 13. [*Ans.* 9009.

3. A number when divided by 5, 7, 9 and 11 leaves respectively the remainders 3, 5, 7 and 9 ; find the remainder when it is divided by $5 \times 7 \times 9 \times 11$. [*Ans.* 3463.

4. A number when divided by 4, 6, 8, 10 and 12 leaves the remainders 2, 4, 6, 8 and 10 ; find the remainder when it is divided by their L. C. M. [*Ans.* 118.

5. A number when divided by 8, 10, 12, 14 and 16 leaves in every case the remainder 1 ; find the remainder when it is divided by their L. C. M. [*Ans.* 1.

6. A number when divided by 16, 20, 24 and 28 leaves the remainders 13, 17, 21 and 25 ; find the remainder when it is divided by their L. C. M. [*Ans.* 1677.

7. Find the greatest number which will divide 14490 and 31530 so as to leave the remainder 6 in each case. [*Ans.* 852.

8. Find the greatest number which will divide 11296 and 13528 so as to leave remainders 11 and 23 respectively. [*Ans.* 185.

9. In finding the G.C.M. of two numbers the last remainder is 35, and the quotients in order are 1, 2, 1, 3. Find the numbers. [*Ans.* 385, 525.

10. Find the greatest number which is such that when 142408, 153599 and 166402 are divided by it, the remainders are all the same. [*Ans.* 31.

11. In a long division sum the quotient consists of two figures ; if the dividend is 40051, and the two remainders are 173 and 294, find the divisor and quotient. [*Ans.* 479, 83.

EXAMINATION PAPERS.

CALCUTTA ENTRANCE PAPERS.

1885.

1. Of what number is $2\frac{1}{6}$ the $\frac{5}{6}$th part? By what fraction must $\frac{1\frac{1}{4}}{1\frac{1}{12}}$ of $\frac{2}{3}+\frac{2\frac{1}{2}-1\frac{5}{6}}{\frac{1}{4}+1\frac{5}{6}}-\frac{8\frac{1}{5}}{7\frac{4}{5}}$ be divided in order to give a quotient $=\frac{2}{3}$?

2. Simplify $\frac{\cdot 12 \text{ of } (\cdot 0104-\cdot 002)+\cdot 36\times\cdot 002}{\cdot 12\times\cdot 12}$; and express your result as a fraction of ·6.

Reduce $\frac{5}{8}$ of 16*s.* $4\frac{1}{2}$*d.* to the decimal of £1. 9*s.* $10\frac{1}{2}$*d.*

3. What circulating decimal multiplied by $\frac{2\frac{8}{9}}{4\frac{1}{3}}$ will give 2 for a product?

If $\cdot\dot{4}2857\dot{1}$ of a barrel of beer be worth ·72 of £2. 10*s.*, what is the value of ·625 of the remainder?

4. Find the price of 10 lbs. 11 oz. 16 dwts. 16 grs. of gold at £3. 17*s.* $10\frac{1}{2}$*d.* per oz. Extract the square roots of $9\frac{1}{4}$ and $\frac{1}{12\cdot 5}$ to 4 places.

5. If 27 men can perform a piece of work in 15 days, how many men must be added to the number that the work may be finished in three-fifths of the time?

I buy a horse for £40 and sell it for £45 at a credit of 8 months. What do I gain per cent., reckoning money worth 6 per cent. per annum?

6. Which is the better investment, bank stock paying 10 per cent. at 319 or 3 per cent. consols at 96?

What will be the cost of £1,500 3 per cent. consols at $89\frac{5}{8}$, brokerage being $\frac{1}{8}$ per cent.? What rate of interest will such investment obtain?

1886.

1. Divide $\frac{1\frac{2}{3}+1\frac{3}{4}}{1\frac{4}{5}+1\frac{5}{6}}+\frac{1\frac{6}{7}+1\frac{7}{8}}{1\frac{8}{9}+1\frac{9}{10}}$ by $\frac{\frac{1}{7}+\frac{1}{8}}{\frac{1}{11}+\frac{1}{18}}+\frac{\frac{1}{17}+\frac{1}{18}}{\frac{1}{19}+\frac{1}{20}}$.

2. Simplify $\frac{3\cdot 125}{2\cdot 16}$ of $\frac{\cdot 2\dot{4}}{\cdot 125}+\frac{2\cdot\dot{2}}{1\cdot 5}$ of $\frac{187\cdot 5}{3\cdot 42}$.

3. Reduce £1. 11*s.* 10½*d.* to the fraction of £7. 18*s.* 6½*d.*

What fraction of £10 must be added to £16. 10*s.* 3*d.* to make it £20?

4. What decimal of 9 mds. 20 sr. is $\frac{3}{8}$ of 7 mds. 5 sr.?

Reduce 5½ sq. yds. to the decimal of an acre.

5. Find the value, by Practice, of 2 tons 15 cwt. 35 lbs. at £13. 6*s.* 8*d.* per ton.

6. What sum of money at 4 per cent. simple interest will secure the same income as *Rs* 25475 at 4½ per cent.?

7. If a rupee is equivalent to 1*s.* 6¾*d.*, what is the price of a sovereign in rupees? If, after buying 250 sovereigns at this price, I sell them again when the rupee is equivalent to 1*s.* 6*d.*, how much shall I gain or lose by the transaction?

1887.

1. Simplify (*a*) $(4\frac{5}{8}-1\frac{1}{8})\times(3\frac{1}{2}-\frac{5}{8})\div(13\frac{1}{3}+7\frac{1}{2})$ of $\frac{3\frac{1}{3}}{1\frac{1}{9}}$.

$$(b)\ \frac{1\cdot8\dot{3}+2\cdot041\dot{6}+\cdot\dot{3}-3\frac{1}{8}}{1\cdot0025+\cdot0627-1\frac{1}{16}}.$$

2. Express $\frac{3}{8}$ of 7*s.* 6*d.* + 1·25 of 5*s.* − $\cdot5\dot{4}\dot{5}$ of 9*s.* 2*d.* as a decimal fraction of £10.

3. (*a*) Find by Practice the value of 5 tons 5 cwt. 2 qrs. 17½ lbs. at £3. 6*s.* 8*d.* per ton.

(*b*) Find the income on which the income-tax at 5*p.* per rupee is *Rs.*52. 1*a.* 4*p.*

4. If 50 men can do a piece of work in 12 days, working 8 hours a day, how many hours a day would 60 men have to work in order to do another piece of work twice as great in 16 days?

5. If *Rs.*450 amount to *Rs.*540 in 4 years at simple interest, what sum will amount to *Rs.*637. 8*a.* in 5 years at the same rate?

6. Extract the square root of 177·1561, and of ·2 to 3 decimal places.

1888.

1. Simplify $\frac{\frac{3}{7}(\frac{5}{16}\text{ of }3\frac{1}{5}-\frac{1}{3}\text{ of }2\frac{1}{8})}{\frac{1}{23}\times1\frac{6}{17}\times1\frac{1}{15}-\frac{7}{36}+2\frac{1}{5}}\div\frac{\frac{1}{2}+\frac{5}{14}-\frac{7}{18}}{\frac{1}{3}\times\frac{1}{4}-\frac{1}{17}\times1\frac{45}{18}}$.

2. Divide 16·016 by ·00143, and extract the square root of 1440·9616.

3. Add together $55\cdot500\dot{2}$, $3\cdot\dot{1}\dot{7}$, $4\cdot\dot{5}0\dot{6}$ and $75\cdot\dot{2}7\dot{1}$, and find the value of ·7365 of £3. 6*s.* 8*d.* + ·504 of £15. 12*s.* 6*d.* + $2\cdot10208\dot{3}$ of £5.

4. Find by "Practice" the value of 2 tons 7 cwt. 3 qrs. 11 lbs. at £21. 12*s.* 6*d.* per cwt.

5. A man can walk 600 miles in 35 days, resting 9 hours each day; how long will he take to walk 375 miles if he rests 10 hours each day, and walks $1\frac{1}{3}$ times as fast as before.

6. If the interest on money be one pie per rupee per month, what is the rate per cent. per annum?

A man holds $15\frac{3}{4}$ shares of a bank, and receives £19. 1*s*. 3*d*. per quarter. If the interest he receives be 5 per cent. per annum, find the value of a share.

1889.

1. Multiply ·0069347 by 7439·6.

2. Divide 2100·006983 by 243·5846 correct to 5 places of decimals.

3. Find in any way the value of 1,347 cwt. 3 qrs. 21 lbs. at £3. 17*s*. $10\frac{1}{2}$*d*. per cwt.

4. Extract the square root of $1+(\cdot 0634)^2$ to 6 places of decimals.

5. Find in English money the value of *Rs*.100,000 at 1*s*. $4\frac{3}{32}$*d*. per rupee.

1890.

1. Simplify $2\frac{3}{7}$ of $\dfrac{13\frac{1}{2}-9\frac{2}{3}}{15\frac{1}{5}-11\frac{7}{30}} \div 3\frac{2}{7} + \dfrac{1\frac{7}{9}}{9\frac{2}{9}-8\frac{4}{9}}$, and find by Practice the value of 3,049 articles at *Rs*.7. 13*a*. 7*p*. each.

2. Divide 27·03 by ·0037, and reduce $\cdot 7\dot{5} - \cdot\dot{1}0\dot{2} - \cdot 2\dot{7}$ to a vulgar fraction.

3. Find the cost of putting a fence round a square field, whose area is 13·225 acres at *Re*.1. 12*a*. per yard.

4. A piece of work can be done in 72 days by 17 men working together. If after 9 days of work, these are joined by 4 others, in how many days will the work be finished?

5. Find the price of $4\frac{1}{2}$ per cent. Government Promissory Notes, when an investment of *Rs*.59,422. 8*a*. produces a monthly income of *Rs*.213. 12*a*.

1891.

1. Simplify the following expressions:—

(*a*) $\dfrac{\frac{3}{5}-\frac{2}{7}}{\frac{5}{6}-\frac{3}{5}} + \dfrac{\frac{5}{7}-\frac{13}{49}}{\frac{1}{5}-\frac{1}{11}}$.   (*b*) $\dfrac{1}{4-\dfrac{1}{2-\dfrac{1}{1-\frac{5}{13}}}}$.

2. Find the value of $2\cdot 4607 \times \cdot 06 - 3\cdot 75 \times \cdot 012 + 2\cdot 163 \div 1\cdot 03$.

3. Find the value of 15 cwt. 3 qrs. 9 lbs. at *Rs*.25. 12*a*. 7*p*. per cwt.

4. If a man walking at the rate of $3\frac{1}{2}$ miles an hour, walks to a place in 4 hours 30 minutes, how long will it take a man walking at the rate of $3\frac{1}{4}$ miles an hour, to walk there and back?

5. A man invests a certain sum in $4\frac{1}{2}$ per cent. Government Paper at 104. The price falling to 101, he sells out and loses *Rs* 600 by the transaction, exclusive of brokerage. Find the sum invested.

6. *A* gives *B* 10 yards' start and *C* 15 yards' start in a race of 100 yards; how much should *B* give *C* in 150 yards?

1892.

1. Simplify $\dfrac{3\frac{6}{9}-1\frac{7}{8} \text{ of } \frac{24}{25}}{11\frac{2}{3} \text{ of } \frac{11}{14} \text{ of } \frac{2}{5}} - \dfrac{4\frac{1}{3}-7\frac{5}{9}+3\frac{2}{9}}{\frac{5}{9} \text{ of } 12}$.

2. Find, to the nearest integer, the value of $\dfrac{39\cdot37 \times 760 \times 13\cdot596}{1\cdot293 \times 12}$.

3. Find the square roots of ·097344, of ·009604, and of ·996004.

4. Find the interest on 10 lakhs of rupees for 10 days at $4\frac{1}{2}$ per cent.

5. £3,000, which I held in the 4 per cents., was sold for me when they were at $82\frac{5}{8}$ by a broker whose commission is $\frac{1}{4}$ per cent.; and the proceeds were re-invested by him in the four and a half per cent. at $98\frac{3}{4}$. What amount of the latter stock did he purchase?

1893.

1. Simplify :—

(1) $1+\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\frac{4}{5}+\frac{5}{6}+\frac{6}{7}$.

(2) $\dfrac{8\frac{1}{7}-1\frac{1}{6}}{\frac{2}{3}+1\frac{1}{2}} - \dfrac{1}{5\frac{3}{4}-1\frac{1}{5}}$.

2. Divide 1·84626 by 23·4. Express $\cdot4\dot{5}\dot{6}$ and $\cdot6\dot{5}\dot{4}$ as vulgar fractions reduced to their lowest terms, and their sum as a circulating decimal.

3. Find the cost of 73 cwt. 3 qrs. 14 lbs. at £4. 13*s*. 6*d*. per cwt.

4. Distinguish between true discount and banker's discount. Find the former in the case of a bill for *Rs*.3486. 6*a*. 8*p*. due 16 months hence, the rate of interest being $5\frac{1}{2}$ per cent. per annum.

5. A man invests *Rs*.163000, part in Govt. 4 per cent. stock at 108 and the remainder in Municipal 5 per cent. Debenture stock at $109\frac{1}{2}$. Find how much he must invest in each that he may have an equal income from the two sources.

1894.

1. In a compound metal containing tin and copper only, the proportion of tin to copper is 7·75 to 92·25. Find to the nearest penny the value of 8 cwt. 3 qrs. of it. Tin costs £140; copper £80 per ton.

2. A rectangular court is 50 yards long and 30 yards broad. It has paths joining the middle points of the opposite sides of 6 feet in breadth and also paths of the same breadth running all round it. The remainder is covered with grass. If the cost of the pavement be 1*s*. 8*d*. per sq. ft. and the turf 3*s*. per sq. yd., find the cost of laying out the court.

3. Find the value of $\cdot 267\dot{1}87\dot{5}$ of £3 in *s.* *d.* and decimal of a penny.

4. Find the square root of $1 - (\cdot 0678)^3$ to four places of decimals.

5. At a cricket match a contractor provided luncheon for 24 and fixed the price to gain $12\frac{1}{2}$ per cent. on his outlay. Three persons were absent. The remaining 21 paid the fixed price and the contractor lost 2 rupees. What was the charge ?

6. Find the simple interest on *Rs.*12345 for 134 days at $2\frac{3}{4}$ per cent.

1895.

1. Find the square root of $1 + \frac{1}{2}(\cdot 0345)^3$ correctly to 4 places.

2. Find the sum of money which put out at simple interest at $2\frac{3}{4}$ per cent. will in 134 days exactly produce *Rs.*124.10*a.* $1\frac{1847}{1825}$*p.* (1 year = 365 days.)

3. If one pound sterling be worth 25 francs and 60 centimes ; and also worth 6 thalers and 20 silver groschen ; how many francs and centimes is one thaler worth ? (*N.B*—One thaler = 30 silver groschen. One franc = 100 centimes.)

4. Simplify $\frac{1\frac{1}{4} - \frac{5}{12}}{1\frac{1}{4} + \frac{5}{12}} + \frac{7}{9}$ of $\frac{9 \times 5}{14 \times 3} - \frac{11\frac{1}{4}}{15}$.

5. I invest *Rs.*12805 in the 4 per cents. at $98\frac{1}{4}$, and when they have risen to $102\frac{5}{8}$, I sell out and invest in the $4\frac{1}{2}$ per cents. at $105\frac{3}{8}$; what is the change in my income ? (Brokerage $\frac{1}{4}$ per cent. on all transactions.)

Or convert $\frac{1247}{1825}$ into a decimal fraction, pointing out accurately the recurring portion (if any).

1896.

1. What greatest number and what least number can be subtracted from 23759143 that the remainders may be divisible by 24, 35, 91, 130 and 150 ?

2. (2) Simplify $\frac{5\frac{5}{8}}{6\frac{4}{7}}$ of $\frac{6\frac{7}{11}}{9\frac{1}{8}} \div \frac{8}{9}\left(2\frac{8}{11} + \frac{13}{22}\right)$ of $\frac{7s.\ 6d.}{12s.\ 6d.}$.

(2) Divide ·0023465 by ·03125.

3. Extract the square root of $5\frac{1}{7}$ correct to 4 places of decimals.

4. Find the simple interest on *Rs.*4235. 12*a.* $9\frac{3}{8}$*p.* for 3 years and 7 months at $3\frac{1}{2}$ per cent. per annum.

5. If by selling a horse for *Rs.*1100, I lose 18 per cent. : how much per cent. should I have gained or lost, had it been sold for *Rs.*1320 ?

6. A man invested the same sum in two different stocks, $3\frac{1}{2}$ per cent. Government Securities at $103\frac{1}{2}$ and 4 per cent. Municipal Debentures at 105 ; his income from one was *Rs.*93 more than from the other : what sum was invested in each stock ?

1897.

1. Reduce $\frac{2\frac{1}{3}-1\frac{1}{4}}{2\frac{1}{3}+1\frac{1}{4}} \times 15\frac{17}{39} + \frac{3\frac{1}{3} \times 3\frac{1}{3} \times 3\frac{1}{3} - 1}{3\frac{1}{3} \times 3\frac{1}{3} + 3\frac{1}{3} + 1}$ of 1 cwt. 3 qrs. 7 lbs. to the decimal of $2\frac{1}{3}$ tons.

(*a*) Find the vulgar fraction equivalent to the recurring decimal $\cdot\dot{1}3\dot{3}$, without assuming any rule.

2. What do you understand by an *aliquot* part of a quantity ? Is an area equal to $15\frac{1}{8}$ sq. yards an aliquot part of an acre ?

Find by Practice the income-tax on *Rs.*1250. 10*a.* 8*p.* at the rate of 5 pies per rupee.

3. What is meant by the *ratio* of one quantity to another ?

What is a *proportion* ?

320 people dine together 4 days a week, but on the remaining 3 days some are absent ; the consumption of food is thus reduced for the whole week, in the ratio of 109 to 112. Find the number of absentees.

4. In what time will *Rs.*3546 amount to *Rs.*7683 at $3\frac{1}{3}$ per cent. simple interest ?

5. A person has stock in the $3\frac{1}{2}$ per cent. Government Securities which yields *Rs.*2856 a year. He sells out half of the stock at $109\frac{7}{8}$, and invests the proceeds in Howrah Mills shares at 153. What dividend ought the latter to pay that he may thereby increase his annual income by *Rs.*330 ?

6. Extract the square root of 3·14159 to 4 decimal places.

1898.

1. What is that least number, which, being divided by 48, 64, 72, 80, 120 and 140, leaves the remainders 38, 54, 62, 70, 110, and 130 respectively ?

2. (*a*) Simplify :—

$$\frac{2\frac{3}{4}}{5\frac{1}{6}} \text{ of } \frac{3}{4}\left(\frac{7}{9}+\frac{1}{12}\right) + \frac{5\frac{7}{8}}{7\frac{1}{4}} \text{ of } \frac{2s.\ \ 5d.}{3s.\ \ 11d.}.$$

(*b*) What decimal of 2*l.* 13*s.* 4*d.* is ·0625 of $2\cdot\dot{6}$ of 1*l.* 6*s.* 8*d.* ?

3. Extract the square root of 54756 ; also of $(4\cdot02)^3$ to 4 places of decimals.

4. What sum will amount to *Rs.*300 in $3\frac{1}{4}$ years at $6\frac{1}{4}$ per cent. per annum simple interest ?

5. A grocer buys 480 mds. of sugar for *Rs.*6135 payable at the end of 3 months, and on the same day sells them at *Rs.*12. 11*a.* per maund ready money; what per cent. does he gain or lose by the transaction, reckoning interest at 9 per cent. per annum?

6. One-third of a certain capital is invested in the $3\frac{1}{2}$ per cent. Government Securities at 105, one-fourth in the 3 per cent. Government Securities at $97\frac{1}{2}$ and the remainder in the $4\frac{1}{2}$ per cent. Calcutta Municipal Debentures at $112\frac{1}{2}$. If the total annual income is *Rs.*830, what is the capital?

1899.

1. Find the greatest number which will divide 1028, 1629 and 2130, leaving the remainders 3, 4 and 5 respectively.

2. (*a*) Simplify

$$\frac{\frac{3}{4}+\frac{7}{8}}{\frac{5}{6}+\frac{9}{10}} \text{ of } \frac{13s.\ \ 5d.}{9s.\ \ 10d.}+\frac{3}{5}\left(\frac{2}{7}+\frac{8}{9}\right) \text{ of } \frac{3 \text{ tons } 3 \text{ cwt.}}{4 \text{ tons } 3 \text{ cwt.}}$$

(*b*) Prove that $\cdot 2\dot{3}\dot{4}=\frac{232}{990}$, without assuming the rule of converting a recurring decimal into a vulgar fraction.

3. Find, by Practice, or otherwise, the value of 7 tons 2 cwt. 2 qrs. at *Rs.*3. 2*a.* per maund, assuming that 1 ton is equal to $27\frac{1}{2}$ maunds.

4. Extract the square root of 51076, and of ·051076.

5. A grocer mixed 20 maunds of one kind of rice at *Rs.*4 a maund, with a certain quantity of a second kind of rice at *Rs.* 3. 8*a.* a maund, and selling the mixed rice at *Rs.*3. 12*a.* a maund, gained *Rs.*10 on the whole. Find how many maunds of the second kind of rice he mixed, and the gain per cent. on his outlay.

6. Find the discount on *Rs.*1218 due six months hence at 3 per cent. per annum, simple interest.

1900.

1. What do you understand by the Greatest Common Measure and the Least Common Multiple of two or more whole numbers?

Nine bells begin to strike simultaneously, and strike at intervals of 1, 2, 3, 4, 5, 6, 7, 8, 9 seconds respectively. After what interval of time will they next strike simultaneously?

2. (*a*) Simplify $\dfrac{16\frac{1}{12}-3\frac{1}{2} \text{ of } 2\frac{1}{8}}{\frac{3}{7} \text{ of } 5\frac{1}{2}+3\frac{3}{4}} \times \dfrac{2\frac{5}{16} \text{ of } 4\frac{3}{4}+\frac{3}{8} \text{ of } 13\frac{3}{8}}{5\frac{3}{4}-4\frac{1}{2} \text{ of } \frac{5}{8}}-\frac{67}{117}\div 1\frac{1}{2}$.

(*b*) Reduce $\cdot 041\dot{6}$ to its equivalent vulgar fraction in its lowest terms, and explain the reason for the process you employ.

3. Find the value of $(1\cdot 25)^3+2\cdot 25\times(1\cdot 25)^2+3\cdot 75\times(\cdot 75)^2+(\cdot 75)^3$, without reducing the decimals to vulgar fractions.

4. The length, the breadth, and the height of a room are 25 ft. 7 in., 20 ft. 5 in. and 14 ft. respectively. Its walls are papered at 3*s.* 6*d.* a sq. yd., and its ceiling painted at 1*s.* 2*d.* a sq. ft. Find the total cost.

5. The subscriptions to a certain memorial fund amounted to *Rs.*976. 9*a.*, and each person subscribed as many annas as there were subscribers altogether. Find the number of subscribers.

6. Explain clearly what you mean by saying that the $3\frac{1}{2}$ per cent Government Securities are at 101.

A person invests *Rs.*19,700 in the $3\frac{1}{2}$ per cent. Government Securities at $98\frac{1}{2}$, and when they rise to $101\frac{1}{2}$, he sells out and invests the proceeds in the $4\frac{1}{2}$ per cent. Calcutta Municipal Debentures at $114\frac{3}{16}$. Find the change in his income.

1901.

1. (*a*) Simplify $\frac{306}{323} \div \frac{204}{221}$ of $\frac{22\frac{5}{11}}{32\frac{9}{11}} - \cdot 58\dot{3} \times \cdot\dot{1}4285\dot{7}$, expressing your answer as a decimal.

(*b*) Reduce £3. 15*s.* 4*d.* to the decimal of *Rs.*100 (£1 = *Rs.*15.)

2. (*a*) What is meant by an *aliquot part* of a number ?

Is $2\frac{3}{4}$ yds. an aliquot part of a mile ?

(*b*) Find by Practice, or otherwise, the value of 25 tons 15 cwt. 3 qrs. $17\frac{1}{2}$ lbs. at £2. 13*s.* 4*d.* per ton.

3. If the four-penny loaf weighs 3 lbs. 9 oz. when wheat is at 9*s.* 4*d.* per bushel, what ought the six-penny loaf to weigh when wheat is at 11*s.* 1*d.* per bushel ?

4. (*a*) Define *Interest.* What do you understand by the expression *Rate per cent. per annum* ?

(*b*) At what rate per cent. per annum simple interest will £200 amount to £236. 13*s.* 4*d.* in 4 years 7 months ?

5. Extract the square root of 7468·4164.

6. A man invests one-third of his capital in the $3\frac{1}{2}$ per cent. Government Securities at $96\frac{1}{2}$, and the remaining two thirds in the $4\frac{1}{2}$ per cent. Calcutta Municipal Debentures at $105\frac{1}{2}$. If the difference of the two annual income be *Rs.*1997, find his capital.

1902.

1. (*a*) How can you ascertain whether a given vulgar fraction can be reduced to a terminating or a recurring decimal, without actually converting it into a decimal ? What kind of decimal will the fraction $\frac{161}{17920}$ produce ?

(*b*) Simplify :—

$$1 - \frac{2}{3 + \frac{4}{5 - \frac{6}{7 + \frac{8}{9}}}} \div 2\cdot08\dot{3} \text{ of } \frac{2 \text{ cwt. } 2 \text{ qrs. } 21 \text{ lbs.}}{10 \text{ cwt. } 2 \text{ qrs. } 11 \text{ lbs.}}$$

and reduce the result to the decimal of $1\cdot1$.

2. The area of a rectangular field whose breadth is 500 yds. is 100 acres. Find the cost of cultivating it at *Rs.*3. 2*a.* 8*p.* per 100 sq. yds. and also the cost of fencing it round at *Rs.*2. 8*a.* per yard.

3. If 12 men and 15 boys can do a piece of work in 30 days, working $7\frac{1}{2}$ hours a day, how many boys must assist 21 men to do a piece of work twice as great in 25 days, working 9 hours a day? (3 men are equivalent to 5 boys.)

4. Extract the square roots of $5\frac{1}{16}$ and $76\cdot195441$.

5. (*a*) *Define Discount.*

(*b*) Find the discount on £700 due 3 years 4 months hence at 5 per cent. per annum simple interest.

6. Which is the better investment, the $3\frac{1}{2}$ per cent. Government Securities at $95\frac{5}{8}$ or the 4 per cent. Calcutta Municipal Debentures at $101\frac{1}{4}$? What will be the difference in the annual income by investing *Rs.*22127 in each of them?

1903.

1. (*a*) Simplify $\frac{\cdot67 \times \cdot67 \times \cdot67 - \cdot001}{\cdot67 \times \cdot67 + \cdot067 + \cdot01} + \frac{\cdot57}{1 + \frac{1}{3\frac{1}{14}}}$.

(*b*) What decimal of a mile is a yard?

2. (*a*) What is meant by the aliquot part of a number? Is an acre an aliquot part of a square mile?

(*b*) Find by Practice, or otherwise, the price of 25 tons 12 cwt. 3 qrs. $17\frac{1}{2}$ lbs. at £6. 13*s.* 4*d.* per ton.

3. Three taps *A*, *B* and *C* can fill a cistern in 5, 6 and $7\frac{1}{2}$ minutes respectively. They are all turned on at once; but after one minute, *A* is turned off. How much longer will *B* and *C* take to fill the cistern?

4. (*a*) Define the square root of a number.

(*b*) Extract the square root of $10\frac{1}{36}$; and of $\frac{22}{7}$ to four places of decimals.

5. A man buys wine at 5*s.* a gallon; he mixes it with water, and by selling the mixture at 4*s.* a gallon gains $12\frac{1}{2}$ per cent. on his outlay. How much water did each gallon of the mixture contain?

6. (*a*) Define *Present worth.*

(*b*) A tradesman marks his goods with two prices, one for ready-money and the other for 3 months' credit, allowing interest at $4\frac{1}{2}$ per cent. per annum. If the credit price be marked at *Rs.*50. 9*a.* what ought to be the cash price?

1904.

1. Define the G. C. M. and the L. C. M. of two or more numbers.

(*a*) Find the greatest number of six digits which is exactly divisible by 27, 45, 60, 72 and 96.

2. Write down the local value of each of the figures in the number ·010203.

(*a*) Simplify $\dfrac{(\cdot01)^3+(\cdot02)^3+(\cdot03)^3}{(\cdot001+\cdot002+\cdot003)^2}-\cdot0208\dot{3}+\dfrac{£2.\ 3s.}{£25.\ 16s.}$ of $\frac{1}{3}$.

3. *A* can do a piece of work in 25 days, *B* in 20 days, and *C* in 24 days. The three work together for 2 days, and then *A* and *B* leave; but *C* continues, and after $8\frac{2}{5}$ days is rejoined by *A*, who brings *D* along with him, and these three finish the remainder of the work in 3 days. In what time would *D* alone have done the whole work?

4. The area of a square cricket field is 9 ac. 3 ro. 8·16 po.; find the length of a side.

5. Define *Discount.*

(*a*) The difference between the interest and the discount on a certain sum for 3 years 4 months at 5 per cent. per annum is £16. 13*s.* 4*d.* Find the sum.

6. A person invests a certain sum in the $3\frac{1}{2}$ per cent. Government Securities when they are at $97\frac{3}{8}$; had he waited till they had fallen to $97\frac{1}{8}$, he would have had *Rs.*400 more of Government Securities. How much money did he invest, $\frac{1}{8}$ per cent. being charged as brokerage in both cases?

1905.

1. When is one number said to be a *measure* of another? What is a *Prime Number*?

A man bought two heaps of mangoes, one for *Rs.*10. 5*a.* and the other for *Rs.*18. 0*a.* 9*p.* If the price of each mango be the same, and not less than two and not more than three annas, find the total number of mangoes he bought.

2. (1) What is the meaning of $\frac{3}{8}$ and of $\frac{1}{4}$ of $\frac{3}{2}$?

(2) Simplify :—

$$\left(5\tfrac{5}{6}-\tfrac{5}{12}\right) \text{ of } \left(\frac{\frac{5}{6}}{3\frac{1}{3}}+\tfrac{7}{8} \text{ of } \tfrac{4}{5}\right)+\tfrac{5}{7} \text{ of } \frac{3 \text{ tons } 3 \text{ cwt.}}{9 \text{ cwt.}}$$

3. Extract the square root of 19·951 and of $\frac{5}{8}$ correct to three places of decimals.

4. Find the cost of paving a pathway 6 ft. wide, round and immediately outside a flower garden, 21 yds. long and 10 yds. broad, at $5\frac{1}{7}$ pies per sq. yd.

5. Find the price of 35 mds. $13\frac{3}{4}$ srs. of rice at *Rs.*3. 2*a.* per maund.

If it is sold at the rate of *Rs.*3. $3\frac{1}{8}$*a.* per maund, what is the profit per cent. ?

6. I pay *Rs.*45900 to a Bank for a Bill of Exchange payable in London. The rate of exchange is 1*s.* 4*d.* for the rupee and the Bank charges me 2 per cent. on the amount payable in England. How much will my agent in London receive ?

1906.

1. (1) When is one number said to be a *multiple* of another? How can you ascertain by inspection whether a given number is a multiple of 3 ?

(2) What is the greatest number consisting of five digits which can be added to 8321 so that the sum may be exactly divisible by 15, 20, 24, 27, 32 and 36 ?

2. (1) What is the meaning of $\frac{3}{4}$ of $\frac{1}{2}$? Give an illustration.

(2) Simplify :—

(1) $12\times\left(\frac{2}{29}-\frac{1}{87}-\frac{1}{58}-\frac{1}{174}-\frac{1}{232}\right)+\frac{323}{361}+\frac{2\frac{5}{8}}{2\frac{5}{7}} \text{ of } \frac{11s.\ 4d.}{12s.\ 3d.}$.

(2) $\dfrac{1{\cdot}59\times15{\cdot}9-{\cdot}41\times4{\cdot}1}{15{\cdot}9-4{\cdot}1}$.

3. The cost of matting a room 16 ft. broad and 12 ft. high at 3*a.* per sq. yd. is *Rs.*7. 9*a.* 4*p.* What will be the cost of papering its walls at the same rate, allowing for six doors, each 6 ft. by 3 ft. ?

4. Extract the square root of $\cdot02\dot{7}$ and of $\frac{3}{8}$ correct to four places of decimals.

5. A book sent from England costs me (including *Re.*1. 2*a.* postage) *Rs.*12. 1*a.* But my bookseller allows me a discount of 2*d.* in the shilling on the published price. What is the published price in English money, the rate of exchange being 1*s.* 4*d.* for the rupee ?

6. Define *Present Worth.*

A man bought a horse for 30 guineas and sold him immediately for £36. 1*s.* payable at the end of 6 months. If interest be reckoned at 6 per cent. per annum, find his gain per cent. upon the transaction.

1907.

1. What do you understand by the G. C. M. and the L. C. M. of two or more integers? What is a prime number? Find the least number which is exactly divisible by 12, 34, 56 and 78.

2. Simplify

(1) $$\frac{\cdot 2 \times \cdot 2 \times \cdot 2 + \cdot 02 \times \cdot 02 \times \cdot 02}{\cdot 6 \times \cdot 6 \times \cdot 6 + \cdot 06 \times \cdot 06 \times \cdot 06} \div \frac{2\frac{1}{3} - 1 \cdot 1\dot{6}}{2 \cdot \dot{3} + 1\frac{1}{6}}.$$

(2) $$\frac{Rs.2.\ 5a.\ 6p.}{Rs.3.\ 12a.} - \frac{1 \text{ hr. } 16\text{m. } 45\text{sec.}}{2 \text{ hr. } 7\text{m. } 55\text{sec.}}.$$

3. Find the price of 8 mds. 16 srs. 2 chks. of rice at *Rs.*5. 9*a.* per maund.

4. How many paving stones, each of them 1 ft. long and 9 in. wide, will be required for paving a street 30 ft. wide, surrounding the outside of a square grass plot, the area of the grass plot being 10 acres?

5. If 8 men or 15 women can earn *Rs.*120 in 30 days, how much can 21 men and 24 women earn in 45 days?

6. The debts of a bankrupt amount to £2134. 10*s.* 6*d.* and his assets consist of property worth £916. 5*s.* 4*d.* and an undiscounted Bill of £513 due 4 months hence, simple interest at 4 per cent. How much in the pound can he pay to his creditors?

1908.

1. (1) When can a vulgar fraction be converted into a terminating decimal? What kind of decimal will the fraction $\frac{119}{136}$ produce?

(2) Simplify :—

$$\frac{2\frac{1}{4} + \frac{3}{4} \text{ of } 2\frac{3}{8} - 1\frac{1}{4}}{3 \cdot 6 + \cdot 15 \times \cdot 4 \div \cdot 24 - 2 \cdot 1} \text{ of } \frac{20 \cdot 1 \text{ of } 1 \text{ md. } 30\frac{1}{4} \text{ srs.}}{17 \cdot 5625 \text{ of } 2 \text{ mds. } 20\frac{1}{2} \text{ srs.}} \text{ of } Rs.2.\ 8a.$$

2. Find by Practice, or otherwise, the value of 5 acres 3 roods 7 poles $5\frac{1}{2}$ sq. yds. of land at £161. 6*s.* 8*d.* per acre.

Or

II. A reservoir is 25 ft. 5 in. long and 12 ft. 10 in. wide; how many gallons of water must be drawn off to make the surface sink one foot? (A cubic foot of water weighs 1000 ounces, and one gallon = 10lbs. avoirdupois.)

3. (1) The hands of a clock coincide after every 66 minutes of correct time. How much is the clock fast or slow in 24 hours?

(2) A race-course is 440 yards long. *A* and *B* run a race and *A* wins by 5 yards : *B* and *C* run over the same course and *B* wins by 4 yards : *C* and *D* run over it and *D* wins by 16 yards. If *A* and *D* run over it, which would win, and by how much ?

4. (1) What number multiplied by itself will produce $4\frac{285}{529}$?

(2) Extract the square root of $\frac{2}{7}$ correct to four places of decimals.

Or

IV. The discount on a certain sum due 2 years hence is *Rs.*638. 8*a.*, and the interest on the same sum for the same time is *Rs.*718. 5*a.* ; find the sum and the rate per cent. per annum.

5. A trader allows a discount of 5 per cent. to his customers. What price should he mark on an article, the cost price of which is *Rs.*712. 8*a.*, so as to make a clear profit of $33\frac{1}{3}$ per cent. on his outlay ?

6. A person invests *Rs.*44100 in the $3\frac{1}{2}$ per cent. Government Securities at 98, and when they rise to $98\frac{1}{2}$, he sells out and invests the proceeds in the 5 per cent. Calcutta Municipal Debentures at $110\frac{13}{16}$. Find the alteration in his income.

1909.

1. Multiply 62031 by 46189, and divide the product by 7429.

2. Simplify :—

(1) $$\frac{10\frac{3}{8}-(5\frac{3}{8}+4\frac{9}{20})}{10\frac{1}{3}-(2\frac{1}{6}-\frac{4}{15})-7} \div \frac{1}{6+\frac{1}{1+\frac{1}{6}}};$$

(2) $2\cdot1\dot{4}2857\dot{1} \div \cdot07\dot{6}9230\dot{7} \times 2\cdot\dot{3}$.

3. Find by Practice, the price of 28 bags of sugar, each weighing 3 cwt. 2 qrs. 8 lbs., at *Rs.*9. 6*a.* $4\frac{1}{2}$*p.* per cwt.

Or

Extract the square root of 137769·395929.

4. The area of a square garden is 10 acres. On the inside of the garden and along four sides of it there is a gravel path 5 feet wide. Find the cost of constructing the path at 1 anna 6 pies per square foot.

Or

On what capital will the interest for 219 days at 4 per cent. per annum amount to £14. 2*s.* 6*d.* ?

5. Among a certain number of children 91509 mangoes and also 83721 oranges may be equally divided. How many are the children ? Give all possible answers.

Or

What profit per cent. is made by selling an article at a certain price, if by selling at two-thirds of that price there would be a loss of 20 per cent.

(Supplementary Examination.)

1909.

1. Multiply 18054 by 43565 and divide 722646078 by 56294.

2. Simplify :—

(a) $\frac{29}{108}+\frac{4}{85}$ of $1\frac{3}{4}-\frac{7}{11}$ of $3\frac{3}{8}+8\frac{1}{6}$.

(b) $\frac{2\cdot4\dot{6}+2\cdot\dot{3}\dot{0}}{\cdot\dot{3}+\cdot1\dot{2}\dot{7}}+\frac{4\frac{1}{6}}{19}$.

3. Find by Practice the cost of 1000 mds. of sugar at *Rs.*8. 13*a.* 4*p.* per maund.

Or

III. Find the price of tiles, each 18 inches by 15 inches, that will be required to pave a square court, whose side is 30 yards, at *Rs.*25 per hundred.

4. If 9 men do $\frac{3}{8}$ of a piece of work in 14 days working 10 hours a day, how many extra men must be employed to finish the work in 5 days more, if all of them are now to work only 8 hours a day ?

Or

IV. In what time will the simple interest on £285 be £84. 15*s.* 9*d.* at $3\frac{1}{2}$ per cent. per annum ?

5. Extract the square root of 184389241.

Or

V. Find the present worth of *Rs.*315 due 10 months hence at 6 per cent. per annum.

MATRICULATION EXAMINATION.

1910.

Compulsory Paper.

1. Multiply 407566 by 800209 ; and divide 507233438305 by 670549.

Or

I. Find the G. C. M. of 253512 and 568512 ; and the L. C. M. of 432, 720, 1152

2. Reduce to its simplest form :—

(1) $$\frac{4\frac{1}{2}\times17\frac{1}{7}\times2\frac{1}{8}}{5\frac{1}{3}\times5\frac{4}{7}\times\left(\frac{1}{4\frac{1}{2}}+\frac{1}{6\frac{1}{2}}\right)};$$

(2) $$\frac{\cdot\dot{8}\dot{1}\times\cdot005}{\cdot45}.$$

Or

II. A contractor engaged to finish six miles of railway in 200 days, but after employing 140 men for 60 days he found that only one and a half miles were completed. How many additional men must be engaged that the work may be finished within the given time?

3. (1) Find by Practice, or otherwise, the value of 458 things at *Rs.*8. 5*a.*·4*p.* each.

(2) In what time will a sum of money double itself at 6 per cent. simple interest per annum?

Or

III. The weight of a cubic inch of water is 253·17 grains and that of a cubic inch of air is ·31 grains. Find to three places of decimals how many cubic inches of water weigh as much as one cubic foot of air.

Additional Paper.

1. Extract the square root of 6256586734489.

Or

I. A cistern contains 243⅜ cubic feet of water. Find the length of the side of a second cistern 4 ft. 4 in. deep, with a square base, which contains 4 times as much water as the first.

2. (1) Calculate, correct to three places of decimals, the value of

$$1+\frac{1}{1.2}+\frac{1}{1.2.3}+\frac{1}{1.2.3.4}+\&c.\text{ to infinity.}$$

(2) A metre is defined to be the ten-millionth part of a quarter of the circumference of the earth, and is equal to 39·37079 inches. Find the circumference of the earth in miles.

1911.

Compulsory Paper.

1. Multiply 87904563 by 7059089; and divide the product by 998875.

Or,

A square grass-plot whose side is 200 yds., is bordered on the outside by a path 10 ft. wide. Find the cost of gravelling the path at *Rs.*2. 8*a.* per 100 square ft.

2. (1) Simplify :—

$$\frac{\frac{1}{2}+\frac{3}{4}}{\frac{5}{6}-\frac{7}{8}+\frac{9}{10}}+\dot{1}4285\dot{7} \text{ of } 10\frac{20}{108}.$$

(2) What decimal of a rupee is a pie ?

Or,

What decimal of an hour is a second ?

3. (1) Find the value of 5 mds. 25 sr. 10 ch. of milk at *Rs.*5. 10*a.* 8*p.* per md.

(2) What sum of money must be put out at $3\frac{3}{4}$ p. c. per annum, simple interest, in order to amount to £248. 18*s.* 9*d.* in $2\frac{1}{3}$ years ?

Or,

A contractor undertakes to execute a certain work in a given time ; he employs 55 men, who work 9 hrs. daily ; when $\frac{3}{4}$ of the time has expired, he finds that only $\frac{3}{7}$ of the work is done ; how many men must he now employ 11 hrs. a day to fulfil his contract ?

Additional Paper.

1. Find the square root of 220191808516,

Or,

2919·46783041.

Or,

A general wishing to arrange his men, who were 335250 in number, into a solid square, found that there were 9 men over. How many men were there in the front ?

2. (1) Find a decimal that is within $\frac{1}{100000}$ of $\frac{355}{113}$.

Or,

Find the value (correct to 5 places of decimals) of

$$1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\frac{1}{9}-\ldots\ldots\ldots\ldots$$

(2) Assuming a metre to be $39\frac{3}{8}$ inches, find the nearest whole number of litres in one cubic foot.

1912.

Compulsory Paper.

1. Multiply 814703 by 703692 ; and divide 246741768 by 75318.

Or,

Reduce to its lowest terms $\frac{142593}{514199}$.

2. (1) Reduce to the simplest vulgar fraction.

$$\frac{2\cdot4\dot{6} - 2\cdot\dot{3}\dot{0}}{\cdot\dot{3} + \cdot1\dot{2}\dot{7}} + \frac{4\frac{1}{3}}{19}.$$

(2) Find the value of $\frac{3 \text{ cwt. } 3 \text{ qrs. } 14 \text{ lbs.}}{2 \text{ cwt. } 1 \text{ qr. } 20 \text{ lbs.}}$ of £7. 18*s*. 8*d*.

Or,

(1) At what rate per cent. simple interest will £440. 6*s*. 8*d*. amount to £511. 17*s*. 9*d*. in 5 years ?

(2) Find the price of 12 maunds 8 seers 4 chattaks of ghee at *Rs*.36. 4*a*. per maund.

3. If the wages of 45 women amount to £207 in 48 days, how many men must work 16 days to receive £76. 13*s*. 4*d*., the daily wages of a man being double those of a woman ?

Or,

A rectangular courtyard 100 ft. long by 80 ft. wide has within it a gravel path 8 ft. wide running round it. Find the area of the path, and the cost of gravelling it at 5*a*. 3*p*. per sq. yd.

Additional Paper.

1. Find the square root of 137769·395929.

Or,

A rectangular court, three times as long as it is broad, is paved with 2028 stones, each 1½ ft. square. Find the length of the court.

2. If a metre be 3·2809 ft. and the length of a line drawn on the earth from the North Pole to the Equator be 10,000,000 metres, find the circumference of the earth to the nearest mile.

Or,

Find, correct to five places of decimals, the value of

$$\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{2^3} + \frac{1}{5}\cdot\frac{1}{2^6} + \frac{1}{7}\cdot\frac{1}{2^7} + \frac{1}{9}\cdot\frac{1}{2^9}.$$

1913.

Compulsory Paper.

1. (1) Multiply 426042 by 90578.

Or,

Divide 5208465 by 754.

(2) Find the G. C. M. of 253512 and 568512.

Or,

Find the L. C. M. of 105, 135, and 210.

2. (1) Simplify $\frac{1}{2}+\frac{2}{3}-\frac{3}{4}+\frac{4}{5}-\frac{5}{6}-\frac{1}{60}$.

(2) Express in decimals the sum of $\cdot 438 \times \cdot 15$ and $\frac{\cdot 063}{\cdot 28}$.

Or,

(1) Find what decimal of a maund is a chattack.

(2) Find the price of 432 pieces of cloth at *Rs.*5. 7*a.* 6*p.* each.

3. (1) If *Rs.*750 amount to *Rs.*873. 12*a.* in 5 years and 6 months, find the simple interest per cent. per annum.

(2) *A* can run 8 yds. in the same time that *B* can run 9. They start together; when *B* has run 252 yds., how far behind is *A*?

Additional Paper.

1. Find the square root of 29·192409.

Or,

Find the cost of fencing a square field of 10 acres at 6*a.* 8*p.* per yd.

2. A room is 20 metres in length and 10 metres in breadth. Find the number of square yards in the area of the floor, taking a metre as equal to 39·37 inches.

Or,

Define a prime number, and state all the prime numbers between 70 and 90.

1914.

Compulsory Paper.

1. Multiply $777\frac{378}{579}$ by 358, and express the result as a whole number and a proper fraction.

2. Find the G. C. M. of 7163 and 13091

Or,

Find the L. C. M. of 48, 72, 80, 108, and 120.

3. Simplify $\dfrac{3\frac{5}{6}+\frac{7}{12}+9\frac{11}{18}}{11\frac{7}{12}-5\frac{13}{18}}+\dfrac{16\frac{2}{3}}{16\frac{2}{3}-2\frac{11}{18}}$.

Or,

Find the price of 273 maunds, 33 seers, 7 chattacks of ghee at *Rs.*53. 8*a.* per maund.

4. Add together 0·022 of £1, 0·946 of a shilling, and 3·48 pence, and subtract the sum from 0·26 of a guinea. Express the answer in pence and the decimal of a penny.

Or,

Find what sum will amount to *Rs.*723. 0*a.* 10*p.* in 6 years and 3 months at 4½ per cent. per annum, simple interest.

Additional Paper.

1. Extract the square root of 7 correct to 3 places of decimals.

Or,

Show that 103 is a prime number.

2. Given one centimetre=0·3937 inches, find in square metres the area of a floor whose length is 21 feet and breadth 10 feet 8 inches.

1915.

Compulsory Paper.

1. (1) Multiply 790463 by 95076.

Or,

Divide 277286112 by 35064.

(2) Find the G. C. M. of 253512 and 568512.

Or,

Find the L. C. M. of 125, 160, and 280.

2. (1) Simplify $2-\cfrac{5}{3+\cfrac{1}{2-\cfrac{1}{1+\frac{1}{2}}}}$.

(2) Multiply 17·55 by 4·004, and divide the product by 0·819. (The results are to be expressed in decimals).

Or,

(1) Express as a recurring decimal $0·4+\frac{1·5}{0·1\dot{3}}$.

(2) Find the price of 729 slabs of marble at *Rs.*7. 11*as.* 3 *pies* each.

3. (1) At what rate per cent. per annum (simple interest) will a sum of money double itself in 10 years?

(2) Find the cost of papering the walls of a room 12 ft. 6 in. long, 7 ft. 6 in. wide, and 12 ft. high, with half-anna postage stamps measuring $\frac{15}{16}$ inch by $\frac{3}{4}$ inch.

Additional Paper.

1. Find the square root of 170·485249.

2. The palace of the King of Babylon contained a thousand rectangular courtyards, each 60 metres long and 54 metres broad. The courtyards were all paved with marble slabs, 18 inches long by 18 inches broad. Required the total number of slabs. (Metre = 39·37 inches).

Or,

Multiply 0·48785 by 0·85963 by a contracted method so as to obtain the result correct to five places of decimals.

1916.

Compulsory Paper.

1. (1) Multiply 560789 by 987065.

Or,

Divide 823479885 by 9897?

(2) Find the G.C.M. of 36176 and 85085.

Or,

Find the least whole number which is exactly divisible by 1, 2, 3, 4, 5, 6, 7, 8, and 9.

2. (1) Simplify $\frac{3\frac{3}{8}}{2\frac{3}{4}} - \frac{2\frac{1}{6} \div 7}{\frac{2}{3} \text{ of } \frac{1}{6}} + \frac{85}{204}$.

(2) What decimal of a sovereign is a penny ?

Or,

(1) Express $\frac{2651}{25600}$ as a terminating decimal fraction.

(2) Find the cost of 153 articles at £1. 2*s.* 8*d.* each.

3. (1) If I have to pay 2 pies as interest on one rupee for one month, what is the rate per cent. per annum ?

(2) If 24 men can do a piece of work in 15 days, working 8¼ hours a day, how many men will be required to do another piece of work twice as great in 17 days, working 6 hours a day ?

Additional Paper.

1. Find the square root of 0·0041409225.

2. Express the value of :—

0·04375 kilogram + 0·3775 gram + 0·72 milligram as the decimal of a pound Avoirdupois.

[1 gram = 15·432 grains, and one pound Avoirdupois = 7000 grains.]

Or,

Divide 2·4494897 by 1·4142135 by a *contracted method*, correct to three decimal places.

1917.

Compulsory Paper.

1. (*a*) Multiply 783256 by 347816.

Or,

The quotient after division of a certain number by 372 is 273 and the remainder is 237. Find the number.

(*b*) Find the G.C.M. of 31752 and 41580.

Or,

The circumferences of the fore-wheel and hind wheel of a carriage are 9 ft. 11 in. and 12 ft. 9 in. respectively. Find the least distance over which the carriage must travel in order that both the wheels may make a complete number of revolutions.

2. (*a*) Simplify $\frac{\frac{5}{8}+\frac{3}{4}}{1\frac{3}{4}+2\frac{5}{12}}+\frac{1}{3\frac{1}{11}}$.

Or,

Find the value of

$\frac{1}{3}$ of 9*s.* 10*d.* $-$ $\frac{1}{4}$ of 6*s.* 9*d.* $+$ $\frac{1}{6}$ of £1. 0*s.* 7*d.*

(Express the answer in shillings and pence.)

(*b*) Simplify $\frac{\cdot 1\dot{7}01 + 16\cdot 2}{\cdot 005 \times \cdot 07}$.

Reduce $\frac{7}{18}$ to recurring decimals.

3. (*a*) Find the cost of 21 tons 5 cwt. 3 qrs. of coal at *Rs.*5 per ton.

Or,

Find the simple interest on *Rs.*892 for 8 months at 6$\frac{1}{4}$ per cent. per annum.

(*b*) By selling goods at *Rs.*240 a merchant gains 25 per cent. How much would he gain per cent. by selling them at *Rs.*216?

Or,

In an examination 52 per cent. of the candidates fail in English and 42 per cent. fail in Mathematics. If 17 per cent. fail both in English and Mathematics, find the percentage of those who pass in both subjects.

Additional Paper.

1. Find the square root of 57592921.

Or,

Find to within one millimetre the length of the side of a square whose area is two square metres.

2. Calculate, to four places of decimals, the value of

$$1+\frac{1}{2}+\frac{1}{2\times4}+\frac{1}{2\times4\times6}+\frac{1}{2\times4\times6\times8}+\ldots\ldots$$

Or,

Divide ·12345678 by ·09876543 correct to four places of decimals.

1918.

Compulsory Paper.

1. (*a*) Either,

Multiply 390626 by 331779.

Or,

Find the G.C.M. of 78657 and 90275.

(*b*) A reservoir contains 218,703 gallons of water. How many cisterns, each holding 37 gallons, can be filled out of it, and how many gallons will be left in it when they are all full ?

2. (*a*) Simplify—$\dfrac{\frac{1}{2}+\frac{3}{4}+\frac{15}{28}}{(\frac{5}{6}-\frac{3}{14})\times1\frac{5}{13}}$.

(*b*) Simplify $(1{\cdot}4-0{\cdot}362)+(0{\cdot}31+0{\cdot}123-0{\cdot}0005)$.

(Express the result in decimals.)

3. (*a*) Either,

What will be the cost of paper 20 in. wide, at $3\frac{1}{2}d.$ a yard, for the walls of a room 21 ft. long, 15 ft. wide, and 10 ft. high ?

Or,

Find the cost of 5 cwt. 2 qrs. 14lb. of butter at £2. 5*s*. 6*d*. per cwt.

(*b*) Either,

What sum of money will amount to *Rs.*1532. 4*as*. in three years at $4\frac{1}{2}$ per cent. simple interest ?

Or,

A garrison of 420 have food enough to last them 35 days. After 5 days they are reinforced by 210 men, bringing no food with them. How much longer will the food last ?

Additional Paper.

1. (1) Find the square root of 1000014129.

(2) Find the dimensions of a tank which is 2·56 metres deep and which holds 3000 litres, the length of the tank being three times the width.

2. Either,

Find the value of

$$\frac{1}{2}+\frac{2}{30}+\frac{3}{400}+\frac{4}{5000}+\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

correct to four places of decimals.

Or,

Find the value of

$$\frac{0\cdot34567\times0\cdot73456}{0\cdot67345},$$

correct to four places of decimals.

1919.

Compulsory Paper.

1. Either,

(1) Multiply 9080076 by 6700809.

(2) Find the G.C.M. of 96577 and 448477.

Or,

(1) Divide 4599559845 by 90705.

(2) Find the L.C.M. of 289, 323 and 361.

2. (1) Simplify $\dfrac{\frac{2}{3}+\frac{3}{4}}{\frac{5}{8}\div\frac{7}{8}}$ of *Re.*1, 10*a.* 8*p.*—$0\cdot125$ of $0\cdot1\dot{6}$ of *Rs.*23.

(2) What decimal of an hour is a second?

3. (1) Find the price of 17 cwt. 3 qrs. 14lb. of sugar at £2. 9*s.* per cwt.

(2) In how many years will *Rs.*5000 amount to *Rs.*6100 at $5\frac{1}{2}$ per cent. per annum simple interest?

Additional Paper.

Either,

1. Multiply 5947·183 by 0·093187 by a contracted method so as to retain four places of decimals only.

Or,

Find the value of the following series correct to three places of decimals.

$$\frac{1}{1}+\frac{1}{1\times5\times9}+\frac{1}{1\times5\times9\times13}+\frac{1}{1\times5\times9\times13\times17}+\ldots\ldots\ldots$$

Either,

2. Find the square root of 0·08042896.

Or,

Find the cost of constructing a path 4 ft. wide round a rectangular courtyard 10 yds. long and 7 yds. broad if each square foot costs 2*a.* 6*p.*

1920.

Compulsory Paper.

Either,

1. (1) Multiply 80070430 by 34070080.

(2) Find the G. C. M. of 47821 and 68191.

Or,

(1) The dividend being 545322774 and the quotient 89706, find the divisor.

(2) Find the least number which is exactly divisible by the first nine integers.

2. (1) Simplify $\frac{6\frac{7}{8}+3\frac{4}{5}}{6\frac{7}{8}-3\frac{4}{5}}+10\frac{17}{41}$ of $\frac{1}{8}$.

(2) Express $0\cdot16$ of 2 cwt. 2 qrs. + $0\cdot1\dot{6}$ of $2\cdot6$ cwt. as the fraction of one ton. Convert the fraction into a recurring decimal.

3. (1) Find the rent of 19 acres 3 roods 20 square poles of land at £4. 5*s.* per acre.

(2) What sum will amount to *Rs.*6375 in 5 years at $5\frac{1}{2}$ per cent per annum simple interest?

Additional Paper.

1. Either, by a contracted method find the value of :—

$0\cdot53209853 \times 0\cdot3429448$ correct to seven places of decimals.

Or,

Find correct to four places of decimals the value of $\frac{\sqrt{7}-\sqrt{5}}{\sqrt{7}+\sqrt{5}}$.

Either,

2. A clock in the kitchen loses at the rate of $6\cdot5$ seconds an hour when the fire is alight, and gains at the rate of $3\cdot9$ seconds an hour when the fire is not burning; but in the whole day it neither gains nor loses. How long in the twenty four hours is the fire burning?

Or,

40 per cent. of the gross receipts of a tramway company is taken up in meeting the working expenses, 40 per cent. of the remainder goes to the reserve fund, and the balance is paid away as dividends to share-holders at the rate of $3\frac{1}{8}$ per cent. on their shares, the total value of which is *Rs.*864000; find the amount of the gross receipts.

1921.

Compulsory Paper.

Either

1. (1) The divisor being 102003 and the quotient 45067, find the dividend.

(2) Find the G. C. M. of 65569 and 94829.

Or,

(1) Divide 6579820764 by 98076.

(2) Four bells begin tolling at the same time, and they toll at intervals of 12, 18, 24 and 30 seconds respectively. After what interval of time will they next toll at the same time?

2. (1) Simplify $\dfrac{2\frac{3}{4}-3\frac{1}{8}+4\frac{5}{6}}{7\frac{17}{30}+1\frac{1}{3} \text{ of } 1\frac{1}{2}}$ of $\frac{1}{21}$.

Expressing the answer (i) as a fraction, and (ii) as a recurring decimal.

(2) A post has half of its length in mud, one third of its length in water and ten feet above water Find the whole length of the post.

3. (1) Find the price of 25 maunds 15 seers 5 chattaks of rice at *Rs* 12. 8*as.* per maund.

(2) At what rate per cent. per annum simple interest will *Rs.*6000 amount to *Rs.*7650 in 5 years?

Additional Paper.

Either,

1. Find the diagonal of a rectangle whose sides are 2·56 cm. and 4·73 cm.

Or,

The third class railway fare in France is 5 centimes per kilometre and in England 1*d.* per mile. Given that 1 yard = 0·9144 metre and £1 = 25·17 francs, find (in English money) the difference of the fares for a journey of 100 miles in the two countries, correct within a farthing.

Either,

2. Find the product of 2·73065 and 0·0094738, preferably by a contracted method, correct to four places of decimals.

Or,

Divide £185 between *A*, *B* and *C* so that *A* may have 0·615 of the whole and *B* 0·615 of what is left, and *C* the remainder (to the nearest pound).

MADRAS MATRICULATION PAPERS.

1885.

1. Explain how the value of a fraction is not altered when its numerator and denominator are multiplied by the same number.

Simplify $\left(\frac{\frac{4}{11}\text{ of }1\frac{4}{7}-\frac{5}{6}+\frac{8}{7}}{\frac{3}{7}\text{ of }1\frac{8}{9}+1\frac{4}{5}}\right)\times 4\frac{8}{10}-\frac{5}{6}\text{ of }\frac{4}{5}$.

2. If the rupee is worth 1*s.* $6\frac{3}{8}d.$, express *Rs.*6-5-4 as a fraction of £1; and find the least number of rupees equal in value to an integral number of pounds.

3. State the rule for converting recurring decimals into vulgar fractions; and find the value of $0\cdot\dot{0}\dot{3}$ of 2·75 of £3-2-6 + $0\cdot\dot{2}8571\dot{4}$ of $1\cdot\dot{3}$ of £7-5-10 — $0\cdot5\dot{9}2\dot{5}$ of £2-16-3.

4. Find by any method the value of 5 cwt. 2 qrs. 21 lbs. of goods at £3-7-6 per cwt.

5. The carriage of $17\frac{1}{2}$ cwt. for 52 miles on a certain railway is 8*s.* 4*d.*; find what will be the cost of carrying $4\frac{3}{4}$ cwt. for 300 miles on a railway on which the rate per mile is 9 per cent. lower.

6. A landlord pays 1 per cent. for collecting his rents and a tax of 7 pies in the rupee on what he receives after paying the collector. He has a clear rental of *Rs.*1,831-8-0. Find his gross rental.

7. A grocer mixes four kinds of tea which cost him 5*s.*, 4*s.*, 3*s.*, 2*s.* per lb. respectively, in the proportions of 2, 3, 4, 7 respectively. Find at what rate he must sell the mixture so as to gain 25 per cent. on the whole.

8. Define the terms *interest*, *discount*, and find in what time £533-6-8 will amount to £672 at $6\frac{1}{4}$ per cent. per annum, Simple Interest.

9. A person invests £4800 in 4 per cent. stock at 96, and after a year sells out at $92\frac{1}{2}$ and invests the proceeds together with the interest for the year in stock at $96\frac{1}{2}$. How much stock does he then purchase?

10. Find to four places of decimals the square root of $\frac{187}{8}$; and calculate the cost of surrounding with a fence a square field of $22\frac{1}{2}$ acres at 3*d.* per yard.

11. The population of a country increases at the rate of 7 per cent. every 10 years. What was the population 20 years ago of a country whose present population is 4,007,150?

1886.

1. State and explain the rule of the Multiplication of Vulgar Fractions.

Simplify $\dfrac{\frac{2}{5}(1\frac{2}{3} - \frac{3}{8} \text{ of } 1\frac{4}{5}) + \frac{4}{17}}{\frac{3}{4} \times 1\frac{1}{7} + 1\frac{1}{2} - \frac{11}{28}} \times \dfrac{\frac{3}{4} + \frac{2}{3}}{\frac{3}{4} - \frac{2}{3}} - 20.$

2. Express £66-14-5$\frac{1}{4}$ as the decimal of *Rs.*1,000, the rupee being worth 1*s.* 4$\frac{1}{2}$*d.*

3. Distinguish between pure and mixed circulating decimals. Find the value of $0{\cdot}9\dot{4}\dot{5}$ of £2-3-6$\frac{1}{2}$ + $0{\cdot}37\dot{2}5\dot{9}$ of £1-8-1$\frac{1}{2}$.

4. Find by any method the rent of 156 ac. 3 ro. 24 po. 11 sq. yd. at *Rs.*25-3-4 per acre.

5. A clock which gains 3*m.* 56*s.* in 24 hours was set correctly at noon on the 1st of January, 1884. Find to the nearest minute the next date at which it indicated correct time.

6. Twenty men are employed to make a tank 40 ft. long, 20 ft. broad and 6 ft. deep. They work for 30 days and have just completed one-third of the work, when it was resolved to increase the length of the tank by 10 ft., the breadth by 4 ft. and the depth by 2 ft. How many additional men must be employed in order that the work may be completed in 30 days more?

7. The difference between the simple and compound interest on a sum of money for 3 years at 5 per cent. is £7-12-6. Find the sum.

8. The capital of a certain railway is £1,000,000 in 20,000 shares of £50 each, fully paid up. The gross annual receipts are £105,000 of which 48 per cent. is absorbed in working expenses, £4,600 goes to the reserve fund, and the remainder to pay dividend. Find what annual income a person will obtain from the investment of £4,500 in the undertaking, the shares being at £62-10-0.

9. Ice is manufactured for 6 pies a pound and sold for 9 pies a pound. Two-thirds of the quantity made is kept for sale at the factory and the remainder is sent to branch shops. If the average loss from melting of the former be 12$\frac{1}{2}$ per cent. and that of the latter be 25 per cent., find the gain on every ton made.

10. The average width and depth of a river at its mouth are 240 yards and 6 feet respectively, the average rate of flow is 3 miles per hour and the amount of sediment per cubic foot of water discharged is 1$\frac{1}{4}$ cubic inches. Find the amount of sediment deposited annually; and the depth of the deposit, supposing it spread uniformly; (*i e.*, to the same depth throughout) over an area of 146 square miles.

1888.

2. Simplify $\dfrac{6\frac{2}{3} - 4\frac{1}{3}}{5\frac{1}{2} - 4\frac{2}{3}} - \dfrac{2\frac{2}{35} + 1\frac{11}{35} + 1\frac{1}{4} - \frac{9}{13} \text{ of } 3\frac{1}{4}}{\frac{3}{5} \times 3\frac{5}{7} - 5\frac{2}{5} + 3\frac{4}{15}} \times 13\frac{1}{2}.$

3. Find the value of $1\frac{3}{17}$ of $\cdot 012\dot{3}\dot{6}$ of *Rs.*5-11-8 ; and taking the rupee as worth 1*s.* $4\frac{1}{2}$*d.*, express the result as the decimal of one shilling.

4. Find by any method the value of 9 tons 17 cwt. 3 qrs. 25 lbs. of coffee at £72-18-4 per ton.

5. When iron is at £3-7-6 a ton, the cost of laying a railway 10 mi. 2 fur. 20 po. in length with rails weighing 270 lbs. each is *Rs.*67,500. Find the cost of laying a railway 25 mi. 220 yds. long with rails of the same length weighing 500 lbs. each, when iron is at £3-14-3 a ton.

6. Find the present value of £482-6-$10\frac{1}{2}$ due 3 years hence at 5 per cent. Compound interest.

7. When exchange is at the rate of 1*s.* $4\frac{1}{2}$*d.* per rupee, a person in Madras orders from a bookseller in England a parcel of books, the published price of which is £5. The bookseller allows discount at the rate of 25 per cent. on the published price, but includes in his bill a charge of 13*s.* for packing, freight, &c. When the books arrive in India, a further sum of *Rs.*2-8 has to be paid on account of landing charges and cost of delivery. If the books can be obtained from a bookseller in Madras at the rate of $9\frac{1}{2}$ annas per shilling of the published price, find how much the person loses by ordering from England.

8. A person holds forty *Rs.*500 shares in a concern which pays dividend at the rate of 6 per cent. per annum. When the shares are at *Rs.*675, he sells out and invests half the proceeds in 4 per cent. stock at 90. With the other half he buys a house, for which he receives an annual rental of *Rs.*1,440 subject to a deduction of 3*a.* 9*p.* per rupee for repairs and taxes. Find the alteration in his annual income.

9. In a certain year a country produces 50,000,000 bushels of wheat. Of this quantity 40 per cent. is available for export at *Rs.*3-2 per bushel. In the following year the acreage under wheat has increased 20 per cent., but the yield produce per acre is only seven-eighths of what it was in the previous year, while the quantity required in the country has increased 5 per cent. If at the same time the export price has fallen to *Rs.*3 per bushel, find the increase in the value of the wheat available for export.

10. The population of a country is 33,264,000 and there are 99 males to 101 females ; 2 out of every 11 boys and one out of every 33 girls of school-age are under instruction. If the boys of school-age form one seventh of the male population and the girls of school-age form one-seventh of the female population, find the total number of pupils under instruction.

1889.

2. Simplify $\frac{(\frac{1}{3}+\frac{1}{4})+(\frac{1}{2}-\frac{1}{5})}{(\frac{1}{3}+\frac{1}{4})+\frac{1}{4}-\frac{1}{5}} - \frac{1\frac{7}{8}+\frac{3}{28}}{\frac{7}{12}-\frac{1}{7}} \quad \frac{\frac{1}{2}\text{ of }\frac{4}{7}}{\frac{1}{2}+\frac{1}{3}}$.

3. Multiply $41·36\dot{5}1\dot{4}$ by $·001\dot{9}$, expressing the result as a decimal; and find the value of $·347\dot{2}$ of £1. 4*s*. – $·03\dot{2}8\dot{8}$ of £2. 6*s*. 3*d*.

4. Find by any method the cost of 79 ca. 17 m. 5 v. 25 pal. of salt at *Rs*.21. 10*a*. 8*p*. per candy.

5. The cost of rice for a family of 2 adults and 3 children from Jan. 1st, 1889, to Dec. 11th, 1889, both days inclusive, during which time rice was selling at 15·4 sr. per *Re*., was *Rs*.70. 7*a*. What will be the cost of rice for a family of 3 adults and 5 children from Dec. 19th, 1889, to May 11th, 1890, both days inclusive, assuming that the price of the rice will be 14·7 sr. per *Re*., and assuming also that the quantity required per day by each adult is the same in both cases, and that in both cases the quantity required by a child is $\frac{2}{5}$ of the quantity required by an adult?

6. On what sum due 1 yr. 4 mo. hence does the true discount amount to £100-18-9, Simple Interest being reckoned at $4\frac{3}{4}$ per cent. per annum?

7. How much 3 per cent. stock must a person sell when the selling price is 91 in order that by investing the proceeds in the $4\frac{1}{2}$ per cents. at $113\frac{3}{4}$ he may derive from the investment an annual income of *Rs*.9817. 8*a*., after paying income-tax at the rate of 5*p*. per rupee?

8. *A* and *B* can do a piece of work in 10 days, *B* and *C* in 15 days, and *C* and *A* in 20 days. They all work at it for 6 days; then *A* leaves and *B* and *C* go on for 4 days more. If *B* then leaves, how long will *C* take to complete the work?

9. In a certain year the total amount received by a railway company for carriage of passengers was *Rs*.2,751,000. Of this sum 6 per cent. was contributed by first class passengers, 5 per cent. by second class, and the remainder by third class. The fares were 18, 6, and $1\frac{3}{4}$ pies per mile for first, second and third class passengers respectively. Assuming that the average distance travelled by each third class passenger was 36 miles and the average distance travelled by each passenger of the other classes was 160 miles, find the total number of passengers carried during the year.

10. The length of a rectangular field is twice its breadth. If the rent of the field at £3. 7*s*. 6*d*. an acre is £151. 17*s*. 6*d*., find the cost of surrounding it with a fence at $4\frac{3}{4}$*d*. per yard.

11. Extract the cube root of 9 to five decimal places.

1890.

1. Reduce 2149908480 sq. in. to acres, &c. If this is the area of a rectangle, the length of which is 5 mi. 7 fur. 5 po. 1 ft. 6 in., find its breadth.

2. Simplify $\frac{1835}{2202} + \frac{5468}{12303} + \frac{147}{441} - 3\frac{1}{4}$ of $\frac{6·25}{5·5}$ of $\frac{·04}{1·\dot{2}8571\dot{4}}$.

3. Find the value of 237 c. 17 mds. 6 v. at *Rs.*4100. 1*a.* 4*p.* per candy.

4. 300 coolies are set to build a tank-bund. In 14 weeks they have done $\frac{7}{10}$ of the work, when rain stops the work for 4 weeks and washes away $\frac{2}{7}$ of what they have done. At the end of that time the work is resumed with only 250 coolies. In what time from the commencement will the work be finished?

5. Find the amount of *Rs.*58,59,375 for 3 years at $4\frac{4}{5}$ per cent. per annum, reckoning Compound Interest.

6. Explain the difference between Discount and Interest. If the discount on £2830. 15*s.* $7\frac{1}{2}$*d.* be equal to the simple interest on £2784. 7*s.* 6*d.* for the same time, find the time, the rate of interest being 5 per cent. per annum.

7. A person invests £34,539 in the 3 per cents. at 87. After receiving one year's dividend he sells out at 89. He then invests the whole in Railway stock paying 5 per cent. at 115. What will the difference in his income be?

8. A cistern 10 ft. 6 in. long by 7 ft. 6 in. wide by 3 ft. 4 in. high is lined inside with lead, 7 lbs. of which cover a square foot. Find the weight of the lead and its cost at 53*s.* 4*d.* per cwt.

9. A cask contains 16 gals. of spirits. Two gallons are drawn off and the cask filled up with water. Two gallons are again drawn off and the cask filled up as before. This is done a third time. Compare the quantities of spirits and water remaining in the cask.

10. Find the square root of 579749833·583241.

1891.

2. Subtract 13 times *Rs.*17. 6*a.* 11*p.* from 17 times *Rs.*13. 6*a.* 11*p.*

3. *Rs.*330. 3*a.* 7*p.* are to be divided among 193 persons, two of whom receive *Rs.*2 each, and ten *Rs* 3 each. The others receive equal shares. Find the value of each share.

4. Find the value of $\frac{\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}}{\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\frac{1}{6}} \times 3\frac{1}{2} + \frac{\frac{1}{15}+\frac{1}{20}+\frac{1}{24}}{\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}-\frac{1}{6}}$ and simplify (without reduction to vulgar fractions if you can)

$$2\cdot03 + 1\cdot3\dot{4}\dot{5} + 27\cdot34 + 16\cdot\dot{2}31\dot{7}.$$

5. How long will it take to walk round a square field 14 acres 1 rood 24 poles in extent at the rate of 3 miles an hour?

6. Find the cost of white-washing a room $22\frac{1}{2}$ ft. by 12 ft. and 11 ft. high, at one anna per square yard, making allowance for four

windows each 4 ft. × $2\frac{1}{2}$ ft. and two doors each $8\frac{1}{2}$ ft. × 4 ft. Find also the cost of a carpet for the same room with 3 ft. border all round, the carpet costing *Rs.*4 per square yard and the border *Rs.*6 per square yard.

7. Find the compound interest on £3143. 6*s*. 8*d.* for 3 years at 3 per cent. per annum.

8. A cistern can be filled by three pipes in 30, 40 and 60 minutes respectively, and emptied by an escape pipe in half an hour. The three taps are turned on at noon, but the escape pipe is at the same time accidentally left open and not closed for a quarter of an hour. At what time will the cistern be full ?

9. I purchase 16 ℔s. of tea at 1*s*. 7*d*. per ℔., 14 at 2*s*. 2*d*. and 17 at 1*s*. 8*d*. Seven pounds of the mixture becoming spoiled have to be sold at a low price, but by selling half the remainder at 2*s*. 4*d*. per ℔. and the other half at 2*s*. $7\frac{1}{2}$*d*., I eventually make a profit of 25 per cent. on the original outlay. At what price per pound was the spoiled tea sold ?

10. A person invests a sum of money in the 4 per cents. at 102. When they have risen to 104, he transfers *Rs.*6000 stock to another investment paying 5 per cent. of which the shares are at 120. When the 4 per cents. fall to par, he transfers the remainder to the 5 per cents., which are still at the same price and now finds his income *Rs.*25 more per annum than it was at first. What was the sum originally invested ?

1892.

2. Simplify $\dfrac{\frac{5}{7} \text{ of } 1\frac{3}{8} - \frac{5}{8} \text{ of } \frac{6}{7}}{1 - \frac{1}{7} \times (\frac{5}{12} + \frac{1}{3})} \times \dfrac{\frac{1}{3} + \frac{1}{7} \div (\frac{1}{7} - \frac{1}{8})}{(\frac{1}{3} + \frac{1}{7}) \div \frac{1}{7} - \frac{1}{9}}$.

3. Find the value of $\cdot 041\dot{6}$ of £33. 7*s*. 6*d*. − $\cdot 03\dot{4}\dot{5}$ of £32. 13*s*. $1\frac{1}{2}$*d*. : and express *Rs.*371. 2*a*. 6*p*. as the decimal of a lakh of rupees.

4. Find by any method the cost of making a road 37 mi. 6 fur. 31 po. 3 yds. long at *Rs.*1785. 3*a*. 4*p*. per mile.

5. Find the present value of £482. 6*s*. $10\frac{1}{2}$*d*. due three years hence at 5 per cent. per annum, Compound Interest.

6. Extract the square root of 13·697142031225 to six places of decimals.

7. The annual rainfall of a district is 49·7 inches. Assuming that the fall is distributed uniformly over the district, and that a cubic foot of water weighs 62·5 ℔s, find the weight in tons of the rain that falls throughout the year on a square mile.

8. When exchange is 1*s*. $2\frac{5}{8}$*d*. per rupee, a Madras bookseller sends to a London publisher a bill for £104 in payment of books ordered. Freight and landing charges amount to *Rs.*37. 8*a*. The

publisher allows the bookseller discount at the rate of 35 per cent. on the published price and the latter sells the books at the rate of $10\frac{1}{2}$ annas per shilling of the published price. Find how much he gains on the transaction.

9. In the year 1891, the cost of rice for a family of 2 adults and 4 children was *Rs.*86. 7*a.* 9*p.* In that year rice was sold at 11·2 seers per rupee, and each child received two-fifths of the amount given to an adult. Assuming that in 1893, the price of rice will be 13·5 seers per rupee, what will be the cost of rice for the same family from January 5 to August 11 both days inclusive, if the allowance of each adult be increased by one-fourth and the allowance of each child be three-sevenths of that of an adult ?

10. The capital of a railway company amounts to *Rs.*18,90,00000, of which one-fourth is 5 per cent. preference stock and one-third $4\frac{1}{2}$ per cent. preference stock. In a certain year the receipts are *Rs.*18,150,000, and the working expenses amount to 55 per cent. of the receipts. Of the net receipts *Rs.*540000 are added to the reserve fund, and the remainder, after paying dividend on the preference stock, is divided among the ordinary shareholders. What rate of interest will they receive ?

11. In the ten years from 1871 to 1881, the population of a country increased at the rate of 9·5 per cent., and in ten years from 1881 to 1891, the rate of increase was 10·5 per cent. If the population in 1891 was 31,023,759, find what it was in 1871.

1894.

2. Simplify $\dfrac{\frac{2}{3}-\frac{1}{2}\text{ of }\frac{7}{8}+\frac{8}{9}}{\frac{4}{9}\div\frac{8}{7}-\frac{1}{5}+\frac{2}{3}}-\dfrac{\frac{6}{11}-\frac{4}{9}}{1-\frac{5}{6}\div\frac{2}{3}}$.

3. Find the value of $2\cdot0475\dot{2}$ of £2. 2*s.* 1*d.*$-1\cdot734375$ of £2. 6*s.* 8*d.*

4. Find by any method the value of 59 ca. 14 m. 7 v. 27 pl. of salt at *Rs.*26. 10*a.* 8*p.* per candy.

5. In a certain year the produce of a tea-estate was sold in London at an average rate of $9\frac{1}{2}d.$ per lb, and the amount realised was remitted at an average rate of exchange of 1*s.* $2\frac{1}{4}d.$ per rupee. In the following year the average price realised was only $8\frac{3}{4}d.$ per lb., but the quantity sold exceeded by $12\frac{1}{2}$ per cent. the quantity sold in the previous year, and the average rate of exchange at which remittances were made fell to 1*s.* $1\frac{1}{2}d.$ If in this year the total amount realised from sales in London was *Rs.*105000, find how much was realised in the previous year.

6. A sum of money was invested for four years, interest payable annually. The rate of interest was 5 per cent. per annum for

the first two years and 4 per cent. per annum for the last two; and the amount at the end of four years was £1,164. 10s. 3⅞d. What was the sum invested?

7. Ice is manufactured for 2⅛ pies per lb. and sold at 6 pies per lb. Of the total quantity made one half is kept for sale at the factory, and the remainder sent to branch shops. The loss from melting is 12½ per cent. in the case of the former and 25 per cent. in the latter; and the agents at the branch shops receive commission at the rate of 15 per cent. on the price of every pound sold by them. Find the profit on every ton of ice manufactured.

8. Two persons, *A* and *B*, set out together on a journey. They walked at the rate of 3 miles an hour; and after they had proceeded for three quarters of a mile, *B* returned, walking at the same rate, to the place of starting. Here he was detained three quarters of an hour. Setting out again he overtook *A* who had been walking all the time, at the end of 2½ hours from the second time of starting. At what rate did he walk?

9. A person sold 25 Bank of Madras shares and invested the proceeds in the Government 3½ per cents., when they were at 3¼ premium. If his net annual income from the investment, after paying income-tax at the rate of 5*p*. in the rupee, be *Rs*.876. 9*a*., find the price at which he sold each of his bank shares.

10. In the year 1891 the population of a country was 35640000 and there were 1025 females to every 1000 males. Of the total population 7·5 per cent. could read and write, but of the females only 1 per cent. could do so. Find what percentage of males could read and write.

11. Extract the square root of 81·13183159704101 to seven places of decimals.

BOMBAY MATRICULATION PAPERS.

1885-86.

1. Reduce $\frac{2\frac{1}{2}-\frac{5}{6}}{2\frac{1}{2}+\frac{5}{6}}$ of 2 gui. + $\frac{7}{12}$ of $\frac{9+\cdot\dot{1}}{14\times 3}$ of 4 cr. − $\frac{\cdot 8\dot{3} \text{ of } 1\frac{5}{7}-\frac{1}{7}}{1\frac{3}{7}}$ of £1 to the decimal of five half-guineas, and prove that $\frac{6+5}{11+7}$ is greater than $\frac{6}{11}$ and less than $\frac{5}{7}$.

2. A man contracts to perform a piece of work in 30 days and immediately employs 15 men on it; at the end of 24 days the work is only half done. How many boys should be given to assist them that the contract may be fulfilled, each boy working two-fifths as much as each man?

3. A person buys 80 tons of coal and after selling them again at 1*s.* 6*d.* per sack finds that he has gained £4; had he sold them for 1*s.* 4*d.* per sack he would have lost £6. Find the weight of each sack and the cost price per ton.

4. A field of 7 acres is sown with wheat, barley and maize, the areas of the crops being respectively as $2\frac{1}{2} : 3\frac{1}{3} : 4\frac{1}{2}$. If the values of an acre of each be also respectively in the same ratios, and an acre of wheat be worth £7, what is the worth of all the crops in the field?

5. If the three per cents. are at $92\frac{3}{4}$ and the four per cents. at $123\frac{1}{4}$, in which should one invest? And how much is one investing when the difference in income is a shilling?

1886-87.

1. Explain carefully the meaning of *prime number*, *factor*, *divisor*, *measure*, *multiple*.

Resolve 5005 into its prime factors.

Add together as decimals $8\cdot1\dot{3}\dot{8}$, $14\cdot6\dot{5}65\dot{1}$, $\cdot205\dot{0}896\dot{3}$.

2. The circumference of the fore-wheel of a carriage is $6\frac{7}{8}$ feet and that of the hind-wheel is $12\frac{5}{8}$ feet. How many feet must the carriage pass over before the wheels shall have made a complete number of revolutions?

3. A vessel is filled with a liquid, 3 parts of which are water and 5 parts syrup. How much of the mixture must be drawn off and replaced with water so that the mixture may be half water and half syrup?

4. (i) The surface of a cube is $308\cdot1\dot{6}$ square feet. Find the length of its edge.

(ii) Extract the cube root of $45\cdot\dot{6}9\dot{8}$ to four places of decimals.

5. If the price of gold be £3. 10*s.* 10½*d.* an ounce and a cubic inch of gold weigh 10 ounces, what is the price of the gold that would be required to gild a dome whose surface is 5,000 square feet, the thickness of the gold gilding being ·0002 of an inch?

6. A person invests in 4 per cent. Government paper so as to receive 4 per cent. clear when the income-tax is 5 pies in the rupee. What percentage will be received if the tax be increased to 7*p.* in the rupee?

1887-88.

1. Simplify $\dfrac{\cdot\dot{1}4285\dot{7} \times \cdot\dot{0}7692\dot{3}}{\cdot\dot{0}1098\dot{9}} + \dfrac{2\cdot7\dot{5} \times 11\cdot25}{6\cdot2}$.

2. If 9 lbs. of rice cost as much as 4 lbs. of sugar, and 14 lbs. of sugar are worth as much as 1½ lbs. of tea, and 2 lbs. of tea worth 5 lbs. of coffee, find the cost of 11 lbs. of coffee, if 2½ lbs. of rice cost 6¼*d*

3. If *Rs.*165. 14*a.* $1\frac{7}{17}$*p.* be the discount of a debt of *Rs.*2820, simple interest being at the rate of 3¾ per cent., how many months before due was the debt paid?

4. The price of gold is £3. 17*s.* 10½*d.* per oz.; a composition of gold and silver weighing 18 lbs. is worth £637. 7*s.*, but if the proportions of gold and silver were interchanged it would be worth only £259. 1*s.* Find the proportion of gold and silver in the composition and the price of silver per oz.

5. By selling 4 dozen mangoes for 13 rupees, it was found that $\frac{3}{10}$ths of the outlay was gained; what ought the retail price per mango to have been in order to have gained 60 per cent.?

1888-89.

1. One clerk has $24\cdot\dot{4}2857\dot{1}$ and a second clerk has $38\frac{4}{7}$ sheets to engross; they call in a third clerk and agree to divide the work equally among the three, and to pay the third clerk at the rate of $\cdot2430\dot{5}$ shilling per sheet; how much will he receive from each of them?

2. If the manufacturer makes a profit of 20 per cent., the wholesale dealer a profit of 25 per cent., and the shopkeeper a profit of 40 per cent., what was the cost of the manufacture of an article bought at a shop for 17*s.* 6*d.*?

3. If 15 men eat 28 shillings worth of bread in 14 days, when wheat is at 52 shillings per quarter; what must be the price of wheat per quarter that 18 shillings worth may provide bread for 13 men for 5 days?

4. Find the value of $\sqrt{(902525080 1\cdot7424)} - \sqrt[3]{(347740371\cdot686161)}$.

5. If the discount on £678. 8*s.* which is due at the end of a year and a half be £38. 8*s.*, what is the rate per cent. of Simple Interest?

1889-90.

1. Simplify : $\dfrac{5\frac{1}{3} \text{ of } \cdot\dot{2} \text{ of } 2\cdot\dot{5}7142\dot{8} - 1 + (\frac{1}{3} + \cdot 5)}{1 - \frac{3}{14} \text{ of } \left\{ \cdot 5 + \frac{1}{2} \text{ of } \dfrac{\cdot 05}{\cdot\dot{1}4285\dot{7} \text{ of } 1\frac{1}{20}} \right\}}$.

2. A rectangular cistern, whose length is equal to its breadth is $5\frac{5}{8}$ feet deep and contains 5 tons of water. If a cubic foot of water weighs 1000 ounces, find the dimensions of the cistern.

3. *A*, *B* and *C* can walk at the rate of 3, 4, and 5 miles an hour; they start from Poona at 1, 2, 3 o'clock respectively; when *B* catches *A*, *B* sends him back with a message to *C*; when will *C* get the message?

4. If I borrow money at 3 per cent. per annum, interest payable yearly, and lend it immediately at 5 per cent. per annum, interest payable half-yearly (receiving compound interest for the second half year), and gain thereby at the end of the year *Rs.*660;' what was the sum of money which I borrowed?

5. A person buys tea at 6 annas per seer and also some at 4 annas per seer. In what proportions must he mix them so that by selling the mixture at $5\frac{1}{4}$ annas per seer he may gain 20 per cent. on each seer sold?

1891-92.

1. Simplify (i) $\dfrac{\frac{7}{18} \text{ of } \frac{91}{128} + \frac{5}{9} \text{ of } \frac{13}{88}}{5\frac{2}{77} \text{ of } \frac{122}{133} - \frac{299}{323} \text{ of } \frac{19}{21}}$.

(ii) $\dfrac{3\cdot 6\dot{4}2857\dot{1} - (\cdot 009923 + \cdot 0102 - \cdot 000123)\dfrac{\cdot 145}{\cdot 0056}}{\sqrt{34\cdot 5744} - \sqrt[3]{9\cdot 663597}}$.

2. Two passengers have together 5 cwt. of luggage and are charged for the excess above the weight allowed 5*s.* 2*d.* and 9*s.* 10*d.* respectively; but if the luggage had all belonged to one of them he would have been charged 19*s.* 2*d.* How much luggage is each passenger allowed to carry free of charge, and how much luggage had each passenger?

3. Two clocks *A* and *B*, whose rates are uniform, at noon yesterday indicated 11 hrs. 55 min. A.M. and 0h. 2m. P. M. respectively. *A* indicated the correct time at 9 P.M. yesterday and *B* at 6 A. M. this morning. When did *A* and *B* last agree and what time did they then indicate?

4. A person borrows two equal sums of money at the same time at 5 per cent. and $3\frac{3}{4}$ per cent. simple interest respectively, and finds that if he repays the former sum with interest on a certain date a year before the latter, he will have to pay in each case the same amount, *viz.* *Rs.*736. Find the amounts borrowed.

1892-93.

1. What decimal of a rupee is ·964 pie? Find the value of *Re.*·97625.

Simplify : $\dfrac{\frac{5}{14}-\frac{3}{7}\text{ of }\frac{1}{2}}{\frac{5}{16}+\frac{7}{12}\text{ of }3\frac{1}{4}-(\frac{7}{8}\text{ of }\frac{27}{21}-\frac{1}{8})}+\dfrac{\frac{1}{8}\text{ of }\frac{1}{3}+\frac{3}{8}\text{ of }5}{9\frac{1}{8}-1\frac{3}{8}}$.

2. How long will two examiners, working 8 hours a day, take to look over the answers to this paper, if four examiners, working 5 hours a day, can do it in 8 days?

3. On a stream, *B* is intermediate to and equidistant from *A* and *C*; a boat can go from *A* to *B* and back in 5 hrs. 15 min., from *A* to *C* in 7 hrs. How long would it take to go from *C* to *A*?

4. What income will a retired officer obtain in England, from one lakh of rupees, Indian Government $4\frac{1}{2}$ per cent. bonds, when for drawing and remitting it, his agents in India charge him 3 per cent. and exchange is at 1*s.* $2\frac{1}{4}$*d.* for the rupee?

5. Three equal glasses are filled with a mixture of spirits and water, the proportion of spirits to water in each glass is as follows: in the first glass as 2 : 3, in the second as 3 : 4, and in the third as 4 : 5. The contents of the three glasses are poured into a single vessel; what is the proportion of spirits to water in it?

1893-94.

(*Set in the Mofussil.*)

1. Divide each of the numbers 2,572,125 and 4,961,250 by 125; and express as a decimal the first quotient divided by the second.

2. Find, by Practice, the value of 5 yds. $22\frac{1}{2}$ in. at £2. 1*s.* 2*d.* a yard.

3. If the carriage of 2 cwt. 1 qr. 18 lbs. of goods for 56 miles be £1. 1*s.*, what weight can be carried at the same rate, 200 miles for £4. 3*s.* 4*d.*?

4. A man invests £3,000 in the 5 per cents. If after deducting an income-tax of 8*d.* in the pound, the man's clear income is £174, what is the price of the 5 per cents.?

5. A cistern is filled by two taps *A* and *B* in 4 hours and 6 hours respectively, and is emptied by a waste pipe *C* in 3 hours. When the cistern is half full, *A* and *B* are closed, and *C* is opened; after one hour, *B* is turned on; and after half an hour more, *A* is turned on. In what time after *C* is first opened, does the cistern become full?

6. A person buys two kinds of tea, at 5*s.* a lb. and 6*s.* a lb. respectively and after mixing them he sells the mixture at 6*s.* 6*d.* a lb. thereby gaining 17 per cent. In what proportion does he mix them?

(*Set at Bombay.*)

1. Reduce to their simplest forms : —

(i) $\dfrac{\frac{1}{2}+\frac{1}{3}-\frac{1}{6}}{\frac{1}{2}\text{ of }\frac{1}{3}\text{ of }\frac{1}{6}}$. (ii) $\dfrac{2}{3+\dfrac{4}{5-\frac{6}{7}}}$.

2. Find, by Practice, the value of 9 cwt. 3 qrs. 24 lbs. at £3. 5*s.* 8*d.* per cwt.

3. If 40 men, 60 women or 80 children can do a work in 6 months, in what time will 10 men, 10 women, and 10 children do $\frac{1}{3}$ of the work ?

4. A person invested £1,000 in the 3 per cents. at $90\frac{5}{8}$; but the price rising to $91\frac{1}{4}$, he sold out, and invested the proceeds in the $3\frac{1}{2}$ per cents. at $97\frac{1}{3}$; find the increase in his income.

5. A cistern can be filled by two pipes, *A* and *B*, in 12 minutes and 14 minutes respectively, and can be emptied by a third *C*, in 8 minutes. If all the taps be turned on at the same moment, what part of the cistern will remain unfilled at the end of 7 minutes ?

6. Two clocks point to 2 o'clock at the same instant on the afternoon of 25th April ; one loses 7 seconds, and the other gains 8 seconds, in 24 hours : when will one be half-an-hour before the other, and what time will each clock then shew ?

1894-95.

1. When the number representing the year is a multiple of four, it is a leap-year consisting of 366 days, except when this number is a multiple of 100, in which case it is an ordinary year consisting of 365 days ; but when the number is a multiple of 400, it is again a leap-year ; on this supposition calculate the number of days from the 1st January, 1495 to 31st December, 1894, both days inclusive.

2. A school of boys and girls consists of 453 children ; the number representing the boys is $\cdot\dot{5}\dot{2}$ of the number of girls. How many boys were there ?

3. Two-thirds of a certain number of poor persons received 1*s.* 6*d.* each, and the rest 2*s.* 6*d.* each ; the whole sum spent being £2. 15*s.*, how many poor persons were there ?

4. If 3 men and 5 women do a piece of work in 8 days, which 2 men and 7 children can do in 12 days, find how long 13 men, 14 children and 15 women will take to do it.

5. *A* sells a house to *B* for *Rs.*4860, thereby losing 19 per cent. ; *B* sells it out to *C* at a price which would have given *A* 17 per cent. profit. Find *B*'s gain.

6. The compound interest on one rupee is one quarter of a rupee at the end of three years ; find the rate per cent. per annum, correct to two places of decimals, and calculate exactly the compound interest at the end of 9 years.

PUNJAB ENTRANCE PAPERS.

1885.

1. Simplify $\frac{\frac{1}{3}-\frac{1}{6}+\frac{2}{3}}{\frac{1}{4}+\frac{1}{6}}+\frac{\frac{2}{3}+\frac{1}{10}}{\frac{1}{5}-\frac{1}{8}\text{ of }\frac{1}{3}}$ of $\frac{16}{69}-\frac{31}{80}$, and find how many times ·027 can be taken from 3·33.

2. Convert $\frac{13}{20\times 8}$ into a decimal ; why is the result a terminating and not a recurring decimal ? Subtract ·03 from $\cdot\dot{0}\dot{3}$ and divide the result by $\cdot 00\dot{7}$.

3. Find, by Practice, the value of 12 maunds 8 seers 4 chataks of ghee at *Rs.*72. 8*a.* per maund.

4. A legacy of £1901. 5*s.* is to be distributed amongst a number of persons, in such a way that each shall receive as many shillings as there are persons ; what will be the portion of each ?

5. Find the Least Common Multiple of 35280 and 592704. What is the smallest number of square yards which can be measured, either by roods or square chains ?

6 Four per cents. are offered at *Rs.*98, five per cents. at *Rs.*$120\frac{10}{28}$; which is the better investment ? How much is the investment when the difference of income is *Rs.*30 ?

1886.

1. Simplify $\frac{4\cdot\dot{4}-2\cdot 8\dot{3}}{1\cdot\dot{6}+2\cdot\dot{6}2\dot{9}}$, and extract the square root of the result to three places of decimals.

2. Reduce $\frac{5}{7-\frac{1}{2-\frac{1}{3}}}$ to a decimal fraction correct to four places.

Is there anything to suggest that the result will be a terminating or recurring decimal ?

3. What fraction of £51,120. 18*s.* is 17·975 of £71. 2*s.* ?

4. A clever housekeeper went out shopping and found that 2 cocoanuts were selling for the same price as 144 plums ; she

bought half a dozen cocoanuts exchanged one of them for 5 lemons, and a couple of lemons for 5 oranges ; she then gave 3 oranges for 42 limes, and finally secured a couple of plums for 5 limes. Has she gained or lost in buying the plums ?

5. Distinguish between Interest and Discount.

Find the Interest and Discount of *Rs.*1,450. 8*a.* for 3 years at $4\frac{1}{2}$ per cent. per annum, Simple Interest.

1887.

1. (*a*) Write in figures—three billions, five millions, four hundred and nine thousand and sixty-two.

(*b*) Write out the measures of length and surface, both English and Indian.

(*c*) Express an acre as the decimal of a *bigha*, a cubit being equivalent to 18 inches.

2. Owing $\frac{4}{17}$ of an estate, I sold $\frac{8}{11}$ of $\frac{3}{8}$ of my share for £$\frac{400}{33}$, what is the value of $\frac{1\frac{2}{3}}{4\frac{1}{4}}$ of $\frac{3}{5}$ of the estate at the same rate ?

3. A merchant having 150 maunds of grain sold 50 maunds at *Rs.*9. 1*a.* $1\frac{1}{3}$*p.* per maund, and thereby gained $7\frac{1}{2}$ per cent. At what rate should he sell the remainder 100 that he may gain 10 per cent. on the whole ?

4. A merchant in trade successively admits three partners at the end of 3 months, 5 months, and 6 months respectively from the opening of the business. The capitals embarked by them were *Rs.*400, *Rs.*450, *Rs* 480, and *Rs.*495 respectively. After 6 months more, the profit was found to be *Rs.*1,518. Divide this rateably between the partners.

5. What sum of money invested in the 4 per cents. at par would realise the same income as *Rs.*10,200 invested in the $4\frac{1}{4}$ per cents. at 102 ?

6. Extract the square root of :—

$$\frac{.0025\times 1.\dot{6}}{3.\dot{6}-2.5} \text{ of } \frac{.42\dot{6}\times 2.625}{12.7-10.2}.$$

1888.

1. Simplify $$\frac{1}{1-\frac{1}{12}}-\frac{1-\frac{7}{22}}{2-\frac{1}{3}}+\frac{1\frac{2}{5}}{4-1\frac{1}{2}}-\frac{6\frac{2}{3}-\frac{3}{4}}{6\frac{1}{4}}\times\left\{\frac{1}{5}-\frac{\frac{1}{2}-\frac{1}{8}}{4\frac{3}{4}-3\frac{5}{6}}\right\}.$$

2. Express the difference between $.\dot{3}7\dot{8}$ of 13*s.* $10\frac{1}{2}$*d.* and $.3\dot{7}\dot{8}$ of 16*s.* 6*d.* as the decimal of

$$.42\dot{6} \text{ of } \frac{3.\dot{3}}{.08} \text{ of } \frac{.3}{.\dot{7}3\dot{5}} \text{ of } \frac{.147\times 4.\dot{4}}{11.1} \text{ of £1. 17s. 6d.}$$

3. Four men working together all day, can finish a piece of work in 11 days ; but one of them having other engagements can work only half time, another only quarter time. How long will it take the men to complete the work ?

4. A merchant sells his goods worth *Rs.*500 directly for *Rs.*600, giving three months' credit. Find his profit per cent., interest being calculated at 12 per cent. per annum.

5. Find the value of $\frac{12+\sqrt{(\cdot 009)}}{1-\sqrt{\cdot 4}}$ correct to 3 places of decimals.

1889.

1. Express 80080080·0975 in words and give the local value of the digits. What decimal of *Rs.*75 is *Rs.*24. 2*a* 6*p.* ?

What is the least number which, when divided by 22, by 88, by 132, and by 198, gives in each case a remainder 7 ?

2. Why is the fraction $\frac{3}{\frac{5}{8}}$ objectionable ?

After walking $4\frac{1}{2}$ miles, a man has accomplished $\frac{2\frac{1}{2}-1\frac{1}{3}\text{ of }2\frac{1}{4}+1\frac{7}{8}}{(2\frac{1}{2}-1\frac{1}{3})\text{ of }(2\frac{1}{4}+1\frac{7}{8})}$ of $\frac{\frac{2}{11}+\frac{1}{3}}{\frac{4}{7}-\frac{3}{11}}$ of his journey ; how far has he still to walk ?

3. Add together $\frac{5\cdot 7}{1\cdot 52}$ and $\frac{\cdot 01\dot{1}\dot{2}}{\cdot 74}$.

Five bells which commence tolling together, toll at intervals of 1·2, 1·5, 1·75, 1·8, 2·1 seconds respectively ; after what interval will they again toll together ?

4. Define "present worth."

A farmer buys 57 sheep for *Rs* 120 payable at the end of 12 months and sells them directly at *Re.*1 12*a.* ready money ; what does he lose by the transaction, supposing the interest of money to be 5 per cent. ?

Which is the better investment, the 3 per cents. at $83\frac{1}{4}$ or the $3\frac{1}{2}$ per cents. at 3 per cent. discount ?

5. Shew which is the greater, $\sqrt{2}$ or $\sqrt[3]{3}$?

1890.

1. Simplify (*a*) $\frac{\frac{\frac{2}{5}}{1-\frac{1}{25}}+\frac{1}{3}+\frac{1}{7}}{1-\frac{1}{7}\left(\frac{\frac{2}{5}}{1-\frac{1}{25}}+\frac{1}{3}\right)}$. (*b*) $\frac{\cdot 47-(\cdot 5-\cdot 0303)}{\cdot 0873-(\cdot 0083+\cdot 07)}$.

2. What part of $\frac{2}{3}$ of 5 cwt. is $\frac{1}{100}$ of a ton ?

Express $\cdot 3\dot{7}\dot{8}$ of 16*s.* 6*d.* as a decimal of $\cdot 42\dot{6}$ of £1. 17*s.* 6*d.*

3. A man bequeathed $\frac{5}{12}$ of his property to one son, 30 per cent. of the remainder to another and the surplus to his widow. The difference of his sons' legacies was £754. How much did the widow receive?

4. A ship with 1200 men on board had sufficient provisions to last 17 weeks. The survivors of a wreck having been taken aboard, the provisions were consumed in 15 days. How many men were taken aboard?

5. At what price must a person invest in the 4 per cent. Government Promissory Note, so that after paying income-tax at the rate of 5 pies in the rupee, he may receive $4\frac{1}{2}$ per cent. on his investment?

6. *A* and *B* travel together 120 miles by rail. *A* takes a return ticket for which he has to pay one fare and a half. Coming back they find that *A* has travelled cheaper than *B* by 4*a.* 2*p.* for every 100 miles. Find the fare per mile.

1891.

1. Simplify :—

$$(1)\ \frac{\frac{1}{7}+\frac{1}{3}+\dfrac{\frac{2}{3}}{1-\frac{1}{28}}}{1-\frac{1}{7}\text{ of }\left(\frac{1}{3}+\dfrac{\frac{2}{3}}{1-\frac{2}{3}}\right)}. \qquad (2)\ \frac{3\sqrt{2}-2\sqrt{3}}{3\sqrt{2}+2\sqrt{3}}+\frac{\sqrt{12}}{\sqrt{3}-\sqrt{2}}.$$

2. Express 2·75 oz. + ·075 cwt. as decimal of $2\cdot2\dot{5}$ of $\cdot\dot{2}\dot{7}$ of a ton.

3. A sum of money invested at 5 per cent. per annum, Simple Interest, amounts in 6 years to *Rs.*1,326; in what time will it amount to *Rs.*1530?

4. What is Discount? Distinguish between True and Commercial Discount.

The interest on a certain sum at 5 per cent. per annum for a certain time is £50 and the discount at the same rate for the same time is £40. Find the sum and time.

5. Nine gallons are drawn from a cask full of wine; it is then filled with water. Nine gallons of the mixture are drawn, and the cask is again filled with water. The quantity of wine now left in the cask is to that of the water in it as 16 : 9. How much does the cask hold?

1892.

1. Find by how much the square root of

$9+\dfrac{1}{1+\dfrac{1}{7+\frac{1}{6}}}$ differs from $\frac{355}{113}$; which of these comes nearest to $3+\frac{1}{10}\sqrt{2}$?

2. Find the value of $\left(\frac{\cdot 0019}{3\cdot 1\dot{6}} \text{ of } \frac{4\cdot\dot{4}}{\cdot 0005}\right) + \left(\frac{8\cdot\dot{8}}{7} \text{ of } \frac{4}{5\cdot 625}\right)$.

3. A stream which flows at a uniform rate of 1·109 miles an hour, is 20 yards wide, the depth of a certain ferry being 6 ft. : how many gallons pass the ferry *in a minute*? (Each gallon contains about 277¼ cubic inches.)

4. A person invests £14,970 in the purchase of the 3 per cents. at 90 and the 3½ per cents. at 97. His total income being £500, how much of each stock did he buy?

5. A spirit merchant buys 80 gallons of whisky at 18*s*. per gallon, and 180 gallons more at 15*s*. per gallon and mixes them. At what price must he sell the mixture to gain 8⅓ per cent. upon his outlay?

1893.

2. Multiply 319·9657 by ·04286.

3. Find the value of $\frac{\sqrt{(2-\sqrt{2})}}{\sqrt{(2+\sqrt{2})}}$ correct to 5 places of decimals.

4. Calculate the income-tax on *Rs*.666. 10*a*. 8*p*. at 5*p*. per rupee.

5. A local train which travels at the rate of 24 miles an hour, leaves Lahore at 20 min. past 8 and reaches Amritsar at 5 min. past 10 the same morning. It stops at Mianmir for 10 min. and at each of three other stations for 5 min. Find the distance between Lahore and Amritsar.

1894.

1. Convert $\frac{4}{7}$ and $\frac{3}{7}$ into circulating decimals and point out the relation between the figures in their periods.

2. The sides of a rectangle are as 3 : 4 and the area is 1452 square feet. Find its length and breadth.

3. Exchange *Rs*.7680 for English money at 1*s*. $3\frac{3}{8}\frac{1}{4}$*d*. per rupee.

4. What is discount? How is it commonly calculated? If a sum of *Rs*.1000 becomes due three months hence, what is its present value as commonly calculated, and what as correctly calculated, interest being reckoned at 5 per cent.?

5. Find the square root of 101 correct to 5 places of decimals.

1895.

1. Divide $\frac{48\frac{4}{7}}{1085\frac{7}{10}}$ by $\frac{71\frac{4}{11}}{174\frac{3}{17}}$, and reduce the quotient to a recurring decimal.

2. The imperial gallon contains 277·27 cubic inches and a cubic foot of water at its maximum density weighs 62·42 lbs.; find the weight of a pint of water correctly to two places of decimals.

3. The capital of a firm consists of £713. 3*s.*; £964. 17*s.*; £2391. 3*s.* subscribed by three partners; divide £2231 among them in proportion to their several capitals.

4. Find the square root of 5 correctly to seven places of decimals.

5. The area of a rectangular field is $\frac{3}{8}$ of an acre; and its length is twice its breadth; determine the lengths of its sides approximately.

1896.

1. Make out a bill for the following articles supplied by Messrs. Mool Chand & Co. to Lala Gujar Mal :—

10 lbs. of tea at *Re.*1. 3*a.* per lb.; 6 seers of sugar at *Rs.*2. 3*a.* per bag of 5 seers; 4 tins of coffee at *Re.*1. 1*a.* per tin; 8 silk handkerchiefs at *Rs.*3. 8*a.* per dozen; 3 mds. 37 sr. of Portland cement at 8 seers per rupee; a child's perambulator, price *Rs.*30. Subtract 10 per cent. discount for cash.

2. Reduce to its lowest terms $\frac{1}{89}$ of $4\frac{1}{3}$ of $\frac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{\frac{1}{4}+\frac{1}{5}+\frac{1}{6}}$.

3. A cubic foot of copper weighs 560 lbs. It is rolled into a square bar 40 ft. long. An exact cube is cut from the bar. What is its weight to four decimals of a pound?

4. The area of a country is 32,300,000 acres. It consists of three kinds of land, the areas of which are in proportion to the numbers 2, 3 and $\frac{2}{3}$. How many acres are there of each kind of land?

5. If the 3 per cent. stock is at $98\frac{1}{2}$, how much money must be invested in the stock to yield an annual income of *Rs.*120?

1897.

1. Define a fraction, and prove that the value of a fraction is not altered by multiplying both its numerator and denominator by the same whole number. Deduce from this principle a rule for the addition of fractions.

2. The sum of £177 is to be divided among 15 men, 20 women and 30 children, in such a manner that a man and a child may together receive as much as two women, and all the women may together receive £60. What will they respectively receive?

3. Find the value of $\frac{\sqrt{(2+\sqrt{2})}}{\sqrt{(2-\sqrt{2})}}$ correct to 7 places of decimals.

4. A garrison of 800 men has provisions sufficient for 10 weeks. How long would they last if the garrison were reduced to 560 men?

5. Find the L. C. M. of $4\frac{1}{2}$, $5\frac{2}{3}$, $6\frac{3}{8}$ and $7\frac{4}{5}$.

1898.

1. State in words the value of the figures 2000690125, and multiply ·056931 by 1879·6958.

2. On what day of the week will December 25 fall next year ?

3. Find the cost of papering the walls of a room 22 ft. long, 18 ft. wide and 20 ft. high, with rolls of paper 21 inches wide, at *Rs.*2. 10*a*. per roll of 12 linear yards.

4. Simplify $\dfrac{\frac{3}{11}(\frac{2}{19} \text{ of } 2\frac{5}{7} + \frac{1}{3} \text{ of } 1\frac{2}{7})}{\frac{2}{13} \times 1\frac{5}{8} \times 1\frac{1}{7} - \frac{1}{11}} + \dfrac{\frac{1}{3}}{\frac{2}{9} \text{ of } 3 - \frac{1}{17} \text{ of } 5\frac{2}{3}}$.

5. A person holding £10,000 in the 3 per cents. sells out at 93⅝, and invests the proceeds in the 4 per cent. stock at 101⅞. Find the change in his income, allowing ⅛ per cent. commission in each transaction.

1899.

1. Express $\cdot 763\dot{9}$ as a non-recurring decimal.

(*a*) Simplify $\cdot 00\dot{6}2\dot{1} + \cdot 1089\dot{1} + 81\cdot\dot{0}56\dot{3} + 21\cdot\dot{0}\dot{2}$ *without reducing the terms to vulgar fractions.*

2. The length of a hall is three times the breadth. The cost of whitewashing the ceiling at 5½*d*. per square yard is £4. 12*s*. $7\cdot\dot{1}$*d*. and the cost of papering the walls at 1*s*. 9*d*. per square yard is £35. Find the height of the hall.

3. Shew that the difference between interest and the true discount on a given sum at a given rate for a given time, is equal to the interest on the discount.

4. A man has £5. 17*s*. consisting of sovereigns, half-crowns and shillings, in the propoition of 2, 3, 11. How many has he of each coin ?

5. Which is the better investment, the 3½ per cents. at 102, or the 3 per cents. at 97 ?

1900.

1. Find the square root of 4001204·090601.

2. Find the present worth of *Rs.*10000 due 8 years hence at 4½ per cent.

3. A rectangular court-yard the sides of which are as 5 : 11, costs *Rs.*144. 6*a*. for paving at 10*a*. 6*p*. per sq. yd. Find the lengths of its sides.

4. Shew that *this* year the 23rd of March, and 23rd of November fall on the same day of the week.

5. Shew that Compound Interest reckoned quarterly at *Re.*1. 3*a*. $7\frac{1}{2}$*p*. per cent. is nearly equal to Interest reckoned yearly at 5 per cent.

1901.

1. Eight bells which toll at intervals of 1, 2, 3, 4, 5, 6, 7, 8 seconds respectively, begin tolling all simultaneously with the clock striking. How many hours must elapse before they all toll simultaneously again with the clock striking? (*N. B.*—The clock is supposed to strike at the hour only.)

2. Find the true discount on a bill for £721. 13*s*. 8*d*. paid 73 days before due, the rate of interest being $3\frac{1}{8}$ per cent. per annum.

3. Divide each of the numbers 4061250 and 2572125 by 125 and express the ratio of the quotients correctly to three places of decimals.

4. A man buys eggs at 1*s*. 3*d*. per dozen and sells them at 11*s*. 8*d*. per hundred. Find his gain per cent.

5. There are four vessels of equal capacity; $\frac{1}{8}$ of the first is filled with spirit, $\frac{1}{6}$ of the second, $\frac{1}{4}$ of the third, and $\frac{1}{3}$ of the last. The first is then filled with water and from this mixture the second is filled up, again from the second mixture, the third is filled up, and in like manner the fourth from the third. What proportion of spirit to water is there in the fourth vessel?

1902.

1. Define a prime number. Find the prime factors of 555,555.

2. A railway truck is 29 ft. 4 in. in length; how many such trucks will be required to fill up the entire length of the line between Lahore and Amritsar, a distance of 32 miles?

3. The difference between the simple and compound interest on a sum of money for 2 years at 5 per cent. per annum is *Rs.*12. Find the sum.

4. If 3 fowls and 4 pigeons cost *Rs.*2. 3*a*. 6*p*., and 5 fowls and 2 pigeons cost *Rs.*2. 12*a*., find what must be paid for 4 fowls and 3 pigeons.

5. A person sold 60 yards of cloth for *Rs.*28. 2*a*. gaining thereby the cost price of 9 yards. Find his gain per cent.

ALLAHABAD ENTRANCE PAPERS.

1889.

1. Define a fraction and show that $\frac{1}{3}=\frac{2}{6}$.

By how much does the difference of $1\frac{1}{26}$ and $\frac{9}{13}$ fall short of their sum ? Express the defect as a decimal.

2. (*a*) Simpiify $\dfrac{3\frac{1}{8}-1\frac{1}{2}\text{ of }1\frac{1}{5}-1\frac{1}{7}}{(3\frac{1}{3}-1\frac{1}{2})\text{ of }(1\frac{1}{8}-1\frac{1}{7})}$.

(*b*) Subtract ·03 from $\cdot\dot{0}\dot{3}$ and divide the result by $\cdot 10\dot{2}$.

3. Find the square root of ·001 to four places of decimals. What number has ·1 for its square root ?

4. What sum of money will amount to *Rs.*1381. 4*a.* in 15 months at 5 per cent. per annum, Simple interest ?

5. How long will it take to walk along the four sides of a square field which contains 16 acres 401 square yards, at 3 miles an hour ?

6. *A* and *B* complete a piece of work in 8 days, *B* and *C* do the same in 12 days, and *A*, *B* and *C* finish it in 6 days. In how many days will *A* and *C* complete the work ?

7. *A* who travels $3\frac{1}{2}$ miles an hour starts $2\frac{1}{2}$ hours before *B* who goes the same road at $4\frac{1}{2}$ miles an hour ; where will he overtake *A* ?

1890.

1. Multiply ·347695 by 2·0066, and divide the product by ·01905.

2. Simplify $1\frac{3}{8}+3\frac{1}{3}-5\frac{5}{6}+2\frac{1}{4}-1\frac{1}{18}$.

3. Find by Practice or otherwise the value of 2345 mds. 27 sr. and 10 ch. of wheat at *Rs.*3. 10*a.* 8*p.* per maund.

4. Extract the square root of $1-(\cdot 00135)^2$ to 5 places of decimals.

5. One cubic inch of water weighs 253·17 grains while one inch of air weighs ·31 grain ; find the number of inches of water (to three places of decimals) that would be equivalent to one cubic foot of air.

6. On measuring a distance of 32 yds. with a rod of a certain length, it was found that the rod was contained 41 times with $\frac{1}{2}$ an inch over. How many inches will there be over in measuring 44 yds. with the same rod ?

1891.

1. Define " Notation," " Numeration" ; and prove that " three times four " = " four times three."

2. Reduce to a single fraction $\frac{919\frac{2}{17}}{79\dot{5}\dot{4}} \times \frac{4\cdot\dot{1}0\dot{0}}{442\frac{80}{87}} \times \frac{7}{11}$ of ·07344.

3. The wine in a pipe when full is worth £19. 9*s.* 9*d.* How much has leaked away, if what is left is worth £9. 16*s.* $7\frac{85}{224}$*d.* ?

4. In discounting a bill, what do you mean by "the Banker's profit ?" If the simple interest on £923. 18*s.* $1\frac{1}{2}$*d.* amounts to £17. 9*s.* $3\frac{3}{4}$*d.* exactly in 138 days, what is the rate of interest per cent. per annum ?

5. Extract the square root of 99,980, 001 ; and of $60\frac{61}{169}$.

1892.

1. How is a fraction affected by adding the same number to the numerator and the denominator ?

Prove that $\frac{3+4}{4+5}$ is greater than $\frac{3}{4}$ and less than $\frac{4}{5}$.

2. (*a*) Divide $\frac{1}{3}[3+\frac{1}{3}\{3+\frac{1}{3}(3+1\frac{1}{2})\}]$ by ·125.

(*b*) Reduce $\frac{1744}{1532}$ and $\frac{1526}{3232}$ to their lowest terms and express their difference as a decimal.

3. Forty men finish a piece of work in 40 days ; if 5 men leave the work after every tenth day, in what time will the whole work be completed ?

4. Find the difference between the Simple Interest and Discount of £330 in 4 years at $2\frac{1}{2}$ per cent. per annum.

5. Extract the square root of $\frac{1000\cdot20001}{1000}$.

1893.

1. Two recurring decimals are added together ; prove that the number of digits in the period of the result, cannot exceed the product of the numbers of the digits in the original periods.

2. Find the value of $\cdot\dot{5}\dot{4}$ of $\cdot30\dot{7}\dot{2}$ of 1 mile 5 fur. 30 poles.

3. Multiply *Rs.*2. 1*a.* by $\frac{\frac{4}{5}+\frac{6}{7}+\frac{13}{15}+\frac{10}{21}}{\frac{1}{5}+\frac{5}{7}+\frac{7}{15}+\frac{13}{21}}$.

4. Find by Practice the cost of 10 cwt. 3 qrs. 23 lbs. 8 oz. at £1. 5*s.* 8*d.* per cwt.

5. A sum of money was divided amongst 5 people ; 4 of them received respectively ·15, $\frac{3}{40}$, ·1, $\frac{3}{8}$ of the whole, while the 5th received £105. 3*s.* 6*d.* What was the sum divided ?

6. An oz. of standard gold, $\frac{1}{12}$ of which is alloy is worth £3. 17*s.* $10\frac{1}{2}$*d.* ; how many sovereigns would be coined from 36 lbs. 8 oz. of pure gold ?

7. Find the square roots of 6246·057024 and $71\frac{154}{225}$.

1894.

1. (*a*) A multiplication sum having been worked is partially rubbed out, the figures that remain are the entire multiplicand 999 and the last three digits 193 in the product. Restore the complete work.

(*b*) Simplify $\frac{1}{1\frac{1}{20}} \times \frac{1+\cdot0025+\cdot05}{1\cdot0025-\cdot05} - \frac{45+\cdot3\dot{5}}{8}$.

2. (*a*) What decimal of *Rs.*100 must be added to $\frac{1}{1\frac{4}{9}\frac{1}{6}}$ of *Rs.*5. 10*a.* 8*p.*, that the sum may be 10 annas?

(*b*) Extract the square root of 25·6.

3. Two trains start at the same time from Mirzapore and Delhi and proceed towards each other at the rates of 16 and 21 miles per hour respectively. When they meet it is found that one train has travelled 60 miles more than the other. Find the distance between the two stations.

4. Two years and six months ago I borrowed a sum which with simple interest at 6 per cent. per annum now amounts to *Rs.*638-4-0 Find the sum.

1895.

1. (*a*) Explain what is meant by the following terms :—

Prime factors ; common measure ; common multiple ; lowest common multiple.

(*b*) A court-yard 452 feet long and 404 feet wide, is to be paved with square stones all of one size. What is the largest size which can be used ?

2. (*a*) Simplify $\frac{5\cdot75}{4\cdot25}$ of $\frac{1}{4} \div \frac{8}{7} + \frac{4}{9} \times \frac{3}{5} + \frac{1}{2}$.

(*b*) Find the square root of 3·1415926 to four places of decimals.

3. The *difference* between the interest for 4 months and the Discount on a certain sum due in 4 months at 4 per cent., is one rupee. What is the sum ?

4. A merchant sells silk of two qualities which cost him *Rs.*5. 5*a.* 4*p.*, and *Rs.*4. 4*a.* 4*p.* per yard respectively. The selling price of the latter is two-thirds that of the former, but the quantity sold is double and the merchant gains 25 per cent. on the whole. Calculate the selling price per yard of each.

5. A policeman goes after a thief who has 100 yards' start ; if the policeman run a mile in six minutes, and the thief a mile in ten minutes, how far will the thief have gone before he is overtaken ?

1896.

1. Simplify :—

(a) $5-5\times\frac{2+1\frac{1}{6}(2+1\frac{1}{6})}{1\frac{1}{6}+2(2+1\frac{1}{6})}$.   (b) $\frac{\cdot125\times(\cdot175 \text{ of } \cdot\dot{2}8571\dot{4})}{\cdot00025}$.

2. (a) Express $\frac{3}{8}$ of 7*s*. 6*d*. + 1·25 of 5*s*. − ·5$\dot{4}\dot{5}$ of 9*s*. 2*d*. as a decimal of £10.

(b) Extract the square root of 40000·400001.

3. What is an aliquot part of a quantity ?

Find, by Practice, the time of building a wall 27 yards long, 1 yard thick and 6 ft. high, of which one cubic yard is built in 3 hours 18 minutes and 45 seconds.

4. How far shall I ride with a friend who leaves Allahabad at 9 A. M. and will drive to Karchana which is 10 miles from Allahabad in one hour, that I may by walking back at the rate of 4 miles an hour, reach home at 11-30 A. M. ?

5. *A* owes *B Rs*.1435 due at the end of 4 months, *Rs*.630 due at the end of 8 months, *Rs*.860 due at the end of a year. *B* wants his money forthwith. What ought *A* to pay him reckoning interest at $7\frac{1}{2}$ per cent. ?

1897.

1. What is the largest number which divides both 2397 and 2491 without remainder ? What is the smallest number which is divisible by both of these numbers ?

2. State and prove the rule for pointing in multiplication of decimals. Why is the removal of the decimal point one place to the right equivalent to multiplication by 10 ? Illustrate your answer by comparing the numbers 23·015 and 230·15.

Find the square root of ·0802$\dot{7}$.

3. A person lent another a sum of money for 72 days at 3 per cent. per annum. At the end of that time he received £293. 12*s*. $0\frac{3}{4}$*d*. What was the sum lent ?

4. The compound interest on a sum of money for 3 years at 5 per cent. is £331. 0*s*. 3*d*. ; what is the simple interest ?

5. If a rupee is worth one shilling and three pence half-penny, and a shilling is worth 1·25 francs, what is the value in francs of 1,365 rupees ?

1898.

1. Define *measure* of a number and find the G. C. M. of :—

(i) *Rs*.2. 4*a*. and 10*a*.   (ii) $\frac{3}{4}$ and $\frac{5}{6}$.

(a) Find the greatest number which will divide 13956 and 14565 and leave a remainder 7 in each case.

2. Simplify :—

(a) $\dfrac{(\frac{1}{2})^2+(\frac{1}{3})^3}{(\frac{1}{3})^2+(\frac{1}{2})^3}+\dfrac{\cdot125 \text{ of } Rs.5.\ 10a.\ 8p.}{\cdot\dot{0}3\dot{7} \text{ of } Rs.7.\ 12a.}$

(b) $\frac{1}{2}-\frac{1}{2}\div\frac{1}{3}$ of $\frac{1}{4}+\frac{1}{5}\times\frac{1}{6}$.

3. Extract the square root of $9+\dfrac{1}{1+\dfrac{1}{7+\frac{1}{6}}}$ and calculate the difference between this square root and $3+\frac{1}{10}\sqrt{2}$ to three places of decimals.

4. Find the cost in English money of travelling from Vienna to Trieste, a distance of 363 English miles, the average cost per German mile being 13 kreutzers. (Given that 1 German mile = $4\frac{1}{2}$ English miles ; £1 = 25·5 francs ; 3 75 francs = 105 kreutzers.)

5. What is the present value of a legacy of £149. 1*s.* 3*d.* due 7 years hence, at $2\frac{3}{4}$ per cent. simple interest ?

1899.

1. Simplify $\dfrac{\frac{1}{2}+\frac{1}{3} \text{ of } \frac{1}{4}+\frac{5}{8}}{\frac{1}{18} \text{ of } (1+5\frac{1}{2})+\frac{5}{6} \text{ of } \frac{1}{23} \text{ of } (7-2\frac{2}{3})-\frac{1}{3}}$ and express $\frac{3}{8}$ of *Re.*1. 5*a.* as the decimal of *Re.*1. 4*a.*

2. A number may be divided by 125 by multiplying it by 8 and then marking off the last three digits as decimals. Explain the reason for this ; and divide 5335 by 125.

3 What is the meaning of an "*aliquot part*" ?

Find by Practice the value of 24 tons 3 cwt. 2qrs. 25 lbs. at £17. 11*s.* 6*d.* per ton.

4. A piece of work can be done in 72 days by 17 men working together. If after 9 days of work, these are joined by 4 others, in how many days will the work be finished ?

5. Extract the square root of 5 and ·5 each to 4 places of decimals ; and shew that the square root of $\cdot\dot{4}$ is $\cdot\dot{6}$.

6. What is the difference between the interest on a bill of £138. 13*s.* 4*d.* for 3 months, at 4 per cent. per annum, and the discount on the same for a quarter of a year, at the same rate ?

7. (*a*) A speculator sells at a profit of 50 per cent. ; but his purchaser fails, and only pays 8*a.* in the rupee. How much per cent. does the speculator gain or lose by his venture ?

(*b*) A person investing in the 4 per cents, receives 5 per cent. for his money. What is the price of stock ?

1900.

1. State the rules for multiplication and division of decimal fractions.

Assuming that the surface of a sphere is 3·1416 times the square of its diameter, and that the earth is a sphere whose diameter is 8000 miles, find what fraction of the whole surface of the earth is the area of India which is 1350000 square miles. Express your result as a decimal fraction.

2. What are circulating decimals? Distinguish between pure and mixed circulating decimals.

(*a*) Add together $\frac{1}{6}$, $\frac{1}{30}$, $\frac{1}{42}$, $\frac{5}{66}$ and express the sum as a mixed circulating decimal.

(6) Reduce $\cdot 041\dot{6} \times \dfrac{\cdot\dot{1}4285\dot{7}}{(\frac{5}{12}+\frac{3}{8}) \times 40}$ of *Rs.* 8. 5*a.* to the fraction of 1*a.*

3. (*a*) Find, by Practice, the price of 100 bags of Rosa sugar, each weighing 4 seers 2 powas and 3 chataks, at 6*a.* 9*p.* per seer.

(*b*) Find the square root of $10 \cdot \dot{0}\dot{2}$ to three places of decimals.

4. What sum of money will amount to *Rs.*3528 in two years at 5 per cent. compound interest? and what will it amount to in two more years?

5. What monthly income will be derived from the investment of one lac of rupees in the $3\frac{1}{2}$ per cent. Government of India paper at $100\frac{13}{16}$?

1901.

1. (*a*) What is the greatest length which is contained a whole number of times exactly in both $25\frac{15}{28}$ feet and $21\frac{9}{20}$ feet?

(*b*) Find the value of

$$\frac{\cdot 49}{2 \cdot 1} \text{ of } \frac{(3\frac{1}{3} - 2\frac{1}{2}) + \frac{5}{6} \text{ of } \frac{3}{8}}{2\frac{2}{3} \div (\frac{1}{3} + \frac{1}{4})} \text{ of } £46.$$

2. (*a*) Express the difference between $\cdot 9\dot{4}2857\dot{1}$ and $\cdot\dot{8}5714\dot{2}$ as a vulgar fraction in its lowest terms.

(*b*) Extract the square root of

$$\frac{\cdot 0253 \times \cdot 365}{8 \cdot 03}$$ to five places of decimals.

3. In a two-mile race *A* wins, *B* being 22 yards behind, and *C*, 106 yards behind *B*. By how much would *B*, beat *C* in a three-mile race?

4. What sum at a compound interest will amount to *Rs.*650 at the end of the first year and to *Rs.*676 at the end of the second year?

5. How much $3\frac{1}{2}$ per cent. Government Securities at $95\frac{1}{4}$ must be sold out in order to purchase enough 5 per cent. Calcutta Municipal Debentures at $119\frac{3}{4}$ to produce an annual income of *Rs.*665 ? (a brokerage of $\frac{1}{4}$ per cent. being charged on each transaction.)

1902.

1. Find the G. C. M. and L. C. M. of 49·383 and ·142569.

2. Simplify $\frac{1\cdot 5}{\cdot 075} \times \frac{3\frac{1}{4}}{1\frac{1}{3}} + \frac{1\cdot 875}{2\cdot 1} \times \frac{3\cdot 5}{3\cdot 75} - \cdot 1\dot{6}$.

3. Find by Practice the value of $246\frac{5}{8}$ maunds of sugar at *Rs.*13. 5*a.* 4*p.* per maund.

4. *A* and *B* have between them 132 horses ; ·25 of *A*'s = $\cdot\dot{1}4285\dot{7}$ of *B*'s. How many had each of them ?

5. Six men and five boys can do a piece of work in 7 days ; they work at it till they have completed $\frac{3}{4}$ of it ; then two of the men leave and two more boys come. How long will the work be in hand, if a boy does half as much work as a man ?

6. If I lend a friend *Rs.*1250 at 4 per cent. Simple Interest and tell him to keep it until principal and interest amount to *Rs.*1666. 10*a.* 8*p.*, how long will he have it ?

ANSWERS.

Ex. I. (pp. 7-8.)

1. 43 ; 79 ; 65 ; 84 ; 58 ; 97 ; 60 ; 87.

2. 449 ; 598 ; 704 ; 405 ; 235 ; 958 ; 725 ; 835.

3. 4000 ; 7804 ; 89063 ; 53223 ; 8046 ; 603240 ; 500505 ; 909009.

4. 341323 ; 200075 ; 707070 ; 500000 ; 80008 ; 402700.

5. 9043602 ; 7859632 ; 3040020 ; 1403000 ; 5500676 ; 11000005. 1378267 ; 1010001.

6. 45387025 ; 92568985 ; 11565437 ; 40040005 ; 96096096.

7. 349004065 ; 100013001 ; 909009099 ; 842246484 ; 3452161 ; 494000000.

8. 99099099 ; 111650050 ; 640064600 ; 500703002 ; 609001208.

9. 2804252097 ; 12036054079 ; 4000900005 ; 6304506506 ; 40280530259.

10. 400000010000 ; 836573244006 ; 900900900009 ; 600060006006.

11. 9405004250 ; 413723009004 ; 5808068080.

12. 8000000207005 ; 3004702164722 ; 1000000300005.

13. 99000090099909 ; 100196400010009.

14. 6543230040210503 01 ; 47526870744103284.

15. 900000400064000365.   **16.** 100000 ; 99999999 ; 6.

17. 88, 89, 90, 91, 92 ; 612, 613, 614, 615, 616, 617, 618, 619 ; 948 949, 950, 951, 952, 953, 954, 955, 956, 957, 958, 959, 960, 961, 962, 963, 964, 965, 966, 967, 968, 969.

18. The correct writing is 5505505 ; hence find out the mistakes.

Ex. II. (p 11.)

1. Seventeen ; twenty-four ; thirty-five ; forty-six ; twenty-seven ; forty-eight ; fifty nine ; seventy-six ; eighty-four ; ninety-five ; sixty-six ; seventy-five ; eighty-nine.

2. Two hundred and seventeen ; three hundred and nineteen ; five hundred and eighty-three ; six hundred and ninety-five ; seven hundred and twenty-five ; three hundred and eight ; four hundred and six ; eight hundred and forty six ; nine hundred and thirty-two ; seven hundred and twenty-five.

3. Three thousand, four hundred and six ; five thousand, two hundred and sixty ; four thousand, two hundred and thirty-six ; three thousand, two hundred and ninety-eight ; five thousand, six hundred

and seventy-eight ; two thousand, four hundred and five ; nine thousand, two hundred and eighty-six.

4. Forty-three thousand, two hundred and one ; eighty-seven thousand, and fifty-four ; thirty-four thousand and two ; forty-nine thousand, eight hundred and three ; fifty-eight thousand and thirty ; seventy-six thousand, five hundred and three.

5. Nine hundred three thousand, seven hundred and fifty-six ; nine hundred three thousand, two hundred and eighty four ; eight hundred twenty-seven thousand, one hundred and nine ; three hundred nineteen thousand, four hundred and twenty ; two hundred forty-three thousand and sixty-five ; one hundred twenty-three thousand, four hundred and fifty-six.

6. Two million, seven hundred fourteen thousand, three hundred and twenty-five ; eight million, forty-seven thousand, three hundred and twenty-eight ; four million, ten thousand and ten ; eight million, four thousand, six hundred and forty ; one million, two hundred thirty-four thousand and seven.

7. Twelve million, eight hundred seventy thousand and forty-five ; twenty million, eighty-four thousand, two hundred and sixteen ; seventy-nine million, thirty thousand, two hundred and eighty four ; forty-three million, two thousand and five.

8. Three hundred twenty-one million, four hundred eight-thousand, six hundred and fifty-three ; four hundred eight million, seventy-six thousand and thirty-two ; three hundred fourteen million, one hundred fifty-nine thousand, two hundred and sixty-five ; one hundred twenty-three million, four hundred fifty-six thousand, seven hundred and eighty-nine.

9. Five hundred seventy-one million, two hundred sixty-eight thousand, four hundred and five ; three thousand, one hundred seventy-nine million, forty thousand, six hundred and one ; three hundred nineteen thousand, six hundred eighty million, two hundred nine thousand and seventy-eight.

10. One billion, two hundred thirty-four thousand, five hundred sixty-seven million, six hundred fifty-four thousand, three hundred and twenty-one ; five billion twenty thousand, forty million, three thousand and sixty ; four billion, three hundred two thousand, five hundred million, seven hundred sixty-four thousand and nine.

11. Two hundred thousand, nine hundred million, six hundred thousand and two ; forty-three billion, two hundred eighty-seven thousand million, six thousand, three hundred and twenty-one ; sixty-four billion, two million, six hundred forty-six thousand and two.

12. Three hundred nineteen thousand, eighty million, two hundred fifty-nine thousand, four hundred and seventeen ; two hundred thirty-six billion, forty-five thousand, nine hundred seventy-eight million, two hundred thirteen thousand, four hundred and seventy-eight.

13. One billion, three hundred twenty-seven thousand, eight hundred seventy-five million, four hundred thirty thousand and twenty-nine ; five billion, four hundred thirty-two thousand, one hundred seventy-six million, nine hundred eighty-nine thousand and seven.

14. 97542 ; 24579. **15.** 90, 5 ; 60, 4 ; 500, 70, 5 ; 8000, 200, 90, 7 ; 40000, 200, 70, 6 ; 3000, 200, 5 ; 400000, 70000, 8000, 200, 90, 6 ; 40000000, 300000, 2000, 600, 5 ; 50000000, 3900, 20, 9 ; 70000000, 300000, 6 ; 9000000000, 700000000, 80000000, 6000000, 2000, 30.

16. Ninety-nine thousand, nine hundred and ninety-nine ; one million. **17.** 234, 243, 324, 342, 423, 432.

Ex. III. (pp. 12-13.)

1. Nineteen thousand, two hundred and thirty seven ; sixty thousand and eighty-one ; forty-nine thousand and twenty-seven ; one lac, sixty-seven thousand, two hundred and eight ; two lacs, seven hundred and fifty-three ; eight lacs, thirty thousand and five.

2. Seventy lacs, ninety thousand, seven hundred and nine ; eighty lacs, one thousand, and tweenty-five ; thirty-nine lacs, five thousand and eighty-six ; two crores, forty lacs, fifty thousand and eight ; forty lacs, one thousand, seven hundred and forty five.

3. Four crores, two lacs, seventeen thousand, eight hundred and fifteen ; four hundred and three crores, twenty-four thousand, three hundred and forty ; four hundred and seventy-eight crores, two lacs, thirty-thousand and sixteen ; two crores, thirty-four lacs, fifty-six thousand.

4. Twelve crores, thirty-four lacs, fifty-six thousand, seven hundred and eighty-nine ; six hundred and forty-five crores and three lacs ; seventy-six crores, two lacs, forty-two thousand and nine hundred.

5. Four hundred and fifty crores, two thousand, four hundred and thirty ; eight hundred crores, seven lacs, and eighty-five thousand ; four hundred and two crores, five lacs, four thousand and eight.

6. 415208 ; 5604029 ; 84374209 ; 800005 ; 7500000 ; 3000708.

7. 21500004 ; 370704012 ; 1451900007 ; 50990405607.

8. 803001011 ; 42951400085 ; 754101409009.

9. 200 ; 1 thousand ; 400. **10.** Forty crores, fifty lacs, seventy-five thousand, nine hundred and four.

11. Sankha ; trillion. **12.** The correct writing is 90504756 ; hence find out the mistakes.

Ex. IV. (p. 14.)

1. 7 ; 17 ; 21 ; 54 ; 34 ; 39. **2.** 95 ; 48 ; 95 ; 214 ; 514 ; 419.

3. 1009 ; 1804 ; 1650 ; 1766 ; 1100000 ; 605.

4. 5555 ; 6550 ; 210440 ; 290540 ; 1000010 ; 2501100.

5. IX, XVI, XXXV, XLVI, LXVIII, LXXV, LXXXIX, XCIX, CV, CXLVIII.

6. XXXII, XXVIII, XLIX, LXIX, LXXVIII, XCV, CCXV, CCCXXVII, CDXXXIII, DXLIX.

7. DCCXLV, CMXXIII, DLXVII, MCCXXXIV, MDLXVII, MDCCCLIII, MCMXVII.

8. MCCXXXI, MCCLXII, MDCCCLXII, MDCCLXXVII, MCMXCIX, MMI, MDCCLXIX.

9. $\overline{XV}$CDXCVII, $\overline{X}$XXV, CCCL, $\overline{DCLIII}$, $\overline{MI}$, MMMMMLD.

Ex. V. (pp. 16-17)

7. 11. 8. 13. 9. 16. 10. 23. 11. 9. 12. 10.
13. 23. 14. 30. 15. 27. 16. 20. 17. 33. 18. 21.
19. 25. 20. 16. 21. 19. 22. 29. 23. 50. 24. 21.
25. 23. 26. 37. 27. 36. 28. 40. 29. 128. 30. 57.

Ex. VI. (pp. 19-22.)

1. (1) 102. (2) 208. (3) 200. (4) 115. (5) 213.
(6) 215. (7) 200. (8) 116. (9) 214. (10) 224.
(11) 276. (12) 222. (13) 503. (14) 2133. (15) 1697.
(16) 14624. (17) 1890. (18) 995. (19) 12345. (20) 2956.
(21) 2489. (22) 29125. (23) 25575. (24) 2239. (25) 780.
(26) 2227. (27) 2435. (28) 6553. (29) 7812. (30) 36092.
(31) 28026. (32) 37667. (33) 16553. (34) 23724. (35) 17764.
(36) 26973. (37) 60714. (38) 336513. (39) 144563. (40) 400257.
(41) 358064. (42) 390370. (43) 3140069. (44) 2329089.
(45) 746506. (46) 23726503. (47) 30421482. (48) 25522084.
(49) 28556362. (50) 20222215. (51) 3388360. (52) 4025738.
(53) 37155818. (54) 260342508. (55) 171357572.

2. (1) 17866. (2) 172846. (3) 42612875.
(4) 518890. (5) 19169327. (6) 12842644.

3. (1) 12891663. (2) 21086067. (3) 531904.
(4) 2689134309. (5) 1788591628.

4. 162209. 5. 1694375. 6. 2230626. 7. 2294129927.
8. 20566726566. 9. 13241749. 10. 15701653985.
11. 1454. 12. 497. 13. 337. 14. 140.
15. 1338. 16. 1464. 17. 9770. 18. *R*42068.
19. 50150009. 20. 17863411. 21. *R*4387. 22. 3554.
23. 6529. 24. 47423136. 25. 1847. 26. 3826.
27. 382169. 28. 365. 29. *R*232031. 30. 6116.

Ex. VII. (pp. 24-25.)

11. 5. 12. 1. 13. 7. 14. 15. 15. 17.
16. 11. 17. 5. 18. 1. 19. 11. 20. 10.

Ex. VIII. (pp. 26-28.)

1. (1) 17. (2) 34. (3) 16. (4) 16. (5) 9. (6) 154. (7) 209. (8) 198. (9) 594. (10) 205. (11) 6239. (12) 2849. (13) 1189. (14) 2886. (15) 4370. (16) 6092. (17) 4960. (18) 18469. (19) 16909. (20) 16449. (21) 14759. (22) 668493. (23) 327699. (24) 127589. (25) 229678. (26) 74819. (27) 286699. (28) 18838478. (29) 246913578. (30) 5101262. (31) 64446566. (32) 100909765. (33) 22591687. (34) 555939946. (35) 4691357. (36) 272886756. (37) 738776598. (38) 66760615. (39) 414866185. (40) 100011. (41) 364179951.

2. (1) 68999 ; 77036 ; 9999. (2) 99099 ; 766899 ; 173706. (3) 60005393 ; 192484228. (4) 10942895 ; 67200757689.

3. (1) 40101 ; 88890 ; 109089. (2) 1288874 ; 28890 ; 520986. (3) 53318 ; 2378 ; 1188988. (4) 823611 ; 1213667908.

4. 4699 ; 2167090875. **5.** 19279548.

6. 809089. **7.** 4091 ; 900000.

8. 1576542 ; 9542315 ; 967599 ; 2387655 ; 4959711 ; 9095493 ; 996535.

9. 754321 ; 179400 ; 8480222 ; 6174909 ; 183227.

10. 304924818. **11.** 619310439. **12.** 748696147.

13. 1. **14.** 1688. **15.** 1796. **16.** 749. **17.** 2327. **18.** 417.

19. 51. **20.** 49. **21.** *R*891. **22.** 12161. **23.** *R*897976.

24. 8630098. **25.** 76. **26.** *A* 23 ; *B* 47 ; *C* 35. **27.** *R*3087.

28. 3361. **29.** 650 ; 858. **30.** 1900 ; 81 years.

Ex. IX. (pp. 29-30.)

1. 4. **2.** 1609. **3.** 92. **4.** 396. **5.** 1034.
6. 3742. **7.** 6140. **8.** 15022. **9.** 1273. **10.** 682.
11. 26 ; 19. **12.** 6 ; 3 ; 57 ; 14 ; 426 ; 2.

Ex. X. (pp. 33-34.)

4. 96. **5.** 88. **6.** 126. **7.** 54. **8.** 81.
9. 162. **10.** 41. ; 60. **11.** 108. **12.** 10 ; 40 : 36.
13. 408. **14.** 217. **15.** 18. **16.** 56 ; 18.
17. 32. **18.** 64. **19.** 22. **20.** 108.

Ex. XI. (pp. 35-36.)

1. 568 ; 4425 ; 11468 ; 18096 ; 24228 ; 404825 ; 396064 ; 231483 ; 1098444.

2. (1) 11698 ; 17547 ; 23396 ; 29245 ; 35094 ; 40943 ; 46792 ; 52641 ; 64339

(2) 115428 ; 192380 ; 269332 ; 346284 ; 423236 ; 500188 ; 538664 ; 577140 ; 731044.

(3) 7740984 ; 19352460 ; 11611476 ; 27093444 ; 15481968 ; 34834428 ; 23222952 ; 30963936 ; 42575412 ; 46445904 ; 58057380.

(4) 52070352 ; 45561558 ; 58579146 ; 71596734 ; 84614322 ; 97631910 ; 110649498 ; 123667086.
(5) 1975308642 ; 2962962963 ; 3950617284 ; 4938271605 ; 5925925926 ; 6913580247 ; 7901234568 ; 8888888889 ; 10864197531 ; 11851851852.

3. (1) 388064 ; 416160 ; 541376 ; 833184.
(2) 346284 ; 10518588 ; 6050000 ; 8224776.
(3) 735675 ; 736944 ; 584212 ; 12019080.
(4) 64536612 ; 87585402 ; 1014848586.

4. (1) 9809890 ; 98098900 ; 980989000 ; 9809890000.
(2) 2161530 ; 2882040 ; 5043570 ; 6484590 ; 7205100.
(3) 1827140 ; 18271400 ; 27407100 ; 456785000 ; 822213000.
(4) 94894200 ; 1186177500 ; 13443345000 ; 1028020500 ; 150249150000.
(5) 720774400 ; 1441548800 ; 12613552000 ; 162174240000

5. 2274. **6.** 74451. **7.** 5555555505. **8.** *R*811224.
9. 214948. **10.** *R*1125. **11.** 166. **12.** (1) 98 miles. (ii) 1204 miles.

Ex. XII. (pp. 39-40.)

1. 57706 ; 77341 ; 42182 ; 79992 ; 281504 ; 308163 ; 1619723 ; 50516.

2. (1) 1287657 ; 1000055 ; 34381488 ; 1531335.
(2) 8539410 : 11216556 ; 46634205 ; 48954719.
(3) 1013736849 ; 145651668 ; 311305816.
(4) 2518028865 ; 757030260 ; 37335129056.
(5) 1973316695 ; 2706262896 ; 4059394344 ; 7442222964.
(6) 24149786524 ; 296988105062 ; 5327809224181.
(7) 28631518784 ; 213248118864 ; 638402785674/2.
(8) 2053737000 ; 59392800000 ; 622439160.
(9) 2851265148 ; 593928000000 ; 5994485049000.
(10) 2299320000 ; 51734700000 ; 24717690000 ; 1408333500000 ; 268445610000000.

3. (1) 416948784. (2) 278178269193. (3) 19948130736.
(4) 78214076605. (5) 3405426851645. (6) 2586568132906.
(7) 2090752670781. (8) 21862529907675.
(9) 22102928324900. (10) 34525538710.
(11) 39835614456377 82. (12) 31260150931584.
(13) 6043567453684 50 (14) 742892741529300
(15) 15241578750190521.

4. (1) 433418175 ; 173367270 ; 337103025.
(2) 2600509050 ; 9101781675 ; 7313243616.
(3) 6535022130 ; 31260150931584 ; 4594091417461.
(4) 2809599487053727789412 ; 5167962635293377516961 65.
(5) 1345336932419572888 3956 ; 2680337934953838015 4320.

5. 0 **6** *R*159152. **7.** 34864128. **8.** 125[illegible]
9. 24941648. **10.** *R*42973403. **11.** [illegible]10212[illegible]0.
12. 9630. **13.** 2482. **14.** [illegible]3374.
15. 41610. **16** 2799992. **17.** [illegible]456.
18. 6428506. **19.** 92005008 miles. [illegible]

Ex. XIII. (p. 41.)

1. (1) 700. (2) 3315. (3) 6840. (4) 36630.
(5) 355752. (6) 517248. (7) 454720. (8) 3797115.
(9) 1441440. (10) 68785605. (11) 57737160.
(12) 58605120. (13) 67886100. (14) 3417187500.
2. 62968425. **3.** 595680000. **4.** 62172. **5.** 23328.
6. 99216. **7.** 720. **8.** 51948. **9.** 4096. **10.** 15360

Ex. XIV. (p. 42.)

1. (1) 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, 256, 289, 324, 361, 400, 441, 484, 529, 576, 625 ;
1521, 2116, 2916, 7396, 9801.
(2) 29584 ; 56169 ; 820836 ; 974169.
(3) 531441 ; 762129 ; 1087849 : 30206016.
(4) 53904964 , 88059456 ; 77369616 ; 1522756.
2. (1) 1, 8, 27, 64, 125, 216, 343, 512, 729, 1000, 1331, 1728, 2197, 2744, 3375, 4096, 4913, 5832, 6859, 8000, 9261, 10648, 12167, 13824, 15625, 50653, 110592, 314432, 456533.
(2) 681472 ; 912673 ; 1860867 ; 94818816.
(3) 29218112 ; 672221376 ; 447697125 ; 997002999.
(4) 961504803 ; 156242452456 ; 963259373376 ; 1879080904.
3. (1) 211309379856 ; 8653650625 ; 949005240561 ; 952857108736
(2) 679740887296 ; 11447803771 2481 ; 41041981460482256 ;
999600059996 0001.
4. (1) 285 (2) 727. (3) 45. (4) 2025. (5) 56. (6) 910.
5. (1) 55. (2) 150. (3) 672. (4) 88. (5) 466. (6) 18.
(7) 14767 (8) 141.

Ex. XV. (pp. 43-44.)

4. 6 times and 6 over ; 13 times ; 13 times and 6 over.
5. 4. **6.** 4 ; 6 ; 12 ; 16. **7.** 17. **8.** 12. **9.** 16.
10. 20. **11.** 11. **12.** 12. **13.** 360. **14.** 7. **15.** 27.
16. 66. **17.** 380 gain. **18.** 10. **19.** 18. **20.** 17.

Ex. XVI. (p. 46.)

1. (1) 154 ; 77 ; 57...6 ; 51...3; 46...2 ; 42 ; 38...6.
(2) 227...1 ; 170...2 ; 113...4 85...2 ; 75...7 ; 62 ; 48...10 ; 45...7.
(3) 1685 ; 1203...4 ; 1053...1 ; 842...5 ; 648...1 ; 526 . 9 ; 495...10 ; 443...8.
(4) 3438 ; 2292 ; 982...2 ; 764 ; 625 1 ; 573 ; 491...2.
(5) 11766 ; 7059...3 ; 3922 ; 5042...4 ; 3529...8 ; 2941...6 : 1961.
(6) 174 ; 1532 ; 69071 ; 16875.
(7) 10768...4 ; 76582 : 11979052 1 ; 315836...7.
(8) 795917072...3 ; 112233444...6 ; 823045...3.
(9) 1974538 ; 2384163 ; 2645753.
(10) 2167022 ; 58097313 ; 54869684...9 ; 51981806...7 ; 49382716...1.
2. 93. **3.** 180. **4.** 9006 ; 9192. **5.** 94.
6. 156. **7.** 1728. **8.** *Rs.*20.

Ex. XVII. (pp. 49-50.)

1. (1) 4021. (2) 2050...8. (3) 1010...6. (4) 7002.
(5) 1849129...40. (6) 2176183. (7) 1367372. (8) 1736834...24.
(9) 1272250...6. (10) 84293. (11) 7629302. (12) 93845796.
(13) 593559...6. (14) 325698042...25. (15) 172956436...7.
(16) 45783975. (17) 987654321.

2. (1) 3409371. (2) 11951629...79. (3) 356995601...29.
(4) 7071. (5) 57096. (6) 103944.
(7) 87997...214. (8) 7341069. (9) 137641...371.
(10) 967427210...61. (11) 87366...6076. (12) 190182.
(13) 19915...5559 (14) 65839...2. (15) 886797...1310.
(16) 3192...4966. (17) 1453...2286. (18) 43349...1140.
(19) 193421...811 (20) 16575. (21) 4938.
(22) 73086413. (23) 37956314. (24) 70080092...7322.
(25) 507001...4221. (26) 85802...2575858. (27) 9640...999821.
(28) 736...30167291. (29) 328...65220054. (30) 2796.
(31) 30305. (32) 987654321. (33) 574585614865.
(34) 800300500. (35) 2837154309.

3. (1) 42439...5498; 26171...7874; 5822...6639; 4167...2898.
(2) 8122...9; 4061 · 9; 2707...19; 2030...29; 1624...29; 1015...29; 902...49.
(3) 3426...4; 856 204; 571...4; 428...204; 380...604.
(4) 98168...426; 6544 . 6826; 24542...426; 165...159826.
(5) 12433128...54; 653266...334; 6566...27734; 42401...18834.
(6) 3216...6886; 229...47832; 3396...5094687.

4. (1) 11. (2) 102. (3) 433. (4) 53 (5) 284074.
5. 72158. **6.** 17. **7.** 13. **8.** 75. **9.** 21. **10.** 420.
11. 973. **12.** 5669. **13.** 1799. **14.** *R*11. **15.** 478. **16.** 201.
17. 1888; 101001. **18.** 66. **19.** 15; 3. **20.** 186746...492.

Ex. XVIII. (pp. 51-52.)

1. (1) 26. (2) 26. (3) 25. (4) 27. (5) 552. (6) —6.
(7) 815. (8) 303. (9) 69. (10) 130. (11) 778.
2. (1) 1036. (2) 1008. (3) 1808. (4) 8969578 (5) 2067.
3 382. **4.** 1000. **5.** 1675. **6.** 3582. **7.** 636.
8. 162. **9.** 3370. **10.** 1999999998. **11.** 1098897.
12. (1) 150. (2) 2034. (3) 83. (4) 172720. (5) 24.
(6) 554. (7) 21. (8) 24. (9) 93. (10) 1764.

Ex. XIV. (pp. 57-58.)

1. 847091. **2.** 38967. **3.** 81757. **4.** 7640. **5.** 7124; 5516.
6. 94; 79. **7.** 905; 521. **8.** Carriage *Rs.*511 : Each horse *Rs.*173.
9. 780; 420. **10.** 105; 63 **11.** Ram 42, Gopal 35, Hari 28.
12. 50 oranges, 35 apples, 27 plums. **13.** *A Rs.*55, *B Rs.*65, *C Rs* 85.
14. *A Rs.*1244, *B Rs.*1460, *C Rs.*1796. **15.** 3507. **16.** 1887378.
17. 9306. **18.** 37246. **19.** 175. **20.** 5878. **21.** 2107.

22. 31352. **23.** 109 ; 278. **24.** 403. **25.** 141. **26** (1) 1830. (2) 5050. (3) 155. (4) 256. (5) 240. (6) 800 (7) 493. (8) 1430.
27. *Rs.*2704. **28.** 960 miles. **29.** 156. **30.** 5193171.
31. 99735 ; 10137. **32.** 100191. **33.** 9999700002.
34. 9999, *rem.* 999. **35.** 10876799.

Ex. XX. (p. 60)

1. 942636 ; 999457944 ; 992140936 ; 9920146.
2. (1) 4352. (2) 10924. (3) 3318939. (4) 3857.
3. (1) 60 ; 6 ; 10277. (2) 29889 ; 3318377. (3) 3349319 ; 36438018. **4.** 1370 ; 25780 ; 17363.
5. (1) 4355516 ; 7121448 ; 8552448 ; 5027211.
(2) 1252562304 ; 1613118976 ; 3769357248.
(3) 7490168136 ; 4334453248 ; 3160438848.
(4) 15735060 ; 461981520 ; 14810510675.
6. (1) 2558192 ; 2295864 ; 77967198 ; 448602324.
(2) 157739562 ; 303794712 ; 4288179204 : 672604515.
7. (1) 49110419796 ; 144872064531 ; 63723226584.
(2) 996275287620 ; 397685408184 ; 5334673883463.
(3) 44818796323449 ; 603155680760244.
8. 1269488804031 ; 25332654572848.

Ex. XXI. (pp. 63-64.)

1. (1) 4396630 ; 21983150 ; 65949450 ; 109915750 ; 549578750.
(2) 959175 ; 2238075 ; 4795875 ; 7993125.
(3) 439556825 ; 2197784125 ; 6593352375 ; 10988920625 ; 54944603125.
(4) 439545546 ; 4435414146 ; 44394100146 ; 443980960146.
(5) 556032 ; 5768832 ; 57885248 ; 57908416.
(6) 96081172 ; 1056892892 ; 11625821812 ; 873412792088.
2. (1) 1369 ; 2025 ; 2304 ; 3025 ; 4225 ; 5625 ; 4096 ; 5041 ; 6889 ; 9216 ; 15625.
(2) 11664 ; 22201 ; 24336 ; 33489 ; 46225 ; 152881 ; 228484 ; 207936 ; 274576.
3. $(59)^2-(6)^2$; $(85)^2-(11)^2$; $(105)^2-(21)^2$; $(221)^2-(24)^2$; $(397)^2-(81)^2$.
4. (1) 150000 ; 400 ; 2140200 ; 2805000
(2) 176384 ; 2892000 ; 7386000 ; 52640
5. (1) 2 ; 9669. (2) 500 ; 4124.
6. 99995356. **7.** 1006434. **8.** 100028544 ; 99991331.

Ex. XXII. (p. 66.)

1. (1) 45[illegible] ; 6210...48 ; 9607.
(2) [illegible]6...23 ; 78556...65 ; 99904...53.
(3) 79732...53 ; 181554...6 ; 450314...66.

(4) 2733534...7 ; 1195921...15 ; 1063041...7 ; 797280...79 ; 154457...242. (5) 120696...136 ; 37663...30 ; 4565...512 ;
(6) 28716695...68 ; 19339815...68 ; 10028052...152 ; 4099...133.

2. (1) 7592...4 ; 759...14 ; 75...464 ; 7...2964 ; 1518...14.
(2) 1750863...1 ; 583621...1 ; 350172...16 ; 250123...11 ; 194540...16 ; 159169 ..21 : 134681...51 ; 116724...16.
(3) 722185......64 ; 515846...139 ; 401214...39 ; 328266...39.
(4) 1237807...1 ; 412602...126 ; 247561...251 ; 176829...501.
(5) 7859...3851 ; 11171...4593 ; 8787...1988.

Ex. XXIII. (pp. 68-69.)

1. 52. **2.** 78. **3.** 21. **4.** *Rs.*288. **5.** 7 hours.
6. *A Rs.*224, *B Rs.*336, *C Rs.*448. **7.** *A* 410, *B* 902, *C* 1312.
8. *Rs.*265. **9.** 33. **10.** Son *Rs.*45000, daughter *Rs.*15000.
11. Each man *Rs.*1050, each woman *Rs.*875, each child *Rs.*175.
12. 144*d.* **13.** *A Rs.*422, *B Rs.*633, *C Rs.*844, *D Rs.*1055.
14. *Rs.*4. **15.** 6.

Miscellaneous Examples I. (pp. 71-76.)

1. 533884537. **2** 379708. **3.** 63550 and 62128.
4. 7222 and 8456. **5.** (i) 119. (ii) 2268. **6.** *Rs.*23006. **7.** 38.
8. 2378. **9.** 172180. **10.** 400. **11.** 3679. **12.** 191.
13 188521 males, 246090 females. **14.** $(4010)^2-(2779)^2$.
15. 22771. **16.** 305. **17.** 17350. **18.** 723 **19.** 1865.
20. 49505296. **21.** 480. **22.** 2220. **23.** 581506 ; 581550.
24. 51 ; 6. **25.** House *Rs.*507200. **26.** 13.
27. 1st 8799 ; 2nd 3557 ; 3rd 5204. **28.** 203. **29.** 400.
30. 52 years. **31.** *Rs.*2914. **32.** 139. **33.** He gains *Rs.*25.
34. Eldest *Rs.*1230928, second *Rs.*706806, third *Rs.*489764.
35. 180440. **36.** 435 seers. **37.** *C* is 3 years older than *D*.
38. *A* has 80, *B* 69, and *C* 157. **39.** 29. **40.** 1429.
41. *Rs.*790. **42.** *Rs.*3327, *Rs.*3201, *Rs.*3070, *Rs.*2949.
43. 99 hours. **44.** $(10208)^2-(8983)^2$. **45.** Quotient = 2442 ; rem. = 102. **46.** *Rs.*8240 **47.** *Rs.*593810.
48. After 9 hours. **49.** *C* had *Rs.*1545 more than *B*.
50. 4997. **51.** 2568307. **52.** *A Rs.*25, *B Rs.*…
53. *A Rs* 5, *B Rs.*10, *C Rs.*15. **54.** 987. **55.** 2664 ; 1590408.
56. 38664. **57.**

2	7	6
9	5	1
4	3	8

58. 286. **59.** 3.
60. *A* 167, *B* 501, *C* 799. **61.** *A Rs.*500, *B Rs.*750 ; *C Rs.*1000, *D Rs.*2750.
62. (1) 1440. (2) 300.
63. 15. **64.** 45. **65.** *Rs.*1250.
66. 49110419796 ; 144872064531 ; 63723226584.
67. 18441216. **68.** *Rs.*670. **69.** 403. **70.** 312.
71. 139. **72.** 1044 miles. **73.** 224 miles. **74.** 57606.
75. 10 days. **76.** *Rs.*11. **77.** 105. **78.** 49.
79. *A Rs.*546, *B Rs.*569, *C Rs.*1160, *D Rs* 1700.
80. He gained *Rs.*4 a head, no. of bullocks = 175.

Ex. XXIV. (pp. 80-81.)

1. (1) 272*a.*; 304*a.*; 672*a.*; 720*a.*; 1104*a.*; 1344*a.*; 1520*a.*
(2) 1392*a.*; 1920*a.*; 3920*a.*; 7360*a*; 156*a.*; 334*a.*
(3) 582*a.*; 861*a.*; 1403*a.*; 1279*a.*; 3755*a.*

2. (1) 6528*p.*; 10752*p.*; 18624*p.*; 28032*p.*; 65664*p.*; 95232*p.*
(2) 16188*p.*; 14736*p.*; 50988*p.*; 154524*p.*; 181512*p.*
(3) 2979*p.*; 1511*p.*; 1541*p.*; 1857*p.*
(4) 81693*p.*; 105647*p.*; 240067*p.*; 969769*p.*; 87731*p*; 6073*p.*; 65959*p.*

3. (1) 3328*ps.*, 9984*p.*; 1216*ps.*, 3648*p.*; 7192*ps.*, 21576*p.*; 2350*ps.*, 7050*p.*; 1315*ps.*, 3945*p.*
(2) 5609*ps.*, 16827*p.*; 11031*ps.*, 33093*p.*; 14438*ps.*, 43314*p.*; 30513*ps.*, 91539*p.*; 50051*ps.*, 150153*p.*; 875*ps.*, 2625*p.*; 13791*ps.*, 41373*p.*

4. (1) 6080*g.*, 24320*c.*; 10880*g.*, 43520*c.*; 17920*g.*, 71680*c.*; 24960*g.*, 99840*c.*; 33600*g.*, 134400*c.*; 27020*g.*, 108080*c.*
(2) 32945*g.*, 131780*c.*; 7975*g.*, 31900*c.*; 129860*g.*, 519440*c.*; 24145*g.*, 96580*c.*
(3) 15550*g.*, 62200*c.*; 17237*g.*, 68948*c.*; 3062696*g.*, 12250784*c.*

5. 68950*c.*; 87805*c.*; 23570*c.*; 7605831*c.*

6. (1) 5600*ps.*, 16800*p.*; 5920*ps.*, 17760*p.*; 6760*ps.*, 20280*p.*
(2) 850*ps.*, 2550*p.*; 24928*ps.*, 74784*p.*; 77280*ps.*, 231840*p.*
(3) 547392*ps.*, 1642176*p.*; 40314*ps.*, 120942*p.*; 1087200*ps.*, 3261600*p.*; 26464*ps.*, 79392*p.*

7. (1) 1458, 2916, 5832; 1850, 3700, 7400; 2456, 4912, 9824; 2854, 5708, 11416; 8486, 16972, 33944; 194806, 389612, 779224.
(2) 1717, 3434, 6868; 19453, 38906, 77812; 146492, 292984, 585968; 115018, 230036, 460072.

8. 2412, 9648; 4242, 16968; 4801, 19204; 96181, 384724.

9. (1) 6400000; 28501; 48604; 40015; 19729.
(2) 169574; 99741; 309206.

10. (1) 6900*s.*; 9960*s.*; 15900*s.*; 28040*s.*; 181720*s.*; 161840*s.*
(2) 1421*s.*; 9818*s.*; 15813*s.*; 69137*s.*; 128067*s.*

11. (1) 15600*d.*; 23520*d.*; 37440*d.*; 97200*d.*; 443760*d.*; 1210320*d.*; 2216640*d.*
(2) 32340*d.*; 47676*d.*; 99972*d.*; 126300*d.*; 222324*d.*
(3) 560*d.*; 9726*d.*; 2733*d.*; 89900*d.*
(4) 157362*d.*; 17201*d.*; 82481*d.*; 305997*d.*

12. (1) 4243*q.*; 7391*q.*; 13393*q.*; 28364*q.*
(2) 97403*q.*; 147042*q.*; 576300*q.*; 80563979*q.*

13. (1) 372, 744; 450, 900; 334, 668; 468, 936; 212, 424; 418, 836.
(2) 1958400, 3916800; 4131840, 8263680; 4192320, 8384640; 348912, 697824; 248472, 496944; 1020144, 2040288.
(3) 38272, 76544; 23035, 46070; 187025, 374050; 422353, 844706.
(4) 749049, 1498098; 1030047, 2060094; 4385659, 8771318.

(5) 935760, 1871520 ; 576840, 1153680 ; 516480, 1032960 ;
313440, 626880 ; 70428, 140856 ; 55664, 111328 ;
48054, 96108 ; 4574304, 9148608 ; 778428, 1556856 ;
61056, 122112 ; 936360, 1872720 ; 327520, 655040 ;
157728, 315456 ; 23796, 47592.

14. (1) 7600, 5700, 3800 ; 10240, 7680, 5120 ; 646080, 484560, 323040 ; 148560, 111420, 74280 ; 726640, 544980, 363320 ; 725760, 544320, 362880.

(2) 936, 702, 468 ; 11588, 8691, 5794 ; 171644, 128733, 85822 ; 341260, 255945, 170630 ; 261188, 195891, 130594.

15. (1) 96615 ; 18033 (2) 5706 ; 3059 ; 2359 ; 28007.
(3) 7959 . 167190. (4) 47855 ; 91000.

16. (1) 72346 ; 90009 ; 696543 (2) 72346 ; 80563979 ; 696543.

17. 260. **18.** 173. **19.** 93 **20.** 101.

Ex. XXV. (pp. 82-83)

1. (1) *Rs*.131. 14*a* 5*p* ; *Rs*.299 8*a*. 5*p* ; *Rs* 265. 13*a* 3*p*. ; *Rs*.3538 0*a* 2*p*. , *Rs*.197 8*a*. 1*p* ; *Rs* 2379 1*a*. 6*p*.

(2) *Rs*.3814. 8*a*. 10*p*. ; *Rs* 3350. 7*a*. 10*p*. ; *Rs*.10324. 11*a*. 5*p*. ; *Rs*.5039. 7*a* 1*p* ; *Rs*.5433 15*a* 8*p*.

2 *Rs*.15436. 10*a*. 1*ps* ; *Rs*.19290 1*a*. 3*ps*. ; *Rs*.8551. 14*a*. 1*ps*. ; *Rs*.13983. 11*a*. ; *Rs*.66528.

3. (1) *Rs* 26 ; *Rs*.6 ; *Rs*.1185. 14*a* 18*g* : *Rs* 637.

(2) *Rs*.32677. 4*a*. 2*ps*. ; *Rs* 16152. 11*a*. ; *Rs*.842.

(3) *Rs* 5357. 7*a*. ; *Rs* 9325. 6*a*. ; *Rs*.739. 15*a*. 2*ps*. ; *Rs* 809 15*a*. 3*ps*. ; *Rs*.902. 11*a*

4. (1) *Rs*.824 ; *Rs* 473 ; *Rs* 316. 4*a*

(2) *Rs* 181 ; *Rs*.93 ; *Rs* 2876 ; *Rs*.19.

5. (1) £288 1*s*. ; £253 19*s*. ; £375 19*s*. 3*d* ; £11. 7*s*. 9*d*. ; £374 11*s*. 8*d*. ; £655. 13*s*. 6*d*.

(2) £153 3*s*. 4*d*. ; £295. 17*s*. 11¼*d* ; £128 8*s*. 6½*d* ; £364 11*s* 8*d*. ; £83920. 16*s*. 2¾*d*.

(3) £360. 1*s*. 7¾*d* £135 8*s*. 7¼*d*. ; £1041. 13*s*. 8¾*d*. ; £295. 18*s* 3¾*d* , £433 1*s*. 2¾*d* ; £3802. 8*s*. 5*d*.

6. (1) £449522. 10*s*. ; £242778 , £4807. 6*s*. ; £110762. 12*s*. 6*d*. ; £178. 12*s*. 1*d* ; £162. 17*s* 3*d*

(2) £252. 5*s* 8*d*. ; £8255. 12*s*. ; £3611. 10*s*. ; £47. 12*s*. 4*d*. ; £2057. 12*s* 2½*d* ; £49649 5*s*

(3) £31920 15*s*. ; £26106. 3*s*. ; £4587. 5*s* 6*d*. ; £3513. 5*s*. 6*d*. ; £42. 0*s*. 5*d*.

7. £156. 7*s*. 4*d* **8.** *Rs*.20 8*a*. 6*p*. **9** *Rs* 52. 9*a*. **10.** *Rs*.1125.

Ex. XXVI. (p. 84.)

1. 60, 120 ; 100, 200 ; 92*gs*. 4*s*., 184*h*. *gs* 4*s*. ; 835, 1670 ; 512*gs*. 8*s*., 1024*h*. *gs*. 8*s*. ; 10217*gs*. 3*s*., 20434*h*. *gs*. 3*s*.

2. 1062, 2124 ; 2359, 4718 ; 1750, 3500 ; 2481, 4962 ; 20759, 41518.

3. 46145*crs* 2*s*. ; 356000 ; 147*crs*. 2*s*. 6*d*. ; 1250*crs*. 2*s*.

4. [illegible]

5. 2752*gs*. 20*s*. ; 12771*gs*. 19*s*. ; 5669*gs*. 1*s*. ; 90*gs*. 10*s*. ; 616*gs*. 4*s*. 11*d*. ; 357*gs*. 19*s*. 0$\frac{3}{4}$*d*.
6. 154*h. gs*. 8*s*. ; 10349*h. gs*. 5*s*. 6*d*. ; 100 ; 6172*h. gs*. 4*s*. 6*d*. ; 35.
7. £2. 11*s*. 9*d*. ; £51. 9*s*. 5*d*. ; £182. 11*s*. 8*d*.
8. *Rs*.327. 4*a*. 8*p*. ; *Rs*.968. 12*a*. ; *Rs*.9038. 12*a*. 4*p*. ; *Rs*.283. 8*a*. ; *Rs*.107 ; *Rs*.21. 4*a*.
9. 147840.
10. *Rs*.8000 ; *Rs*.6030 *Sic*.

Ex. XXVII. (pp. 85-88.)

1. (1) *Rs*.2. 11*a*. (2) *Rs*.2. 6*a*. 5*p*. (3) *Rs*.2. 5*a*. 9*p*.
(4) *Rs*.2. 12*a*. 6*p*. (5) *Rs*.2. 0*a*. 8*p*. (6) *Rs*.2. 8*a*. 6*p*.
(7) *Rs*.29. 5*a*. 3*p*. (8) *Rs*.57. 0*a*. 1*p*. (9) *Rs*.313. 2*a*. 7*p*.
(10) *Rs*.361. 11*a*. 1*p*. (11) *Rs*.50. 6*a*. 5*p*. (12) *Rs*.102. 0*a*. 1*p*.
(13) *Rs*.206. 7*a*. 2*p*. (14) *Rs*.308. 7*a*. 7*p*. (15) *Rs*.1205. 14*a*. 2*ps*.
(16) *Rs*.912. 15*a*. (17) *Rs*.2813. 5*a*. 4*p*. (18) *Rs*.10256. 9*a*. 10*p*.
(19) *Rs*.18632. 2*a*. 5*p*. (20) *Rs*.25536. 9*a*. 8*p*.
(21) *Rs*.28141. 14*a*. 11*p*. (22) *Rs*.346001. 0*a*. 8*p*.
2. (1) £2. 10*s*. 2*d*. (2) £3. 1*s*. 4*d*. (3) £3. 18*s*. 2$\frac{1}{4}$*d*.
(4) £136. 17*s*. 7*d*. (5) £150. 8*s*. 6$\frac{1}{2}$*d*. (6) £200. 17*s*. 10$\frac{3}{4}$*d*.
(7) £4908. 1*s*. 4$\frac{3}{4}$*d*. (8) £59667. 3*s*. 7$\frac{3}{4}$*d*. (9) £10798. 5*s*. 9$\frac{3}{4}$*d*.
(10) £16004. 11*s*. 7*d*.
3. (1) *Rs*.44626. 11*a*. 11*p*. (2) *Rs*.34717. 4*a*. 8*p*.
(3) *Rs*.38349. 10*a*. 2*p*. (4) £23506. 14*s*. 9$\frac{1}{4}$*d*.
(5) £30485. 17*s*. 4$\frac{1}{2}$*d*. (6) £24780. 12*s*. 6$\frac{1}{2}$*d*.
4. £113. 6*s*. 5. £1667. 0*s*. 3*d*. 6. *Rs*.368. 3*a*. 3*p*.
7. *Rs*.9691. 11*a*. 6*p*. 8. £311. 11*s*. 10$\frac{3}{4}$*d*.

Ex. XXVIII. (pp. 89-91.)

1. (1) *Rs*.8. 6*a*. 11*p*. (2) *Rs*.28. 12*a*. 11*p*. (3) *Rs*.10. 12*a*. 5*p*.
(4) *Rs*.25. 14*a*. 3*ps*. (5) *Rs*.43. 10*a*. 2*ps*. (6) *Rs*.125. 15*a*. 1*p*.
(7) *Rs*.511. 8*a*. 10*p*. (8) *Rs*.79. 8*a*. 1*p*.
(9) *Rs*.2163. 9*a*. 8*p*. (10) *Rs*.424. 13*a*. 10*p*.
(11) *Rs*.2183. 14*a*. 9*p*. (12) *Rs*.447. 6*a*. 10*p*.
(13) *Rs*.518. 13*a*. 10*p*. (14) *Re*.1. 1*a*. 1*p*.
(15) *Rs*.485. 14*a*. 9*p*. (16) *Rs*.898. 5*a*. 4*p*.
(17) *Rs*.1364. 4*a*. 11*p*.
2. (1) 6*s*. 0$\frac{3}{4}$*d*. (2) 11*s*. 8$\frac{1}{4}$*d*. (3) 4*s*. 0$\frac{3}{4}$*d*.
(4) 6*s*. 7$\frac{3}{4}$*d*. (5) 5*s*. 7$\frac{1}{2}$*d*. (6) £39. 10*s*. 7$\frac{3}{4}$*d*.
(7) £58. 7*s*. 10$\frac{1}{4}$*d*. (8) £288. 3*s*. 11$\frac{3}{4}$*d*. (9) £49. 19*s*. 11$\frac{1}{2}$*d*.
(10) £30. 14*s*. 9$\frac{1}{2}$*d*. (11) £4. 7*s*. 8$\frac{3}{4}$*d*. (12) £118. 18*s*. 5$\frac{3}{4}$*d*.
(13) £467. 4*s*. 11$\frac{1}{4}$*d*. (14) £179. 16*s*. 10$\frac{3}{4}$*d*. (15) £446. 14*s*. 9$\frac{1}{2}$*d*.
(16) £478. 15*s*. 7$\frac{3}{4}$*d*. (17) £76. 16*s*. 9$\frac{3}{4}$*d*.
3. (1) *Rs*.4727. 15*a*. 10*p*. (2) *Rs*.[illegible]262. 12*a*. 10*p*.
(3) *Rs*.1085. 13*a*. 10*p*. (4) £1184. 14*s*. 10$\frac{1}{2}$*d*.
(5) £1027. 16*s*. 6$\frac{3}{4}$*d*. (6) *Rs*.32924. 11*a*. 11*p*.
(7) [illegible]9. 15*a*. 6*p*. (8) £264. 17*s*. 6*d*.
4. £197. 16*s*. 6$\frac{1}{2}$*d*. 5. *Rs*.294. 11*a*. 1*ps*. 6. £3. 1*s*. 0$\frac{3}{4}$*d*.
7. £5. [illegible] 8. *Rs*.590. 2*a*. 11*p*. 9. *Rs*.57. 2*a*. 1*p*.

10. *Rs.*56. 10*a.* 6*p.* **11.** *Rs.*365. 6*a.* **12.** £2. 0*s.* 2¼*d.*
13. *Rs.*250. 12*a.* 11*p.* **14.** *A* £26. 15*s.* 7*d.*; *B* £43. 9*s.* 10*d.*; *C* £37. 6*s.* 3*d.* **15.** *Rs.*16. 13*a.* 4*p.* **16.** £18. 16*s.* 11¾*d.*

Ex. XXIX. (pp. 93-94.)

1. (1) *Rs.*37. 0*a.* 8*p.*; *Rs.*127. 15*a.* 6*p.*; *Rs.*541. 12*a.*
(2) *Rs.*206. 14*a.* 4*p.*; *Rs.*475. 0*a.* 3*p.*; *Rs.*348. 15*a.* 6*p.*
(3) *Rs.*328. 15*a.* 4*p.*; *Rs.*931. 13*a.* 7*p.*
(4) *Rs.*440. 5*a.* 6*p.*; *Rs.*1191. 12*a.*
(5) £159. 8*s.* 10*d.*; £41. 15*s.* 9*d.*; £874. 9*s.* 9½*d.*
(6) £341. 18*s.* 1½*d.*; £2622. 9*s.* 11¾*d.*
(7) £6046. 10*s.* 1¼*d.*; £943. 7*s.* 4¾*d.*
(8) £995. 19*s.* 2½*d.*; £1493. 18*s.* 9¾*d.*; £1991. 18*s.* 5*d.*; £2489. 18*s.* 0¼*d.*; £2987. 17*s.* 7½*d.*; £3485. 17*s.* 2¾*d.*; £3983. 16*s.* 10*d.*; £4481. 16*s.* 5¼*d.*; £4979. 16*s.* 0½*d.*; £5477. 15*s.* 7¾*d.*; £5975. 15*s.* 3*d*
(9) *Rs.*1333. 5*a.* 6*p.*; *Rs.*2000. 0*a.* 3*p.*; *Rs.*2666. 11*a.*; *Rs.*3333. 5*a.* 9*p.*; *Rs.*4000. 0*a.* 6*p.*; *Rs.*4666. 11*a.* 3*p.*; *Rs.*5333. 6*a.*; *Rs.*6000. 0*a.* 9*p.*; *Rs.*6666. 11*a.* 6*p.*; *Rs.*7333. 6*a.* 3*p*; *Rs.*8000. 1*a.*
(10) £86. 1*s.* 5*d.*; £92. 13*s.* 10*d.*; £105. 18*s.* 8*d.*; £119. 3*s.* 6*d.*; £132. 8*s.* 4*d.*
(11) *Rs.*1362. 1*a.* 5*p.*; *Rs.*1466. 13*a.* 10*p.*; *Rs.*1571. 10*a.* 3*p.*; *Rs.*1781. 3*a.* 1*p.*; *Rs.*1990. 11*a.* 11*p.*; *Rs.*2095. 8*a.* 4*p.*
(12) £1297. 19*s.* 8¼*d.*; £1384 10*s.* 4*d.*; £1471. 0*s.* 11¾*d.*; £1557. 11*s.* 7½*d.*; £1644. 2*s.* 3¼*d.*

2. (1) *Rs.*4668. 14*a.*; *Rs.*6739. 7*a.* 6*p.*
(2) *Rs.*37559. 5*a.* 4*p.*; *Rs.*18750. 3*a.*
(3) *Rs.*72415. 2*a.*; *Rs.*10005. 7*a.* 6*p.*
(4) *Rs.*22168. 11*a.*; *Rs.*20182. 8*a.*
(5) *Rs.*49492. 12*a.* 6*p.*; *Rs.*3147603. 7*a.* 6*p.*
(6) £9495. 12*s.*; £2392. 18*s.* 10¼*d.*; £3676. 13*s.* 10¼*d.*
(7) £7625. 9*s.* 8¾*d.*; £537. 1*s.* 3¼*d*; £1818. 11*s.* 6*d.*
(8) *Rs.*59901. 12*a.*; *Rs.*275661. 2*a.* 8*p.*

3. (1) *Rs.*1774. 5*a.* 8*p.*; *Rs.*865. 6*a.* 8*p.*; £309. 9*s.* 0¼*d.*
(2) *Rs.*267. 4*a.* 10*p.*; *Rs.*283. 4*a.* 2*p.*; *Rs.*315. 2*a.* 10*p.*
(3) *Rs.*27527. 6*a.* 6*p.*; *Rs.*23537. 14*a.* 10*p.*; *Rs.*16356. 13*a.* 10*p.*
(4) £11653. 12*s.* 5½*d.*; £12177. 7*s.* 7½*d.*; £14796. 3*s.* 5½*d.*
(5) £63881. 15*s.* 8¾*d.*; £67116. 6*s.* 1¾*d.*; £105930. 11*s.* 1¾*d.*
(6) *Rs.*7473. 6*a.* 6*p.*; *Rs.*16662. 5*a.* 6*p.*
(7) *Rs.*10964. 9*a.* 8*p.*; *Rs.*50451. 6*a.* 4*p.*
(8) *Rs.*37[illegible]207. 10*a.* 6*p.*; *Rs.*221737 8*a.*
(9) £4656820. 11*s.* 0¾*d.*; £4823315. 15*s.* 3*d*; £13950280. 8*s.* 5¼*d.*

4. (1) £127903. 1*s.* 5*d.*; £1993. 7*s.* 9*d.*
(2) *Rs.*61892. 13*a.*; *Rs.*86151. 14*a.*
(3) £44154. 18*s.* 5*d.*; £225382. 12*s.* 3½*d.*

(4) Rs.127023. 13a. 8p.; Rs.299362. 0a. 8p.
(5) £4959308. 18s.; £56051090. 3s.

5. (1) Rs.3. 12a. 2p. (2) Rs.112. 2a. (3) Rs.56. 8a.
(4) Rs.308. 7a. (5) £361. 2s. 6d. (6) £278. 6s. 2d.
(7) Rs.3889. 14a. (8) £1046. 17s. 6d. (9) Rs.5342. 3a. 3p.
(10) Rs.7223. 8a. 5p. (11) £1462. 17s. 8$\frac{1}{2}$d. (12) £1162. 5s.

6. (1) Rs.3655. 6a. 4p. (2) Rs.487. 3a. 8p. (3) £16. 2s. 8d.

7. £1393. 8s. 10$\frac{3}{4}$d. **8.** Rs.2905. 4a. 4$\frac{1}{2}$p.; Rs.219. 11a. 7$\frac{1}{2}$p.

9. £160. 10s. **10.** He ought to pay Rs.49. 6a.

11. £860 0s. 7$\frac{1}{2}$d. **12.** £68423.

Ex. XXX. (pp. 97-98.)

1. (1) Rs.5. 14a. 10p.; Rs.56. 4a. 4p.; Rs.65. 12a. 8p.
(2) Rs.1720. 8a. 5p. rem. 1p.; Rs 48. 14a. 10p.; Rs.39. 10a. 8p.
(3) Rs.674. 0a. 2p.; Rs.48. 0a. 8p.; Rs.122. 15a. 5p.
(4) £13. 7s. 7$\frac{3}{4}$d.; £9. 15s. 2$\frac{1}{4}$d.; £37. 14s 7$\frac{3}{4}$d.
(5) £6. 12s. 9$\frac{3}{4}$d.; £9. 16s. 9$\frac{1}{4}$d.; £8. 12s. 11$\frac{1}{4}$d.
(6) £3. 16s. 7$\frac{3}{4}$d.; £47. 7s. 10$\frac{1}{2}$d.; £1. 13s. 4$\frac{1}{2}$d.
(7) Rs.492. 6a. 6p. rem. 6p.; Rs 164. 2a 2p. rem. 6p.; Rs 66. 6a. 11p. rem. 7a. 6p.
(8) Rs.649. 9a. 10p. rem. 1a. 4p.; Rs 209. 3a 2p. rem. 1a 4p.; Rs.37. 15a. 7p. rem. Re 1. 3a. 7p.
(9) Rs.141. 7a. 7p. rem. 7p.; Rs.37. 6a. 5p. rem. 9p.; Rs.4. 9a 1p. rem. Re.1. 11a. 8p.
(10) £9 0s. 11$\frac{1}{2}$d; £7. 2s. 6d.
(11) 14s. 7$\frac{1}{2}$d.; £35. 14s. 11$\frac{1}{2}$d. rem. 2d.
(12) £8 15s. 9$\frac{1}{4}$d. rem. 10s. 4$\frac{1}{4}$d.; £16. 18s. 10$\frac{3}{4}$d. rem. 13s. 8d.

2. (1) £119. 17s. 2$\frac{1}{4}$d.; £79. 18s. 1$\frac{1}{2}$d.; £59. 18s. 7d. rem. 2q.; £47. 18s. 10$\frac{1}{2}$d.; £39 19s. 0$\frac{3}{4}$d.; £34. 4s. 10$\frac{3}{4}$d. rem. 5q.; £29. 19s. 3$\frac{1}{2}$d. rem. 2q.; £26. 12s. 8$\frac{1}{2}$d.; £23. 19s. 5$\frac{1}{4}$d.; £21. 15s. 10d. rem. 10q.; £19. 19s. 6$\frac{1}{4}$d. rem. 6q.
(2) Rs.544. 6a.; Rs.362. 14a. 8p.; Rs.272. 3a.; Rs.217. 12a.; Rs.181. 7a. 4p.; Rs.155. 8a. 6p. rem. 6p.; Rs.136. 1a. 6p.; Rs.120. 15a. 6p. rem. 6p.; Rs.108. 14a.; Rs.98. 15a. 7p. rem. 7p.; Rs.90. 11a. 8p.
(3) Rs.134. 1a. 8p.; Rs.153 3a. 2p.; Rs 351. 2a. 6p.
(4) £47. 18s. 10$\frac{1}{2}$d.; £22. 15s. 0$\frac{1}{2}$d.; £8. 14s. 7$\frac{1}{4}$d.

3. (1) Rs.14 12a. 8p.; 12a. 4p.; Rs.39. 1a. 2p.
(2) Rs.44. 14a. 10p.; Rs.24. 2a. 10p.; Rs 40. 0a. 2p.
(3) £12. 17s. 8$\frac{3}{4}$d. rem. 2$\frac{1}{4}$d.; £23. 14s. 6$\frac{1}{4}$d. rem. 4d.; £5. 17s. 9$\frac{3}{4}$d.
(4) £3. 9s. 5$\frac{1}{4}$d; £4. 0s. 0$\frac{1}{4}$d.; £3. 18s. 1$\frac{3}{4}$d.

4. (1) Rs.262. 8a. 2p. (2) £17. 13s. 8d. (3) Rs.33. 15a 4p.
(4) 14s. 8$\frac{3}{4}$d. (5) £8. 15s. 2$\frac{3}{4}$d.
(6) Re.1. 2a. 5p. rem. 8a. 6p.
(7) £9. 13s. 2d. rem. 10s. 7$\frac{1}{2}$d. (8) Rs.11. 7a. 2p.
(9) Rs.8. 10a. 6p.; 9a. 8p.; Rs.4. 11a. 10p.
(10) 8s. 7$\frac{3}{4}$d.; 3s. 3$\frac{3}{4}$d.; 2s. 8$\frac{1}{2}$d

5. (1) *Rs*.87. 0*a*. 10*p*. rem. 6*a*. ; *Rs*.80. 5*a*. 11*p*. rem. 15*a*. 8*p*. ; *Rs*.8. 13*a*. rem. *Rs*.4. 10*a*. 8*p*.
(2) *Rs*.215. 11*a*. 8*p*. rem. 4*a*. 10*p*. ; *Rs*.147. 9*a*. 4*p*. rem. 14*a*. 2*p*. ; *Rs*.10. 12*a*. 2*p*. rem. *Rs*.32. 5*a*. 4*p*.
(3) *Rs*.6. 1*a*. 1*p*. rem. *Rs*.14. 6*a*. 11*p*.
(4) *Rs*.9. 13*a*. 3*p*. rem. *Rs*.14. 5*a*. 2*p*.
(5) £2579. 0*s*. $0\frac{3}{4}$*d*. (6) £166. 14*s*. $5\frac{1}{2}$*d*. rem. £1. 3*s*. 5*d*.
(7) £191. 10*s*. $2\frac{1}{2}$*d*. (8) £1473. 2*s*. $9\frac{3}{4}$*d*.

6. (1) *Rs*.3. 11*a*. 8*p*. (2) *Rs*.5. 4*a* 5*p*. (3) 15*a*. 4*p*.
(4) £12. 10*s*. $4\frac{3}{4}$*d*. (5) £3. 4*s*. $7\frac{3}{4}$*d* (6) *Rs*.11. 2*a*. 2*p*.
(7) *Rs*.5. 3*a*. 9*p*. (8) £4. 11*s*. $9\frac{1}{2}$*d*. (9) £151. 1*s*.
(10) £2. 3*s*. $2\frac{3}{4}$*d*. **7.** *Rs*.153. 3*a*. 2*p*. **8.** *Rs* 107. 10*a*.

9. 5*s*. $7\frac{1}{2}$*d*. **10.** 15*a*. 10*p*.
11. 19*s*. $8\frac{1}{4}$*d*. **12.** *Rs*.13. 4*a*. 6*p*.

Ex. XXXI. (p. 99.)

1. (1) 98 ; 41. (2) 69 ; 156. (3) 45 ; 478. (4) 184 ; 290.
(5) 345 ; 648. (6) 347 ; 247.
2. (1) 9. (2) 729. (3) 800. (4) 278. (5) 155.
3. (1) 365 rem. 2*a*. (2) 22 rem. *Rs*.1. 1*a*. 1*p*.
(3) 399 rem. 2*s*. 2*d*. (4) 74 rem. 3*s*. 7*d*.
4. 1142. **5.** 13. **6.** 121. **7.** 921. **8.** 480. **9.** 96. **10.** 4 gals.

Ex. XXXII. (pp. 102-105.)

(*Indian Bazar and Avoirdupois Weights.*)

1. (1) 52060 kan., 65075 to. ; 31720, 39650 ; 194108, 242635.
(2) 119408, 149260 ; 64000, 80000 ; 116000, 145000.
2. (1) 78578 kan. ; 117153. (2) 538571 ; 564291.
3. 1610 mds. 21 sr. 14 ch. ; 11799 mds. 3 sr. 11 ch. ; 4695 mds. 15 sr. 15 ch. 1 to. ; 30476 mds. 18 sr. 3 po. ; 6750 mds. 36 sr. 2 ch. ; 1363 mds. 11 sr. 5 ch. 3 to. ; 4890 mds. 26 sr.
4. 2616 ; 110880 ; 167895.
5. 873600 ; 65118176 ; 77414400 ; 361576.
6. 8162 kan. 18 mds. 5 vis. 1 sr. 2 to. ; 62 kan. 10 mds. 8 sr. 41 ta. ; 1 m. 4rat. 1 dh. ; 13kan. 13mds. 10 ta.; 5kan. 14 mds. 5vis. 1sr.
7. (1) 20895 oz., 5584 oz. (2) 216160 oz. ; 10708 lbs.
8. (1) 1441331 ; 2149817 ; 1741872 ; 6912.
(2) 31497 ; 4301 ; 534793.
9. (1) 39 tons 2 cwt. 2 qrs. 14 lbs. ; 10 tons 11 cwt. 26 lbs. 11 oz. ; 2 tons 19 cwt. 7 lbs. 6 oz. 3drs. ; 2 cwt. 1 qr. 3 lbs. 9 oz. 13drs.
(2) 4 tons 1cwt. 3qrs. 7lbs. 5 oz. 12drs. ; 28 tons 3cwt. 2qrs. 1[illegible]. ; 6 tons 12 cwt. 1 qr. 1lb. 15 oz. ; 372831 tons 5 cwt. 7 st.
(3) 9 tons 13 cwt. 17 lbs. ; 494 tons 19 cwt. 1 qr. 24 lbs. 13 oz. ; 9 tons 7 cwt. 3 qrs. 26 lbs. 14 oz. 6 drs.
1245 tons 1 cwt. 1 qr. 3 lbs. 8 drs.

10. (1) 451 mds. 6 sr. 10 ch. (2) 573 mds. 24 sr. 10 ch. 2 kan.
(3) 1177 mds. 36 sr. 2 ch. 3 kan. (4) 694 tons 5 cwt. 27 lbs. 7 oz.
(5) 120 cwt. 2 qrs. 6 lbs. 2 oz. (6) 150 tons 13 cwt. 1 qr. 26 lbs.
11. (1) 133 mds. 22 sr. 13 ch. (2) 544 mds. 27 sr. 11 ch. 1 kan.
(3) 214 mds. 24 sr. 11 ch. 2 kan. (4) 14 cwt. 18 lbs.
(5) 33 tons 12 cwt. 2 qrs. 25 lbs. (6) 53 cwt. 1 qr. 23 lbs. 14 oz.
12. (1) 2652 mds. 18 sr. ; 3978 mds. 27 sr. ; 7957 mds. 14 sr.
(2) 32482 mds. 9 sr. 8 ch. ; 28421 mds. 38 sr. 5 ch ; 81205 mds. 23 sr. 12 ch.
(3) 2645 tons 13 cwt. 16 lbs. ; 2866 tons 2 cwt. 2 qrs. 8 lbs.
(4) 3734 tons 3 cwt. 1 qr. 4 lbs ; 44809 tons 19 cwt. 1 qr. 20 lbs.
(5) 144 tons 13 cwt. 3 qrs. 26 lbs. 14 oz. 10 drs. ; 233 tons 2 cwt. 2 qrs. 4 lbs. 7 oz. 9 drs. ; 377 tons 16 cwt. 2 qrs. 3 lbs. 6 oz. 3 drs. ; 1069 tons 3 cwt. 1 qr. 13 lbs. 12 oz. 1 dr.
13. (1) 4 mds. 2 ch. 2 kan. rem. 58 kan. ; 3 mds. 2 ch. rem. 4 ch.
(2) 10 mds. 39 sr. 7 ch. 3 kan. rem. 44 kan. ; 3 mds. 26 sr. 7 ch. 3 kan. rem. 268 kan.
(3) 18 mds. 28 sr. 9 ch. rem. 46 kan. ; 31 sr. 3 ch. rem. 714 kan.
(4) 5 tons 8 cwt. 2 qrs. 18 lbs. 9 oz. ; 2 tons 17 cwt. 2 qrs. 3 lbs. 3 oz. 13 drs. rem. 3 drs. ; 1 cwt. 3 qrs. 23 lbs. 1 oz. 1 dr. rem. 476 drs.
(5) 4 cwt. 1 qr. 2 lbs. 13 oz. 3 drs. rem. 80 drs. ; 2 qr. 23 lbs. 12 oz. 13 drs. rem. 800 drs. (6) 28 ; 27 ; 17.
14. £1. 5*s*. 7½*d*. **15.** 2*s*. 10¾*d*. **16.** 150.

(*Indian Jeweller's and Troy Weights*).

1. 9939 ; 57956 ; 62534.
2. 146 to. 11 m. 3 ra. 2 dh. ; 556 to. 8*a*. 2 ra. ; 97 to. 7 m. 3 ra. ; 338 to. 1*a*. 4 ra.
3. (1) 74294 ; 91697 ; 52272. (2) 92160 ; 4750 ; 79480.
4. 2 lbs. 4 oz. 6 dwts. 16 grs. ; 25 lbs. 4 oz. 16 dwts. 10 grs. ; 1670 lbs. 5 oz. 3 dwts. ; 561 lbs. 7 oz. ; 151 lbs. 7 oz. 16 dwts. 17 grs.
5. (1) 383 to. 8 m. 3 ra. (2) 402 to. 6*a*. 1 ra. 3 dh.
(3) 140 lbs. 9 oz. 3 dwts. 14 grs.
6. (1) 202 to. 7 m. 5 ra. 3 dh. (2) 198 to. 12*a*. 3 ra. 3 dh.
(3) 66 lbs 10 oz. 16 dwts. 23 grs. (4) 57 lbs. 8 oz. 15 dwts. 20 grs
(5) 1 lb. 11 dwts. 19 grs. (6) 187 to. 12*a*. 3 ra. 2 dh.
7. (1) 8325 to. 11 m. 2 ra. ; 9251 to. 4 ra.
(2) 27805 to. 15*a*. ; 30123 to. 1*a*. 3 ra. 2 dh.
(3) 1448 lbs. 9 oz. 1 dwt. 8 grs. ; 2173 lbs. 1 oz. 12 dwts.
(4) 2159 lbs. 10 oz. 7 dwts. 10 grs. ; 10882 lbs. 4 oz. 14 dwts. 7 grs. ; 21349 lbs. 5 oz. 2 dwts. 13 grs.
8. (1) 8 to. 6 m. 2 ra. 2 dh. rem. 115 dh. ; 7 to. 9 m. 6 ra. 1 dh. rem. 139 dh.
(2) 5 to. 14*a*. 5 ra. 2 dh. rem. 170 dh. ; 1 to. 15*a*. 3 ra. 3 dh. rem. 342 dh.

(3) 4 lbs. 7 oz. 10 dwts. 23 grs. rem. 7 grs. ;
1 lb. 2 oz. 11 dwts. 1 gr. rem. 360 grs.
(4) 9 oz. 15 dwts. 16 grs. rem. 96 grs. ;
4 oz. 17 dwts. 20 grs. rem. 96 grs.
(5) 365. **9.** £3. 17*s*. 10½*d*. **10.** 80.

(*Native Physicians' and Apothecaries' Weights.*)

1. 8275 ; 10478 ; 2862.
2. 15 to. 2 m. 4 ra.; 41 to. 2 m.; 1332 to. 6 m. 6 ra. 3 dh. ; 30 to. 6 ra.
3. 17599 ; 11800 ; 104932.
4. 46lbs. 10 oz. 5 drs. 1 scr. 3 grs.; 93 lbs. 8 oz. 2 drs. 2 scr.;
16 lbs. 2 scr.; 9 lbs. 2 ʒ.; 3 lbs. 5 ʒ. 19 grs.
5. (1) 304 to. 4 m. 1 dh. (2) 5 lbs. 9 oz. 1 dr. 1 scr. 3 grs.
(3) 143 lbs. 9 ℥. 5 ʒ. 1 gr. **6.** (1) 66 to. 6 m. 6 ra. 2 dh.
(2) 15 lbs. 10 oz. 7 drs. 12 grs. (3) 25 lbs. 8 ℥. 4 ʒ. 1 ℈.
7. (1) 4337 to. 3 m. 5 ra.; 4698 to. 7 m. 1 ra. 1 dh.
(2) 729 lbs. 11 oz. 4 drs. 2 scr. 8 grs.; 2919 lbs. 10 oz. 3 drs. 12 grs.
(3) 374 lbs. 5 oz. 2 drs. 1 scr. 2 grs.; 501 lbs. 7 oz. 2 drs. 1 scr.
14 grs.; 17662 lbs. 3 oz. 7 drs. 1 scr.
8. (1) 7 m. 3 ra. 1 dh. rem. 10 dh. ; 6 m. 5 ra. rem. 46 dh.
(2) 1 lb. 1 scr. 18 grs. rem. 112 grs. ;
10 oz. 7 drs. 2 scr. 11 grs. rem. 100 grs.
(3) 5 lbs. 10 oz. 4 drs. 1 scr. 9 grs. rem. 26 grs.
5 lbs. 1 oz. 1 dr. 2 scr. 16 grs. rem. 12 grs. ; 3 oz. 1 dr. 2 scr. 18 grs.
(4) 252 ; 28. **9.** 140 pills + 80 grs.

Ex. XXXIII. (pp. 106-108.)

1. 8 ; 245 ; 528 ; 6. **2.** (1) 237600 ; 373140 ; 660600 ; 94860.
(2) 132401045 ; 66796027623½ ; 1155000.
3. 24 lbs. 3 oz. 13 dwts. 8 grs.; 6 drs. 1 scr. 4 grs.; 12 dwts. 12 grs.;
5 mds. 10 sr. **4.** 36 ; 216 mds. 3 vis. 19 pal. 1 to. 117 grs.;
3760460 mds. 10 sr. 10 ch.
5. 19 mds. 27 sr. 8 ch.; 14 mds. 35 sr.; 98 mds. 17 sr. 8 ch.;
107 mds. 25 sr.; 6 mds. 22 sr. 8 ch.
6. 5 cwt. 16 lbs.; 11 cwt. 2 qrs. 26 lbs.; 6 cwt. 3 qrs.; 7 cwt. 2 qrs.
24 lbs.; 1 ton 19 cwt. 24 lbs.; 16 cwt. 8 lbs.
7. 936lbs.; 394lbs. 11oz. 11 dwts. 16 grs.; 703lbs. 5 oz. 18 dwts. 13grs.
8. 585 Mad. mds.; 648 Bom. mds. and 9 lbs. over ; 4 tons 12 cwt.;
1 ton 11 cwt. 1 qr. **9.** 442. **10.** 50 lbs. 9 oz. 16 dwts. 3 grs.
11. 6600. **12.** 274 mds. 35 sr. **13.** 144.
14. 8773 ka. 7 mds. 24 sr.; 5848 ka. 18 mds. 16 sr.;
12761 ka. 5 mds. 24 sr.; 13950 ka. 5 mds. 20 sr.
15. 117 tons 18 cwt. 1 qr. 16 lbs. **16.** 35 sr.; 14 mds. 4 sr. 6 ch.;
12 lbs. 8 oz. **17.** 700lbs.; 48lbs.; 6 drs. 2 scr.
18. 54 tolas ; 384 lbs. Troy. **19.** 25. **20.** 801 tons 8 cwt.
3 qrs. 21lbs. **21.** 154. **22.** 3 mds. 2 ch. rem. 4 ch.
23. 13797 ka. 8 mds. **24.** 1943. **25.** 2 mds.

26. 7895 ka. 6 mds. 32 sr. ; 3838 ka. 10 sr. ; 82243 ka. 2 mds. 20 sr. ; 26061 ka. 2 mds. 10 sr.
27. (1) 3 sr. 27 ta. (2) 12 mds. 15 sr. 4 pal.
28. 84.
29. 213.
30. 114 grs.
31. 187.
32. 1290 cwt. 9 lbs. 13 oz.
33. 845.
34. 8 lbs. 9 oz. ; 5 dwts.
35. Feathers ; 1240 grs.

Ex. XXXIV. (pp. 110-111.)

1. 124032, 2976768 ; 215120, 5162880 ; 1616, 38784 ; 2063, 49512 ; 83503, 2004072.
2. 713 gaj. 1 ht. 2 gi. 2 an. ; 5912 gaj. 1 ht. 5 gi. ; 8757 gaj. 1 ht. 2 gi. 1 an. ; 439 gaj. 1 ht. 3 gi.
3. (1) 28624 ; 324003 ; 18465 ; 760320.
(2) 185184 ; 2573426. (3) 2000000 ; 1635033.
(4) 1600555 ; 205862.
4. 13600 yds. ; 15620 yds. ; 29005 ft. ; 5031 poles.
5. (1) 32 mi. 4 fur. 33 po. 1 yd. 1 ft. 6 in. ; 349 mi. 7 fur. 18 po. 1 ft. ; 4 mi. 1 fur. 36 po. 1 yd. 1 ft. 7 in. ; 57 mi. 6 fur. 5 yds. ; 10 mi. 1 fur. 56 yds.
(2) 17 mi. 110 yds. ; 131 mi. 2 fur. 31 yds. ; 8 mi. 1 fur. 86 yds. 4 in. ; 6 mi. 6 fur. 150 yds. ; 31 mi. 4 fur. 115 yds. 1 ft. 8 in.
6. 11 lea. 1 mi. 6 fur. 110 yds. ; 2399 mi. 2 fur. 5 po 4 yds. 1 ft. 8 in.
7. (1) 333. (2) 4400. (3) 716. (4) $736\frac{3}{4}$. (5) 1000. (6) 550.
(7) 280000. (8) 40000. (9) 75. (10) $24\frac{3}{4}$.
8. (1) 80 yds. 1 qr. 3 nl. $1\frac{1}{4}$ in. (2) 7098 bi. 18 ka.
(3) 699 dan. 2 haths 12 ang 2 yab. (4) 3096 gaj. 6 tasu.
(5) 56 mi. 1440 yds. (6) 33 kros 3520 haths. (7) 383999.
9. (1) 323yds. 4in. (2) 43po. 1 ft. 9in. (3) 195mi. 1fur. 23po. $4\frac{1}{2}$yds.
(4) 46mi 6fur. 2po. 5yds. 8in. (5) 167yds. 1na. (6) 142ells. 1na.
10. (1) 12 mi. 2 fur. 29 po. (2) 1 fur 18 po. 5 yds.
(3) 2 mi. 7 fur. 2 po. $2\frac{3}{4}$ yds. (4) 9 yds. 2 qrs. 1 nl.
11. (1) 12 lea. 1 mi. 4 fur. 16 yds. 8 in. ; 1705 yds.
(2) 585 lea. 1 mi. 6. fur. 10 po. ; 1352 lea. 1 mi. 2 fur. 25 po.
(3) 1446 mi. 1 fur. 7 po. 3 yds. 10 in. ; 2143 mi. 3 fur. 7 po. 3 yds. 2 ft. 4 in.
(4) 81 dan. 3 ha. 4 gi. ; 102 dan. 1 ha. 3 gi. 245 dan. 2 ha. 4 gi.
12. (1) 69 lea. 2 fur. 36 po. ; 44 yds. 2 nls. 2 in. rem. $\frac{1}{4}$ in.
(2) 4 mi. 7 fur. 36 po. 7 in. rem. 1 ft. 4 in. 34 po. 4 yds. 1 ft. 11 in. rem. 12 po. 4 yds. 2 ft. 9 in.
(3) 61 mi. 6 fur. 111 yds. 1 ft. 3 in. rem. 9 in. ; 5 mi. 5 fur. 72 yds. 1 ft. 7 in. rem. 2 yds. 1 ft. 6 in.
(4) 91 dan. 1 gaj 1 ha. 4 gi. ; 55 dan. 4 gi. ; 27 dan. 1 gaj 2 gi. ; 22 dan. 1 gaj 1 ha. 7 gi.
(5) 58.
13. 34 yds. 3 qrs. 4 in.
14. 11 mi. 3 fur. 14 po.
15. 29 kros 155 dan. 1 gaj 1 ha. 5 gir.
16. 61.

Ex. XXXV. (pp. 113-115.)

1. 33080 ; 291020 ; 164895 ; 867520 ; 2725717 ; 191040.

2. 1117 bi. 11k. 12ch.; 1bi. 12k. 12ch. 6gan.; 1301bi. 13ch. 1·5sq. cub.; 730bi. 12k. 3ch.; 392bi. 17k. 8ch.
3. 198000 ; 1093940 ; 690800 ; 3388255.
4. 349360012800 ; 62585395200 ; 107494250688 ; 29376720.
5. 38160 ; 61875 ; 30604. 6. 65343 ; 235500 ; 16912.
7. (1) 30 bi. 15 bisv. 2 bisvansi 13 kachvan.
(2) 86282 bi. 3 pands. 16 ka.
(3) 5 caw. 5 grounds 533 sq. ft. 98 sq. in.
(4) 8400 cha. 3 ruk. 1 bi. 2 pands. 14 ka.
(5) 97 sq. mi. 164 caw. 4 man.
(6) 6069 ghu. 1 bi. 1 kanal. 11 marla. 2 sar.
8. (1) 23184 ; 4827 ; 37584 ; 71240 ; 5399316.
(2) 107183736 ; 9531756 ; 5595452 ; 355433005.
(3) 242868780 ; 12702096 ; 12043468800 ; 26109864 ; 263450880.
9. (1) 103 ac. 1 ro. 33 po.; 2 ac. 3 ro. 12 po. 5 sq. yds.; 3 ac. 3 ro. 25 po. 3 sq. yds. 108 sq. in.; 518 ac. 2 ro. 13 po.; 185 ac. 2 po. 26 sq. yds. 4 sq. ft. 72 sq. in.; 62 ac. 32 po. 5sq. ft.
(2) 19 ac. 2 ro. 29 po. 2 sq. yds. 5 sq. ft. 81 sq. in.; 15 ac. 10 po. 15 sq. yds. 1 sq. ft. 138 sq. in.; 72 ac. 2 ro. 17 po. 23 sq. yds. 88 sq. in.; 11480 ac. 2 ro. 11 po. 28 sq. yds. 6 sq. ft. 116 sq. in.; 56 ac. 2 ro. 25 po. 37 sq. yds. 5 sq. ft. 73 sq. in.
10. (1) 1400000 sq. links. (2) 2 sq. mi. 523 ac.
(3) 1 sq. mi. 434 ac. 1840 sq. yds. (4) 268468992 sq. in.
(5) 351 sq. chs. and 5 sq. po. (6) 1035760 bi.
11. 1800 ac.; 7744 bi.; 3200 ac.; 43560 bi.; 2400 ac.; 279389 bi.
12. 332800 ; 9600 ; 80195 ; 1452000 ; 29160000.
13. (1) 136 bi. 13 k. 11 ch. (2) 134 sq. yds. 6 sq. ft. 53 sq. in.
(3) 148 ac. 4 po. (4) 30 ac. 2 ro. 14 po. 10 sq. yds.
(5) 98 ac. 2 ro. 18 po. 23 sq. yds.
(6) 86 ac. 1 ro. 32 po. $2\frac{1}{2}$ sq. yds. 4 sq. ft. 109 sq. in.
14. (1) 48 bi. 15 k. 12 ch. (2) 66 ac. 1 ro. 29 po.
(3) 5 ac. 2 ro. 10 po. $28\frac{1}{4}$ sq. yds.
15. (1) 5915 bi. 16 k. 10 ch.; 19371 bi. 5 k. 8 ch.
(2) 1255 ac. 3 ro. 32 po.; 4 ac. 5 po. 4 sq. yds. 5 sq. ft. 36 sq. in.; 6 ac. 2 ro. 35 po. 7 sq. yds. 5 sq. ft. 108 sq. in.
(3) 302 ac. 3 ro. 39 po. 16 sq. yds. 3 sq. ft. 130 sq. in.; 2840 ac. 2 ro. 15 po. 21 sq. yds. 7 sq. ft. 21 sq. in.
16. (1) 1 bi. 2 k. 13 ch. rem. 7 ch. ; 15 bi. 1k. 14 ch. rem. 4 k. 3 ch.
(2) 1 bi. 3 k. rem. 12 ch. ; 1 ac. 13 po. rem. 2 ro. 28 po.
(3) 94 ac. 3 ro. 38 po. 17 sq. yds. 7 sq. ft. 25 sq. in.; 3 ac. 1 ro. 36 po. 1 sq. yd. 3 sq. ft. 72 sq. in. rem. 9 sq. in.
(4) 23 ; 17. 17. 297. 18. 653 sq. mi. and 439 ac.

Ex. XXXVI. (pp. 115-116.)

1. 2742 ; 5630 ; 9376 ; 9216.
2. 1131984 ; 841082 ; 599616 ; 1074088.
3. 4 cub. yds. 7 cub. ft. 1280 cub. in.; 2 cub. yds. 26 cub. ft. 57 cub. in.; 3 cub. yds. 1 cub. ft. 25 cub. in.; 18 cub. yds. 13 cub. ft. 33 cub. in.

4. 566386 chouk. 4 cub. yds. 6 cub. cubits ;
67324 chouk. 2 cub. yds. 4 cub cubits ;
13470 chouk. 2 cub. yds. 1 cub. cubit.
5. 932014080 cub angulis. 6. (1) 464 chouk. 7 cub. yds.
(2) 189 c. yds. 22 c. ft. 431 c. in. (3) 2627 c. yds. 3 c. ft. 27 c. in.
7. (1) 9 c. yds. 18 c. ft. 1534 c in. (2) 90 c. yds. 1 c. ft. 727 c. in.
(3) 247 c. yds. 25 c. ft. 1470 c. in.
8. (1) 24 c. yds. 5 c. ft. 832 c. in. ; 50 c. yds. 16 c. ft. 640 c. in.
(2) 15435 c. yds. 11 c. ft. 88 cub. in.
9. (1) 5 c. yds. 21 c. ft. 1639 c. in. rem. 20 c. in.
(2) 354 c. yds. 18 c. ft. 1025 c. in. rem. 1 c. in. ;
13 c. yds. 6 c. ft. 822 c. in. rem. 246 c. in.
(3) 26 times and 11 c. ft. 700 c. in. over. 10. 7.

Ex. XXXVII. (pp. 117-119.)

1. 1603 ; 890 ; 3655680 ; 80520 ; 5735200 ; 29560.
2. 6 mds. 2 ch. ; 1573 mds. 1 do. 1 pa. 1 rek. ;
76190 mds. 5 do. 1 rek. 15 ch. ; 1140 kah. 3 bis. 8 arh. 6 kat. ;
7711 mds. 7 do. ; 43347 do. 1 pa.
3. 576000 ; 47923200 ; 52190 ; 2166 mudas 12 ph. 13 paylis.
1 sr. 1 tipari ; 1465 ka. 7 ph. 2 paylis 2 sr. 28 tanks.
4. 65600 ; 40152 ; 9270 ; 10 garces 4 markals 5 padis 4 ollaks ;
385 ph. 4 markals ; 58 ph. 6 markals ; 15431 markals 2 pad.
5. 188 ; 1158 ; 3518 ; 41528.
6. 6480 ; 9240 ; 43832 ; 91380 ; 3936.
7. 9354 qrs. 7 bus. ; 10007 lasts 1 qr. 1 co. 2 bus. ;
1606 co. 2 bus. 1 pot. ; 246688 qrs. 1 str.
8. 561 lds. 1 bus. 1 pk. ; 22 lds. 7 bus. 1 pk. 2 qts. 1 pt. ;
278 lds. 1 qr. 2 bus. 3 pks. 3 qts. ; 6250 lds. ;
38 lds. 2 pks. 1 gal. 2 pts. ; 13 lds. 3 qrs. 2 pks. 1 gal.
9. 3136 ; 20160 ; 8616 ; 28032 ; 225516
10. 1032 ; 1400 ; 284463 ; 44284. 11. 10256 ; 459705 ; 24040.
12. 55 pipes 110 gals. 1 pt. ; 992 tuns 16 gals. ; 659 gals 3 qts. 1 pt. ;
1960 gals. 2 qts. 1 pt. 1 gill ; 1816 qrs. 6 bus. 1 pk. 1 gal. 1 qt. 1 pt.
13. 48 tuns 1 butt 1 hhd 1 fir. 8 gals. 2 qts. 1 pt. ;
6 pipes 5 gals 1 pt. 1 gill. ; 83 butts 2 kil. ;
2852 lasts 1 ld. 4 qrs. 2 bus. 2 pks. 1 gal. 3 qts. 1 pt. ;
23211 hhds. 8 gals.
14. 11 *C.* 3 *O.* 19 fl. oz. 23 m. ; 760 *C.* 6 *O.* 5 fl. oz. 1 fl. dr. 32 m.
15. 1 cwt. 1 qr. 3 lbs. 12 oz. 16. 16 kan.
17. 21000 lbs. ; 26 tons 15 cwt. 2 qrs. 24 lbs.
18. (1) 623 mds. 1 rek. (2) 187 gals. 1 qt. 1 pt. 3 gills.
(3) 21 lds. 3 qrs. 1 pk. 1 gal. (4) 191 gals. 1 qt. 1 pt.
(5) 178 lds. 3 qrs. 2 bus. (6) 8 *C.* 6 *O.* 7 fl. oz.
19. (1) 30 gals. 2 qts. 1 pt. 3 gills. (2) 99 gals. 1 pt.
(3) 1 tun 2 hhds. 30 gals. 4 pts. (4) 3 lds. 3 qrs. 5 bus. 2 pks. 1 gal.
(5) 17 bus. 1 pk. 1 gal. (6) 3 *C.* 4 *O.* 14 fl. oz. 3 fl. dr. 35 m.
20. (1) 856 qrs. 3 bus. 1 pk. ; 1760 qrs. 3 bus. 1 gal.
(2) 1006 gals. 3 qts. 1 pt. ; 6600 gals. 2 qts. 1 pt.

21. (1) 732 gals. 3 qts. 1 pt. 3 gills. rem. 1 pt. 3 gills. ;
78 gals. 1 pt. 1 gill. rem. 2 gals. 1 qt.
(2) 1477 lds. 7 bus. 3 pks. rem. 3 gals. ;
110 lds. 1 qr. 1 bus. 2 pks. rem. 21 gals.
(3) 67 qrs. 1 bus. 3 pks. 1 gal. ; 79 ; 12 fl. oz. 3 fl. dr. 2 m.
22. 128. 23. 172 days and 4 pts. over. 24. 283 gals. 2 qts. 25. 96.

Ex. XXXVIII. (pp. 123-124.)

1. (1) 1188000. (2) 20738400. (3) 798660000.
(4) 16395480000. (5) 450542400. (6) 9720000.
(7) 1179360000. (8) 155520000. (9) 137700000.
(10) 2210711189860.
2. (1) 16815600 ; 23132 ; 3283200. (2) 107362800 ; 6637437.
(3) 190310 ; 10969200 ; 94867200.
3. 7921587 ; 170120 ; 2030400.
4. (1) 156 da. 4 pr. 1 dan. 12 pals.;
4 da. 33 dan. 14 pals 5 bip. 32 anupals.
(2) 313 pr. 3 dan. 20 pals 45 bip.; 345 dan. 31 pals 15 bip. 8 anu.
(3) 52 da. 18 pals 14 bip. 3 anu.; 1686 bat. 11 ma. 10 da. 5 dan.
5. (1) 428 wks. 4 da. 15 hrs.; 4 wks. 3 da. 7 hrs. 45 min. 59 sec.;
26 days 18 hrs. 47 min.
(2) 34 hrs. 17 min. 36 sec.; 40 da. 13 min. 14 sec.;
1 yr. 37 days 11 hrs. 34 min.
6. 2 yrs. 101 da. 20 hrs. 25 min.;
41 yrs. 97 da. 16 hrs. 35 min. 10 sec. ; 22 yrs. 216 da.
20 hrs. 50 min. 10 sec.; 13 yrs. 49 da. 22 hrs. 37 min.
7. (1) 131 sap. 6 da. (2) 212 dan. 38 pals 9 bip. 33 anu.
(3) 229 din 24 dan. 41 pals 46 bip. 7 anu.
(4) 152 hrs. 20 min. 45 sec. (5) 285 da. 13 hrs. 47 min. 52 sec.
(6) 224 wks. 2 da. 2 hrs. 10 min.
8. (1) 108 sap. 5 da. 6 pr. (2) 29 din 54 dan. 49 pals 44 bip.
(3) 9 da. 7 hrs. 28 min. 55 sec. (4) 19 da. 17 hrs. 27 min. 39 sec.
(5) 2 wks. 3 da. 22 hrs. (6) 3 yrs. 220 da. 18 hrs. 51 min. 48 sec.
9. (1) 524 din 48 dan. 53 pals 30 bip. ; 992 din 50 dan. 35 pals.
(2) 418 ba. 11 ma. 23 da. 55 dan. 35 pals 51 bip. 16 anu.;
652 ba. 10 ma. 18 da. 15 pals 24 bip. 4 anu.
(3) 3371 days 3 hrs. 45 min. ; 6435 days 20 hrs. 15 min.
(4) 158 wks. 6 da. 1 hr. 8 min. 24 sec ;
1394 wks. 3 da. 7 hrs. 20 min. 24 sec.
(5) 2491 yrs. 247 da. 2 hrs. 16 min. 48 sec.
10. (1) 21 din 5 pr. 5 dan. 30 pals.
(2) 23 ba. 7 ma. 19 da. 37 dan. 45 pals.
(3) 2 da. 13 hrs. 6 min. 37 sec. rem. 47 sec.
(4) 97 wks. 4 da. 9 hrs. 44 min. 10 sec. rem. 6 sec.;
15 wks. 4 da. 19 hrs. 59 min. 36 sec.
11. (1) 212 days. (2) 366 days. (3) 162 days. (4) 659 days.
12. 78894000 bip. 13. (1) 20927½ sec. (2) 65472½ sec.
14. 262. 15 Wednesday.

Ex. XXXIX. (p. 125.)

1. (1) 619705. (2) 991826. (3) 519158. (4) 205665.
2. (1) 57° 17′ 45″. (2) 2 rt. ang. 60° 23′ 28″. (3) 2 rt. ang. 19° 31′ 16″. (4) 7 rt. ang. 8° 21′. (5) 14 rt. ang. 43° 25′.
3. 951° 28′ 6″. **4.** 127° 42′ 51″. **5.** 1016° 35′ 12″; 692° 39′.
6. 1° 33′ 20″ rem. 12″; 6° 34′ 43″ rem. 21″. **7.** 26880.
8. 86829755. **9.** 5701 kahans 7 pans 6 ga. 1 unit; 15 doz.
10. 752 kahans 11 pans 15 ga.; 3176 kahans 2 pans 15 ga.

Ex. XL. (pp. 125-127.)

1. £297. 10*s*. **2** 154; 370843. **3.** 2723; 778.
4. 3100. **5.** £461. 16*s*. 6*d*.; £860. 0*s*. $7\frac{1}{2}$*d*.
6. £219. 13*s*. 9*d*. **7.** £4. 4*s*. $3\frac{3}{4}$*d*. **8.** 11 mi. 3 fur. 14 po.
9. 9 yrs. 131 days 18 hrs. 12 min. 54 sec.
10. 1767 din 6 pr. 2 dan. 8 pals 45 bip 15 anupals.
11. 48 kros 1582 dan. 1 hath 1 big. 1 mush. 3 ang.; 2340 bi. 8 k. 3 ch. 11 gan.
12. 18783 times, and 18 in. over **13.** 1705 yds. **14.** 6*d*.
15. 2*s* 6*d*. **16.** 41 lbs. 6 oz 11 dwts. **17.** 7 da. 13 hrs. 30 sec.
18. £23. 6*s*. 8*d*. **19.** *Rs*.8. **20.** 7. **21.** 144062.
22. 564394385 grs. **23.** 46 ac 3 ro. 27 po. 13 sq. yds. 8 sq. ft. 65 sq. in. **24.** 2 tons 4 cwt 2 qrs. 16 lbs.
25. 1 lb of sugar. **26.** £3. 17*s*. $10\frac{1}{2}$*d* **27.** 57.
28. £4. 17*s*. 10*d*. **29** 91 mi. 3 fur. 15 po. 5 yds. 1 ft. 3 in.
30. 25 yrs. 200 da 9 hrs **31.** 118 cwt. 3 qrs. 23 lbs. 12 oz.; $25\frac{1}{2}$.
32. 108; 275. **33.** (i) 21120. (ii) 53712. **34.** 272. **35.** 245.
36. 2161; 11040. **37.** Friday; Monday. **38.** Friday; Wednesday.

Ex. XLI. (p. 129.)

1. £49. 4*s*. **2.** £31. 17*s*. 6*d* **3.** £1. 4*s*. $7\frac{1}{2}$*d*. **4.** £1. 5*s*. $7\frac{1}{2}$*d*.
5. *Rs*.196. 8*a*. **6.** *Rs*.21. 6*a*. 9*p*. **7.** *Rs*.54. 10*a*. **8.** *Rs*.930. 4*a*.
9. 72 days. **10.** 100 miles. **11.** 160. **12.** *Rs*.69. 14*a*.
13. *Rs*.315. **14.** 75 ac. 2 ro. 10 po. **15.** 25 men.
16. *Rs*.11. 13*a*. 3*p*. **17.** 22 mds. 15 sr. **18.** 6 cwt. 3 lbs.
19. £18. 16*s*. 8*d*. **20.** 168 days.

Ex. XLII. (pp. 130-131.)

1. 3 miles 960 yds. **2.** 202 miles. **3.** 2880. **4.** 4 yds. 1 ft.
5. 4 mi. 5 fur. 10 po. 2 yds. 2 ft. 2 in. **6** 6400. **7.** 7 ft. 9 in.
8. 4675. **9.** 10240. **10.** 36 mi. 2 fur. 18 po. 1 yd.
11. 4 yds. 1 ft. 4 in. **12.** 5 yds. 2 ft. 5 in. **13.** 4 yds. 1 ft.
14. 2 ft. 9 in. **15.** 6 fur. 2 po. 2 yds. 1 ft.

Ex. XLIII. (p. 133.)

1. *Rs*.36. 5*a*. 2*p*. rem. 6*p*. **2.** (1) *Rs*.76. 6*a*. (2) *Rs*.63. 10*a*. 4*p*.
3. $1\frac{3}{4}$*d*. **4.** *Rs*.18. 7*a*. 6*p*. and 1*a*. over. **5.** 54*s*. $6\frac{3}{4}$*d*.

6. *Rs*.3339. 10*a*. 11*p*. 7. 12359 ft. 8. 14*s*. 7¾*d*.
9. 6*a*. 8*p*. 10. *Rs*.3740. 2*a*.

Ex. XLIV. (pp. 134-135.)

1. *Rs*.125. 2. *Rs*.40. 3. *Rs*.171. 4*a*. gain. 4. £86. 13*s*. 11½*d*.
5. *Rs*.467. 8*a*. 6. *Rs*.2. 13*a*. 7. *Rs*.104. 10*a*. 8. 3*a*. 8*p*.
9. 96 books. 10. 226 gals. 11. *Rs*.595 ; *Rs*.37.
12. 12 chairs. 13. £10. 10*s*. 8*d*. 14. £3. 11*s*. 15. 10 gals.

Ex. XLV. (p. 136.)

1. 3550. 2. 1775. 3. 115. 4. Receives *Rs*.136. 14*a*.
5. *Rs*.100000. 6. 128 lbs. 7. 720 yds. 8. 1771.
9. 72 lbs. 10. £2. 3*s*. 4¼*d*. 11. *Re*.1. 7*a*. 4*p*. 12. 185.

Ex. XLVI. (pp. 137-138.)

1. 20. 2. 25. 3. 240. 4. 418. 5. 19. 6. 585.
7. 45. 8. *Rs*.64 ; 128 *half Rs*. ; 320*qr. Rs*. ; 512 two-anna pieces.
9. 15 mds. 34 sr. 6 ch. 10. 88.

Ex. XLVII. (pp. 139-140.)

1. *Re*.1. 6*a*. 8*p*. 2. 5½*d*. 3. *Re*.1. 7*a*. 8*p*. 4. 5 lbs.
5. 3*a*. 4*p*. 6. 2*s*. 10½*d*. 7. 13. 8. 5*d*. 9. 4½*d*. 10. 14 gals.

Ex. XLVIII. (pp. 140-141.)

1. *Rs*.28748. 7*a*. 2. *Rs*.9. 12*a*. 3. *Rs*.5520. 10*a*.
4. *Rs*.1252. 8*a*. 5. £253. 16*s*. 3*d*. 6. £100. 3*s*. 1½*d*
7. £256. 8. *Rs*.8750. 9. 2*s*. 6*d*. 10. *Rs*.431. 5*a*. 2*p*.

Ex. XLIX. (p. 142.)

1. *A Rs*.3. 10*a*. 4*p*. ; *B Rs*.6. 15*a*. 8*p*. ; *C Rs*.13. 15*a*. 4*p*.
2. *Rs*.48. 13*a*. 8*p*. ; *Rs*.24. 6*a*. 10*p*.
3. *Rs*.47. 5*a*. 2*p*. ; *Rs*.54. 1*a*. 4*p*. 4. £14 7*s*. 6*d*. ; £5. 15*s*.
5. *A Rs*.10448 9*a*. 8*p*. ; *B Rs*.8661. 13*a*. 8*p*. ; *C Rs*.5404. 8*a*. 8*p*.
6. *A Rs*.278. 14*a*. ; *B Rs*.836. 10*a*. ; *C Rs*.1394. 6*a*.
7. £73. 11*s*. 10½*d*. ; each of the other two £57. 16*s*. 10½*d*.
8. *Rs* 17. 8*a*. 9. *A* gets *Rs*.60 ; *B Rs* 180 ; *C Rs*.450.
10. *Rs*.152. 11*a*. 4*p*. ; *Rs*.129. 12*a*. 10*p*. ; *Rs*.91. 10*a*. ; *Rs*.61. 1*a*. 4*p*. ; *Rs*.38. 2*a*. 10*p*. 11. £2. 17*s*. 8½*d*. ; £3. 16*s*. 5½*d*.
12. *Rs*.16053. 15*a*. 4*p*. ; *Rs*.23343. 0*a*. 8*p*. ; *Rs*.29030. 3*a*. 4*p*.

Ex. L. (pp. 143-144.)

1. Each man receives 13*s*. 10½*d*. ; each woman 4*s*. 7½*d*.
2. *Rs*.2. 1*a*. 3. A man *Rs*.1426. 4*a*. ; a woman *Rs*.427. 14*a*. ; a boy *Rs*.142. 10*a*. 4. A man *Rs*.660. 3*a*. ; a woman *Rs*.330. 1*a*. 6*p*. ; a boy *Rs*.110. 0*a*. 6*p*. 5. 6*s*. 4½*d*.
6. *Rs*.29. 12*a*. 7. *Rs*.5. 8. 40 men. 9. *Rs*.25. 10*a*. 4*p*.
10. A goat *Rs*.4. 4*a*. 7*p*. ; a lamb *Rs*.2. 5*a*. 5*p*. ; a calf *Rs*.6. 0*a*. 1*p*.

Miscellaneous Examples II. (pp. 146-152.)

1. *Rs.*11. 4*a.* **2.** 1065. **3.** 587. **4.** 2*s.* 3*d.*; £11250.
5. *Rs.*50. **7.** *A Rs.*12. 4*a.*; *B Rs.*10. 6*a.*; *C Rs.*15. 2*a.*
8. *Rs* 32. 8*a.* **9.** *Rs.*331. 4*a.* 2*p.* **10.** *Rs.*1074. 12*a.* 10*p.*
11. *Re.*1. 7*a.* 4*p.* **12.** 247 sr. 8 ch. tea.; 1897 sr. 8ch. sugar; 2530 sr. rice; 690 sr. coffee. **13.** *Rs.*4. 6*a.*
14. *Rs.*49573. 5*a.* 4*p.* **15.** 54 gals. **16.** *Rs.*61. 12*a.* **17.** 589 days.
18. *Rs.*230. 12*a.* **19.** *Rs.*2. 7*a.* **20.** *A* 10 ac.; *B* 1 ro.; *C* 1 po. $9\frac{3}{4}$ sq. yds. **21.** Thursday. **22.** 42 gals.
23. 1600 yds. **24.** £2993. **25.** *Rs.*52. 4*a.*
26. The first gains 1*a.* 4*p.* more than the 2nd. **27.** *Rs.*3. 1*a* 1*p.*
28. 80 half *Rs.*; 240 qr. *Rs.* **29.** *A* shall pay *B Rs.*72. 8*a.*
30. 23043 days. **31.** *Rs.*1250. **32.** Piano costs *Rs.*372.9*a.*4*p.*; table *Rs.*174. 12*a.* 8*p.*; carpet *Rs.*85. 6*a.* **33.** 100lbs.; *Re.*1. 5*a.* 4*p.* **34.** *A Rs.*37. 14*a.*; *B Rs.*25. 4*a.*
35. *Rs.*25; 50 half *Rs.*; 100 four-anna bits; 200 two-anna bits.
36. *C* shall have to pay *A Rs.*35. 4*a.*, and to *B Rs.*26. 2*a.*
37. *Rs.*1025. 12*a.* 6*p* **38.** *A. Rs.*2630; *B Rs.*1252. 8*a.*: *C Rs.*4007. 8*a.* **39.** 50. tolas. **40.** 2*a.* **41.** Tuesday.
42. 5 mds. 8 sr.; *Rs.*96. 8*a.* **43.** 25 days. **44.** Loss *Rs.*7. 8*a.* 8*p.*
45. 1st *Rs.*3785. 5*a.* 4*p.*; 2nd *Rs.*2534. 1*a* 4*p.*; 3rd *Rs.*3937. 5*a* 4*p.*
46. A man 10*a.*; a woman 6*a.*; a boy 4*a.*
47. 14 double pice; 36 single pice. **48** 7th Feb. 1833, Thursday.
49. First £400; 2nd £800; 3rd £1600. **50.** *A* gainer by *Rs.*85. 2*a.*
51. A boy *Rs.*2; a woman *Rs.*4. 2*a.*; a man *Rs.*6. 6*a.*
52. Monday, 16th March, 1863. **53.** *Rs.*953; *Rs.*422. 12*a.* 6*p.* *Rs.*405. 0*a* 6*p.* **54.** *Rs.*59. **55.** *Re.*1. 12*a.* **56.** 20.
57. 20.; 40. **58.** 1. mi 7 fur. 22 po. 5 yds. **59.** 3*s.* $7\frac{1}{2}$*d.*
60. £494812. 10*s.* **61.** 941070 sec. **62.** 91. **63.** 45 miles.
64. 1 mi. 904 yds. 2 ft. **65.** 26 days 20 min.; 40000. **66.** 10 sr.
67. *Rs.*48243. **68.** 107 tons 2 cwt. 3 qrs. 12 lbs.
69. 1 sr. 2 ch. 2 to. **70.** First *Rs.*1920: 2nd *Rs.*3840; 3rd *Rs.*5760; 4th *Rs.*7680. **71.** 671 lbs. 4 oz. **72.** *Rs.*140. 2*a.*
73. 10 ten-seer weights; 30 five-seer weights; &c.
74. 10 sov.; 20 cr.; 50 half-cr.; 80*s.*; 120 six-pences.
75. 2nd 16 mds. 5 sr.; 3rd 48 mds. 15 sr.

Ex. LI. (pp. 156-157.)

2. (1) $2^6.5$; $2^2.5.23$; $2.3.7.11.$; $3^2.5.7$; $2^2.3^2.17$; $5.11.13$; $2.3^2.47$; $3^3.5.7$; $3.5.7^2$.
(2) $2^2.3^3.11$; $7.11.17$; $3^2.7.29$; 11^3 ; $2^4.7.13$; $3^3.5.11$; $3.5^2.7^2$; $2^2.3.5.7.11$.
(3) $2.3.5^3.7$; [illegible]$3.5.7.131$; $2^3.3^3.7.11$; $7^2.31^2$; $7.13.19.31$; $3.5^2.7.13^2$; $3^2.5^2.7^2$.
(4) $2.5^3.11^2.17$; $7^2.13^2.23$; $17^2.29.31$; $11^2.13.17.19$; $7^2.17^2.293$; $2.5.7^4.13^2$.
(5) $2^5.3.5^4.7.11.13$; $13^2.17^3.89$; $2^8 \times 3^4 \times 7^2 \times 11$; $2.3^5.5.7^2.11.13$; $2^8.3^6.5^3.7^3$.

3. (1) 31, 53, 167 are primes ; 2×43 ; $2^5\times3$; $2^2\times3\times11$; $5^2\times11$; $2^5\times3\times5$; $2^3\times107$; $3^2\times97$.
(2) 397, 461, 727, 953, 971, 997 are primes ; 17^2 ; 23×29 ; 23×37.
(3) 1009, 3389 are primes ; 37×41 ; $7\times13\times19$; $2^5\times11\times13$; 41×61 ; 53×89.

4. (1) 3, 5, 11 ; 2, 3, 4, 6, 8, 9, 12 ; 2, 3, 4, 6, 9, 12 ; 5 ; 3, 9 ; 2, 3, 4, 6, 8, 9, 12, 13 ; 2, 4, 7 ; 2, 4, 8 ; 3, 5
(2) 2, 4, 7 ; 2, 4, 7 ; 2, 3, 4, 6, 7, 12 ; 3 ; 7 ; 2, 3, 4, 6, 11, 12 ; 13.
(3) 5, 13 ; none ; 3, 9 ; 2, 3, 4, 6, 8, 12 ; 3, 7, 11, 13 ; 2, 3, 5, 6, 9, 10.
(4) 2, 4, 8, 11 ; 2, 3, 6, 11 ; 3, 7, 9 ; 2, 4, 13.

5. (1) 18. (2) 9. (3) 16. (4) 4. (5) 8. (6) 30.

6. 4, 8, 11, 16, 22, 44, 88 ; 7, 11, 14, 22, 28, 44, 77, 154 ; 8, 9, 11, 12, 18, 22, 24, 33, 36, 44, 66, 72, 88, 99, 132, 198, 164, 396.

Ex. LIII. (p. 158.)

1. 9. **2.** 32. **3.** 24. **4.** 28. **5.** 9. **6.** 37. **7.** 41.
8. 25. **9.** 46. **10.** 14. **11.** 22. **12.** 19. **13.** 8. **14.** 124.
15. 9. **16.** 81. **17.** 442. **18.** 2. **19.** 4. **20.** 2.

Ex. LIV. (pp. 159-160.)

1. 6. **2.** 38. **3.** 6. **4.** 2. **5.** 28. **6.** 39. **7.** 27.
8. 17. **9.** 12. **10.** 39. **11.** 53. **12.** 131. **13.** 113. **14.** 173.
15. 147. **16.** 55. **17.** 221. **18.** 1536. **19.** 25. **20.** 105.
21. 142857. **22.** 142. **23.** 3. **24.** 1246 **25.** 37. **26.** 571.
27. 2476099. **28.** 1031. **29.** 2003. **30.** 11. **31.** 57. **32.** 8221.
33. 31. **34.** 17. **35.** 23. **36.** 21. **37.** 163. **38.** 6862.
39. 3432. **40.** 84.

Ex. LV. (pp. 160-161.)

1. (1) Yes. (2) Yes. (3) Yes. (4) Yes. (5) No. (6) Yes. (7) Yes. (8) Yes.
2. (1) 37. (2) 1287. (3) 3432. (4) 257. (5) 37.

Ex. LVI. (pp. 162-163.)

1. 6*p*. **2.** 8. **3.** 6912. **4.** 40 grs. **5.** 58, 870 ; 174, 754 ; 290, 638 ; 406, 522. **6.** 57 min.
7. 11. **8.** 5, 1140 ; 15, 380 ; 20, 285 ; 95, 60.
9. 24060 and 21672 ; 13, 3, 1. **10.** 24720 and 4155.
11. 20 pairs. **12.** 2 pairs. **13.** 19. **14.** 9. **15.** 4*d*.
16. *Rs.*7 a head ; *A* 97, *B* 843, *C* 962. **17.** 9971, 10140.
18. 28. 9. **19.** 23. **21.** 9920, 10044. **22.** 999984, 100149 ; 753.

Ex. LVII. (pp. 163-164).

2. (1) 144. (2) 240. (3) 168. (4) 1680.

(5) 2100. (6) 810 (7) 540540 (8) 1225449.
(9) 43344. (10) 2286900. (11) 11025. (12) 25200.
(13) 4204200. (14) 236876164. (15) 21600. (16) 27720.

Ex. LVIII. (p. 164.)

1. 5491. 2. 3396 3. 3556. 4. 7980. 5 734877.
6. 67868155. 7. 4040400. 8. 117544483. 9. 14664.
10. 39648. 11. 159137. 12. 2880. 13. 4029051.
14. 61688187. 15. 7258671. 16. 119268375.

Ex. LIX. (p. 165.)

1. 240. 2. 80. 3. 525. 4. 180. 5. 120. 6. 68. 7. 144.
8. 12600. 9. 810. 10. 1680. 11. 840. 12. 11088. 13. 1260.
14. 7560. 15. 7560. 16. 352800. 17. 6300. 18. 10800.
19. 285120. 20. 95040. 21. 180180. 22. 9828.
23. 120929760. 24. 5040. 25. 9009. 26. 98280.
27. 6630. 28. 602910. 29. 138600. 30. 27720 ; 720720.

Ex. LX. (pp. 166-168).

1. 77. 2. £10. 10*s*. 3 6 min. 4. 982832. 5. 144 lbs. Avoir.
6. 7 hours ; 210, 24, 60, 42, 35, 30. 7. 42 min.
8. 13 min. past 1 P. M. 9. £1. 10. 2108. 11. 875. 12. 842.
13. 960 pages ; 25 chap. 14. 60 min. 15. 9883. 16. 1197.
17. 6193. 18. 2521. 19 4199 ; 2431. 20. 288 ; 432 ; 576.
21. *Rs*.105. 22. 48 hours. 23. 840 hrs. 24. 218 yds. 2 ft. 3 in.
25. 22 ft. 26. 2 mds. 10 sr. 27. 740 miles.
28. (i) 10117800. (ii) 94500.

Ex. LXI. (p. 171.)

1. $\frac{11}{13}$; $\frac{101}{1000}$. 2. $\frac{1}{7}$; $\frac{1}{4}$; $\frac{7}{2}$; $\frac{34}{3}$; $\frac{45}{79}$; $\frac{7}{8}$; $7\frac{1}{2}$; $9\frac{7}{9}$; $16\frac{4}{21}$; $200\frac{3}{11}$; $94\frac{5}{17}$.

3. Two-thirds ; three-fourths ; eight-ninths ; fifteen-seventeenths ; eighty-sevenone hundredths ; three, and four-sevenths : eight, and eleven-sixteenths ; twenty-four, and twenty-seven forty-fourths ; one hundred twenty-five, and ninety-one one-thousandths.

4. (1) $\frac{8}{5}$; $\frac{12}{5}$; $\frac{20}{5}$; $\frac{28}{5}$; $\frac{36}{5}$; $\frac{44}{5}$; $\frac{48}{5}$; $\frac{52}{5}$; $\frac{72}{5}$.

$\frac{48}{37}$; $\frac{72}{37}$; $\frac{120}{37}$; $\frac{168}{37}$; $\frac{216}{37}$; $\frac{264}{37}$; $\frac{288}{37}$; $\frac{312}{37}$; $\frac{432}{37}$.

(2) $\frac{540}{23}$; $\frac{1020}{23}$; $\frac{1200}{23}$; $\frac{1425}{23}$; $\frac{1680}{23}$; $\frac{2355}{23}$;

$\frac{3132}{109}$; $\frac{5916}{109}$; $\frac{6960}{109}$; $\frac{8265}{109}$; $\frac{9744}{109}$; $\frac{13659}{109}$.

5. (1) $\frac{5}{14}$; $\frac{5}{21}$; $\frac{5}{35}$; $\frac{5}{49}$; $\frac{5}{63}$; $\frac{5}{77}$; $\frac{5}{84}$; $\frac{5}{91}$; $\frac{5}{126}$;

$\frac{8}{26}$; $\frac{8}{39}$; $\frac{8}{65}$; $\frac{8}{91}$; $\frac{8}{117}$; $\frac{8}{143}$; $\frac{8}{156}$; $\frac{8}{169}$; $\frac{8}{234}$.

(2) $\frac{19}{1332}$; $\frac{19}{2516}$; $\frac{19}{2960}$; $\frac{19}{3515}$; $\frac{19}{4144}$; $\frac{19}{5809}$;

$\frac{26}{5724}$; $\frac{26}{10812}$; $\frac{26}{12720}$; $\frac{26}{15105}$; $\frac{26}{17808}$; $\frac{26}{24968}$.

Ex. LXII. (p. 172.)

$\frac{39}{13}$; $\frac{65}{13}$; $\frac{91}{13}$; $\frac{104}{13}$; $\frac{195}{13}$; $\frac{234}{13}$; $\frac{260}{13}$; $\frac{325}{13}$.

2. $\frac{388}{13}$; $\frac{2925}{25}$; $\frac{4875}{33}$. **3.** $\frac{60}{120}$; $\frac{80}{120}$; $\frac{90}{120}$; $\frac{95}{120}$; $\frac{75}{120}$; $\frac{56}{120}$.

4. $\frac{756}{918}$; $\frac{756}{1080}$; $\frac{756}{1152}$; $\frac{756}{980}$. **5.** (i) Each $=\frac{4}{9}$. (ii) Each $=\frac{5040}{11340}$.

6 $\frac{4}{5}$; $\frac{27}{40}$; $\frac{7}{15}$; $\frac{11}{14}$; $\frac{18}{20}$.

Ex. LXIII. (pp. 173-174.)

2. (1) $\frac{1336}{11}$; $\frac{602}{11}$; $\frac{540}{13}$; $\frac{2095}{17}$; $\frac{2038}{13}$; $\frac{1525}{15}$; $\frac{359}{16}$.

(2) $\frac{4139}{15}$; $\frac{38576}{108}$; $\frac{10000}{29}$; $\frac{1733}{17}$; $\frac{4337}{87}$; $\frac{1899}{99}$; $\frac{2605}{59}$.

(3) $\frac{88716}{126}$; $\frac{8407}{680}$; $\frac{69057}{465}$; $\frac{60389}{2400}$; $\frac{684525}{999}$; $\frac{97596098}{9879}$.

4. (1) $33\frac{11}{13}$; $127\frac{4}{19}$; $130\frac{27}{29}$; $322\frac{17}{31}$; $823\frac{20}{37}$; $22\frac{17}{28}$; $16\frac{27}{45}$; 25.

(2) $13\frac{82}{217}$; $26\frac{73}{171}$; $14\frac{250}{481}$; $91\frac{452}{572}$; $339\frac{109}{152}$; $28\frac{283}{347}$; $136\frac{1}{144}$.

(3) $76\frac{921}{999}$; $83\frac{9573}{9991}$; $116\frac{477}{7816}$; $40\frac{8227}{99999}$; $50\frac{10211}{30125}$; $10010\frac{101}{100001}$

5. $2\frac{1}{7}$; $3\frac{4}{15}$; $3\frac{14}{17}$; $68\frac{74}{100}$; $39\frac{22}{37}$; $46\frac{13}{93}$; $341\frac{15}{152}$; $47\frac{1159}{1231}$.

6. (a) $\frac{10050}{240}$; $\frac{6220}{240}$; $\frac{2385}{240}$; $\frac{2508}{240}$. (b) $\frac{30150}{720}$; $\frac{18660}{720}$; $\frac{7155}{720}$; $\frac{7524}{720}$.

7. (1) $5\frac{2}{3}$; 12 ; $44\frac{2}{5}$; 178 ; 76 ; $28\frac{1}{5}$.

(2) $\frac{5}{49}$; $\frac{2}{19}$; $\frac{9}{120}$; $\frac{13}{205}$; $\frac{4}{7}$; $\frac{11}{17}$.

Ex. LXIV. (p. 175.)

1. $\frac{3}{5}$; $\frac{2}{3}$; $\frac{4}{25}$; $\frac{35}{16}$; $\frac{5}{6}$; $\frac{50}{21}$. **2.** $\frac{176}{45}$; $\frac{40}{7}$; 25 ; $\frac{15}{2}$; $\frac{25}{8}$.

3. $\frac{2}{21}$; $\frac{3}{7}$; $\frac{2}{3}$. **4.** $\frac{100}{21}$; $\frac{3}{64}$. **5.** $\frac{17}{6}$. **6.** 231. **7.** $\frac{65860}{27}$.

8. $\frac{616}{9}$. **9.** 9. **10** $\frac{6}{5}$. **11.** $\frac{64672}{35}$. **12.** $\frac{1}{11}$.

13. 120. **14.** 6. **15.** 6. **16.** $\frac{6}{11}$.

Ex. LXV. (p. 176)

2. (1) $\frac{5}{11}$; $\frac{29}{45}$; $\frac{13}{47}$; $\frac{1}{4}$; $\frac{12}{19}$; $\frac{37}{41}$; $\frac{8}{27}$; $\frac{41}{49}$.

(2) $\frac{119}{117}$; $\frac{123}{127}$; $\frac{169}{193}$; $\frac{15}{23}$; $\frac{43}{51}$; $\frac{71}{94}$; $\frac{713}{986}$; $\frac{50}{99}$.

(3) $\frac{13}{17}$; $\frac{15}{17}$; $\frac{103}{397}$; $\frac{3}{4}$; $\frac{8}{9}$; $\frac{51}{107}$; $\frac{98}{99}$.

(4) $\frac{9}{11}$; $\frac{4}{7}$; $\frac{46}{57}$; $\frac{2}{7}$; $\frac{295}{304}$; $\frac{79}{82}$; $\frac{8}{9}$ (5) $\frac{50}{161}$; $\frac{67}{111}$; $\frac{5}{7}$; $\frac{7}{61}$; $\frac{11}{18}$; $\frac{7}{8}$.

(6) $\frac{121}{456}$; $\frac{311}{2311}$; $\frac{49}{1330}$; $\frac{376}{643}$; $\frac{58}{73}$; $\frac{24681}{50003}$.

3. (1) $\frac{16}{39}$; $\frac{2}{5}$; $\frac{3}{28}$; $\frac{7}{108}$; $\frac{28}{99}$. (2) $\frac{27}{80}$; $\frac{1}{63}$; $\frac{2}{11}$; $\frac{2}{11}$.

Ex. LXVI. (p. 178.)

1. (1) $\frac{10}{15}$, $\frac{12}{15}$. (2) $\frac{9}{63}$, $\frac{14}{63}$. (3) $\frac{33}{44}$, $\frac{36}{44}$. (4) $\frac{35}{70}$, $\frac{56}{70}$, $\frac{60}{70}$.

(5) $\frac{154}{231}$, $\frac{99}{231}$, $\frac{341}{231}$. (6) $\frac{195}{390}$, $\frac{260}{390}$, $\frac{234}{390}$, $\frac{270}{390}$.

(7) $\frac{1155}{3465}$, $\frac{1386}{3465}$, $\frac{1485}{3465}$, $\frac{1540}{3465}$, $\frac{1575}{3465}$. (8) $\frac{140}{105}$, $\frac{231}{105}$, $\frac{330}{105}$.

(9) $\frac{396}{495}$, $\frac{1265}{495}$, $\frac{1575}{495}$. (10) $\frac{4312}{616}$, $\frac{385}{616}$, $\frac{6720}{616}$, $\frac{16104}{616}$.

(11) $\frac{15}{40}$, $\frac{14}{40}$, $\frac{18}{40}$.

2. (1) $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$. (2) $\frac{16}{24}$, $\frac{18}{24}$, $\frac{21}{24}$. (3) $\frac{95}{105}$, $\frac{96}{105}$.

(4) $\frac{189}{1008}$, $\frac{384}{1008}$, $\frac{560}{1008}$. (5) $\frac{98}{210}$, $\frac{110}{210}$, $\frac{161}{210}$. (6) $\frac{75}{90}$, $\frac{70}{90}$, $\frac{48}{90}$, $\frac{81}{90}$.

(7) $\frac{432}{504}$, $\frac{441}{504}$, $\frac{448}{504}$, $\frac{462}{504}$. (8) $\frac{24}{36}$, $\frac{27}{36}$, $\frac{30}{36}$, $\frac{32}{36}$.

(9) $\frac{105}{180}, \frac{102}{180}, \frac{130}{180}, \frac{135}{180}, \frac{84}{180}$. (10) $\frac{2016}{2160}, \frac{2025}{2160}, \frac{2040}{2160}, \frac{2052}{2160}, \frac{2070}{2160}, \frac{2080}{2160}$. (11) $\frac{30}{60}, \frac{40}{60}, \frac{45}{60}, \frac{48}{60}, \frac{50}{60}$. (12) $\frac{18}{12}, \frac{40}{12}, \frac{51}{12}, \frac{74}{12}$.
(13) $\frac{23688}{7560}, \frac{6993}{7560}, \frac{10152}{7560}, \frac{4480}{7560}, \frac{105210}{7560}$. (14) $\frac{144}{304}, \frac{665}{304}, \frac{2280}{304}$.

3. (1) $\frac{12}{24}, \frac{12}{16}, \frac{12}{15}, \frac{12}{42}$. (2) $\frac{1144}{1430}, \frac{1144}{1287}, \frac{1144}{1248}, \frac{1144}{1320}$
(3) $\frac{126}{216}, \frac{126}{210}, \frac{126}{135}, \frac{126}{270}$. (4) $\frac{63}{147}, \frac{63}{117}, \frac{63}{154}, \frac{63}{78}$.
(5) $\frac{936}{12636}, \frac{936}{1728}, \frac{936}{1560}, \frac{936}{1209}$. (6) $\frac{48}{108}, \frac{48}{72}, \frac{48}{57}, \frac{48}{52}, \frac{48}{78}$.

Ex. LXVII. (pp. 179-180.)

1. $\frac{4}{5}$; $\frac{7}{9}$; $\frac{19}{26}$; $\frac{15}{32}$; $\frac{8}{21}$, $\frac{15+8}{19+8}$. 2. $\frac{17}{45}$; $\frac{5}{12}$; $\frac{85}{108}$; $\frac{31}{63}$; $\frac{1}{2}$ of $\frac{3}{7}$; $\frac{15-8}{19-8}$.

3. (1) $\frac{11}{12}$ and $\frac{1}{18}$; $\frac{3}{5}$ and $\frac{4}{9}$; $\frac{48}{63}$ and $\frac{15}{42}$; $\frac{4}{9}$ and $\frac{7}{27}$; $\frac{11}{12}$ and $\frac{11}{15}$.
(2) $\frac{4}{5}$ and $\frac{15}{14}$; $\frac{9}{10}$ and $\frac{2}{3}$; $\frac{15}{17}$ and $\frac{26}{51}$; $\frac{11}{24}$ and $\frac{5}{27}$.
(3) $\frac{2}{3}$ of $\frac{5}{7}$ of 6 and $\frac{1}{2}$ of $\frac{5}{6}$ of 4; $\frac{13}{42}$ and $\frac{1}{24}$.
(4) $4\frac{1}{5}$ of $3\frac{3}{4}$ of 10 and $\frac{2}{5}$ of $\frac{5}{8}$ of $\frac{8}{10}$; $7\frac{3}{8}$ and $\frac{1}{2}$ of $\frac{3}{7}$.
(5) $\frac{17}{27}$ and $\frac{7}{16}$; $\frac{9}{22}$ and $\frac{18}{35}$; $\frac{5}{8}$ and $\frac{4}{5}$.

4. (1) $\frac{19}{20}, \frac{11}{12}, \frac{7}{8}, \frac{2}{3}$. (2) $\frac{7}{10}, \frac{3}{25}, \frac{7}{65}, \frac{8}{50}$. (3) $\frac{10}{21}, \frac{7}{15}, \frac{16}{35}$
(4) $\frac{1}{3}$ of $2\frac{5}{8}$, $\frac{5}{21}$ of $3\frac{1}{2}$, $1\frac{1}{7}$ of $\frac{1}{4}$. (5) $\frac{3}{8}$ of $2\frac{2}{7}$, $1\frac{2}{5}$, $3\frac{1}{2}$ of $\frac{5}{14}$ of $\frac{8}{9}$.
(6) $\frac{15}{16}$, $\frac{13+15}{14+16}$, $\frac{13}{14}$. (7) $\frac{9}{10}$, $\frac{7}{8}$, $\frac{5+6+7+9}{6+7+8+10}$, $\frac{6}{7}$, $\frac{5}{6}$.

5. (1) $\frac{4}{5}, \frac{3}{4}, \frac{2}{3}, \frac{1}{2}$. (2) $\frac{11}{12}, \frac{9}{10}, \frac{8}{9}, \frac{6}{7}$. (3) $\frac{45}{46}, \frac{13}{14}, \frac{7}{8}, \frac{5}{6}$.
(4) $\frac{99}{100}, \frac{89}{90}, \frac{15}{16}, \frac{11}{12}$. (5) $\frac{74}{75}, \frac{61}{62}, \frac{49}{50}, \frac{47}{48}$. (6) $\frac{24}{25}, \frac{19}{20}, \frac{17}{18}, \frac{15}{16}$.

6. (1) $\frac{7}{8}, \frac{2}{3}, \frac{1}{4}, \frac{2}{5}$. (2) $\frac{21}{22}, \frac{15}{16}, \frac{14}{33}, \frac{5}{48}$. (3) $\frac{2}{3}, \frac{5}{9}, \frac{4}{17}, \frac{1}{18}$.
(4) $\frac{19}{20}, \frac{15}{16}, \frac{9}{10}, \frac{23}{30}$. (5) $\frac{12}{13}, \frac{16}{19}, \frac{2}{3}, \frac{8}{13}, \frac{4}{9}$. (6) $\frac{13}{15}, \frac{3}{5}, \frac{5}{9}, \frac{6}{13}$.

Ex. LXVIII. (pp. 181-182.)

2. (1) $\frac{37}{90}$; $\frac{24}{35}$; $\frac{62}{63}$; $1\frac{11}{72}$; $1\frac{45}{136}$; $\frac{41}{56}$; $1\frac{59}{112}$.
(2) $8\frac{7}{10}$; $16\frac{11}{70}$; $17\frac{29}{30}$; $62\frac{15}{143}$; $12\frac{9}{56}$.
(3) $10\frac{17}{36}$; $3\frac{209}{234}$; $13\frac{14}{15}$; $9\frac{41}{72}$; $4\frac{31}{48}$; $\frac{307}{315}$.
(4) $2\frac{1}{4}$; $\frac{314}{385}$; $2\frac{39}{70}$; $1\frac{17}{18}$; $3\frac{43}{120}$. (5) $54\frac{13}{16}$; $10\frac{287}{360}$; $49\frac{593}{693}$; $20\frac{53}{60}$.
(6) 2; 2; $2\frac{77}{390}$. (7) $4\frac{1}{5}$; $19\frac{7}{36}$; $5\frac{86}{105}$. (8) $5\frac{2663}{3465}$. (9) $14\frac{87}{1320}$.
(10) $15\frac{71}{360}$. (11) $23\frac{3}{4}$. (12) $10\frac{139}{270}$. (13) 25. (14) $3\frac{5}{7}$.
(15) $8\frac{11}{30}$; $2\frac{4}{5}$. (16) $2548\frac{41}{60}$; $14\frac{64}{135}$. (17) 5000. (18) $3\frac{8}{9}$.
(19) 4. (20) $492\frac{2}{3}$. (21) 1.

Ex. LXIX. (p. 184.)

2. (1) $\frac{13}{30}$; $\frac{22}{63}$; $\frac{5}{99}$; $\frac{3}{10}$; $\frac{7}{15}$; $\frac{1}{60}$. (2) $\frac{1}{84}$; $\frac{166}{399}$; $7\frac{5}{9}$; $6\frac{9}{13}$; $1\frac{61}{264}$; $3\frac{31}{72}$.
(3) $6\frac{8}{77}$; $\frac{674}{675}$; $615\frac{23}{55}$; $110\frac{24}{35}$. (4) $39\frac{11}{30}$; $1\frac{709}{924}$; $61\frac{43}{72}$.

3. (1) $\frac{1}{4}$. (2) $\frac{9}{560}$. (3) $1\frac{7}{24}$. (4) 2. (5) $3\frac{13}{45}$. (6) $2\frac{8}{23}$. (7) $9\frac{1}{50}$.
(8) $\frac{9}{10}$. (9) $9\frac{13}{126}$. (10) $10\frac{27}{88}$. (11) $7\frac{97}{560}$. (12) $1\frac{29}{56}$.
(13) $21\frac{9}{17}$. (14) $4\frac{107}{168}$. (15) $15\frac{13511}{70200}$.

Ex. LXX. (p. 185.)

2. (1) $23\frac{1}{3}$; $37\frac{1}{3}$; $38\frac{2}{11}$. (2) $2\frac{6}{7}$; $8\frac{4}{7}$; $11\frac{3}{7}$; $11\frac{2}{3}$.
(3) $11\frac{5}{7}$; $46\frac{6}{7}$; $61\frac{1}{2}$. (4) $41\frac{1}{2}$; $56\frac{13}{22}$; $79\frac{5}{23}$; 498.
(5) $188\frac{1}{3}$; $263\frac{2}{3}$; $376\frac{2}{3}$. (6) $26\frac{3}{4}$; $80\frac{1}{4}$; $86\frac{11}{26}$; $240\frac{3}{4}$.
(7) $1912\frac{1}{2}$; $37389\frac{7}{11}$; $111\frac{3}{5}$; $58727\frac{5}{7}$; $168726\frac{1}{13}$.

3. (1) $599\frac{3}{5}$; $799\frac{1}{5}$; $1498\frac{1}{2}$; $1798\frac{1}{5}$; $2497\frac{1}{2}$. (2) $98994\frac{1}{2}$; $549969\frac{4}{5}$.
(3) $12499\frac{12}{13}$; $24999\frac{11}{13}$; $37499\frac{10}{13}$; $49999\frac{9}{13}$; $74999\frac{7}{13}$;
$99999\frac{5}{13}$; $124999\frac{3}{13}$. (4) $3223\frac{11}{17}$; $32399\frac{25}{106}$; $998999\frac{2947}{4545}$.

Ex. LXXI. (pp. 186-187.)

2. (1) $\frac{6}{35}$; $\frac{5}{24}$; $\frac{36}{72}$; $\frac{5}{28}$; $\frac{1}{15}$; $\frac{27}{200}$. (2) $\frac{8}{45}$; $\frac{7}{48}$; $\frac{55}{112}$; $\frac{16}{55}$; $\frac{15}{56}$.
(3) $18\frac{7}{9}$; $864\frac{8}{77}$; $91\frac{1}{2}$; $61\frac{4}{9}$; 17. (4) $\frac{133}{540}$; $414\frac{16}{81}$; $3\frac{3}{4}$.
(5) 26; $1267\frac{5}{7}$; $5\frac{1}{7}$.

3. (1) $\frac{7}{80}$; $\frac{45}{56}$; $\frac{4}{7}$; $\frac{80}{81}$; 2. (2) $\frac{1}{5}$; $\frac{40}{77}$; 2; $\frac{13}{40}$ (3) $242\frac{17}{24}$; $498\frac{10}{11}$.
(4) $70\frac{5}{16}$. (5) $\frac{53}{90}$.

4. (1) $\frac{8}{75}$. (2) 3. (3) $\frac{88}{129}$. (4) $1\frac{18}{29}$. (5) 1. (6) $\frac{175}{6336}$.

5. (1) $66\frac{1}{7}$; $33\frac{8}{63}$; $\frac{13}{210}$. (2) $\frac{13019}{55440}$; $3\frac{1}{12}$; $\frac{4}{21}$.
(3) $22\frac{9}{175}$; $3\frac{17}{140}$; $14\frac{23}{35}$. (4) $5\frac{7}{36}$. (5) $7\frac{1}{2}$. (6) $\frac{3}{5}$.

Ex. LXXII. (p. 189.)

2 (1) $\frac{5}{67}$; $\frac{5}{134}$; $\frac{5}{201}$; $\frac{10}{603}$. (2) $\frac{29}{1710}$; $\frac{29}{5130}$; $\frac{5}{1026}$.
(3) $\frac{1}{6}$; $\frac{1}{30}$; $\frac{1}{54}$. (4) $\frac{6}{7}$; $\frac{114}{175}$; $\frac{57}{112}$.

4. (1) $1\frac{17}{32}$; $2\frac{1}{3}$; $2\frac{4}{5}$; $1\frac{3}{16}$. (2) $1\frac{17}{35}$; $1\frac{3}{10}$; $1\frac{1}{14}$; $4\frac{1}{3}$.
(3) $\frac{27}{44}$; $1\frac{25}{56}$; $\frac{45}{56}$; $\frac{81}{160}$. (4) $3\frac{7}{9}$; $\frac{425}{513}$; $\frac{170}{171}$; $3\frac{4}{27}$.

Ex. LXXIII. (p. 189.)

1. (1) $\frac{16}{21}$; $1\frac{7}{20}$; $\frac{26}{33}$; $\frac{28}{33}$; $\frac{4}{21}$; $1\frac{13}{171}$; 156.
(2) $\frac{99}{145}$; $\frac{7}{9}$; $2\frac{76}{333}$; $4\frac{1}{2}$; $1\frac{11}{104}$. (3) $14\frac{7}{15}$; 8; $19\frac{87}{95}$. (4) $3\frac{31}{75}$.

2. (1) $\frac{9}{327}$. (2) $1\frac{1}{2}$. (3) $7\frac{3833}{5400}$. (4) $9\frac{1333}{5400}$. (5) $7\frac{17}{24}$. (6) $1\frac{1}{4}$.
(7) $7\frac{1}{5}$. (8) $1\frac{1}{4}$. (9) 1. (10) $\frac{3}{4}$. (11) $1\frac{155}{256}$ (12) $1\frac{1}{6}$.

Ex. LXXIV. (pp. 191-192.)

1. (1) $\frac{95}{116}$; $\frac{49}{88}$; $\frac{3}{5}$; $3\frac{1}{2}$; $\frac{16}{21}$; $\frac{133}{185}$; $1\frac{3}{13}$; $\frac{28}{725}$; $\frac{3}{7}$.
(2) $2\frac{1}{5}$; $\frac{99}{175}$; $\frac{63}{116}$; $\frac{1}{195}$; $1\frac{1}{3}$. (3) $\frac{28}{33}$; $\frac{5}{27}$; $\frac{3}{4}$; $2\frac{29}{80}$.
(4) 3; $\frac{1125}{5117}$; 1. (5) $\frac{25}{424}$; $2\frac{1}{455}$; $\frac{719}{40803}$.

2. (i) $\frac{12}{36}$, $\frac{297}{36}$, $\frac{400}{36}$, $\frac{610}{36}$. (ii) $\frac{3\frac{3}{4}}{5}$; $\frac{10}{11\frac{3}{7}}$. 3. The *first* is the *least* and the *second* is the *greatest*.

4. (1) 15. (2) $10\frac{10}{33}$. (3) $\frac{15}{16}$. (4) $5\frac{1}{5}$.

5. (1) $4\frac{37}{72}$. (2) $\frac{109}{525}$. (3) 7.

6. (1) $5\frac{2}{3}$. (2) $10\frac{1}{2}$. (3) $\frac{27}{35}$. (4) $1\frac{7}{18}$. (5) $\frac{38}{415}$. (6) $7\frac{19}{35}$. (7) 26. (8) 55. (9) $3\frac{9}{13}$.

Ex. LXXV. (pp. 192-194.)

1. (1) $72\frac{81}{13}$. (2) $16\frac{140}{401}$. (3) $25\frac{46}{87}$. (4) $194\frac{88}{45}$. (5) $70\frac{89}{180}$. (6) $182\frac{77}{181}$; $92\frac{811}{910}$. (7) $378\frac{3}{8}$; $31\frac{58}{96}$. (8) $2108\frac{783}{837}$.

2. (1) $2\frac{8}{35}$. (2) $\frac{8}{43}$. (3) $\frac{22}{63}$. (4) $2\frac{31}{117}$. (5) $3\frac{11}{28}$. (6) $4\frac{1}{13}$. (7) $14\frac{29}{64}$. (8) $3\frac{727}{1636}$. (9) $1\frac{5425}{58861}$. (10) $\frac{1297}{2062}$. (11) $2\frac{2}{5}$. (12) $1\frac{2903}{9907}$. (13) $4\frac{120023}{252885}$. (14) $4\frac{831}{890}$. (15) $6\frac{23}{30}$. (16) 2. (17) $3\frac{307}{500}$. (18) $\frac{226}{265}$. (19) $1\frac{348064}{421515}$. (20) $\frac{15}{163}$.

3. (1) $16\frac{41}{52}$. (2) $101\frac{1}{2}$. (3) $19\frac{1}{4}$. (4) $15\frac{1}{4}$. (5) $3\frac{119}{180}$. (6) $1\frac{7}{10}$. (7) $5\frac{7}{36}$. (8) $\frac{14}{81}$. (9) $\frac{14}{81}$. (10) 5.

Ex. LXXVI. (pp. 195-197.)

1. $1\frac{29}{56}$. **2.** $\frac{60}{169}$. **3.** $4\frac{25}{49}$. **4.** $7\frac{11}{12}$. **5.** $1\frac{28521}{64704}$. **6.** 2. **7.** $7\frac{33}{35}$. **8.** 9. **9.** 2. **10.** $1\frac{8}{108}$. **11.** $\frac{1}{2}$. **12.** 1. **13.** 10. **14.** $\frac{1}{273}$. **15.** 1. **16.** $\frac{7}{22}$. **17.** $\frac{889}{1152}$. **18.** 3. **19.** $\frac{7}{8}$. **20.** 1. **21.** $\frac{315}{8784}$. **22.** $\frac{1}{2}$. **23.** 2. **24.** 1. **25.** $2\frac{8}{21}$. **26.** $1\frac{3}{5}$. **27.** $1\frac{1}{110}$. **28.** $1\frac{7}{8}$. **29.** $\frac{3}{11}$. **30.** $\frac{8}{705}$. **31.** $4\frac{23}{64}$. **32.** $\frac{121}{168}$. **33.** 902. **34.** $\frac{7}{10}$. **35.** $4\frac{341}{2810}$. **36.** $20\frac{1}{11}$. **37.** $36\frac{31}{39}$. **38.** $\frac{33}{112}$. **39.** $2\frac{1}{2}$. **40.** $1\frac{1}{2}$. **41.** 11. **42.** $14\frac{46}{51}$. **43.** 4. **44.** 3. **45.** $1\frac{1}{2}$. **46.** 1. **47.** 2. **48.** $\frac{4}{9}$. **49.** $\frac{7}{9}$. **50.** $3\frac{657}{2479}$; $\frac{1}{2}$. **51.** 2 ; $\frac{2}{21}$. **52.** $\frac{1}{4}$. **53.** $5\frac{79}{81}$. **54.** $\frac{1}{2}$. **55.** $\frac{16}{105}$; $2\frac{2}{3}$. **56.** $\frac{683}{2000}$.

Ex. LXXVII. (pp. 198-199.)

1. (1) $\frac{1}{18}$; $13\frac{1}{3}$. (2) $\frac{1}{60}$; $51\frac{1}{3}$. (3) $\frac{1}{144}$; 1. (4) $\frac{1}{36}$; $25\frac{2}{3}$. (5) $\frac{2}{215}$; 24. (6) $\frac{1}{420}$; 60. (7) $\frac{11}{84}$; $18\frac{6}{7}$. (8) $\frac{1}{48}$; 720. (9) $\frac{13}{300}$; $1801\frac{4}{5}$. (10) $\frac{13}{2310}$; $70\frac{10}{11}$. (11) $\frac{3}{140}$; 1890. (12) $\frac{1}{504}$; 5060.

2. $3\frac{1}{2}$ in. **3.** 211. **4.** $33\frac{2}{5}$. **5.** 10 times. **6.** 6 min. **7.** $4042\frac{1}{2}$ ft. **8** 720.

Ex. LXXVIII. (pp. 200-201.)

1. $1\frac{11}{135}$; $1\frac{11}{135}$. **2.** $\frac{62}{231}$. **3.** $\frac{1}{7}$. **5.** $\frac{700}{990}$, $\frac{847}{990}$. **6.** $\frac{5}{12}$. **7.** 60 ; $2\frac{24}{35}$. **8.** $\frac{7}{120}$. **9.** $492\frac{898}{905}$. **10.** $13\frac{47}{3600}$. **11.** 2500. **12.** $2\frac{1}{10}$. **13.** $\frac{1}{128}$. **14.** $\frac{167}{432}$. **15.** 1 ; $\frac{3}{8}$. **16.** $\frac{1}{2}$. **17.** $\frac{1}{360}$. **18.** $\frac{8}{11}$. **19.** $\frac{85}{174}$. **20.** $5\frac{1619}{1760}$. **21.** $109\frac{17}{22}$. **22.** $\frac{25}{198}$. **23.** $\frac{1}{5}$. **24.** $\frac{13}{15}$. **25.** $20\frac{1}{11}$. **26.** 99. **27.** $\frac{583}{720}$. **28.** $\frac{4}{15}$; $\frac{1}{12}$. **29.** $\frac{4}{21}$. **30.** *B* receives $\frac{1}{30}$; *C* receives $\frac{11}{720}$; *B* receives $\frac{1}{12}$ of *A*'s money after loss, and *C* receives $\frac{11}{288}$ of *A*'s money after loss. **31.** $\frac{8}{35}$. **32.** $\frac{29}{420}$; $\frac{391}{420}$.

Ex. LXXIX. (pp. 202-203.)

1. (1) *Rs.*28. 15*a*. $9\frac{18}{38}$*p*. (2) *Rs.*87. 15*a*. $8\frac{5}{8}$*p*. (3) £32. 12*s*. $9\frac{8}{63}$*d*.
(4) £70. 10*s*. $11\frac{43}{78}$*d*. (5) 1 lb. 9 oz. 14 dwts. $4\frac{22}{24}$ grs.
(6) 19 cwt. 1 qr. 17 lbs. $7\frac{7}{40}$ oz. (7) 2 fur. 8 po. 5 yds. $1\frac{17}{210}$ in.

2. (1) *Rs.*5. 1*a* $5\frac{17}{24}$*p*. (2) *Rs.*10. 15*a*. $2\frac{29}{48}$*p*. (3) £13 13*s*. $8\frac{14}{15}$*d*.
(4) £98. 19*s*. $1\frac{29}{48}$*d*. (5) 6 cwt. 1 qr. $18\frac{22}{35}$ lbs. (6) 9 cwt. $10\frac{74}{98}$ lbs.
(7) 3 fur. 10 po. 2 yds. $9\frac{6}{13}$ in. (8) 7 hrs. 54 min. $46\frac{17}{24}$ sec.

3. (1) *Rs.*74. 0*a*. $9\frac{1}{11}$*p*. ; *Rs.*101. 13*a*. $0\frac{1}{2}$*p* ; *Rs.*416. 8*a*. $3\frac{8}{22}$*p*. ;
*Rs.*1286. 9*a*. $1\frac{21}{22}$*p*.
(2) £302. 15*s*. $0\frac{4}{5}$*d*. ; £1135. 6*s*. 6*d*. ; £6660. 11*s*. $5\frac{3}{5}$*d*. ;
£7266. 1*s*. $7\frac{1}{5}$*d*.
(3) 8 tons 19 cwt. 3 qrs. 2 lbs. $0\frac{4}{7}$ oz. ; 17 tons 19 cwt. 2 qrs.
4 lbs. $1\frac{1}{7}$ oz. ; 71 tons 18 cwt. 16 lbs. $4\frac{4}{7}$ oz.
(4) 408 mds. 10 sr. $2\frac{1}{2}$ ch. ; 1088 mds. 27 sr. $1\frac{1}{3}$ ch. ;
1587 mds. 12 sr. $2\frac{11}{18}$ ch.
(5) 21 mi. 6 fur. 7 po. 2 yds. $3\frac{1}{4}$ in. ; 47 mi. 7 fur. 8 po. 1 yd. $10\frac{3}{4}$ in.

4. (1) *Rs.*20. 9*a*. $1\frac{11}{13}$*p*. ; *Rs.*17. 10*a*. $1\frac{25}{56}$*p*. ; *Rs.*9. 7*a*. $10\frac{97}{104}$*p* ;
*Rs.*4. 4*a*. $1\frac{41}{232}$*p*.
(2) £38. 7*s*. $7\frac{1061}{1352}$*d*. ; £18. 16*s*. $6\frac{2725}{2756}$*d*. ;
£11. 17*s*. $7\frac{1165}{4368}$*d*. ; £6. 17*s*. $7\frac{5798}{7540}$*d*.
(3) 112 lbs. 13 oz. $4\frac{94}{217}$ drs. ; 52 lbs. 10 oz. $7\frac{199}{325}$ drs. ;
11 lbs. 12 oz. $9\frac{27}{35}$ drs.
(4) 2 mds. 29 sr. $7\frac{187}{6861}$ ch. ; 1 md. 12 sr. $7\frac{3877}{13889}$ ch.
(5) $2\frac{4261}{10936}$; $7\frac{8029}{4545}$. (6) 41 ; $10\frac{32887}{180081}$.

Ex. LXXX. (pp. 204-205.)

1. $3\frac{1}{5}$*a*. ; $4\frac{4}{7}$*a*. ; $13\frac{1}{3}$*a*. ; $4\frac{4}{15}$*a*. ; $\frac{16}{35}$*a*. ; 15g. 2. $34\frac{2}{7}$*d*. ; $106\frac{2}{3}$*d*. ; $11\frac{1}{4}$*d*. ; $194\frac{9}{14}$*d*.
3. $\frac{1}{50}$; $\frac{1}{336}$; $\frac{1}{66}$. 4. $\frac{7}{30}$; $\frac{63}{160}$; $\frac{12}{7}$. 5. $\frac{21}{2}$; $\frac{1}{3136}$; $\frac{565}{2}$.
6. $\frac{14}{19}$; $\frac{6}{1595}$. 7. $\frac{15}{16}$; $\frac{25}{2}$. 8. $\frac{70}{9}$; $\frac{7}{60}$; 210 ch.
9. $\frac{64}{81}$; $\frac{1}{1386}$. 10. $147\frac{19}{33}$ po. ; $2660\frac{5}{8}$ yds. 11. $\frac{3}{50}$; $\frac{1}{900}$; $\frac{1}{9000}$.
12. $\frac{16}{285}$. 13. $\frac{4}{27}$ ac. ; $6091\frac{2}{3}$ yds. ; $\frac{24}{5}$. 14. $\frac{61}{8640}$.

Ex. LXXXI. (p. 207.)

1. (1) 13*a*. 4*p*. ; 9*a*. 4*p*. ; *Rs* 16. 14*a*. ; *Rs.*6. 9*a*. $7\frac{1}{5}$*p*. ;
*Rs.*6 12*a*. $9\frac{3}{5}$*p*.
(2) *Rs.*364 ; *Re.*1. 5*a*. 4*p*. ; *Rs.*4. 10*a* $6\frac{2}{5}$*p*. ; $3\frac{3}{4}$*p*.
(3) 12*s*. ; $6\frac{1}{2}$*d*. $\frac{2}{3}$*q*. ; 5*s*. 10*d*. ; 16*s*. 8*d*. ; 1*s*. 4*d*. ; 10*s*. $7\frac{1}{2}$*d*.
(4) £3. 5*s*. ; 2*s*. 8*d*. ; 9*s*. $4\frac{1}{2}$*d*. ; £3. 0*s*. 8*d* ; £3. 12*s*. 6*d*.
(5) £2. 6*s*. 8*d*. ; 10*s*. $3\frac{1}{4}$*d*. $\frac{5}{7}$*q*. ; 8*s*. ; £4. 17*s*. 6*d*.
(6) 2 qrs. 24 lbs. ; 10 lbs. ; 6 oz. ; 2 qrs. 26 lbs. 4 oz. ;
21 cwt. 1 qr. 9 lbs. 5 oz. $5\frac{1}{3}$ drs.

(7) 4cwt. 2qrs. 8lbs. ; 16*s.* 8*d.* ; 1*s.* $6\frac{1}{4}$*d.* $\frac{1}{2}$*q.* ; 4 fur. 26po. 3yds. 2ft.
(8) 6 oz. 17 dwts. $3\frac{3}{7}$ grs., 9 oz. $2\frac{2}{7}$ drs.; 6 oz 6 drs. 2 scr. $11\frac{3}{7}$grs. 2 lbs. 5 oz. 13 dwts. 8 grs.
(9) 1 yd. 1 ft. $10\frac{1}{2}$ in. ; 1 po. 1 yd. 1 ft. $5\frac{3}{7}$ in. ; 2 ro. 16 po. ; 37 po. 24 sq. yds. 6 sq. ft. 108 sq. in.
(10) 3 qrs. ; 1 da. 22 hrs. 40 min. ; 3 da. 17 hrs. 36 min.
(11) 1 pk. ; 1 pk. 1 gal.; 1gal. 1qt. $1\frac{3}{7}$pts.; 3 lds. 7bus. 3pks. $1\frac{7}{9}$pts.
(12) 148 c. yds. 12 ft. $1036\frac{4}{5}$ in. ; 27 mds. 10 sr. 13 ch. $1\frac{2}{3}$ to. ; 145 tons 16 cwt. 2 qrs. 18 lbs. 10 oz $10\frac{2}{3}$ drs.
(13) 5 hrs. 36 min. ; 7 hrs. 12 min.; 2 pipes 115 gals. 2 qts.
(14) £7 ; 109 da. 13 hrs. 20 min. ; 4*s.* 8*d.*
(15) 3 cwt. 1 qr. 6 lbs ; 3 da. 11 hrs. 13 min. ; £4. 14*s.* 11*d.*

3. (1) £4. 1*s.* 8*d* ; £2. 18*s.* 4*d.* (2) 19*s* $6\frac{1}{2}$*d.* (3) £1. 2*s.*
(4) 12 cwt 2qrs. 14lbs. 10 oz. $10\frac{2}{3}$ drs. ; 2qrs. 17 lbs. 1oz. $3\frac{11}{13}$drs.
(5) 4 da 23 hrs. 31 min. $31\frac{3}{7}$ sec. (6) 2 mi. 6 fur. 22 po. 3 in. ; 17 mds. 27 sr. $5\frac{1}{3}$ ch. (7) *Rs.*5. 14*a.* (8) £50.

Ex. LXXXII. (pp. 208-209.)

1. (1) £1. 12*s.* $10\frac{1}{2}$*d.* $\frac{3}{7}$*q.* ; £4. 15*s.* $11\frac{189}{208}$*d.* ; £39. 11*s.* $11\frac{7}{18}$*d.*
(2) *Rs.*486. 10*a.* 6*p.* ; *Rs.*951. 7*a.* $11\frac{2}{5}$*p.* ; *Rs.*2687. 9*a.* $3\frac{9}{13}$*p.* ; *Rs.*7898. 2*a.* $6\frac{9}{13}$*p.*
(3) £82. 18*s.* $0\frac{1}{4}$*d.* $\frac{1}{3}$*q.* ; £98. 19*s.* $11\frac{1}{2}$*d.* ; £402. 17*s.* $5\frac{1}{4}$*d.* $\frac{9}{11}$*q.*
(4) 25 tons 3 cwt. 3 qrs. 7 lbs. $8\frac{8}{9}$ oz. ; 106 tons 6 cwt. 2 qrs. 16 lbs. $14\frac{6}{11}$ oz. ; 242 tons 9 lbs. $4\frac{4}{15}$ oz. 1100 tons 9 cwt. 1 qr. 5 lbs. $6\frac{3}{7}$ oz.
(5) 10 da. 10 hrs. 36 min. $5\frac{1}{65}$sec. ; 119 ac. 1ro. 1 po. $21\frac{65}{252}$sq. yds.

2. (1) *Rs.*1766. 12*a* 11*p.* ; *Rs.*491. 15*a.* $1\frac{1}{3}$*p.* ; £1. 8*s.* $6\frac{8}{44}$*d.*
(2) £42. 7*s.* $3\frac{3}{4}$*d.* $\frac{2}{3}$*q.* , £46. 4*s* $0\frac{1}{2}$*d.* $\frac{3}{4}$*q.* , £3. 12*s.* $4\frac{1}{2}$*d.* $\frac{10}{11}$*q.*
(3) *Rs.*65 ; *Rs.*36. 7*a* $10\frac{6}{19}$*p* ; *Rs.*17. 11*a.* $7\frac{7}{11}$*p.* ; *Rs* 8. 5*a.* 4*p.*
(4) 3 cwt. 3 qrs. 18 lbs. $12\frac{1}{8}$ oz. ; 20 ac. $23\frac{3}{5}$ po.
(5) 1 da. 19 hrs. 29 min. $46\frac{6}{7}$ sec. ; 7 c. yds. 11 c. ft. $1210\frac{1}{3}$ c. in.
(6) 2 fur. 124 yds. 2 ft. ; 4 mds. 29 sr.

3. (1) *Rs.*3. 4*a.* 6*p.* ; *Re.*1. 12*a.* 6*p.* ; *Rs.*26. 4*a.* ; 8*a.*
(2) *Rs.*25. 5*a.* 4*p.* ; *Rs.* 133. 4*a.* ; *Rs.*1575. 5*a.* 4*p.*
(3) £4. 1*s.* $8\frac{1}{2}$*d.* ; 3*s.* ; 4*s.* (4) £12. 3*s.* ; £8. 14*s.* $6\frac{3}{4}$*d.* ; *Rs.*40. 8*a.*
(5) 11 mds. 11 sr. 5 ch. ; 9 cwt. 2 qrs.
(6) 4 ft. $6\frac{42}{80}$ in. ; 3 sq. ft. 96 sq. in. ; 1 c. ft. 512 c. in.
(7) 102 mi. 4 fur. 11 po. $3\frac{7}{80}$ yds. ; £40. 4*s.* 2*d.*
(8) 26 cwt. 24 lbs. 4 oz. ; 36 po.
(9) 32 da. 9 hrs. 49 min. ; *Rs.*79. 9*a* $3\frac{3}{15}$*p.*
(10) 2 ro. $18\frac{1}{2}$ po. ; £22. 16s.

4. (1) *Rs.*56. 4*a* (2) *Rs.*85. 5*a.* $11\frac{3}{5}$*p.* (3) £5. 12*s.* 6*d.*
(4) £6. 10*s.* $3\frac{3}{7}$*d.* (5) £50. (6) £5. 12*s.* $8\frac{1}{4}$*d.*
(7) *Rs.*62. 11*a.* $9\frac{3}{11}$*p.* (8) 6 mds. 18 sr.
(9) 7 yrs. 169 da. 34 min. (10) 1 po. 34 yds. 1 ft. $5\frac{1}{1}\frac{9}{3}$ in.

Ex. LXXXIII. (pp. 210-211.)

1. [illegible] ; $\frac{17}{20}$; [illegible].
2. [illegible].

3. £$1\frac{44}{63}$; £$3\frac{63}{64}$; £$37\frac{53}{62}$; £$5\frac{485}{514}$.
4. *Rs.*$3\frac{2}{3}$; *Rs.*$8\frac{1}{3}$; *Rs.*$15\frac{127}{192}$; *Rs.*$81\frac{63}{128}$.
5. $\frac{67}{560}$; $\frac{287}{288}$; $\frac{19}{256}$. 6. $\frac{1}{6}$; $\frac{261}{3520}$; $\frac{73}{80}$. 7. $\frac{890}{1920}$; $\frac{1159}{10368}$.
8. $4\frac{15}{16}$ mds. ; $2\frac{49}{64}$ bi. ; $\frac{11}{576}$. 9. $\frac{47}{64}$. 10. $\frac{79}{1460}$; $\frac{59}{64}$.
11. $\frac{19}{96}$; $\frac{11}{1920}$. 12. $\frac{6}{1823}$; $\frac{17}{80}$.

Ex. LXXXIV. (pp. 212-215.)

1. (1) $\frac{17}{23}$; $\frac{5}{21}$. (2) $\frac{89}{105}$. (3) $\frac{7}{12}$; $\frac{123}{284}$. (4) $\frac{7}{25}$. (5) $\frac{4}{15}$; $\frac{257}{5460}$.
(6) $\frac{772}{3795}$. (7) $\frac{1037}{168}$. (8) $\frac{25}{659}$.
2. (1) $\frac{19}{50}$; $\frac{158}{211}$. (2) $\frac{418}{125}$. (3) $\frac{133}{400}$; $\frac{256}{1137}$. (4) $\frac{18}{13}$.
(5) $\frac{103}{2240}$; $\frac{21}{128}$. (6) $\frac{23}{135}$. (7) $\frac{1}{45}$. (8) $\frac{35}{36}$; $\frac{21}{80}$.
(9) $\frac{64}{25}$. (10) $\frac{107}{160}$.
3. $\frac{7}{21}$. 4. $\frac{20899}{24528}$. 5. $\frac{3025}{6596}$. 6. $\frac{347}{1280}$. 7. $\frac{84525}{1248}$; $\frac{6}{49}$.
8. $\frac{2841}{3248}$. 9. $\frac{307}{4050}$; $\frac{192}{175}$. 10. $\frac{481}{600}$; $\frac{81}{184}$. 11. $\frac{8}{15}$; $\frac{1}{3360}$.
12. $\frac{125}{308}$. 13. (1) $\frac{7}{9}$. (2) $13\frac{1}{2}$. (3) $\frac{5}{88}$. (4) $2\frac{78}{121}$.
14. $\frac{166}{1351}$; $\frac{5}{4}$. 15. $\frac{5}{24}$. 16. (1) $\frac{629}{1440}$. (2) $\frac{23}{48}$; $\frac{1}{48}$. (3) $\frac{107}{840}$.
(4) $\frac{79}{810}$; $\frac{1}{30}$. (5) $\frac{11}{40}$. (6) $\frac{3}{70}$. (7) $\frac{65}{318}$.
17. In order of value the fractions will stand thus :—
(1) $\frac{8}{35}$ of a cr., £$\frac{1}{10}$, $\frac{1}{20}$ of a gui. (2) $\frac{1}{22}$ of a gui., £$\frac{1}{21}$, $\frac{1}{4}$ of 3*s.* $9\frac{1}{2}$*d.*
(3) $\frac{7}{8}$ of *Rs.*10 = $\frac{5}{6}$ of *Rs.*10. 8*a.*, $\frac{2}{3}$ of *Rs.*7. 13*a.*
(4) $\frac{1}{5}$ of a md., $\frac{5}{6}$ of 3 sr. 6 ch., $\frac{1}{6}$ of 14 sr.
(5) $\frac{2}{3}$ of 5 days, $\frac{6}{7}$ of 20 hrs., $\frac{4}{5}$ of 59 min.
18. $\frac{63}{400}$. 19. $\frac{233}{20}$. 20. $\frac{13}{320}$. 21. $\frac{70}{1123}$. 22. $\frac{13}{56}$. 23. $\frac{761}{10560}$.
24. $\frac{2256}{3675}$. 25. *Re.* 1. 5*a.* $5\frac{1}{7}$*p.* 26. 1 fur. 27. *Rs.*10., $\frac{375}{28}$.
28. 7 cwt. 11 lbs.

Ex. LXXXV. (pp. 215-216.)

1 3 days 10 hrs. 25 min. 3 sec. 2. 25 mds. 30 sr.
3. £3. 13*s.* $8\frac{1}{2}$*d.* 4. 5 po. 10 in. 5. 31 years 203 da. 18 hrs.
6. $242\frac{1}{2}$ days. 7. $344\frac{82784}{100305}$. 8. $2440\frac{167}{3696}$.
9. 79 wks. 1 da. 22 hrs. 49 min. 48 sec. 10. $\frac{12\frac{1}{2}}{76\frac{12}{13}}$; $\frac{\frac{1}{3}}{5\frac{1}{3}}$.

Ex. LXXXVI. (pp. 218-220.)

1. *Rs.* 32. 6*a.* 2. $55\frac{1}{6}$. 3. £1. 2*s.* 6*d.* 4. *Rs.*2. 12*a.*
5. £44. 17*s.* $8\frac{1}{2}$*d.* $\frac{86}{205}$*q.* 6. £4. 16*s.* 7. £200.
8. $111835\frac{3}{7}$ metres. 9. 6*a.* $3\frac{1}{4}$*p.* 10. *Rs.*225. 11. $\frac{3}{7}$.
12. $\frac{25}{128}$; *Rs.*31250. 13. 9 men. 14. $7\frac{7}{10}$ days. 15. $41\frac{1}{3}$ days.
16. $15\frac{15}{23}$ days. 17. 7 miles. 18. 60 cwt. 19. 2 lbs. 3 oz.
20. 4 wks. 2 da. 21. 85 days. 22. 12800. 23. *Rs.*4. 8*a.*
24. 75. 25. 100 days. 26. *Rs.*43. 8*a.* 27. 10 yds. 11 in.
28. *Rs.*46. 10*a.* 8*p.*

Ex. LXXXVII. (pp. 221-222.)

1. 12*a*. 8*p*. **2.** £3791. 11*s*. $3\frac{1}{4}$*d*. **3.** 6*a*. $3\frac{1}{3}$*p*. **4.** *Rs* 2376.
5. 10*a* $9\frac{3}{8}$*p* **6.** 12*a*. ; *A Rs*.3867. 3*a*. ; *B Rs*.3052 8*a* ; *C Rs*.2200.
7. *Rs*.3000. **8.** £722. 13*s*. 4*d*. **9.** £412. **10.** 3*s*. 4*d*.
11. *Rs*.2790. 10*s*. **12.** Debts=£25230 ; assets=£15558. 10*s*.
13. *Rs*.37350. **14.** £8000. **15.** Debts=*Rs*.13600 ; assets=*Rs*.9350. **16.** £6666. 13*s*. 4*d*.

Ex. LXXXVIII. (pp. 223-224.)

1. *Rs*.1752. 12*a*. **2.** £631. 10*s*. **3.** *Rs* 360. **4.** £598.
5. £451. 10*s*. **6.** *Rs*.12000. **7.** £1040. **8.** 6*p*.
9. £108. **10.** *Rs*.11480. **11.** £70. **12.** *Rs* 960.
13. £3642. **14.** £280.

Ex. LXXXIX. (pp. 226-227.)

1. 4200. **2.** *Rs* 2160. **3.** 120 gals **4.** 210. **5.** *Rs*.16. 13*a* 2*p*.
6. *Rs*.22500. **7.** 3 ro. 24 po. ; 1 ac. 2 ro. 30 po. **8.** £7. 8*s*.
9. £1. 5*s*. **10.** 38 yds. 1 ft. 6 in. **11.** *A Rs* 22. 14*a*. 8*p*. ; *B Rs*.20. 13*a*. 4*p*. **12.** Eldest *Rs*.12000 ; younger *Rs*.4500 ; wife *Rs*.2700. **13.** *Rs*.16. 4*a* **14** *Rs* 15000; *Rs*.4500 each.
15. 960. **16.** £4000. **17** *A Rs*.114. 4*a* $6\frac{6}{7}$*p*. ; *B Rs*.128. 9*a*. $1\frac{5}{7}$*p*. ; *C Rs*.157. 2*a*. $3\frac{3}{7}$*p*. **18.** *Rs*.3333. 5*a*. 4*p*. ; $\frac{1}{30}$.

Ex. XC. (pp 229-230.)

1. 6 hrs 40 min. **2.** 24 hrs. **3.** 3 hrs. $38\frac{4}{7}$ min. **4.** 12 min.
5. $3\frac{159}{187}$ hrs. **6.** $\frac{7}{8}$. **7.** 12 min. past 5. **8** 58 min. 40 sec. ; $\frac{19}{120}$.
9. $5\frac{305}{2299}$ hrs. **10.** 30 min. **11.** $1\frac{1}{2}$ min. ; $237\frac{1}{2}$ gals. **12.** 4 min. 42 sec.

Ex. XCI. (pp. 233-235.)

1. $6\frac{19}{78}$ days. **2.** 4 days. **3.** The whole. **4.** 36 days.
5. 15 days. **6.** $3\frac{1}{3}$ days. **7.** $10\frac{2}{3}$ days. **8** *A* 8 days ; *B* 24 days ; *C* $4\frac{4}{5}$ days. **9.** $18\frac{14}{81}$ days. **10.** $\frac{4}{9}$.
11. 120 days. **12.** *A* $45\frac{1}{2}$ days ; *B* $10\frac{18}{53}$ days. **13.** $1\frac{1}{2}$ hours.
14. $3\frac{3}{4}$ hours. **15.** $12\frac{1}{2}$ days ; *A* $4\frac{4}{5}$ ac. ; *B* $5\frac{1}{5}$ ac. **16.** *A* $4\frac{8}{13}$ da. ; *B* $8\frac{4}{7}$ da. ; *C* $3\frac{9}{17}$ da. **17.** $16\frac{1}{14}$ days. **18.** 18 days.
19. 8 days. **20.** $4\frac{4}{5}$ days. **21.** *A* 12 da. ; *B* 24 da.
22. $14\frac{34}{45}$ days. **23.** *A* 4 days ; *B* 12 days. **24.** 26 days.
25. $4\frac{116}{175}$ days. **26.** $9\frac{3}{5}$ days. **27.** $11\frac{1}{4}$ weeks. **28** $23\frac{1}{3}$ hrs.

Ex. XCII. (p. 237.)

1. 56. **2.** 112. **3.** 90. **4.** 15. **5.** 172. **6.** 3456 ; 2304.
7. $\frac{43}{120}$; $\frac{23}{40}$; $\frac{11}{13}$; $\frac{7}{12}$. **8.** *Rs*.150. **9.** *Rs*.24. **10.** *Rs* 2400.
11. 36 gals. **12.** 60000 men.

Ex. XCIII. (pp. 239-240.)

1. *A Rs*.1696. 15*a*. 10*p*. ; *B Rs*.2196. 1*a*. 8*p*. ; *C Rs*.2595. 6*a*. 4*p*.
2. *A Rs*.15. 8*a*. ; *B Rs*.22. 4*a*. ; *C Rs*.37. 12*a*.

3. *A Rs.*600 ; *B Rs.*480 ; *C Rs.*320.
4. *A Rs.*58. 12*a.* 2*p.* ; *B Rs.* 117. 8*a.* 4*p.* ; *C Rs.*176 4*a.* 6*p.*
5. *A Rs.*1200 ; *B Rs.*400 ; *C Rs* 200.
6. *A Rs.*3762 ; *B Rs.*2280 ; *C Rs* 6498.
7 *A Rs.*768 ; *B Rs* 672 ; *C Rs.*560
8. Man, *Rs* 7 ; woman, *Rs.*3. 8*a.* ; child, *Re.*1. 2*a.* 8*p.*
9. *A* £50 ; *B* £100 ; *C* £300 ; *D* £1200.
10. *A Rs.*$2782\frac{6}{19}$; *B Rs.* $3130\frac{2}{19}$; *C Rs.*$2347\frac{11}{19}$.
11. *A Rs.*6048 ; *B Rs.*5760 ; *C Rs* 5670 ; *D Rs* 5600.
12. *A Rs.*8505 ; *B Rs.*7276. 8*a.* ; *C Rs* 9172.

Ex. XCIV. (pp. 241-242.)

1. 14 min. 24 sec. 2. 6 hrs. 3. $36\frac{1}{2}$ days. 4. $7\frac{1}{5}$ hours.
5 72 hours 6. 258 hrs. 7. 258 hrs.
8. (i) 120 hrs. (ii) 120 hrs.

Ex. XCV. (pp. 243-244.)

1. $5\frac{109}{288}$ yds. 2. 8*a.* 3. 200. 4. 10 guiden. 5. 2601.
6. 1296. 7. 82212. 8. *Rs.*1066. 10*a.* 8*p.*
9. £11760. 10. $113\frac{4}{628}$ grs. 11. £802. 10*s.* $0\frac{1}{4}$*d.* $\frac{4}{5}$*q*
12. 2*s.* $0\frac{3}{4}$*d. nearly.*

Ex. XCVI. (pp. 246-248.)

1. $\frac{1}{20}$. 2. $\frac{2}{21}$. 3. 16. 4. 21. 5. *Rs.*400. 6. *Rs.*3000.
7. 429 8. $\frac{1}{10}$ 9. 3 mds. 10 sr. 10. $3\frac{1}{2}$. 11. *A Rs* 2. 13*a.* *B Rs.*3 12*a.* ; *C* 15*a.* 12. *A Rs.*10 ; *B Rs.*20 ; *C Rs.*10
13. *Rs* 9984 14. *Rs.*53 2*a.* 15. *Rs.*32.
16 *A* $13\frac{48}{106}$ days ; *B* $40\frac{8}{35}$ days ; *C* $67\frac{1}{21}$ days ; *D* $93\frac{12}{15}$ days.
17. 9*a* 6*p.* ; 12*a.* 6*p.* ; 15*a.* 6*p.* ; *Re* 1. 2*a.* 6*p* ; *Re* 1. 5*a.* 6*p.*
18. *Rs.*5. 19 *A Rs.*2 8*a* ; *B Re* 1. 8*a.* ; *C* 8*a.* 20. $20\frac{398}{512}$ in.

Miscellaneous Examples III. (pp.248-253.)

1. $2\frac{4}{9999}$; $\frac{400}{9999}$. 2. $12\frac{11}{34}$. 3. $\frac{27}{32}$ 4. *A Rs.*6666. 10*a.* 8*p.* ; *B Rs.*5000 ; *C Rs.*4000 ; *D Rs.*3333. 5*a* 4*p.*
5. *A Rs.*1344 ; *B Rs* 1053. 6. $40\frac{10}{11}$ days.
7. 80 apple ; 60 pear ; 48 cherry ; 40 filbert ; 12 walnut trees.
9. *Rs.*9. 2*a.* 8*p.* 10. *Rs.*2102. 8*a.* 11. $2\frac{1}{10}$ days.
12. *Rs.*4. 9*a.* 6*p.* 13. $20\frac{5}{11}$ days. 14. $\frac{359}{2376}$. 15. $\frac{2779}{5180}$.
16. $6\frac{6}{59}$ days. 17. 600. 18 (*a*) $25\frac{241}{56161}$ (*b*) $1\frac{8}{9}$.
19. 37610528. 20. 54 days. 21. £22. 8*s.* 22. Yes ; $\frac{1}{280}$.
23. $7\frac{73}{181}$ hours. 24 *Rs.*5040 25. $107\frac{85}{198}$ days.
26. 300 gals. 27. $7\frac{3}{13}$. 28. *B*'s share = *Rs.*30847. 8*a.* ; value of mine = *Rs.*98712.
29 $\frac{9}{14}$ and $\frac{18}{63}$; $2\frac{17}{32}$.
30. 14 each. 31. 50 yds. 32. $\frac{2}{27}$. 33. *Rs.*4200
34. *Rs.*6111. 10*a.* 8*p.* 35. *Rs.*484. 36. 10*s.* 10*d.* to be added.
37. *A Rs.*200 ; *B Rs.* 100 ; *C Rs.*700 ; *D Rs.*3200. 38. 13*a.* 4*p.*
39. *A*'s money 18*s.* ; *B*'s 6*s.* 8*d.* ; *C*'s 12*s.* 40. $\frac{1}{3}$.
41. *Rs.*7. 7*a.* 7*p.* 42. 8*a.* 6*p.* to be subtracted. 43. 10 weeks.

44. *Rs.*132. **45.** $10\frac{4}{5}$ days. **46.** $3\frac{58}{109}$ days. **47.** £80
48. 16720 tons. **49.** $\frac{2}{25}$. **50.** $16\frac{2}{3}$ days.
51. *A Rs.*6. 1*a.* $1\frac{5}{7}$*p.*, *B Rs.*6. 6*a.* $10\frac{2}{7}$*p.* **52.** 50 days. **53.** $2\frac{1}{4}$ days.
54. $5\frac{1}{4}$ days. **55.** 4 hrs. $7\frac{1}{2}$ min. **56.** 4 men. **57.** 30 men.
58. 169 ; 260. **59.** 17. **60.** 3024. **61.** 12 A. M. ; 7-30 P. M.
62. 12 days. **63.** 4 men. **64.** 1st cask $3\frac{1}{2}$ mds. ;
2nd $1\frac{1}{2}$ mds. ; 3rd $1\frac{1}{3}$ mds. ; 4th 2 mds.

Ex. XCVII. (p. 256.)

1. (1) ·4 ; ·18 ; ·006. (2) 3·7 ; 1·54. (3) 24·079 ; ·000001.
(4) ·579. (5) ·0008 ; ·00007 ; ·0000005. (6) 315·00805.

2. (1) Five-tenths ; thirty-five hundredths ; three hundred and twenty-six thousandths ; one-tenth ; one-hundredth ; one ten-thousandth ; five and thirty-seven hundredths ; twenty-five ten-thousandths.
(2) Forty-three thousand two hundred and sixty-eight hundred-thousandths ; three thousand four hundred and five millionths ; four hundred and fifty-six ten millionths ; ninety-eight and seven millions six hundred fifty-four thousands three hundred and twenty-one ten-millionths ; one hundred thousand and one millionths.

3. (1) $\frac{7}{10}$; $\frac{71}{100}$; $\frac{751}{1000}$; $\frac{1461}{10000}$; $\frac{3}{100}$; $\frac{37}{10000}$; $\frac{57}{1000000}$; $\frac{457}{100000000}$.
(2) $\frac{87}{10}$; $\frac{5219}{100}$; $\frac{478913}{1000}$; $\frac{27600482}{100000}$; $\frac{147520457092}{10000000}$.

4. (1) $\frac{1}{5}$; $\frac{1}{2}$; $\frac{3}{4}$; $\frac{5}{8}$; $\frac{3}{16}$; $\frac{64}{25}$; $\frac{54}{125}$; $\frac{1}{160}$; $\frac{8476}{15625}$.
(2) $\frac{35}{8}$; $\frac{13}{16}$; $\frac{5907}{125}$; $\frac{69}{16}$; $\frac{260017}{20000}$; $\frac{11}{10000}$;
(3) $\frac{13}{128}$; $\frac{43}{40}$; $\frac{483}{160}$; $\frac{4483}{640}$; $\frac{66563}{5120}$;

5. (1) $5\frac{1}{5}$; $6\frac{3}{125}$; $41\frac{3}{200}$; $72\frac{3}{4}$; $9\frac{2079}{5000}$; $307\frac{251}{1000}$; $8\frac{73}{80}$.
(2) $376\frac{11}{40}$; $970\frac{1}{25000}$; $7321\frac{101}{2000}$; $23\frac{275}{4096}$.

6. (1) ·3 ; ·7 ; 1·3 ; 4·7 ; ·13 ; ·407 ; 8·01 ; 55·9 ; ·057.
(2) ·0017 ; ·00965 ; ·0303 ; ·000009 ; ·0000057 ; 48·76.
(3) 2·13 ; ·0213 ; ·000213 ; ·0300507 ; ·00076359 ; 2040·3005.
(4) 1·6 ; ·75 ; ·031 ; ·0004 ; ·0275 ; ·000079 ; ·0001001 ; ·00000079 ; ·00261 ; ·000000000328.

Ex. XCVIII. (p. 257.)

1. (1) 8 ; 80 ; 800 ; 80000 ; 8000000.
(2) ·53 ; 53 ; 5300 ; 530000. (3) 130·014 ; 1300·14 ; 13001·4 ; 130014 ; 13001400. (4) 800·3065 . 80030·56 ; 80030560.

2. (1) ·071 ; ·0071 ; ·000071 ; ·00000071.
(2) ·07358 ; ·007358 ; ·00007358 ; ·000007358.
(3) ·00007 ; ·000007 ; ·00000007 ; ·000000007.
(4) ·001 ; ·0000001 ; ·00057426.

Ex. XCIX (p. 258.)

1. (1) 756·2407. (2) 4009·360826. (3) 538·6422021.
(4) 433·416458. (5) 442·10265. (6) 4852·77877.

(7) 634·5529699. (8) 100. (9) 61·58125. (10) 100·1111111.

2. (1) 1589·6738. (2) 1152·445834. (3) 3033·9704833.
(4) R1783·378. (5) £878·368. (6) 2623·952038 cwt.
(7) 2170·0457 yds. (8) 6·171256.

Ex. C. (p. 259.)

1. (1) ·18971 ; 11·203 ; 52·5564. (2) ·117864 ; ·806423 ; ·09914.
(3) 8·00001 ; ·03035039 ; ·51635. (4) ·09 ; ·0001 ; ·623967.

2. (1) 27·849 ; 7040·322 ; 31·580901. (2) 1·0473 ; 3·021975 ; ·0324.
(3) 4·911002 ; ·1431 ; 7·09484.

3. (1) 786·214 ; 11·206157 ; ·00099.
(2) R6·417 ; R2·23224 ; R3·340526.
(3) £·12084 ; £54·3249 ; £·20994.
(4) 6·399936 mds. ; 22·5124 tons ; 1·43856 ft.

4. 9·413. 5. 86·7358. 6. (1) 2·6686 ; 2·102.
(2) 644·447. (3) 5·45414. (4) 527·02949.

7. ·2218487 ; ·999544 ; 1·345679 ; 457·575 ; 1·101 ; ·99999.

8. 3·1415927. 9. ·54321.

Ex. CI. (pp. 260-261.)

1. (1) ·40926 ; ·04032 ; 9 ; ·7777. (2) 100·17484 ; 21·983472 ;
8·2940365. (3) 230·625593 ; 4155·84 ; ·00003738028.
(4) ·123123 ; ·0019610652875 ; ·0000008.
(5) ·0000003217 ; 6·7294023544 ; 5·548075.
(6) ·00128 ; ·000000015625 ; ·0000017084592.

2. (1) ·26325 ; ·0001410816 ; ·00177089 ; 6·240312.
(2) 120·89115 ; ·0011238573 ; ·147168.
(3) 20·796875 ; ·026649 ; 133·46322.
(4) 108·243216 ; 9·12850715. (5) ·0672 ; ·001024.
(6) ·000000001 ; 9·56709.

3. (1) 21·09304541972. (2) 210·795537. (3) 12·66806 ;
23·68676 ; ·001. (4) 678·593664 ; ·14726336 ; 9·141474375.

4. ·0177775 ; ·001.

Ex. CII. (pp. 263-264.)

1. (1) 156·7 ; 31·34 ; 6·268 ; 1·2536 ; ·12536.
(2) 128·947 ; 59·514 ; 9·919 ; 4·578 ; 1·526 ; ·763.
(3) ·00001108 ; ·00000554 ; ·00000277 ; ·0000006925.
(4) 5·12 ; ·427 ; 2·417. (5) 70 ; 7900 ; 3250 ; 22500.
(6) ·24172 ; 2·421875 ; ·0000746.
(7) 10 ; 10·01 ; 1545 ; 110000 ; 10.
(8) 281·315 ; ·039. (9) 1·092 ; ·1092 ; 1092 ; ·0001092.
(10) 15·67 ; 313·4 ; ·6268 ; 1·2536 ; 12·536 ; 125·36 ; ·000012536.
(11) ·007853 ; ·0032 ; 602400. (12) 300.

2. (1) 3·42465 ; 6·26829 ; 17404·27807 ; ·00261.

(2) ·00650 ; 3·21685 ; 00025 ; 1·31579.
(3) 6·03604 ; 91·33632 ; 36542894·03280.

3. (1) 12·096 ; 5·184 ; 4·032 ; ·576 ; ·288 ; 19·2 ; 151·2.
(2) ·85 ; 5·1 ; ·0075 ; ·00425 ; 2·5 ; ·01.

4. (1) ·00134 ; ·13405 ; 1·34057 ; ·00521 ; 5·21333 ; 14·89523
(2) 76293021842·10526 ; 2167022·5625.

5. (1) ·30685 ; 147 ; ·0007. (2) 32236·38 ; 2296.
(3) 20500 ; 1·569995. (4) ·00011 ; 15·845686.

Ex. CIII. (p. 265.)

1. (1) ·5 ; ·25 ; ·75 ; ·125 ; ·625 ; ·1875 , ·16 ; 3·1171875 ; ·008125 ; ·072 ; ·7578125.
(2) ·006640625 ; 03808 ; 4·0064 ; 3·16 ; 4·3125 ; 6·0005859375 ; 7·425.
(3) 1·4 ; ·053125 ; 1·687841796875 ; 1·0693359375 ; 7·00048 ; 3·00224 ; 4·0010624.

2. (1) 12·75 ; 76·234375 ; 8·75 ; 2·59375.
(2) 1·25 ; ·0176 ; 326·4 ; ·1128 ; ·6 ; 1·37.
(3) 83·03355 ; ·007080078125. (4) 89·395 ; ·0029.

3. ·333333 ; ·285714 ; ·272727 ; ·384615 ; ·269230 ; ·309523 ; 2·916666 ; 8·642857 ; 24·352941 ; 7·326086.

4. (1) $\frac{5}{8}$, $\frac{2}{5}$, $\frac{3}{8}$. (2) $\frac{3}{4}$, $\frac{7}{16}$, $\frac{5}{12}$. (3) $\frac{5}{8}$, $\frac{7}{32}$, $\frac{5}{24}$. (4) $\frac{7}{8}$, $\frac{5}{6}$, $\frac{4}{5}$, $\frac{3}{4}$

5. (1) 120. (2) ·9. (3) 85·1. (4) 10·125. (5) ·0000125.
(6) 6400. (7) 3. (8) 562·1 ; 7·174970942857142...

Ex. CIV. (p. 266.)

1. (1) ·36. (2) ·2223. (3) 1·115. (4) ·0094. (5) ·004. (6) ·003.
(7) ·16. (8) ·0008

2. (1) 1260 (2) 12·6. (3) 1466·4. (4) 5319·6. (5) 14360·58.
(6) 8433621·6646.

Ex. CV. (p. 267.)

1. $\frac{13}{16}$; $\frac{9}{48}$; $\frac{11}{25}$; $\frac{81}{80}$; $\frac{19}{64}$; $\frac{91}{560}$; $\frac{18941}{88836}$ **2.** 2, 4, 5, 8, 10, 16, 20.

Ex. CVI. (pp. 268-269.)

1. $\cdot\dot{2}$; $\cdot\dot{2}\dot{7}$; $\cdot\dot{1}\dot{3}$; $\cdot\dot{1}3\dot{5}$; $2\cdot3\dot{4}\dot{5}$; $1\cdot\dot{6}$; $4\cdot\dot{8}$; $1\cdot\dot{9}\dot{0}$; $\cdot\dot{7}1428\dot{5}$;
$7\cdot\dot{4}6153\dot{8}$; $\cdot\dot{1}2698\dot{4}$; $9\cdot3\dot{0}\dot{9}$.

2. $\cdot\dot{3}0769\dot{2}$; $\cdot\dot{1}219\dot{5}$; $\cdot\dot{1}50943396226\dot{4}$; $3\cdot61458\dot{3}$;
$4\cdot803\dot{5}7142\dot{8}$; $7\cdot18\dot{9}\dot{3}$; $5\cdot\dot{2}87\dot{1}$; $3\cdot35\dot{6}\dot{7}$.

3. $\cdot01\dot{2}\dot{6}$; $5\cdot\dot{7}8\dot{3}$; $\cdot2\cdot733\dot{1}0\dot{8}$; $1\cdot085069\dot{4}$, $1\cdot2957\dot{3}17\dot{0}$;
$\cdot75240\dot{3}8461\dot{5}$; $3\cdot45\dot{5}609\dot{7}$; $1\cdot0\dot{0}19\dot{8}$.

4. $2\cdot\dot{4}5700\dot{2}$; $3\cdot\dot{7}8000\dot{3}$; $1\cdot\dot{7}6114\dot{5}$; $\cdot5\dot{6}34\dot{9}$; $\cdot725\dot{0}\dot{9}$;
$\cdot4\dot{9}9077\dot{4}$; $\cdot907\dot{9}6703\dot{2}$.

Ex. CVII. (pp. 269-270.)

1. $3{\cdot}25252\dot{2}\dot{5}$; $4{\cdot}7777\dot{7}$; $290{\cdot}0222\dot{2}$; ${\cdot}3666\dot{6}$; $21{\cdot}14141\dot{4}$;
$\cdot 0352\dot{3}5\dot{2}$; ${\cdot}7065\dot{5}$; ${\cdot}046\dot{3}6\dot{3}$; $3{\cdot}4534\dot{5}3\dot{4}$.
2. ${\cdot}5\dot{7}77777\dot{7}$, $2{\cdot}\dot{3}43434\dot{4}$, ${\cdot}0\dot{6}4564\dot{5}$.
3. ${\cdot}1\dot{2}323232323\dot{3}$, ${\cdot}1\dot{2}342342342\dot{3}4$, ${\cdot}\dot{1}23412341234\dot{4}$.
4. ${\cdot}84615\dot{3}$; ${\cdot}0\dot{7}1428\dot{5}$; ${\cdot}\dot{2}94117647058823\dot{5}$; ${\cdot}\dot{1}05263157894736842$
${\cdot}\dot{3}478260869565217391304\dot{4}$; ${\cdot}\dot{0}344827586206896551724137931$
${\cdot}\dot{1}9354838709677\dot{4}$; ${\cdot}\dot{1}88679245283\dot{0}$.
5. (1) $3{\cdot}07\dot{6}6666\dot{6}$, $9{\cdot}24\dot{5}2452\dot{4}$, ${\cdot}20\dot{3}0303\dot{0}$.
(2) ${\cdot}\dot{8}8888\dot{8}$, ${\cdot}\dot{8}7878\dot{7}$, ${\cdot}\dot{8}7687\dot{6}$
(3) ${\cdot}4\dot{1}41414141414\dot{4}$, ${\cdot}0\dot{3}52352352352\dot{2}$, $6{\cdot}1\dot{0}13101310131\dot{1}$.
(4) ${\cdot}550\dot{7}7777777777\dot{7}$, ${\cdot}046\dot{3}6363636363636\dot{6}$.
$1{\cdot}41\dot{3}41341341341\dot{3}$, ${\cdot}706\dot{5}70657065706\dot{5}$.
(5) ${\cdot}78\dot{5}48548\dot{8}$, ${\cdot}59\dot{5}9595\dot{9}$. $14{\cdot}57\dot{7}77777\dot{7}$, ${\cdot}00\dot{4}5454\dot{5}$.
(6) $9{\cdot}7012\dot{1}21212121212\dot{2}$, $4{\cdot}4033\dot{3}33333333333\dot{3}$,
$10{\cdot}84921\dot{3}7137137137\dot{7}$, ${\cdot}2186\dot{5}18651865186\dot{6}$.

Ex. CVIII. (p. 272.)

1 (1) $\frac{5}{9}$; $\frac{1}{37}$; $\frac{178}{333}$; $\frac{1421}{3333}$; $\frac{56}{99}$; $\frac{7}{27}$; $\frac{73}{101}$; $\frac{508}{819}$.
(2) $\frac{67}{185}$; $\frac{8958}{8325}$; $\frac{1}{18}$; $\frac{11}{2220}$; $3\frac{6}{11}$; $\frac{13}{66}$; $\frac{16611}{99900}$; $\frac{13}{24}$
(3) $\frac{8}{185}$; $\frac{12157}{555}$; $\frac{71}{105}$; $\frac{88}{9768}$; $\frac{357}{440}$; $\frac{2054}{4625}$.
(4) $\frac{731}{3030}$; $1\frac{3}{70}$; $2\frac{9}{14}$; $3\frac{1536}{1776}$; $13\frac{49}{352}$.
(5) $6\frac{10}{13}$; $50\frac{3}{13}$; $4\frac{49}{325}$; $\frac{1}{81}$; $\frac{181}{650}$.
2. ${\cdot}\dot{1}$; $4{\cdot}37$; ${\cdot}458$; 26 ; $15{\cdot}9$; $3{\cdot}79$; 60 ; ${\cdot}01$.
3. 10 and 21 ; $\frac{1}{333331}$.

Ex. CIX. (p. 273.)

1. (1) $4{\cdot}94\dot{3}3269\dot{6}$; ${\cdot}8502\dot{5}88315\dot{5}$; $2{\cdot}738\dot{1}3\dot{7}$.
(2) $45{\cdot}76782\dot{9}\dot{7}$; $115{\cdot}18\dot{2}7\dot{5}$.
(3) $44{\cdot}5012\dot{7}7983\dot{7}$.
(4) $41{\cdot}2553\dot{4}2469\dot{1}$.
(5) $40{\cdot}1413\dot{8}1378\dot{7}$.
2. (1) $13{\cdot}3848250$; $16{\cdot}9602603$.
(2) $611{\cdot}4024344$.
(3) $275{\cdot}0304909$.
(4) $47{\cdot}4114553$.

Ex. CX. (p. 274.)

1. (1) $4{\cdot}75\dot{1}\dot{7}$. (2) ${\cdot}072\dot{2}\dot{0}$. (3) ${\cdot}11\dot{8}72508\dot{8}$. (4) $4{\cdot}\dot{3}50896\dot{6}$.
(5) $216{\cdot}29\dot{5}4\dot{3}$. (6) $8{\cdot}61\dot{9}\dot{8}$. (7) $3{\cdot}661\dot{9}2328\dot{6}$.
(8) ${\cdot}7131\dot{4}93550\dot{6}$. (9) $6{\cdot}114\dot{2}3\dot{7}$.

2. (1) ·032712. (2) 58·628447. (3) 33·022222.
(4) 133·874328. (5) 44·494309. (6) 1·817687.
3. ·954$\dot{3}\dot{6}$; ·9$\dot{2}$1$\dot{0}$; 74·35$\dot{7}$962$\dot{9}$. 4. (1) 4·9838$\dot{1}$93$\dot{3}$.
(2) 157·577$\dot{9}$68$\dot{4}$. (3) 186·853$\dot{8}$18$\dot{7}$. (4) 23·307$\dot{9}\dot{8}$.

Ex. CXI. (p. 275.)

1. (1) 3·3878 ; 4·140$\dot{6}$; 13·92$\dot{7}$; 6·382$\dot{7}$9$\dot{8}$.
(2) 38·0375 ; 2715·121$\dot{3}$69$\dot{3}$; 31·8236520$\dot{6}$3$\dot{9}$.
(3) 2·8$\dot{4}$; ·086$\dot{2}$; ·51014$\dot{6}$3$\dot{0}$; 36·1$\dot{7}$5$\dot{3}$.
(4) 13·36 ; 731·164$\dot{3}$9$\dot{9}$; 71·880317$\dot{8}$23$\dot{5}$.
2. (1) 1·$\dot{1}$8$\dot{5}$; 67·$\dot{4}\dot{5}$; 580·$\dot{9}\dot{0}$; ·1$\dot{2}\dot{0}$.
(2) 3·$\dot{6}\dot{3}$; ·$\dot{6}\dot{3}$; 46178·683$\dot{6}$2$\dot{1}$. (3) ·569$\dot{4}$; ·2 ; 14·$\dot{7}$; ·32386$\dot{3}$.
(4) ·10284...... ; ·$\dot{5}$; 69·951003086419$\dot{7}\dot{5}$.

Ex. CXII. (p. 276.)

1. (1) ·0$\dot{6}$; ·$\dot{0}$4761$\dot{9}$; 7·$\dot{4}$1$\dot{7}$; ·$\dot{8}$24157490$\dot{0}$; ·002660$\dot{4}$; ·0$\dot{1}$5384$\dot{6}$.
(2) ·43224431$\dot{8}$; 5·376$\dot{5}$4897$\dot{7}$; 4·8181639$\dot{6}$1038$\dot{9}$.
(3) ·490578$\dot{7}$0$\dot{3}$; 10·$\dot{6}$8109$\dot{6}$; 6·$\dot{2}$6936$\dot{0}$; 1·52$\dot{7}$.
(4) ·00200212...... ; ·4$\dot{7}$8$\dot{3}$; 280·$\dot{5}$836139169472502$\dot{8}\dot{0}$.
2. (1) 1·$\dot{4}$206$\dot{6}$; 3·$\dot{2}\dot{7}$; 8·$\dot{1}\dot{8}$; 47·$\dot{3}$; 2·52$\dot{2}\dot{7}$.
(2) 28·65789 ; ·24 ; 2·$\dot{9}$71428$\dot{5}$; ·4$\dot{4}$5$\dot{9}$.
(3) ·689921875 ; ·02674$\dot{5}$4$\dot{9}$; ·5 ; 92·5.
(4) 21 ; 6·7$\dot{0}$4761$\dot{9}$; 8·187$\dot{4}\dot{6}$.

Ex. CXIII. (p. 277.)

1. 7·$\dot{1}$3$\dot{7}$. 2. 1·6875. 3. 1·$\dot{8}$5$\dot{1}$. 4. ·1$\dot{0}$. 5. 13·1349$\dot{3}$.
6. 2715. 7. 1·2823. 8. 283·$\dot{5}$. 9. 1461·91304...
10. ·88$\dot{6}\dot{3}$. 11. ·0$\dot{2}$45700$\dot{0}$. 12. $4\frac{101}{1875}$. 13. ·00$\dot{7}$.
14. 20. 15. 25. 16. 5·62682... 17. ·3125. 18. ·84575.
19. 7. 20. 1·22974$\dot{5}$37$\dot{0}$. 21. 4·21689$\dot{3}$. 22. 14·765625.
23. 5. 24. 11344·$\dot{6}$. 25. 3·78$\dot{3}$. 26. ·125.
27. ·4. 28. 41·$\dot{6}$. 29. 26·4$\dot{1}\dot{5}$. 30. 55·1625.

Ex. CXIV. (p. 279.)

1. 5·7*d.* ; ·9*d.* ; 851·472*d.* ; 1·5*d.* ; 67·5*d.*
2. 1·5*q.* ; 859*q.* ; 1·12*q.* ; 452*q.* ; 368*q.*
3. 961·2*p.* ; 530·96256*p.* ; 744·6*p.* ; 585·408*p.*
4. 62·2$\dot{6}$*ps.* ; 26·$\dot{2}$*ps.* ; 43·6*ps.* ; 40·6*ps.* ; 21·79$\dot{4}\dot{5}$*ps.*
5. 1408·3328 oz. ; 7655·2 yds. ; 47491·92 sec. ; 219321 sec.

6. 3570 grs. ; 6840 min.

7. 17847·016 sq. yds. ; ·38 pt. ; 1455·376 ch. ; 4356 yds.

8. £·03$\dot{5}$; £·1675 ; £·0265625 ; £·00042375.

9. ·000439$\dot{6}$ da. ; ·086945 ac. ; 1260 po.

10. ·021309375 ton ; ·911458$\dot{3}$ oz. ; ·18$\dot{3}$ ton.

11 ·5$\dot{9}$5238$\dot{0}$ gui. ; 1·$\dot{1}$9047$\dot{6}$ half-gui.

12. ·29965875 mi. ; 7·15$\dot{8}$57142$\dot{2}$ oz. Av. ; ·00055803$\dot{5}$71428$\dot{8}$ cwt.

Ex. CXV. (pp. 280-281.)

1. (1) *Rs.*5. 1*a.* 1·68*p.* ; *Rs.*80. 1*a.* 2·4*p.* ; 3·2*p.* ; *Rs.*227. 11*a.* 7·6$\dot{3}$*p.*
(2) 9*s.* ; 10*s.* 1½*d.* ; £14. 4*s.* 3*d.* ; 11*s.* 4⅓*d.* ; 11·616*d.* ; 15*s.* 6¾*d.* nearly.
(3) 6*s.* 9⅞*d.* ; 7¼*d.* ·52*q.* ; 9*s.* 8*d.* nearly ; £5. 12*s.* 3*d.* ; 8*s.* 9*d.*
(4) 7*s.* 10½*d.* ; £1. 1*s.* 6¼*d.* ·2*q.* ; 15*s.* 3¼*d.* ; 5½*d.*
(5) 1qr. 14lbs.; 2qrs. 3nails; 13ac. 1ro. 14po.; 15hrs. 43min. 12sec.
(6) *Rs.*5. 12*a.* 9·4*p.* ; 17 cwt. 1 qr. 20 lbs. 8 oz. $8\frac{8}{13}$ drs. ; 2 days 12 hrs. 55 min. 21 sec. ; £47. 5*s.* 7½*d.*
(7) 2¼ in. ; 12 lbs. ; 15 po.
(8) 4 ch. ; 1*s.* 3¾*d.* ; 4 ac. 3 ro. 16 po.

2. (1) ·4375*s.* ; ·0625*s.* ; 8·9375*s.* ; 1·2708$\dot{3}$*s.* ; 34·875*s.*
(2) £·628125 ; £·790625 ; £·853125 ; £·007291$\dot{6}$; £2·790625.
(3) ·46875 ; ·515625 ; ·84765625 ; 53·8541$\dot{6}$
(4) ·902$\dot{7}$; ·09975. (5) ·57291$\dot{6}$; 1·43625 ; ·6$\dot{1}$.
(6) ·628125 ; ·00628125 ; 628·125. (7) ·88289930$\dot{5}$; ·7265.
(8) ·455921875 ; 4·34375 ; ·89533203125.
(9) ·$\dot{5}$3846$\dot{1}$; ·7882$\dot{5}$71428$\dot{8}$; ·089$\dot{2}$8571$\dot{4}$.
(10) ·00625 ; ·4125 ; ·00390625.
(11) ·005563... ; ·30016741... ; ·02161458$\dot{3}$.
(12) ·807244$\dot{3}$1$\dot{8}$; ·9471$\dot{5}$9$\dot{0}$; ·50983$\dot{7}$96$\dot{2}$.

Ex. CXVI. (pp. 282-283.)

1 (1) *Rs.*3. 1*a.* 4*p.* ; *Rs.*15. 13*a.* 4*p.* ; *Rs.*38. 12*a.* 6*p.*
(2) 6*s.* 2*d.* ; 4*s.* 10¾*d.* ·68*q.* ; 11*s.* 5¾*d.* ; £4. 13*s.* 9*d.*
(3) 1½*d.* ; 10*s.* 1¾*d.* ; 3*s.* 6*d.*
(4) *Rs.*246. 4*a.* 1·99999488*p.* ; 12*a.* 6*p.* ; 4 bi. 8 kat. 2 ch.
(5) 2 sr. 4 ch. ; 19 mds. 30 sr. 6·48ch. ; 1 fur. 17 po. 3 yds. 10·8 in.
(6) 9*s.* 10·16*d.* ; 3 tons 11 cwt. 2 qrs.; 61 mi. 377 yds. 2 ft. 6·96 in.
(7) £1. 9*s.* 7½*d.* ; 1*d.* ·8594*q.* ; 7*s.* 7½*d.*

(8) £9. 4*s*. 0¾*d*. ·65625*q*. ; £1640.
(9) £23. 10*s*. 10*d*. ; £19. 2*s*. 9¾*d*.
(10) *Rs*.1175. 1*a*. 9·45*p*. ; *Rs* 51612. 13*a*. 7·56*p*. ; *Rs* 17. 11*a*. 4*p*.
(11) 3 sq. ft. 67·5 sq. in. ; 2 tons 17 cwt. 1 qr. 27 lbs. 7 oz. 4 drs.
(12) 25 days 21 hrs. 3·0028...min. ; 12 lbs. 2 oz. 1·152 drs.

2. (1) *Re*.1. 5*a*. 4*p*. ; *Rs*.6. 5*a*. ; *Re*.1. 2*a*. $9\cdot\dot{3}$*p*.
(2) 7*s*. 6*d*. ; £1. 9*s*. $1\cdot 4\dot{7}\dot{8}$*d*. ; *Rs*.31. 10*a*. 8*p*.
(3) £1. 14*s*. 4*d*. $\cdot 8\dot{2}9\dot{6}$*q*. ; *Rs*.67. 8*a*. ; 2 448*p*.
(4) 1 cwt. 2 qrs. 6 lbs. 6 oz. ; 52 mi. 2 fur. 25 po. 3 yds.
(5) 20 lbs. ; 15*s*. 4*d*. ; £1. 14*s*, 7½*d*. ; 24 bi. 6⅓ k. ; 16 qrs. 4 bus.
(6) 7*a*. 2·4*p*. ; 13*a*. 5·616*p*. ; *Rs*.189. 15*a*. 4*p*. ; *Rs*.2. 7*a*. ·1*p*.

3. £4. 13*s*. 8¼*d*. $\frac{11}{87}$*q*. ; *Rs* 12. 4. 1 oz. 17 dwts. 22 grs.

5 (1) *Rs*.22. 14*a*. 6·4*p*. (6) *Rs*.6. 1*a*. 2*p*. (3) 10*s*. 11*d*.
(4) 6*s*. 6*d*.; 16*s*. 11¾*d*. (5) *Rs*.21. 12*a*. 1·8*p*. (6) 5 mds. 9 sr. 4ch.
(7) 19*s*. 1½*d*. (8 £1. 18*s*. 7*d*. (9) £15. 14*s*. 10$\frac{2}{7}$*d*.
(10) 1 ton 17 cwt. 2 qrs. 4 lbs.

Ex. CXVII. (pp. 284-285.)

1. (1) ·0625. (2) $\cdot 4\dot{7}2\dot{9}$. (3) ·675. (4) $\cdot 057291\dot{6}$. (5) ·375.
(6) ·875. (7) $\cdot 4447\dot{2}$. (8) 1·32531 .. (9 $\cdot\dot{0}1587\dot{3}$.
(10) 4878. (11) ·875. (12) $\cdot\dot{0}7142\dot{8}$. (13) 14·49.
(14) $\cdot 1088541\dot{6}$. (15) $\cdot 01\dot{0}3\dot{6}$. (16) ·0625. (17) ·95.
(18) 3·4375. (19) $71\cdot\dot{7}0\dot{3}$. (20) ·046875. (21) ·36.
(22) ·003. (23) $1\cdot 8\dot{6}$. (24) $\cdot\dot{7}$. 2. $\cdot 82\dot{9}\dot{3}$. 3. 59 52.
4. ·6875. 5. $\cdot 630\dot{5}$. 6. $\cdot 171\dot{2}9\dot{6}$. 7. $\cdot 081\dot{6}$. 8. ·208.
9. 3·28. 10. ·45. 11. ·0625. 12. ·5.

Ex. CXVIII. (pp. 289-291.)

1. (1) ·3010300. (2) 11704·013860. (3) 5·74. (4) ·392754.
(5) 46287·739. (6) ·95424. (7) 985·428337. (8) 1·1038.
(9) 229·92815. (10) ·60941. 2. (1) 14·26. (2) ·1495.
(3) 2115·1921. (4) 285119·0879907. (5) 30·799. (6) ·13059.
(7) ·318310. (8) ·7157788. (9) 7·037. (10) ·26544.

3. (1) ·25. (2) $\cdot 1\dot{6}$. (3) 2. (4) 1·4106861.
(5) ·202733. (6) ·009706. (7) 3·141593. (8) $\cdot 4\dot{6}$.
(9) 4·89898. (10) $\cdot 58\dot{3}$. 4. (1) 324048, rem. 6 ; 8340153, rem. 5.
(2) 521695, rem. 96 ; 72773, rem. 34.
(3) 7211, rem. 772 ; 581, rem. 425.
(4) 59, rem. 1667 ; 739, rem. 2423.
(5) 23, rem. 6939 ; 7205, rem. 40081.

Miscellaneous Examples IV. (pp. 292-295.)

1. 961·683232i. **2.** ·0002938. **3.** 360. **4.** The latter.
5. 3. **6.** ·02525. **7.** ·23957... **8.** *Rs* 7206. 6*a*. 4·8*p*.
9. *Rs*.285120. **10.** 44. **11.** 861 **12.** 4·8. **13.** ·$\dot{5}$.
14. 40. **15** £6. 1*s*. 6*d*. **16** *A Rs*.360 ; *B Rs*.270 ; *C Rs*.240.
17. ·54*p*. **18.** *Rs*.2571. 15*a*. **19.** 168. **20.** 5·077881.
21. *A*'s property exceeds *B*'s by *Rs*.49000. **22.** 2530·6.
23. ·075. **24.** ·225625. **25** ·00256256 ; 256·256 ; ·0256256 ; ·$\dot{6}\dot{3}$.
26. 1·414 **27.** ·03. **28** ·00$\dot{2}9\dot{7}$. **29.** *Rs*.17. 1*a*. 8*p*.
30. 6*p*. **31.** $\frac{9}{80}$. **32** *A* $28\frac{28}{151}$ days ; *B* $35\frac{161}{197}$ days ; *C* $38\frac{178}{181}$ days. **33.** ·32. **34.** $13\frac{29}{60}$ = 13·48$\dot{3}$.
35 *Rs*.88200. **36.** 200 gals. **37.** 120 runs. **38.** (1) $\frac{103}{104}$. (2) ·708. (3) 4·4. **39.** 126 sec. **40.** 6·75. **41.** 20 hrs. 30 min.
42. 69 eggs. **43.** *Rs*.247. 8*a*. **44.** 35584345 ; rem. is ·0035 in.
45. ·02291$\dot{6}$. **46.** *Rs*.2428. 12*a*. ; *Rs*.1238. 10*a*. 7·2*p* ; *Rs*.1190. 1*a*. 4·8*p*. **47.** *A*'s inc. = $\frac{1}{7}$ of *B*'s inc. ; *Rs*.4200.
48. *Rs*.15. **49** *Rs*.137. 8*a*. **50** £419 19*s*. 3*d*.

Ex. CXIX. (p 299.)

1. *Rs*.541. 11*a*. 6*p*. **2.** *Rs*.542. 4*a*. 10*p*. **3.** *Rs*.965. 5*a*. 4*p*.
4. *Rs*.427. **5.** *Rs*.1975. **6.** £54. 4*s*. $7\frac{1}{4}d$. **7** £144. 10*s*. $7\frac{1}{2}d$.
8. £138. 14*s*. 4*d*. **9.** £5429. 0*s*. $4\frac{3}{4}d$. **10** *Rs*.6839. 9*a*. 4*p*.
11. *Rs*.3750. **12.** *Rs*.122415. 7*a* $3\frac{1}{2}p$. **13.** *Rs*.9065. 4*a*.
14. £6893. 19*s*. $8\frac{3}{4}d$. **15.** £1156. 0*s*. $7\frac{1}{2}d$. **16.** £730. 10*s*. $10\frac{1}{2}d$.
17. £7620. 18*s*. $0\frac{3}{4}d$. **18.** £13903. 10*s*. 6*d*. **19.** £2357. 16*s* 10*d*.
20 *Rs*.109320. 7*a*. $6\frac{1}{4}p$. **21.** *Rs* 1689. 3*a*. $7\frac{3}{7}p$. **22.** £35978. 18*s*. $1\frac{1}{2}d$.
23. £20504. 0*s*. $1\frac{1}{2}d$. **24.** *Rs* 315597. 3*a*. 2*p*. **25.** *Rs*.16532. 8*a*. $8\frac{4}{7}p$.
26 *Rs*.202. 13*a*. 10*p* **27.** *Rs* 3379. 5*a*. 2*p*. **28.** *Rs* 50324. 3*a* 6*p*.
29. £5392. 1*s*. $9\frac{1}{4}d$. $\frac{1}{2}q$. **30.** £1873. 18*s*. $3\frac{1}{4}d$. $\frac{1}{8}q$.
31. £625. 19*s*. $0\frac{1}{4}d$ $\frac{1}{2}q$. **32.** *Rs*.12506.
33. £2683. 0*s*. $9\frac{1}{2}d$. $\frac{1}{4}q$. **34.** *Rs*.7. 2*a*. 10·6*p*.
35. £491. 7*s*. $11\frac{1}{2}d$ $\frac{7}{8}q$ **36.** £470. 2*s*. $0\frac{1}{8}d$
37. *Rs*.102781. 6*a*. $9\frac{1}{5}p$. **38.** *Rs*.593. 11*a* $1\frac{1}{3}p$.
39. *Rs*.5708. 1*a*. $7\frac{1}{5}p$. **40.** *Rs*.4706. 10*a* 9*p*.
41. *Rs*.17739. 4*a*. 1·6*p*. **42.** £831. 3*s*. $2\frac{1}{4}d$. $\frac{1}{7}q$.
43. £12126. 7*s* $11\frac{1}{2}d$. $\frac{3}{8}q$. **44.** *Rs*.208336. 5*a*.
45. *Rs*.124479. 2*a*. 8*p*. **46.** £24617. 0*s*. $1\frac{1}{2}d$. $\frac{12}{13}q$.

Ex. CXX. (pp. 301-303)

1. *Rs* 198. 2*a*. $1\frac{1}{5}p$. **2.** *Rs*.55. 3*a*. 10*p*. **3.** *Rs*.506. 4*a*. $0\frac{1}{3}p$.
4. *Rs*.31619. 1*a* 6·4*p*. **5.** *Rs*.9404. 3*a*. 0·10625*p*. **6.** £18. 14*s*. 11*d*.
7. £316. 17*s*. $3\frac{3}{4}d$. **8.** £13. 4*s* $4\frac{1}{48}d$. **9.** £140. 18*s*. $5\frac{13}{112}d$.
10. £89. 6*s*. $1\frac{1}{8}d$. **11.** £1052. 13*s*. $11\frac{1}{2}d$. $\frac{5}{7}q$.
12. £496. 1*s* $9\frac{27}{128}d$. **13.** *Rs*.221. 0*a*. $7\frac{17}{121}p$.
14. £2. 3*s*. $0\frac{3}{4}d$. $\frac{2}{9}q$. **15.** *Rs*.71369 12*a*. $6\frac{3}{8}p$.

16. *Rs.*61. 14*a.* $0\frac{3}{4}$*p.*
17. *Rs* 1014. 5*a.* 0·75*p.*
18. *Rs.*1058. 11*a.* 6·9*p.*
19. *Rs.*13881 5*a.* $2\frac{1}{4}$*p.*
20. £2493. 4*s.* $3\frac{1}{4}$*d.* $\frac{11}{16}$*q.*
21. £105. 4*s.* $7\frac{1}{2}$*d.* $\frac{3}{16}$*q.*
22. £46. 4*s.* $10\frac{1}{2}$*d.*
23. £275. 2*s.* 0*d.* $\frac{19}{32}$*q.*
24. £2395. 7*s.* $7\frac{1}{2}$*d.* $\frac{8}{11}$*q.*
25. £23264. 13*s.* $3\frac{1}{2}$*d.*
26. £8095. 4*s.* $2\frac{1}{4}$*d.*
27. £16. 18*s* 6·09*d.*
28. *Rs.*568. 5*a.* $10\frac{1}{2}$*p.*
29. *Rs.*1548. 1*a.* 3·765*p.*
30. £897. 6*s.* 6*d.*
31. *Rs.*185. 9*a.* $0\frac{4}{7}$*p.*
32. *Rs.*430. 10*a.*
33. £33. 8*s.* $3\frac{3}{4}$*d.* $\frac{181}{216}$*q.*
34. £44 10*s.* $5\frac{1}{2}$*d.* $\frac{13}{32}$*q.*
35. *Rs.*23. 11*a.* $10\frac{7}{32}$*p.*

Ex. CXXI. (pp. 303-304.)

1. *Rs.*2277. 13*a.* 6*p.* **2.** *Rs.*12973. 6*a.* 6*p.* **3.** £44 3*s.* $0\frac{19}{20}$*d*
4. £11870. 3*s.* $4\frac{1}{4}$*d.* ·97*q.* **5.** £836 16*s.* $4\frac{7}{8}$*d.* **6.** *Rs.*7. 11*a.* $5\frac{1}{4}$*p.*
7. *Rs.*26882. 0*a.* 6*p.* **8.** £1144. 0*s.* $11\frac{29}{80}$*d.* **9.** £117. 0*s.* 3*d.*
10. 53703 lbs. 18 dwts. 6 grs. **11.** 182 miles 3 fur. $143\frac{124}{225}$ yds.
12. *Rs.*448. 7*a.* **13.** 181 mds. 34 sr. 4 ch.
14. *Rs.*11237. 9*a.* 4*p.* **15.** *Rs.*1707. 1*a.*
16. *Rs.*6089. 9*a.* $5\frac{1}{4}$*p.* **17.** *Rs.*12659. 6*a.* $8\frac{1}{3}$*p.*
18. £428. 9*s.* $1\frac{13}{16}$*d.* **19.** *Rs* 470. **20.** 235 lbs. Tr. 4 oz. $19\frac{1}{16}$ grs.

Ex. CXXII. (p. 306.)

1. *Rs.*23. 0*a.* 6*p.* **2.** *Rs.*9. 4*a.* $6\frac{1}{2}$*p.* **3.** *Rs.*22. 10*a* 2*p.*
4. £8. 19*s.* $11\frac{3}{4}$*d.* **5.** *Rs.*293. 4*a.* 4*p.* **6.** *Rs.*197. 6*a.* $5\frac{1}{22}$*p.*
7. *Rs* 850. 5*a.* 8*p.* **8.** *Rs.*274. 7*a.* 8*p.* **9.** *Rs.*40. 0*a.* 11*p.*
10. *Rs.*41. 7*a.* 4*p.*

Ex. CXXIII. (p. 309.)

1. (1) 29791. (2) 168210432. (3) 855625. (4) 31255·875
(5) 38385223. (6) $21\frac{204}{2401}$. (7) 100875809. (8) $14\cdot88559\dot{3}$.
(9) 124618464. (10) 355999·6... (11) 53·841087.

2. (1) 12321. (2) 388129 ; 38812900.

3. (1) 7 ; 14 ; 17 ; 19 ; 18 ; 16 ; 11 ; 20 ; 12.
(2) 25 ; 23 ; 30 ; 36 ; 132 ; 252 ; 315.
(3) 825 ; 6930 ; 3528 ; 1575 ; 7560

4. 7 ; 77 ; 91 ; 385 ; 6 ; 14 ; 3157 ; 30030. **5.** 900.

Ex. CXXIV. (p. 312.)

1. (1) 26 ; 38 ; 127 ; 145 ; 537 ; 999 ; 267.
(2) 45 ; 832 ; 234 ; 907 ; 9878 ; 3163.
(3) 5746 ; 6772 ; 2403 ; 7925 ; 2005.
(4) 80700 ; 76008 ; 309000 ; 62573.
(5) 90888 ; 9688669 ; 887145333.

2. (1) 4·7 ; 5·8 ; 3·29 ; 4·78 ; 13·69 ; ·359.
(2) ·08 ; ·073 ; ·0231 ; ·0195 ; 60·49.

(3) 34·12 ; 370·09 ; 490·07 ; 15·367.
(4) 203·975 ; ·0708069 ; ·0007008.

3. 19. 4. 144. 5. 210. 6. 124.

Ex. CXXV. (pp. 313-314.)

1. (1) 1·4142 ; 1·7320 ; 2·2360 ; 2·4494 ; 2·6457 ; 2·8284 ; 3·4641 ; 3·6055 ; 4·2426 ; 4·4721 ; 5·6568 ; 6·1644.
(2) 6·6332 ; 7·1414 ; 8·4852 ; 8·9442 ; 9·7467 ; 25·2586 ; 28·2134 ; 28·3019 ; 31·6227.
(3) 75·5843 ; 19·0525 ; 187·4033 ; 94·0053 ; 906·9983 ; 8513·0157 ; 86·2090.

2. (1) ·3162 ; ·4472 ; ·5477 ; ·6324 ; ·7071 ; ·7745 ; ·8366 ; ·8944 ; ·9486 ; 1·0954 ; 1·2649.
(2) ·2236 ; 2·2583 ; 2·2135 ; ·4 ; ·1264 ; ·1 ; ·7141 ; ·2258 ; 2·0074.
(3) ·0447 ; ·4743 ; ·1449 ; ·5773 ; 2·1343 ; 5·9033 ; 17·9368 ; 4·$\dot{3}$.
(4) 1·7724 ; 13·2382 ; 3·5449 ; 5·4233 ; 88·8516.
(5) 1·5367 ; 2·2390 ; 2·0074 ; 7·0003 ; ·2659 ; 7·5878 ; ·1410.

3. (1) ·0415692193 ; 3·1288975694 ; ·2613426869 ; ·5888123640 ; 6·6546224536 ; 1·2529964086 ; 8·6802649729.
(2) ·9219544457 ; ·2645751311 ; 1·7320508075 ; 9·8944428848 ; ·0059430631 ; 5·2700094876 ; ·1910497317.

Ex. CXXVI. (p. 315.)

1. (1) $\frac{2}{5}$; $\frac{13}{16}$; $\frac{19}{87}$; $\frac{29}{82}$; $\frac{48}{59}$; $\frac{7}{8}$; $\frac{189}{126}$.
(2) $2\frac{1}{6}$; $3\frac{2}{7}$; $18\frac{3}{8}$; $5\frac{5}{7}$; $5\frac{2}{3}$; $6\frac{7}{15}$; $23\frac{13}{17}$.
(3) $55\frac{8}{15}$; $268\frac{1}{3}$; $175\frac{8}{17}$; $122\frac{8}{11}$; $393\frac{5}{13}$.

2. (1) ·7905 ; 3·0822 ; 2·5298 ; ·32 ; ·2625 ; 8·7649 ; 20·4939 ; ·0029.
(2) ·7745 ; 4·6612 ; 2·0586 ; ·9607 ; ·9701 ; 4·6150 ; ·3717 ; 5·2164.
(3) ·6060 ; ·5678 ; ·3118 ; ·2373 ; 16·9595 ; ·0574 ; 19·1646 ; 27·5$\frac{1}{12}$.

3. (1) ·$\dot{3}$; ·1$\dot{6}$; 4·$\dot{3}$; 1·8$\dot{3}$; 5·$\dot{3}$; ·01$\dot{6}$; 68·8$\dot{3}$.
(2) ·03$\dot{6}$; ·2$\dot{6}$; 7·$\dot{3}$; ·0$\dot{6}$; 2·$\dot{3}$; ·1$\dot{3}$; ·$\dot{2}$.

Ex. CXXVII. (pp. 317-318.)

1. (1) 11 ; 25 ; 36 ; 13 ; 57 ; 49. (2) 89 ; 97 ; 321 ; 247 ; 473.
(3) 956 ; 5836 ; 8888. (4) 2·6 ; 5·1 ; 7·9 ; 40·1 ; 2·65.
(5) ·197 ; ·957 ; ·101 ; ·0299. (6) $\frac{4}{7}$; $\frac{9}{64}$; $3\frac{3}{5}$; $19\frac{5}{8}$; $7\frac{3}{4}$; $3\frac{6}{7}$.
(7) 10·1120 ; 1·8081 ; 1·7099 ; ·4286 ; 1·5135.
(8) ·7539 ; ·8505 ; ·8939 ; 1·9813 ; 2·6487 ; 2·6536.
(9) ·12599210 ; ·14422495 ; ·23513346 ; ·28844991 ; 1·29802461.

2. (1) ·5$\dot{3}$; ·425 ; 5·$\dot{3}$; ·425.
(2) 15·$\dot{6}$; ·$\dot{3}$; 11·$\dot{6}$; ·9283 ; ·6463 ; ·6876 ; 4·3411.

3. 37 ft. 4. 6·16 ; 1·232.

Ex. CXXVIII. (p. 319.)

1. 18 ; 35 ; 45·6. **2.** 12 ; 1·7 ; 5·1. **3.** 11 ; 3·47032 ; ·49374 ; 1·65970. **4.** 4 ; ·8 ; 12. **5.** $2\frac{1}{7}$; 2·1 ; 799.

Miscellaneous Examples V. (pp. 319-322.)

1. $5 \cdot 3^2 \cdot 11^2 \cdot 7^2 \cdot 13^2$; 5. **2.** 8*p*. **3.** 100000. **4.** $\frac{7}{1860}$.
5. *Rs*.603. 13*a*. 9*p*. **6.** 3759. **7.** The second ; $\cdot 0708\dot{3}$.
8. $\frac{2}{3}$. **9.** *Rs*.51738. 3*a*. 3*p*. **10.** £6. 16*s*. 5*d*. **11.** 1.
13. Wife *Rs*.14508 ; eldest son *Rs*.12090 ; youngest *Rs*.12896 ; daughter *Rs*.9672. **14.** £213. 12*s*.
15. 63 ac. ; *A* 90 days ; *B* 63 days ; *C* 70 days.
16. (1) 6·6140... (2) ·2752057... **17.** $2333283\frac{1}{3}$ francs.
18. 1055. **19.** *Rs*.5888. **20.** 35 measures.
21. (1) $5 \cdot 9902\dot{7}$. (2) 0. (3) 7·7. **22.** *Rs* 4. **23.** *Rs*.5. 11*a*.
24. $6\frac{1}{4}$; ·0505. **25.** 105570 and 950370. **26.** 1·5795.
27. 3*a*. 6·24*p*. **28.** 80. **29.** 151249·7. **30.** $2 \cdot \dot{5}7142\dot{8}$.
31. £2. 15*s*. $2\frac{3}{4}$*d*. **32.** The first is greatest ; second is least.
33. $128\frac{84}{97}$ years. **34.** $\frac{1}{25}$. **35.** 645 ; 151 ; 567.
36. $\cdot 00\dot{3}0769\dot{2}$. **37.** 1 qr. 13 lbs. to be subtracted.
38. 9975 and 9925 ; 10000 and 10015. **39.** $7575\frac{3}{4}$ hrs **40.** $\frac{1}{3}$.
41. *Rs*.42. 14*a*. $0\frac{69}{320}$*p*. **42.** $1\frac{96}{121}$; $7\frac{17761}{108800}$. **43.** 420 rix-dollars.
44. $11\frac{513}{625} = 11 \cdot 8208$. **45.** 8·175 ; $\cdot 81\dot{6}$; 27 ; ·75 ; 135·1940625.
46. 300 miles. **47.** *Rs*.258. 9*a*. $7\frac{7}{8}$*p*. ; *Rs*.86. 12*a*. 8*p*.
48. £55. 6*s*. $10\frac{1}{2}$*d*. **49.** ·0030422... **50.** $\frac{5}{27}$. **51.** $\frac{752}{1225}$.
52. $1\frac{1}{2}$*d*. ; 160 oz. **53.** 1·000049... ; 266·6. **54.** 67 ft. ; 125.

Ex. CXXIX. (pp. 326-329.)

1. (1) 47 sq. yds. 2 ft. (2) 52 sq. yds. (3) 25 sq. yds. 8 ft. 48 in.
(4) 35 sq. yds. 7 ft. 32 in. (5) 11 sq. yds. 3 ft. 129 in.
(6) 42 sq. yds. 1 ft. 50 in. (7) 24 sq. yds. 4 ft. 54 in.
(8) 36 sq. yds. 2 ft. $115\frac{1}{2}$ in.
2. (1) 683 sq. yds. 2. ft. 25 in. (2) 10 sq. yds. 5 ft. 90 in.
(3) 43 sq. yds. 5 ft. 128 in. (4) 16 sq. yds. 6 ft. 27 in.
3. (1) 2 yds. 1 ft. 5 in. (2) 11 ft. 11 in. (3) 2 ft. 9 in.
(4) 13 ft. 1 in. (5) 7 yds. 2 ft.
4. 55 sq. yds. 7 ft. 73 in. **5.** 96 sq. ft. 93 in.
6. 7 ft. 10 in. ; 15 yds. 1 ft. 8 in. ; 15 ft. 5 in. **7.** 2 ft. $6\frac{3}{4}$ in.
8. 2 ac. 4 po. **9.** 5760 sods. **10.** 84. **11.** 29400.
12. $72\frac{3}{37}$. **13.** 1707. **14.** 142 sq. yds. 90 in. **15.** 759.
16. 85 sq. ft. 56 in. **17.** 3 chs. 50 lks. off the length. **18.** $16\frac{1}{2}$ yds.
19. 131 ft. **20.** 6160 acres. **21.** (1) *Rs*.7468. 12*a*.
(2) *Rs*.678. 5*a*. $4\frac{6}{8}$*p*. (3) *Rs*.68. 10*a*. $8\frac{1}{11}$*p*. (4) £451. 14*s*. $4\frac{1}{2}$*d*.
22. (1) $52\frac{1}{2}$ yds. (2) 142 yds. 1 ft. $11\frac{1}{3}$ in. (3) 26 yds. 4 in.
23. (1) *Rs*.114. 13*a*. 4*p*. (2) *Rs*.228. 9*a*. 6*p*. (3) *Rs*.130.
(4) £21. **24.** 48 ft. **25.** 14 ft. 3 in. **26.** 20 ft.

27. *Rs.*2. 12*a*. 28. 2 ft. 29. *Rs.*135. 30. 15 ft. 6 in.
31. $1406\frac{1}{4}$ sq. ft. 32. 195 sq. yds. 33. 14 yds. 1 ft. 34. 1310.
35. 41 yds. ; $20\frac{1}{2}$ yds. 36. *Rs.*1050.

Ex. CXXX. (pp. 331-333.)

1. (1) 875 sq. ft. 126 sq. in. (2) $605\frac{1}{2}$ sq. ft. (3) 798 sq. ft.
(4) 791 sq. ft. 132 sq. in.
2. (1) 250 yds. (2) 248 yds. $2\frac{9}{11}$ ft. (3) 648 ft. (4) 115 yds.
3. (1) *Rs.*28. 6*a*. (2) £13. 19*s*. $10\frac{1}{3}$*d*.
(3) *Re.*1. 9*a*. (4) £3. 10*s*.
4. (1) *Rs.*76. 2*a*. 4*p*. (2) *Rs.*61. 15*a*. 8*p*. (3) £9. 9*s*. $8\frac{7}{18}$*d*.
5. *Rs.*47. 1*a*. 4*p*. 6. *Rs.*294. 14*a*. $2\frac{101}{108}$*p*. 7. £8. 15*s*.
8. £5. 10*s*. 9. 113400. 10. *Rs.*12. 6*a*. 4*p*.
11. 37 sq yds. 5 ft. 12. 8 ft. $10\frac{2}{3}$ in. 13. 11 ft. $6\frac{1}{5}$ in.
14. 10 ft. 15. 16 ft. 16. Height 10 ft. ; breadth 12 ft.
17. *Rs* 37. 8*a*. 18. 77824.
19. 13 ft. 6 in. 20. 12 ft. 6 in. 21. *Rs.*125.
22. $20\frac{1}{2}$ ft. ; *Rs.*42. 11*a*. 4*p*. 23. 15 ft.

Ex. CXXXI. (pp. 335-336)

1. 6 ac. 3 ro. 30 po. $22\frac{1}{2}$ sq. yds. 2. 1913 sq. yds. 3 ft.
3. *Rs.*125. 8*a*. 4*p*. 4. 60750. 5. *Rs.*312 12*a*. 6. *Rs.*10464.
7. 300 stones. 8. £117. 15*s*. 9. £176. 17*s*. $9\frac{9}{28}$*d*.
10. 352 or 360. 11. $34\frac{3}{4}$ sq. ft. 12. *Rs.*1965.
13. 55 sq. ft. 80 in. ; *Rs.*9. 4*a*. $1\frac{7}{9}$*p*. 14. 8*a*. 15. *Rs.*274. 4*a*. $5\frac{1}{3}$*p*.
16. $378\frac{3}{16}$ sq. cub. ; 75 mds. 25 sr. 8 ch. ; *Rs.*374. 3*a*.

Ex. CXXXII. (pp. 339-342.)

1. (1) 4 cub. yds. 8 ft. 648 in. (2) 38 cub. yds. 22 ft. 592 in.
(3) 59 cub. yds. 19 ft. 764 in. (4) 260 cub. yds. 4 ft. 1053 in.
(5) 69 cub. yds. 16 ft. 1026 in. 2. 2552 cub ft. 1088 in.
3. *Rs.*246. 15*a*. 3*p*. 4. 2 yds. 5. $\frac{209}{318}$ in.
6. 123 ft. $11\frac{1172}{1185}$ in. 7. 1 ft. 6 in. 8. 5044 9. 2 ft. 6 in.
10. 11 ft. $6\frac{5}{8}$ in. 11. 5·07 ft. 12. 29 cub. yds. 17 ft.
13. 1 ton 3 cwt. 1 qr. $0\frac{1}{6}$ lb. 14. 2304.
15. £4. 1*s*. $10\frac{3}{4}$*d*. $\frac{1}{2}$*q*. 16. 28 tons 9 cwt. 1 qr. 11 lbs. $14\frac{4}{5}$ oz.
17. 3 ft. 6 in. 18. 16 sr. $11\frac{13}{33}$ ch.
19. 115 cub. ft. 30 in. 20. 1950. 21. $101\frac{127}{448}$ tons,
22. 32 ft. 23. 114. 24. $4\frac{73}{378}$ yds. 25. *Rs.*276 5*a*. 3*p*. ; 31440.
26. 34 yds. ; *Rs.*3984. 6*a*. 27. Depth $1\frac{1}{2}$ ft. ; breadth 3 ft. length 6 ft. ; *Rs.*55. 2*a*.
28. *Rs.*2. 6*a*. $11\frac{2}{9}$*p*.
29 2 tons 1 cwt. 2 qrs. $14\frac{4982}{5978}$ lbs. 30. *Rs.*389. 9*a*. 4*p*.
31. 2 ft. 3 in. 32. ·0000459...in. 33. $\frac{6}{7}$ mile per min.
34. $1\frac{7}{11}$ miles. 35. 22 min. 24 sec. 36. *Rs.*41. 2*a*. 4*p*.
37. 30 cub. ft. 1404 in. ; 33 mds. 28 sr. 8 ch. 38. 135 times.
39. 11 mds. $26\frac{2}{3}$ sr. ; *Rs.*64. 2*a*. 8*p*. 40. $277\frac{1}{3}$ cub. in.
41. 4 sq. in. 42. 8 yds. ; 3 hrs.

Ex. CXXXIII. (p. 344.)

1. (1) 5 yds. 2 ft. $3\frac{1}{3}$ in. ; 8 yds. 1 ft. $11\frac{5}{8}$ in. ; 48 yds. 2 ft. $6\frac{12}{78}$ in.
   (2) 2 sq. yds. $138\frac{2}{3}$ in. ; 24 sq. yds. $72\frac{7}{8}$ in. ; 30sq. yds. 4ft. $58\frac{1}{18}$ in.
   (3) 20 cub. ft. 1377 in.; 11cub. ft. 675 in.; 3cub. yds. 2ft $934\frac{12}{18}$ in.
   (4) 8yds. 1ft. $3\frac{3}{4}$in. ; 2sq. yds. 6ft. 135 in, ; 86cub. yds. 19ft. $800\frac{2}{3}$ in.

2. (1) 13 ft. 5′. 3″ ; 59 ft. 6′. 6″. 8‴ ; 22 ft. 6′. 8″ ; 29 ft. 4′. 9″.
   (2) 50 sq. ft. 5′. 8″ ; 47 sq. ft. 5′. 3″ ; 300 sq. ft. 2′. 4″. 3‴.
   (3) 4 cub. ft. 7′. 6″. 8‴ ; 42 cub. ft. 2′. 3″. 10″ ; 39 cub. ft. 7′. 9″. 4‴. 6^{iv} ; 18 cub. ft. 11′. 6″. 8‴.

Ex. CXXXIV. (pp. 345-346.)

1. (1) 182 sq. ft. 66 in. (2) 200 sq. ft. 58 in. (3) 212 sq. ft. 7 in.
   (4) 27 sq. ft. 1′. 8″. 8‴. (5) 51 sq ft. 10′. 4″. 0‴. 4^{iv}.
   (6) 336 sq. ft. 9′. 6″. 8‴. 8^{iv}. (7) 154 sq. yds. 6 ft. 7′. 6″. 4‴. 6^{iv}.
   (8) 2203 sq. yds. 2 ft. 8′. 2″. 2‴. 9^{iv}. (9) 216 sq. ft 6′. 0″. 10‴. 6^{iv}.
   (10) 147 sq yds. 6 ft. 2′. 11″. 6‴. 10^{iv}. (11) 4′. 3″. 9‴. 9^{v}. 8^{v}.
   (12) 2 sq. ft. 4‴. 7^{iv}.

2. (1) 170 cub. ft. (2) 59 cub. ft. 2′. 8″. 4‴. (3) 167 cub ft. 6′.
   (4) 180 cub. ft. 9′. 2″. (5) 132 cub. yds. 4 ft. 10′. 4″. 9‴. 6^{iv}.

3. (1) 5 sq. yds. 8. ft. $82\frac{25}{36}$ in. (2) 1696 sq. yds. 6 ft. $38\frac{17}{84}$ in.

4. (1) 42 cub. ft. 1512 in. (2) 56 cub. yds. 22ft $1196\frac{1687}{1728}$ in.
   (3) 44 cub. yds. 16 ft. $298\frac{17}{27}$ in. **5.** 18 ft. 9. in.

Ex. CXXXV. (pp. 346-347.)

1. (1) 6 bi. (2) 4 bi. (3) 1 bi. 5 kat. 9 ch. 12 ga.
   (4) 32 bi. 17 kat. 6 ch. 8 ga. (5) 36 bi. 6 kat. 1 ch. 12 ga.
   (6) 136 bi. 14 kat. 12 ch. $16\frac{1}{4}$ ga. (7) 71 bi. 18 kat. 12 ch. 16 ga.
   (8) 55 bi. 2 kat. 6 ch. 8 ga. (9) 136 bi. 10 kat. 15 ch. $2\frac{1}{2}$ ga
   (10) 109 bi. 6 kat. 10 ch. $19\frac{3}{4}$ ga. (11) 15 bi. 8 kat. 12 ch.
   (12) 28 bi. 1 kat. 1 ch. 12 ga.

2. (1) 8046 c. cubits. (2) 8736 c. cubits. (3) 163680 c. cubits.
   (4) 26970 c. cubits. (5) 2592 c. yds. (6) 2900 c yds.

Ex. CXXXVI. (pp. 349-350.)

1. *Rs.*7. 8*a.* **2.** 150. **3.** 11268. **4.** *Rs.*182. 0*a.* 8*p.* **5.** 660.
6. 75. **7.** 15 cwt. **8.** 4 days. **9.** 30. **10.** *Rs.*491. 4*a.*
11. 30. **12.** 100. **13.** *Re.*1. 8*a.* **14.** 3lbs. 11 oz. $7\frac{3}{4}$ drs.
15. *Rs.*505. **16.** 9 months. **17.** 6. **18.** 2 days.

Ex. CXXXVII. (pp. 351-352.)

1. Horse *Rs.*230 ; cow *Rs.*160. **2.** Tea *Re.*1. 5*a.* 4*p.* ; coffee 12*a.*
3. Turkey 15*s.* ; fowl 3*s.* 4*d.* **4.** $1\frac{1}{13}$ days. **5.** $33\frac{1}{3}$ days.
6. 3 days. **7.** 2 boys. **8.** Man $7\frac{1}{5}$ hrs. ; boy 18 hrs. ; together $5\frac{1}{7}$ hrs.
9. 35 days. **10.** Horse *Rs.*24 ; cow *Rs.*12.
11. Man 5*a.* 3*p.* ; woman 3*a.* 5*p.* **12.** 12.

Ex. CXXXVIII. (p. 353.)

1. 1263. **2.** 839. **3.** 903. **4.** 8398. **5.** 1078.
6. 119. **7.** 892430 ; 5390. **8.** 1662. **9.** 5.
10. 737 ; 17. **11.** 9999988. **12.** 10004390.

Ex. CXXXIX. (pp. 354-355.)

1. 39 yds. **2.** 25 yds. **3.** $39\frac{17}{22}$ yds. **4.** 4. ($4\frac{2}{7}$). **5.** $31\frac{24}{121}$ yds.
6. $106\frac{2}{3}$ yds. **7.** 9 points. **8.** 4min. 47sec. **9.** *A* 3 min. $37\frac{7}{11}$ sec. ; *B* 3 min. 48 sec. ; *C* 4 min. **10.** 350 yds. **11.** 10 points.
12. $5\frac{1}{6}$ min. **13.** *C*. **14.** 5 points. **15.** *C* by $\frac{1760}{8741}$ yd. **16.** $5\frac{1}{4}$ min.
17. *C* ; 40. **18.** *A* 7 min. $9\frac{6}{11}$ sec. ; *B* 9 min. $15\frac{16}{17}$ sec.

Ex. CXL. (pp. 357-358.)

[The numbers denote minutes past the hour.]

1. (1) (i) $5\frac{5}{11}$. (ii) $21\frac{9}{11}$; $54\frac{6}{11}$. (iii) $38\frac{2}{11}$.
(2) (i) $10\frac{10}{11}$. (ii) $27\frac{3}{11}$; 3 o'clock. (iii) $43\frac{7}{11}$.
(3) (i) $16\frac{4}{11}$. (ii) 3 o'clock ; $32\frac{8}{11}$. (iii) $49\frac{1}{11}$.
(4) (i) $21\frac{9}{11}$. (ii) $5\frac{5}{11}$; $38\frac{2}{11}$. (iii) $54\frac{6}{11}$.
(5) (i) $27\frac{3}{11}$. (ii) $10\frac{10}{11}$; $43\frac{7}{11}$. (iii) 6 o'clock.
(6) (i) $32\frac{8}{11}$. (ii) $16\frac{4}{11}$; $49\frac{1}{11}$. (iii) 6 o'clock.
(7) (i) $38\frac{2}{11}$. (ii) $21\frac{9}{11}$; $54\frac{6}{11}$. (iii) $5\frac{5}{11}$.
(8) (i) $43\frac{7}{11}$. (ii) $27\frac{3}{11}$; 9 o'clock. (iii) $10\frac{10}{11}$.
(9) (i) $49\frac{1}{11}$. (ii) 9 o'clock ; $32\frac{8}{11}$. (iii) $16\frac{4}{11}$.
(10) (i) $54\frac{6}{11}$. (ii) $5\frac{5}{11}$; $38\frac{2}{11}$. (iii) $21\frac{9}{11}$.
(11) (i) 12 o'clock. (ii) $10\frac{10}{11}$; $43\frac{7}{11}$. (iii) $27\frac{3}{11}$.
(12) (i) 12 o'clock. (ii) $16\frac{4}{11}$; $49\frac{1}{11}$. (iii) $32\frac{8}{11}$.

2. (1) (i) $5\frac{5}{11}$; $27\frac{3}{11}$. (ii) $43\frac{7}{11}$; $54\frac{6}{11}$. (2) (i) $10\frac{10}{11}$; $32\frac{8}{11}$.
(ii) $49\frac{1}{11}$; 5 o'clock. (3) (i) $21\frac{9}{11}$; $43\frac{7}{11}$. (ii) $5\frac{5}{11}$; 7 o'clock.
(4) (i) $27\frac{3}{11}$; $49\frac{1}{11}$. (ii) 7 o'clock ; $10\frac{10}{11}$.

3. 3 times. **4.** 3 times. **5.** 2 times. **6.** $20\frac{20}{23}$ min. past 8.
7. $36\frac{12}{13}$ min. past 4. **8.** $24\frac{5}{11}$ min. past 6. **9.** $18\frac{6}{13}$ min. past 8.
10. $10\frac{10}{143}$ min. gain. **11.** $\frac{1}{3}$ min. div. put back.
12. (i) $4\text{-}3\frac{188}{319}'$, or 4 $37\frac{187}{319}'$ P.M. (ii) $4\text{-}54\frac{114}{319}'$ P.M. (iii) $4\text{-}20\frac{160}{319}'$ P.M.
13. (i) $4\text{-}20\frac{52}{55}'$ P.M. (ii) $3\text{-}49\frac{29}{55}'$ P.M. (iii) $1\text{-}59\frac{31}{55}'$ P.M.
14. 9 hrs. 18 min. $22\frac{1502}{3949}$ sec. P.M. ; 4 min. $27\frac{1}{2}$ sec.

Ex. CXLI. (pp. 360-362.)

1. $887\frac{1}{2}$ days. **2.** 10 hrs. 40 min. $36\frac{7}{48}$ sec. **3.** 48 min. $7\frac{1}{2}$ sec.
4. 60 days ; 12 hrs. 14 min. ; 11 hrs. 44 min. **5.** $5\frac{715}{1201}$ min. before 4.
6. 5 hrs. 10 min. $20\frac{20}{29}$ sec. **7.** 11 o'clock P.M. **8.** 5 hrs. 10 min.
9. 35 min. $2\frac{662}{719}$ sec. past noon. **10.** 5 hrs. 48 min. **11.** 26 sec. loss.
12. 4 hrs. 32 min. **13.** $1\frac{3}{32}$ min. to 12. **14.** On Tuesday evening, when one clock marks 9 hrs. 11 min. and the other 8 hrs. 54 min. 30 sec. **15.** $\frac{1}{11}$ min. **16.** $13\frac{29}{31}$ min.
17. 7 o'clock. **18.** 1 hr. $28\frac{4}{11}$ min.

19. At 3 P. M. on Dec. 3. **20.** 1 min $52\frac{1}{2}$ sec. to 9 P. M
21. $40\frac{320}{721}$ min. past 8 P. M. **22.** Loses 29[illegible] sec.
23. $8\frac{2}{11}$ sec. **24.** 10 o'clock. **25.** At 11 P. M.
26. 1008 days. **27.** 12 days before ; 528 days.
28. 3 hrs. 24 min. $19\frac{17}{37}$ sec. **29.** May 21, at 6 P. M.
30. 1 day 16 hrs. from the time the second was put right.

Ex. CXLII. (p. 363.)

[The numbers denote degrees.]

1. (1) (i) $4\frac{4}{9}$. (ii) $5\frac{5}{9}$. (2) (i) 20. (ii) 16. (3) (i) $47\frac{7}{9}$. (ii) $38\frac{8}{9}$. (4) (i) $71\frac{1}{9}$. (ii) $56\frac{8}{9}$. (5) (i) $-33\frac{1}{3}$. (ii) $-26\frac{2}{3}$.
2. (1) (i) 68. (ii) 16. (2) (i) 113. (ii) 36. (3) (i) $136\frac{2}{3}$. (ii) $46\frac{2}{3}$. (4) (i) $197\frac{2}{3}$. (ii) $73\frac{2}{3}$. (5) (i) 14. (ii) -8.
3. (1) (i) $110\frac{3}{4}$. (ii) $43\frac{3}{4}$. (2) (i) 167. (ii) 75. (3) (i) 203. (ii) 95. (4) (i) $9\frac{1}{2}$ (ii) $-12\frac{1}{2}$. (5) (i) $-24\frac{1}{4}$. (ii) $-31\frac{1}{4}$.

Ex. CXLIII. (pp. 369-375.)

1. (i) 45 min. (ii) 4 hrs. **2.** $2\frac{1}{2}$ hrs. : $13\frac{1}{3}$ miles
3 At 1 A. M. ; 17 and 14 miles. **4.** $10\frac{4}{5}$ miles.
5. At 1 P. M. ; 210 miles. **6.** At 3 P. M.
7. At 12·35 P. M. ; $89\frac{7}{12}$ miles. **8.** $11\frac{7}{12}$ hrs. from *A*'s starting
9. $3\frac{9}{73}$ miles per hour. **10.** 6 30 A M. : $\frac{1}{4}$ mi **11.** *A* 7 miles, *B* 6 miles 352 yds. **12.** $11\frac{2}{3}$ hrs. ; $134\frac{1}{6}$ miles. **13.** $3\frac{3}{5}$ miles.
14. $11\frac{1}{7}$ hours. **15.** *B* 2 mi. 1540 yds. ; *C* 5 mi.
16. 1 mi. ; 40 yds. ; *B* 9 yds. ; 1 hr. 15 min. **17.** 4 hrs. $19\frac{1}{11}$ min
18. 90 miles. **19.** 58 min. **20.** 35 mi ; 25 mi. **21.** 5 hours.
22. $60\frac{5}{22}$ seconds ; 530 yds. **23.** 3 min. **24.** 19 miles.
25. 19 mi. ; 35 mi. **26.** $4\frac{4}{5}$ mi. **27.** 400 miles.
28. $\frac{11}{60}$ mile. **29.** $31\frac{1}{2}$ miles per hour. **30.** $2\frac{2}{5}$ hours
31. $2\frac{1}{2}$ hours. **32.** 3 and $1\frac{1}{2}$ miles per hour. **33** 16 hours.
34. 300. **35.** 600. **36.** 56 yds. **37.** $28\frac{9}{22}$ miles. **38.** $746\frac{2}{3}$ yds.
39. $9\frac{93}{220}$ miles. **40.** (i) $55\frac{1}{2}$ sec. (ii) $7\frac{1}{2}$ sec.
41. 15 hrs. 35 min. **42.** 9 hrs 48 min. **43.** $463\frac{3}{17}$ hrs.
44. 16. min. $32\frac{4}{13}$ sec. **45.** 9 sec. **46.** $22\frac{1}{2}$ miles per hour
47. $7\frac{5}{7}$ sec. **48.** 1 min. $7\frac{1}{2}$ sec. **49.** 50 and 30 miles per hour
50. 297 ft. ; 231 ft. **51.** 3 hrs. 1 min. 30 sec.
52. 15 mi. 180 yds. ; 14 yds. and 1 yd. per sec.
53. 22 mi. 280 yds ; 20 ft and 60 ft. per sec.
54. $17\frac{1}{2}$ miles. **55.** $4\frac{1}{2}$ miles per hour. **56.** 15 miles
57. 2 miles. **58.** 165 miles. **59.** (i) 1 sec. (ii 4 sec.
60. $13\frac{1}{3}$ miles. **61.** $2\frac{1}{5}$ ft. **62.** 2740 yds. ; $13\frac{48}{59}$ miles
63 25 and 30 miles per hour. **64.** $79\frac{1}{2}$ miles

Miscellaneous Examples VI. (pp. 378-385.)

1. 66150140. **2.** *Rs.*628. 0*a.* $10\frac{2}{3}$*p.* **3.** $87\frac{1}{40}$ yds.
4. £365. **5.** *Rs.*182. 4*a.* **6.** 25 sec.

7. 8100 · *Rs.*50. 12*a* $9\frac{1}{3}$*p*. **8** $\frac{1}{8064}$ in.
9. £286492500. **10.** *Rs.*2164 6*a.* $2\frac{1}{2}$*p*. **11.** 121·98.
12. $7\frac{23}{80}$ ft. **13.** 396 or 406. **14.** 64800.
15 18. **16.** 875. **17.** He walked up $6\frac{1}{2}$ miles.
18. 9 hrs $37\frac{8}{11}$ min. **19** 1109894 **20.** 1009805
21. *Re.*1. 2*a.* **22.** 45 miles **23.** *Rs.*3. **24.** 31116.
25. 3 times, or 4 times if we reckon the starting point.
26. 75 days. **27.** 240 men. **28** 942 **29** 1683 and 2431.
30. 60 pieces ; 5·6304 in. **31.** *Rs.*18. 10*a.* 8*p*. **32.** *Rs.*420.
33. $1\frac{1}{2}$ cwt. **34.** *Rs.*963. **35** 3 hrs. 30 min.
36. $\frac{1}{3}$ mile. **37.** ·000004 in. **38.** 2*s.* 4*d.* per stone.
39. 36 and 24 miles per hour. **40.** 60.
41. 20 miles per hour. **42.** *A Rs.*12 ; *B Rs.*9.
43. $3\frac{3}{4}$ min. to 6 A. M. ; 6 o'clock A. M. **44.** 171.
45. 25′.26″·$\dot{6}$ to be subtracted. **46.** 960 seers.
47. $9\frac{887}{1207}$ days. **48.** 53 min. $30\frac{390}{601}$ sec. past 1.
49 9*a* $3\frac{1}{5}$*p*. **50** 567. **51.** 792 yds. or 1572 yds.
52. *A* 12 days ; *B* 15 days ; *C* 20 days. **53.** $27\frac{1}{2}$ yds.
54. 6 min. **55.** $2\frac{179}{210}$ hrs. **57.** At 4·25 P. M. **58** $2\frac{1}{2}$ miles.
59. 80 and 75. **60** 38, 40 and 45. **61.** *Rs.*458.
62. *A Rs* 2400 ; *B Rs.*900 ; *C Rs* 240 ; *D Rs.*60 **63.** 45 and 35.
64. 23 years. **65.** 14 beggars. **66.** 44 days.
67. 45 men. **68.** 5 yards. **69.** $2\frac{1}{5}$ miles.
70. 564 miles. **71.** It will be more advantageous to employ the boys. **72.** *Rs.*4. 6*a.* **73.** $4\frac{4}{5}$ miles per hour.
74. It loses $1\frac{140}{141}$ min. per day. **75.** 30 days.
76. After 8 days. **77.** At $23\frac{71}{148}$ min. past 8 ; $7\frac{12}{35}$ miles.
78. 48 centres, 31 outers. **79.** $611\frac{1}{9}$ hrs. **80.** 10 ft. ; *Rs.*296.
81. *Rs.*446. 10*a.* : $\frac{1}{9}$ ft. **82.** 124 yds.

Ex CXLIV. (pp. 387-388.)

1. (1) $\frac{3}{5}$; $\frac{1}{3}$; $\frac{3}{7}$; $\frac{4}{3}$; $\frac{3}{4}$; $\frac{5}{8}$. (2) $\frac{1}{5}$; $\frac{5}{1}$; $\frac{1}{12}$; $\frac{4}{15}$; $\frac{7}{4}$; $\frac{8}{5}$.
(3) $\frac{8}{1}$; $\frac{26}{31}$; $\frac{85}{24}$; $\frac{14}{15}$; $\frac{2500}{1}$. (4) $\frac{23}{1}$; $\frac{1}{23}$; $\frac{1}{4}$. (5) $\frac{160}{47}$; $\frac{237}{320}$; $\frac{4}{7}$.
(6) $\frac{9}{2}$; $\frac{1501}{2160}$; $\frac{7}{80}$.
2. (1) 7 : 11 ; 17 : 23 ; 34 : 27. (2) 17 : 18 ; Equal : $1\frac{1}{2}$: $\frac{5}{12}$.
(3) 17 : 18 ; $\frac{2}{5}$: $\frac{1}{2}$: $\frac{4}{5}$: $2\frac{1}{2}$.
3. (1) 8 : 9 ; $2\frac{4}{7}$: $3\frac{1}{12}$. (2) ·75 : ·96 ; 4 : 7.
(3) 3 gal. 3 qts. : 42 pts.
4. The first and second are increased and the third is diminished.
5. The first is increased and the second and third are diminished.
6. (1) 4 : 35 ; 1 : 7. (2) 45 : 364 ; 100 : 483. (3) 9 : 50 ; 10 : 21.
7. $10\frac{2}{3}$. **8.** 5·2. **9.** 2 ro. : 13 po. **10.** 25 : 39.
11. 91 : 81. **12.** 16 : 24 : 30 : 35.

Ex. CXLV. (pp. 391-392.)

1. (1) 36. (2) $509\frac{1}{11}$. (3) 57. (4) $22\frac{1}{2}$. (5) 22. (6) 5·15.
(7) 26 208. (8) *Rs.*116. 10*a.* 8*p.* (9) *Rs.*405.
(10) 11 tons 15 cwt. 2 qrs. (11) 40 tons 10 cwt. (12) 17*s.* $0\frac{3}{4}d.$
2. 63 : $31\frac{1}{5}$. 3. 50 ; $74\frac{3}{5}$.
4. The 3rd term is $\frac{3}{680}$; the second term is ·0000406.
5. (1) 8. (2) 25. (3) ·171. (4) $\frac{5}{27}$. (5) 6. (6) ·01.
6. (1) 72. (2) $2\frac{5}{56}$. (3) $\frac{8}{9}$. (4) 1·3. (5) $8\frac{1}{4}$. (6) ·05.
7. (1) 6400. (2) $\frac{15}{49}$. 8. 70 : 105 : 126 : 144.
9. 1 : 3. 10. 2 : 15. 11. $1\frac{1}{3}$ gals. 12. 217 : 865.
13. 9 : 8. 14. 19 : 17. 15. 739 : 257.

Ex. CXLVI. (pp. 397-401.)

1. *Rs.* 1059. 6*a.* 2. 60 days. 3. £189. 3*s.* $11\frac{1}{4}d.$
4. 25 days. 5. *Rs.*56. 4*a.* 6. 8 lbs. 3 oz.
7. 162 mds. 8. *Rs.*10. 11*a.* 4*p.* 9. *Rs.*263 4*a.*
10. £61. 9*s.* 1*d.* 11. £27. 16*s.* $3\frac{1}{2}d.$ 12. 26 hours.
13. *Rs.*516. 4*a.* 14. £1590. 0*s.* $2\frac{1}{4}d$ 15. 25 mds. 29 sr.
16. *Rs.*857. 8*a.* 17. £118. 13*s.* 6*d.* 18 2 tons 3 cwt. 3 qrs.
19. *Rs.*17184. 15*a.* $2\frac{2}{3}p.$ 20. *Rs.*101. 5*a.* 8*p.* 21 365 days.
22. 15 days. 23. 75 ft. 6 in. 24. 6 yards.
25. £126. 12*s.* 10*d.* 26. *Rs.*15. 8*a.* 27. 8 months.
28. *Rs.*4. 12*a.* 29. 2*a* $9\frac{3}{5}p.$ 30. *Rs.*1084
31. *Rs.*8707. 8*a.* 32. £23. 3*s.* $3\frac{3}{13}d.$ 33. 54756.
34. £257. 10*s.* 35. 9*a.* $7\frac{1}{5}p.$ 36. 65 hours.
37. *Rs.*10500. 38. *Rs* 7. 14*a.* 6*p.* 39. *Rs.* 11452. 13*a.*
40. *Rs.*3650. 41. *Rs.* 14000 ; $2\frac{4}{7}p.$ 42. 20 gals.
43. £1386. 44. £2185. 16*s.* 3*d.*
45. He will lose *Rs.*10. 6*a.* 8*p.* 46. $74\frac{11}{12}$ days.
47. *Rs.*236. 15*a.* 8*p.* ; 16 yds. 48. *Rs.*50.
49. £61. 12*s.* 50. £3. 7*s.* 10*d.* 51. 12*a.* 7·2*p.*
52. 9*a.* 53. $25\frac{2}{3}$ days. 54. 8min. $45\frac{21}{23}$sec. past 8.
55. $47\frac{1}{17}$ days. 56. 89 ft. $2\frac{8}{11}$ in. 57. 28.
58. 12800. 59. March 8, Sunday at 4 P. M.
60. £1. 10*s.* 61. *Rs.*17. 14*a.* 62. *Rs.*535. 12*a.* 6*p.*
63. 10 days ; $12\frac{6}{7}$ days. 64. 9 min.
65. 101 days. 66. $124\frac{36}{61}$ days. 67. 6 months.
68. $2\frac{2}{5}$ ch. 69. 150 leaps. 70. 8 ch. 2 to.
71. 368 men. 72. Brandy 18*s.* ; rum 14*s.* ; gin 10*s.* 6*d.*
73. *Rs.*68. 4*a.* $11\frac{17}{565}p.$ 74 4 miles.
75. 21 days. 76. *Rs.*87. 7*a.* $8\frac{92}{623}p.$ 77. $5\frac{2}{3}$ days.
78. 48 men. 79. £1306. 10*s.* 80. $202\frac{1}{2}$ miles.

Ex. CXLVII. (pp. 404-409.)

1. 6 days. 2. *Rs.*282. 14*a.* 3. 1878 men. 4. 5 months.
5. 63 tons. 6. 174 miles. 7. 974 mds. $38\frac{2}{513}$ sr. 8. 1*s.* 6*d.*
9. £4. 0*s.* 10*d.* 10. 7 men. 11. 2250 men. 12. 9 days.

13. *Rs.*564. 2*a.* 8*p.* **14.** 26 lbs. $0\frac{1}{4}$ oz. **15.** 300 men. **16.** 540 guns.
17. 48 days. **18.** 300 men. **19.** 156 men. **20.** 17 cwt. 2 qrs. $9\frac{3}{8}$ lbs.
21. 19·36 days. **22.** 9*s.* $0\frac{3}{4}d.$ $\frac{1}{5}q.$ **23.** $25\frac{5}{7}$ days. **24.** $67\frac{107}{885}$ days.
25. 24 times. **26.** 9 days. **27.** 32 days. **28.** $16\frac{2}{3}$ hours.
29. $6\frac{2}{3}$ oz. **30.** 14 women. **31.** 18 men. **32.** $12\frac{8}{9}$ hours.
33. 542 bi. 11 kat. 12 ch. **34.** *Rs.*392. 4*a.* 8*p.* **35.** 45 men.
36. *Rs.*168. 12*a.* **37.** $7\frac{1}{9}$ hours. **38.** 125 men. **39.** 27 days.
40. 180 men. **41.** $2\frac{1}{2}$ ft. **42.** 15 men. **43.** 100 men.
44. $1515\frac{3}{7}$ bricks. **45.** 1250 men. **46.** 6·48 miles. **47.** 4 dozen.
48. $222\frac{3}{4}$ days. **49.** *Rs.*330. 12*a.* **50.** *Rs.*5096. **51.** 84 men.
52. 4 adults. **53.** *Rs.*39. 1*a.* **54.** $39\frac{3}{8}$ weeks. **55.** $16\frac{1}{8}$ ft.

Ex. CXLVIII. (pp. 412-414.)

1. $4\frac{3}{4}d.$ **2.** $19\frac{13}{17}$ days. **3.** *Rs.*4200. **4.** 10 days.
5. 14 men. **6.** £3. 5*s.* $6\frac{1}{2}d.$ $\frac{1}{2}q.$ **7.** 3 lbs. 11 oz. $7\frac{3}{7}$ drs.
8. 174 miles. **9.** 178 mds. 20 sr. **10.** *Rs.*204. 12*a.* **11.** 240 men.
12. 11 months. **13.** 13·4*s.* **14.** 229 yds. 1 ft. **15** 7 hours.
16. $1\frac{4}{13}$ hours. **17.** 20 cannon. **18.** 180 men. **19.** 5 : 2 ; $4\frac{5}{6}$ ac.
20. 1*s.* $7\frac{3}{4}d.$ $\frac{27}{47}q.$ **21.** 40000 men. **22.** 6 men. **23.** $131\frac{1}{4}$ days.
24. 120 days. **25.** $29\frac{1}{2}$ days.

Ex. CXLIX. (pp. 415-417.)

1. (1) $\frac{1}{8}$. (2) $\frac{3}{20}$. (3) $\frac{3}{400}$. (4) $\frac{61}{80}$. (5) $1\frac{3}{10}$. (6) $\frac{1}{6}$.
2. (1) 21·6. (2) *Rs.*18. (3) 58 mds. 20 sr. (4) *Rs.*109. 6*a.* 4·8*p.*
(5) 187 bi. 17 kat. 8 ch. (6) 171 yds. (7) *Rs.*450. 6*a.* 3*p.*
(8) 2 hrs. 12 min. 36 sec.
3. (1) $28\frac{4}{7}$. (2) $49\frac{1}{3}$. (3) 8·985375. (4) $26\cdot\dot{5}\dot{1}$. (5) 2·530....
(6) 9·5. (7) 103. (8) $11\frac{1}{3}$ (9) $5\frac{65}{87}$.
4. (1) $37\frac{1}{2}$. (2) $16\frac{2}{3}$. (3) 460. (4) 125. (5) $366\frac{2}{3}$. (6) 590.
5. 20 p. c. **6.** 25 p. c. **7.** 43797. **8.** 75844. **9.** 38896200.
10 58000. **11.** $33\frac{1}{3}$ p. c. **12** *Rs.*2607. **13.** 91·2 p. c. **14.** 600.
15. 640. **16.** 3608. **17** 120 ; 156 ; 174 ; 150.
18. $66\frac{2}{3}$ gold ; $33\frac{1}{3}$ silver. **19.** *Rs.*3379. 11*a* $3\frac{8}{51}p.$ **20.** *Rs.*2500.
21. *Rs.*15000. **22.** *Rs.*1500. **23.** $\frac{5}{7}$ p. c. gain. **24.** *Rs.*18000. **25.** 200.

Ex. CL. (pp. 418-419.)

1. (1) 27. (2) $738\cdot\dot{5}7142\dot{8}$. (3) $13\frac{67}{72}$ (4) 7·099794.
(5) 26·9625. (6) 10·154875.
2. 18·552...yrs. **3.** *Rs.*36. 5*a.* $2\frac{6}{7}p.$ **4.** 10 st. $10\frac{1}{2}$ lbs. ; 9 st. $6\frac{1}{2}$ lbs.
5. 1541. **6.** 25311·84. **7.** 8·667...years.
8. $60\frac{1}{4}$ years. **9.** 60. **10.** $14\cdot6\dot{3}$ yrs **11.** 13 st. 2 lbs.
12. 51°. **13.** *Rs.*30. **14.** 78. **15.** 13 yrs. **16.** $21\frac{3}{4}$.

Ex. CLI. (pp. 420-424.)

1. 12 per cent. **2.** *Rs.*656. 8*a.* **3.** *Re.*1. 10*a.* 10*p.*
4. *Rs.*275. **5.** 7*s.* **6.** *Re.*1. 9*a.* $10\frac{10}{231}p.$ **7.** *Rs.*850.

8. 4*s.* $1\frac{1}{2}$*d.* **9.** *Rs.*9 9*a.* 4*p.* **10.** *Rs.*9. 2*a.* 8*p.*
11. 40 per cent. **12.** Gain, $7\frac{1}{2}$ p. c. **13.** $42\frac{6}{7}$ p. c.
14. *Rs* 18. 4*a.* **15.** $62\frac{1}{2}$ p. c. **16.** $96\frac{4}{11}$ p. c.
17. *Rs.*958. 5*a.* 4*p.* **18.** *Rs.*270. **19.** *Rs.*7. 5*a.* 4*p.*; 60 p. c.
20. *Re.*1. 10*a.* 2*p.* **21.** 5 p. c. **22.** Loss, 65 p. c.
23 $30\frac{5}{81}$ p c. **24.** 32. **25.** $3\frac{19}{27}$ p. c.
26. *Rs.*206. 4*a.* ; 25 per cent. **27.** *Rs.* 2500 ; 30 p. c.
28. 400. **29.** 8 cwt. **30.** 16 per cent.
31. 10*a.* **32.** £35. **33.** 1 cwt. 73 lbs. 10 oz. **34.** 6 scores.
35. *Rs* 3 2*a.* **36.** *Rs.*5. 13*a.* 4*p.* **37.** *Rs.*4. 10*a.* $6\frac{6}{13}$*p.*
38. 10*s.* 5*d.* **39** *Rs.*750 **40.** *Rs* 7. 8*a.*
41. 3 cwt. 2 qrs 23 lbs. **42.** *Rs.*100.
43. *Rs.*250 **44.** *Rs.*400. **45.** £48. **46.** $12\frac{1}{2}$ per cent.

Ex. CLII. (pp. 426-427.)

1. (1) 224 ; 336 ; 448. (2) *Rs.*2500 ; *Rs.*3750 ; *Rs* 8750 ; *Rs.*10000.
(3) *Rs* 33000 ; *Rs.*22000 , *Rs.*16500 ; *Rs.*13200.
(4) £320 ; £370 ; £384. (5) 180 bi. 12 k. 6 ch.; 236 bi. 3 k 14 ch. ; 277 bi. 17 k. 8 ch.
(6) *Rs.*50 ; *Rs.*12. 8*a.* ; *Rs.*5 ; *Rs.*2.8*a.* ; *Re.*1. 10*a.* 8*p.*
(7) 264 lbs. ; 59 lbs. ; 40. lbs. (8) £121. 5*s.* 6*d.* ; £179. 11*s.* ; £292. 12*s.*

2. Saltpetre 336 sr. ; sulphur 44·8 sr. ; charcoal 67·2 sr.
3. 3 mds. 12 sr. **4.** 270 lbs. ; 198 lbs. (nitre).
5. N. $851\frac{1}{3}$ sr. ; S. $106\frac{2}{3}$ sr. ; C. $162\frac{2}{3}$ sr.
6. Men *Rs.*12. 8*a.* ; women *Rs.*7. 8*a.* ; boys *Rs.*6.
7. *A Rs* 4242 ; *B Rs* 7070 ; *C Rs.*7777.
8 *C* £630 ; *B* £600 ; *A* £400.
9. *A Rs.*1312. 8*a.* ; *B Rs.*3500 ; *C Rs.*7875. ; *D Rs.*2100.
10 *Rs.*433. 5*a* 4*p.* **11.** 30 ; 48 ; 72.
12. *A Rs.*90 ; *B Rs.*120 ; *C Rs* 180.
13. Men 5400 ; women 675 ; boys 150 ; girls 45. **14.** £83. 17*s.* $10\frac{11}{13}$*d.*
15. *A* 200 mds ; *B* 300 mds. ; *C* 525 mds. ; *D* 1155 mds.
16. 56 half-crs. ; 40 florins ; 24 shillings
17. 12 half-crs. ; 24 pence ; 36 sovs. ; 96 shillings.
18. 50 sovs. ; 60 half-sovs. , 200 florins.
19. *A Rs.*1920 ; *B Rs.*1440 ; *C Rs.*1350 ; *D Rs.*4800.
20. 78 rupees ; 132 half-rupees ; 168 quarter-rupees.

Ex. CLIII. (pp. 429-430.)

1. *Rs.*150 ; *Rs.*200 ; *Rs.*250. **2.** *Rs.*22500 ; *Rs.*15000 ; *Rs.*7500.
3. *A Rs.*1500 ; *B Rs.*1312. 8*a.* ; *C Rs.*1187. 8*a.*
4. The share of each is *Rs.*1190. **5.** *A* £540 ; *B* £900.
6. *A Rs.*2240 ; *B Rs.*2360. **7.** *A Rs.*14337 ; *B Rs.*14458. 8*a.*; *C Rs.*11340. **8.** *A Rs.*160 ; *B Rs.*150 ; *C Rs.*400.
9. *A* £168 ; *B* £280. **10.** *A Rs.*35000 ; *B Rs.*49000 ; *C Rs.*70000 ; *D Rs.*84000. **11.** *A Rs.*1790. 10*a.* 8*p.* ; *B Rs.*2574. 1*a.* 4*p.*

12. *A* £8. 11*s*. 6*d*. ; *B* £13. 9*s*. 6*d*.
13. £486. 13*s*. 4*d*. ; £730 ; £2433. 6*s*. 8*d*.
14. *Rs*.1600. 15. $23\frac{7}{16}$ months. 16. $22\frac{8}{11}$ days.
17. $10\frac{2}{7}$ months. 18. $45\frac{5}{7}$ days.

Ex. CLIV. (pp. 431-432.)

1. $5\frac{9}{14}$ months. 2. 9 months. 3. $7\frac{3}{8}$ months.
4. 28th May. 5. $92\frac{4}{23}$ days.
6. 7 months. 7. 80 months. 8. 24th April.

Ex. CLV. (pp. 434-436.)

1. *Rs*.7. 3*a*. $1\frac{25}{27}$*p*. 2. *Re*.1. 2*a*. $9\frac{81}{416}$*p*. ; *Re*.1. 1*a*. $5\frac{7}{10}$*p*.
3. 7 : 4. 4. 3 : 3 : 1. 5. 3 : 3 : 4 : 18 ; or 18 : 4 : 3 : 3.
6. 2 : 13. 7. 3 mds., 3 mds., 5 mds. 8. 2 : 7.
9. 28 lbs. ; 56 lbs. ; 56 lbs. ; 28 lbs. ; or 42 lbs. of each.
10. 3 lbs. of 1st ; 1 lb. of 2nd ; 12 lbs. of 3rd.
11. 9 gals. ; 18 gals. ; 27 gals. ; 9 gals. 12. $4\frac{4}{5}$ lbs.
13. 3 lbs. ; 3 lbs. ; 3 lbs. ; 2 lbs.
14. 50 lbs. of brass ; 200 lbs. of pewter. 15. 185 lbs. 10 oz.
16. 115 oz. ; 85 oz. 17. 7 oz. gold ; 2 oz. silver ; 2 oz. copper.
18. 20 : 7 ; 5*s*. $1\frac{1}{2}$*d*. 19. 40 apples ; 20 pears.
20. Gold £3. 17*s*. $10\frac{1}{2}$*d*. ; silver 5*s*. $1\frac{1}{2}$*d*.

Ex. CLVI. (pp. 437-438.)

1. 32 oxen. 2. $9\frac{8}{11}$ days. 3. 70 oxen. 4. 32 oxen.
5. 190 oxen. 6. 3 ac. 7. 40 oxen. 8. 25 oxen.
9. 4 taps. 10. 7 sr. 13 ch. 11. 9 min. 12. 21 days.

Miscellaneous Examples VII. (pp. 441-446.)

1. 5*a*. 2*p*. 2. 9 per cent. 3. *Re*.1. 3*a*. per 100.
4. 22 women. 5. *A Rs*.375 ; *B Rs*.500 ; *C Rs*.333. 5*a*. 4*p*.
6. $33\frac{1}{3}$ p. c. 7. 1500000. 8. Loss 7 p. c.
9. Rich 15, poor 85. 10. 18*s*. 11. 1000 and 800.
12. 131 lbs. 13. $28\frac{4}{7}$ p. c. 14. 625. 15. £248.
16. $11\frac{1}{5}$ carats. 17. 150 at *Rs*.4 ; 250 at *Rs*.3.
18. Tea, 2*s*. 6*d*. ; sugar, 6*d*. 19. Tea, *Re*.1. 14*a*. ; coffee, 13*a*. 4*p*. 20. $38\frac{215}{239}$ per cent. 21. 2 : 1.
22. 3 : 7. 23. 200 cub. ft. 24. £96. 11*s*. 8*d*.
25. *Rs*.3. 4*a*. 26. £89. 8*s*. 9*d*. 27. *A* 120 ; *B* 75 ; *C* 125.
28. *Rs*.16500. 29. 200. 30. *A Rs*.30 ; *B Rs*.20.
31. *Rs*.84 ; *A Rs*.44 ; *B Rs*.40. 32. Men, *Rs*.72. 14*a*. $2\frac{2}{3}$*p*. ; women, *Rs*.85. 5*a*. 4*p*. ; boys, *Rs*.81. 12*a*. $5\frac{1}{3}$*p*.
33. 2 : 1. 34. *Rs*.30. 35. $9\frac{53}{188}$ p. c. gain ; 1200.
36. $5\frac{155}{188}$ p. c. gain. 37. *Rs*.375. 38. 3 : 2. 39. 42, 36 and 30 years. 40. 70 oz. 41. 300. 42. *A Rs*.850 ; *B Rs*.846 ; *C Rs*.1182. 43. Copper, 12*a*. ; brass, 9*a*. ; 5 : 6. 44. 390 ; 150.
45. 40 horses ; 25 oxen. 46. 15 carats. 47. 24.

48. 35 ; 15. **49.** Mangoes, 12 ; plantains, 24.
50. 12, 28 and 44 years. **51.** *Rs.*3. 4*a.* ; *Rs.*6. 2*a.*
52. 3 mds. 30 sr. ; 4 mds. 35 sr. **53.** *Rs.*60.
54. $\frac{5}{19}$ p. c. increase. **55.** $16\frac{4}{11}$ days. **56.** 200.
57. *Rs* 200 ; *A Rs.*20 ; *B Rs.*40 ; *C Rs.*60 ; *D Rs.*80.
58. *Rs.*125. *A Rs* 50 ; *B Rs.*41. 10*a.* 8*p.* ; *C Rs.*33. 5*a.* 4*p.*
59. 21 gals. from the 1st ; 8 gals. from the 2nd. **60.** 44 ; 91.

Ex. CLVII. (pp. 447-448.)

1. (1) *Rs.*10. 14*a.* (2) *Rs.*21. 8*a.* (3) *Rs.*130. 12*a.* 6*p.*
(4) *Rs.*104. 1*a.* (5) *Rs.*1209. 6*a.* (6) *Rs.*481. 4*a.*
(7) £32. 3*s.* $1\frac{1}{2}$*d.* (8) £103. 11*s.* $10\frac{1}{2}$*d.*
2. £10. **3.** *Rs.*10. 2*a.* **4.** *Rs.*60. **5.** *Rs.*50.

Ex. CLVIII. (pp. 449-450.)

1. (1) *Rs.*272. (2) *Rs.*49 3*a.* 9·6*p.* (3) *Rs.*88. 1*a.* 9·6*p.*
(4) *Rs.*169. 13*a.* 3·8*p.* (5) *Rs* 28. 14*a.* 9·6*p.* (6) £118. 3*s.* 3*d.*
(7) £159. 17*s.* 3*d.* (8) £24. 6*s.* 2·4*d.* (9) £140. 14*s.* 4·5*d.*
(10) £2. 3*s.* $3\frac{1}{4}$*d.*
2. (1) *Rs.*91. 14*a.* (2) *Rs.*1444. 14*a.* 4·95*p.* (3) *Rs.*647. 3*a.*
(4) *Rs.*890. 10*a.* (5) *Rs.*117. 1*a* 5·28*p.* (6) *Rs* 474. 3*a.* 0·675*p.*
(7) £221. (8) £102. 9*s.* $9\frac{1}{4}$*d.* ·2*q.* (9) £11. 17*s.* 6*d.*
(10) £75. 10*s.* 3*d.* ·6*q.* (11) £1356. 10*s.* $6\frac{3}{4}$*d.* ·4475*q.*
(12) £273. 12*s*
3. (1) *Rs.*22. 6*a.* 10*p.* (2) *Rs.*107. 15*a.* 4*p.*
(3) £36. 16*s.* $7\frac{1}{2}$*d.* (4) £73. 15*s.* $1\frac{1}{8}$*d.* (5) *Rs.*9792. 12*a.* 3*p.*
(6) £81. 10*s.* $3\frac{1}{4}$*d.* (7) *Rs.*39. 2*a.* 2*p.* (8) *Rs.*1714. 14*a.* 8*p.*
(9) £3. 15*s.* $3\frac{3}{4}$*d.* (10) £2. 19*s.* $8\frac{1}{2}$*d.*
4. (1) *Rs.*5916. 2*a.* 8*p.* (2) *Rs.*3873. 12*a.* 9·6*p.*
(3) £959. 13*s.* $8\frac{1}{4}$*d.* $\frac{189}{800}$*q.* (4) *Rs.*2701. 3*a.* $9\frac{38}{78}$*p.*
(5) *Rs.* 18702. 14*a.* 4*p.*

Ex. CLIX. (pp. 452-454.)

1. *Rs.*4688. **2.** *Rs.*1205. **3.** *Rs.*3250. **4.** $3\frac{1}{8}$ p. c. **5.** $2\frac{1}{2}$ p. c.
6. $5\frac{1}{2}$ p. c. **7.** *Rs.*12163. 2*a.* $6\frac{1}{19}$*p.* **8.** £678. 3*s.* $1\frac{1}{107}$*d.*
9. *Rs.*8489. 4*a.* 9*p.* *nearly.* **10.** $4\frac{1}{3}$ yrs. **11.** 16 yrs **12.** 20 yrs.
13. $4\frac{1}{2}$ p. c. **14.** 6 p c. **15.** 6 yrs. 8 mo. **16.** $4\frac{1}{2}$ p. c. *nearly.*
17. 16 yrs. **18.** $2\frac{2}{3}$ p. c. **19.** *Rs* 1140. 10*a.* **20.** *Rs* 43312. 8*a.*
21. 15 yrs. **22.** $12\frac{1}{8}$ p. c. **23.** $6\frac{1}{4}$ yrs. **24.** £640.
25. *Rs.*6878. 9*a.* **26.** $5\frac{1}{4}$ p. c. **27.** *Rs.*20855. 10*a.*
28. *Rs.*3333. 5*a* 4*p.* **29.** £10950. **30.** $2\frac{1}{2}$ p. c.
31. $6\frac{2}{3}$ p. c. **32.** 3 and 5 yrs. **33.** *Rs.*40.
34. *Rs.*156. 4*a.* **35** *Rs.*1600 ; *Rs.*2400.

Ex. CLX. (pp. 457-458.)

1. (1) *Rs.*406. 0*a.* 10*p.* (2) *Rs.*262. 11*a.* $9\frac{3}{8}$*p.*
(2) *Rs.*1495. 11*a.* 6*p.* (3) *Rs.*3454. 4*a.* 2*p.*

(5) *Rs.*171. 2*a.* 4*p.* (6) £934. 3*s.* $5\frac{1}{4}$*d.*
(7) £1237. 19*s.* $2\frac{1}{2}$*d.* (8) £553. 0*s.* $4\frac{1}{4}$*d. nearly.*
(9) *Rs.*1553. 11*a.* 8*p.*

2. (1) *Rs.*459. 1*a.* (2) *Rs.*297. 6*a.* 8*p.* (3) *Rs.*1285. 10*a.* 10*p.*
(4) £33. 13*s.* $10\frac{3}{4}$*d.* ·20384*q.*

3. (1) *Rs.*204. 0*a.* 7·7184768*p.* (2) *Rs.*446. 5*a.* 4·884168*p.*
(3) *Rs.*961. 13*a.* 8*p.*

4. (1) *Rs.*8104. 10*a.* 3*p.* (2) *Rs.*3193. 1*a.* 8*p.*
(3) *Rs.*7155. 15*a.* 11*p.* (4) £5140. 10*s.*
(5) *Rs.*19422. 6*a.* 4·67712*p.* (6) £2714. 9*s.* 9·472*d.*

5. *Rs.*147. 4*a.* 4*p. nearly.* 6. £18. 14*s.* $8\frac{1}{4}$*d. nearly.*

7. 874278. 8. *Rs.*15540. 6*a.* 9·8*p. nearly.*

Ex. CLXI. (pp. 460-461.)

1. *Rs.*40000. 2. £333333. 6*s.* 8*d.* 3. £133. 6*s.* 8*d.*
4. *Rs.*42500. 5. 2 per cent. 6. 5 per cent.
7. $2\frac{1}{2}$ years. 8. $2\frac{1}{2}$ years. 9. 3 years.
10. *Rs.*14080. 10*a.* 11. £800. 12. £430. 14*s.* 6·096*d.*
13. 10 p. c. ; *Rs.*2000. 14. £3497. 5*s.* $4\frac{16}{61}$*d.* 15. *Rs.*125428. 8*a.* 2*p.*

Ex. CLXII. (pp. 462-463.)

1. (1) *Rs.*8500. (2) *Rs.*1200. (3) *Rs.*7212. 8*a.*
(4) *Rs.*3150. (5) £45. 11*s.* $0\frac{1}{2}$*d.* $\frac{30}{329}$*q.* (6) £350.
(7) £367. 1*s.* $6\frac{8}{53}$*d.* (8) *Rs.*10445. 15*a.* $2\frac{6}{7}$*p.* (9) £237. 10*s.*
(10) £8000.

2. (1) *Rs.*1200. 2*a.* (2) *Rs.*350. (3) £5. 5*s.* (4) £15. 3*s.* 9*d.*
(5) £70. 17*s.* 6*d.* (6) *Rs.*482. 14*a.* 8*p.* (7) £12. 16*s.* 8*d.*
(8) £10. 10*s.* 10*d.* (9) *Rs.*622. 11*a.* $10\frac{2318}{18607}$*p.*
(10) £296. 13*s.* $4\frac{1}{4}$*d.*

Ex. CLXIII. (p. 464.)

1. (1) 7 p. c. (2) 6 p. c. (3) 4 p. c. (4) $2\frac{2}{3}$ p. c.
(5) 4 p. c. (6) 5 p. c.

2. (1) 8 months. (2) 3 yrs. (3) $3\frac{1}{2}$ yrs. (4) 4 mo.
(5) $3\frac{1}{2}$ yrs. (6) $1\frac{2}{3}$ yrs.

3. 3 per cent. 4. 6*s.* $0\frac{1}{4}$*d.* $\frac{4}{5}$*q.* 5. *Nothing.*

Ex. CLXIV. (pp. 466-468.)

1. *Rs.*8134. 8*a.* 2. £808. 1*s.* 4*d.* 3. £231. 15*s.*
4. $6\frac{1}{2}$ yrs. 5. $2\frac{1}{2}$ per cent. 6. $7\frac{1}{2}$ yrs.
7. £$6\frac{1}{4}$ p. c. 8. *Rs.*7768. 14*a.* $2\frac{2}{3}$*p.* 9. *Rs.*180.
10. *Rs.*250. 11. *Rs.*5746. 8*a.* ; $6\frac{1}{4}$ p. c.
12. *Rs.*337. 14*a.* 13. £1598. 6*s.* 8*d.*; $1\frac{75}{37}$ per cent.
14. 80 ; 83 ; *Rs.*320. 15. (i) *Rs.*45. 13*a.* 4*p.* (ii) *Rs.*13. 1*a.* $6\frac{2}{7}$*p.*
16. *Rs.*322. 1*a.* $5\frac{25}{27}$*p.* 17. 17*s.* $4\frac{16}{33}$*d.* 18. 10 months.

19. £130. 20. *Rs*.9. 9*a*. $10\frac{2}{13}$*p*. 21. £2500.
22. Disc. $16\frac{2}{3}$ p. c. ; Int. 20 p. c. 23. 16 copies.
24. $8\frac{1}{3}$ p. c. 25. *Rs*.1200. 26. $18\frac{14}{27}$ p. c.
27. *B*'s by *Rs*.33. 5*a*. 4*p*. 28. *Rs*.7014. 5*a*. 6*p*. *nearly*.
29. *Rs*.2115. 1*a*. 4*p*. *nearly* ; 4·66 mo. 30. *Rs*.6078 *nearly*.

Ex. CLXV. (pp 470-471.)

1. *Rs*.318. 10*a*. 6*p*. 2. *Rs*.11685. 5*a*. 4*p*. 3. *Rs*.8. 14*a*. 1·725*p*.
4. 2*s*. 6·625*d*. 5. *Rs*.9. 6. *Rs*.121. 8*a*. 0·534...*p*.
7. *Rs*.255. 8. *Rs*.336. 1*a*. $9\frac{51}{73}$*p*. 9. *Rs*.1585. 8*a*. $9\frac{27}{73}$*p*.
10. £547. 2*s*. 2*d*. $\frac{70}{73}$*q*.; 7*s*. $9\frac{3}{4}$*d*. $\frac{9}{73}$*q*. 11. *Rs*.51238. 14*a*. $1\frac{331}{365}$*p*.
12. *Rs*.57. 0*a*. $5\frac{8711}{9125}$*p*. ; *Re*.1. 0*a*. $5\frac{8711}{9125}$*p*. 13. £24. 5*s*. $11\frac{68}{101}$*d*.
14. £6020. 15. £61. 15*s*. 16. *Rs*.617624. 13*a*.$0\frac{84}{169}$*p*.
17. 31st May. 18. $11\frac{1}{9}$ p. c.

Ex. CLXVI. (pp. 473-474.)

1. *Rs*.65. 12*a*. 4*p*. 2. *Rs*.3507. 7*a* $0\frac{3}{5}$*p*. 3. 9*s*. $1\frac{1}{2}$*d*.
4. *Rs*.128. 0*a*. 4*p*. 5. £4488. 15*s*. 6. *Rs* 969. 4*a*. 9·6*p*.
7. £768. 2*s*. 6*d*. 8. *Rs*.16170 9. *Rs*.5105. 6*a*.
10. *Rs* 693333. 5*a*. 4*p*. 11. $4\frac{1}{2}$ per cent.
12. *Rs*.23149. 5*a* $4\frac{1}{10}$*p*. ; *Rs*.766666. 2*a*. 3*p*. 13. *Rs*.38. 10*a*. 11*p*.
14. £486. 3*s*. $10\frac{1}{2}$*d*. $\frac{1768}{38777}$*q*. 15. *Rs*.68806. 7*a*. $5\frac{159}{201}$*p*.

Ex. CLXVII. (pp. 479-480.)

1 *Rs*.18772. 10*a*. $9\frac{33}{38}$*p*. 2. 17511 fr. $4\frac{1}{3}$ c.
3. 2648 fr. 69·71 c. ; 6294 fr. 6·61875 c. 4. 11474 dol. $38\frac{1}{2}$ c.
5. 124·2675...fr. or $124\frac{1}{4}$ fr. *nearly*.
6. 30*d*. per rouble ; gain £20. 16*s*. 8*d*. 7. 25 fr. 73 . c.
8. 33*s*. 4*d*. 9. (i) $9\frac{3}{8}$*d*. in £1. (ii) $11\frac{23}{32}$ fr. in 300 fr.
10. £10. 14*s*. $3\frac{3}{7}$*d*. 11. 996·87 fr. 12. £1011. 12*s*. 6·7*d*.
13. £2000. 14. £250. 15. 1 dol. = 4·22429...*s*. = 4*s*. $2\frac{3}{4}$*d*. *nearly*.
16. 25·001...fr. or 25 fr. *very nearly*.
17. Gain £5. 9*s*. $0\frac{1}{4}$*d*. *nearly*. 18. 20 marcs $42\frac{1}{2}$pf.

Ex. CLXVIII. (pp. 481-482.)

1. $3\frac{29}{37}$ per cent. 2. $28\frac{4}{7}$ years. 3. *Rs*.1600. 4. 4 p. c. 5. 5 p. c.
6. 25·96...years. 7. £2. 7*s*. $4\frac{4}{17}$*d*. 8. $25\frac{1}{2}$ years.

Ex. CLXIX. (pp. 487-488.)

1. (1) *Rs*.3960. (2) £5418. 3*s*. 9*d*. (3) £408. 5*s*. $11\frac{1}{4}$*d*. $\frac{187}{200}$*q*.
(4) *Rs*.37660. 2*a*. $2\frac{22}{25}$*p*. (5) £588. 5*s*. (6) *Rs*.29808. 10*a*. $3\frac{8}{25}$*p*.
(7) *Rs*.9175. 10*a* (8) £1226. 1*s*. $10\frac{1}{2}$*d*.
2. (1) *Rs*.14100. (2) *Rs*.8065. 14*a*. $2\frac{4}{25}$*p*. (3) *Rs*.8051. 14*a*. $9\frac{8}{201}$*p*.
(4) £15573. 8*s*. $10\frac{1}{2}$*d*. (5) *Rs*.70850. 1*a*. $3\frac{3}{5}$*p*.
3. (1) *Rs*.9000. (2) £1000. (3) £2966. 1*s*. $1\frac{3}{4}$*d*. $\frac{5}{29}$*q*.
(4) *Rs*.49788. 6*a*. $9\frac{7}{9}$*p*. (5) *Rs*.24200. 14*a*. $7\frac{115}{333}$*p*.

(6) £1015. 1*s*. $0\frac{3}{4}$*d*. $\frac{19}{31}$*q*. (7) *Rs*.8670. 7*a*. $1\frac{75}{97}$*p*.
(8) £566. 13*s*. 4*d*.
4. £10039. 2*s*. $2\frac{3}{4}$*d*. $\frac{553}{601}$*q*. 5. *Rs*.238. 14*a*. 6. £50 16*s*. 8*d*.
7. *Rs*.22699. 2*a* $2\frac{14}{29}$*p*. 8 $112\frac{7}{32}$. 9. *Rs* 6250.
10. *Rs*.7500. 11. £175. 12. $93\frac{3}{8}$.
13. *Rs*.624 11*a*. $10\frac{50}{2401}$*p*.

Ex. CLXX. (pp. 489-491.)

1. (1) £165. (2) *Rs*.3155. 4*a*. (3) £108 3*s*. 3*d*. (4) *Rs* 2878. 12*a*.
2. (1) *Rs*.2400. (2) £159. 12*s*. (3) *Rs*.1368. 8*a*.
(4) *Rs*.1093. 13*a*. $2\frac{134}{179}$*p*. (5) *Rs*.483 9*a*. $5\frac{3}{13}$*p*.
(6) *Rs*.9311. 5*a*. $6\frac{364}{387}$*p*.
3. (1) *Rs*.17000. (2) £1785. (3) *Rs*.44072. 1*a*. 4*p*. (4) £5805.
4. *Rs*.52. 5*a*. $4\frac{8}{79}$*p*. 5. £83. 6*s*. 8*d*. 6. £122 3*s*. 4*d*.
7. $91\frac{1}{4}$. 8. *Rs*.54309. 10*a*. $10\frac{2}{31}$*p*.
9. £15 3*s*. $0\frac{4}{11}$*d*. increase. 10. *Rs*.20 gain 11. £26 13*s*. 4*d*.
12. £34. 10*s*. gain. 13. £5 12*s*. 6*d*. loss. 14. 90.
15. *Rs*.47250. 16. *Rs*.455. 5*a*. $8\frac{4}{7}$*p*. 17. $85\frac{1}{2}$.
18. $94\frac{1}{2}$. 19. *Rs*.20000. 20. $90\frac{7}{13}$.
21. *Rs*.25 gain. 22. £5300. 23. *Rs*.9080. 8*a*. $5\frac{1}{19}$*p*.
24. £52. 10*s* ; £58. 6*s*. 8*d*. more.

Ex. CLXXI. (pp. 493-496.)

1. (1) $4\frac{308}{423}$ p. c. (2) $3\frac{27}{91}$ p. c. (3) $3\frac{13}{49}$ p. c. (4) 5 p. c.
2. (1) 80. (2) $87\frac{1}{2}$. (3) $77\frac{7}{9}$.
3. $95\frac{7}{8}$. 4. $85\frac{3}{8}$. 5. $83\frac{4}{7}$. 6. 4 p. c. 7. $3\frac{1}{2}$ p. c.
8. *Rs*.13800. 9. $83\frac{11}{12}$. 10. *Rs*.125000. 11. *Rs*.18240.
12. £5631. 5*s*. ; £90. 6*s*. 3*d*. gain.
13. *Rs*.138548. 12*a*. 14. $117\frac{1}{4}$. 15. 3 p. c. ; £582 10*s*.
16. £3000. 17. *Rs*.96. 5*a*. 8*p*. 18. $106\frac{1}{3}$.
19. *Rs*.17414. 15*a*. $5\frac{23}{49}$*p*. ; *Rs*.13109. 8*a*. $4\frac{4}{7}$*p*. 20. $94\frac{5}{8}$.
21. £9880. 22. *Rs*.14150. 15*a*. $1\frac{7}{53}$*p*. 23. *Rs*.49700.
24. *Rs*.19. 13*a*. 4*p*. less. 25. *Rs*.25200.
26. *Rs*.27000 ; *Rs*.185 more. 27. *Rs*.16000 ; *Rs*.24000.
28. *Rs*.645000. 29. $109\frac{1}{11}$. 30. *Rs*.27000.
31. £32. 5*s*. more. 32. £257. 5*s*. 5*d*. 33. 6*d*. in the £.
34. *Rs*.192000. 35. $85\frac{1}{3}$. 36. *Rs*.4800 ; *Rs*.5200.
37. *Rs*.4000 ; *Rs*.12000. 38. *Rs* 16000000. 39 15 years.
40. *Rs*.6750 ; *Rs*.5250. 41. *Rs*.41066. 0*a*. 8*p*. ; *Rs*.45600.
42. £1000. 43. £400 ; £1600.
44. £3000. 45. £242914 19*s*. $7\frac{85}{247}$*d*.

Ex. CLXXII. (pp. 498-499.)

1. 197 ft. 2. 45·09 ft. 3. 38 ft. 9 in. ; 54 ft. 9·6 in. *nearly*.
4. 3365 yds. 5. 14·02 ft. 6. 153 miles. 7. 273·642102 miles.
8. 520·195...ft. 9. 12 ft. 10. 19. 11. 35 ft. 12. 1045 yds.

13. (1) 1. (2) 1. (3) ·25 (4) 100. (5) 5725.
(6) $1\frac{25}{36}$. (7) $4\frac{61}{180}$. (8) $3\frac{16}{21}$.
14. 3·162... **15.** 5 ft. $7\frac{1}{2}$ in.

Ex CLXXIII. (p., 502.)

1. (1) 1·36602. (2) 6·67423. (3) ·50401. (4) ·71070.
(5) ·82185. (6) ·43417. (7) 4·82825. (8) 4·05143.
(9) 2·61803. (10) 1. (11) 14. (12) 1·38742.
(13) 1·57313. (14) ·26795. (15) 0.
2. (1) $\sqrt[3]{3}$. (2) $\sqrt[4]{(19)}$. (3) $\sqrt[4]{(15)}$. **3.** $\frac{2}{81}$. **4.** 294·151.

Ex. CLXXIV. (pp 504-508.)

1. (1) 17·030003 kilom.; 1703000·3 centim.
(2) 50·000037 kilom.; 5000003·7 centim.
2. (1) 5330003·000002 sq. m.; 5330003000002 sq. millim.
(2) 7·25 sq. m. ; 7250000 sq. millim.
3. (1) 116·075 gr. ; ·116075 kilog. (2) 530·45 gr. ; ·53045 kilog
4. 76300·05 ares. **5.** 5300004·65 decil.
6. (1) 24·14 fr. ; 2414 c. (2) 480·0575 fr. ; 48005·75 c.
(3) 5·08 fr. ; 508 c. (4) 55·536 fr. ; 5553·6 c.
7. 127 fields. **8.** 12. **9.** 180 fr. 74 c. ; £2. 16*s.* $10\frac{1}{4}$*d. nearly*.
10. *A* will have 24 hectar. 80 ares ; *B* 22 hectar. 4 ares ; *C* 33 hectar. 16 ares. **11.** $7\frac{1}{2}$ days. **12.** 30 miles *nearly* ; $1\frac{3}{4}$*d. nearly*.
13. 297 fr. 50 c. **14.** (1) 6 c. (2) Rather more than 1*s.* $2\frac{1}{4}$*d.*
15. 2 fr. 91 c. **16.** 1600 fr. **17.** 6 p. c. **18.** $\frac{3}{17}$.
19. 197·81...met. **20.** 158 of each of two first coins and 79 of each of three last coins. **21.** (1) 34 fr. 7 c. (2) 2 fr. 37 c.
(3) 29 fr. 87 c., or £1. 3*s.* 5*d. nearly*. **22.** £110. 1*s.* 3*d. nearly*.
23. *A*'s share = 24 hectar. 75 ares ; *B*'s share = 13 hectar. 50 ares ; *C*'s share = 47·25 hectar. **24.** 15 fr. 32·68 c. **25.** 8 years.
26. £19. 7*s.* 10*d. nearly*. **27.** 3419 fr. 58 c.
28. (1) 500 fr. $26\frac{1}{4}$ c. (2) 446 fr. $6\frac{1}{4}$ c. **29.** 1785 fr. 50 c.
30. 1 hr. 25 min. 20 sec. **31.** 1st. 3 fr. 25 c. ; 2nd, 4 fr. 63 c.
32. The 5 per cents. ; £21. 4*s.* 11*d. nearly*. **33.** £1. 19*s.* 2·15*d*
34. 27153 fr. $78\frac{3}{4}$ c. **35.** (1) 7*s.* $1\frac{1}{4}$*d.* (2) 8 fr. 94 c.
36. 82 fr. 50 c. **37.** 259 fr. $70\frac{5}{8}$ c. **38.** 6 ares, 6 deciar. 2 centiar. *nearly* ; 791·79...sq. yds. **39.** 120·59...fr.
40. 2 decim. 4 centim. **41.** 9 kilog. 126 grs. **42.** $ 864·8.
43. 15 fr. 14 c. *nearly*. **44.** 1 m. 6 decim.
45. No. of boxes = 14 ; no. of oranges = 588.
46. 7 Nap. 15 fr. 85 c. *nearly*.

Ex. CLXXV. (p. 509.)

1. (1) 59170 m. ; £976. 3fl. 5c. 8m. (2) 18065 m. ; 96239 m.
(3) 14435 m. ; £254. 5fl. 2c. 5m.

2. (1) 1 c. $8\frac{3}{4}$ m. ; 4 c. ·$\frac{2}{3}$ m. ; 2c. 2·916m. ; 2fl. 5c. ; 3fl 1 c. $6\frac{3}{8}$m. ;
1 fl. 5·208$\dot{3}$ m. ; 1 fl. 5 c. $3\frac{1}{8}$ m.
(2) 2 fl. 1 c. $2\frac{1}{2}$ m. ; 7 fl. 8 c. $4\frac{3}{8}$ m. ; £12. 6 fl. 4 c. 8·958$\dot{3}$ m. ;
£4. 4 fl. 2 c. ·8$\dot{3}$ m.
(3) 7 fl. 8 c. $3\frac{1}{8}$ m. ; 6 fl. 4 c. 8·958$\dot{3}$ m. ; £5. 4 fl. 6 c. 9·791$\dot{6}$ m.;
£6. 8 fl. 9 c. 7·91$\dot{6}$ m.

3. (1) 18s. $1\frac{1}{2}$*d.*; £1. 9*s.* 9*d.*; £1. 15*s.* 8*d.*; 18*s.* $1\frac{1}{2}$*d.* 0864*q.*
(2) £3. 12*s.* $5\frac{1}{4}$*d.*, £7. 16*s.* $5\frac{1}{4}$*d.* ; £29. 17*s* 6·24*d.* ; £25. 18*s.* $10\frac{3}{4}$*d.*
4. £990 6 fl. 7 c. 3 m. **5** £243. 2 fl. 6 c. 4 m.
6. £863. 2 fl. 8 c. 5 m. **7.** £14. 3 fl. 2 c. 5 m. , £6. 2 fl 7 c. 4 m.
8. £174. 2 fl. 3 c. 7 m. **9.** £196. 6 fl. 5 c.; £11012. 4 fl.
10. £3051. 7 fl. 2 c. 1 m. **11.** £76. 8 fl. 3 m. **12.** 7 fl. 8 c. 2·2359 m.
13. £2 9 fl 9 c. 9 m. ; £46. 3 fl. 6 c. **14.** 789
15. ·0732 ; ·001464.

Miscellaneous Examples VIII. (pp. 512-519.)

1. *Rs.*2140. **2.** 11th March. **3.** $11\frac{1}{9}$ p. c. **4.** £273. 8*s.* 9*d.*
5. £2376. 5*s.* **6.** £2577. 12*s.* 1*d.* **7.** 10*a* 8*p.* **8.** £$81\frac{613}{1848}$.
9. 155 : 147. **10.** £390. 2*s.* $5\frac{1}{4}$*d.* **11.** 224 seers.
12. £160. 14*s.* $3\frac{3}{7}$*d* **13.** 4 p.c. loss. **14.** First *Re.*1. 8*a.* per lb.
second *Re.*1. 12*a.* per lb. **15** £250.
16. £$192307\frac{9}{13}$; £$57692\frac{4}{13}$. **17.** £2200. **18.** *Rs.*3535.
19. £8. 15*s.* $2\frac{5}{8}$*d.* **20.** $933\frac{1}{3}$ lbs **21.** £5977. **22.** $46\frac{2}{13}$ sr.
23. £2450 ; £2205. **24.** $6\frac{26}{125}$*s.* **25.** 600 sq. ft. **26.** *Rs.*264.
27. The 3 per cents. ; 19*s.* $7\frac{3}{8}$*d* gain **28.** £1500. **29** *Rs.*39440 ;
*Rs.*1560. **30.** *Rs.*6776. **31.** £10. 16*s.* decrease.
32 $3\frac{3}{8}$ p. c. **33.** £1200. **34.** *Rs.*280 **35.** 96 : 19.
36. 25 for 11*d.* **37.** 5 : 9. **38.** 60 ft. **39.** 17*s.* $2\frac{1}{4}$*d.*
40. First £10. 13*s.* 4*d.* ; second £11. 13*s.* 4*d.*
41. *Rs.*72900000. **42** 32·25 p.c. **43.** *A* 28 yrs. ; *B* 32 yrs.
44. $4\frac{1}{2}$ p.c.
45. *A* 32 yrs. ; *B* 28 yrs **46.** Circuitously, by 35·985 milrees.
47. 5·51 p.c. *nearly.* **48.** 4 p. c. **49.** $\frac{2}{3}$. **50.** 12 yds. from *B.*
51. $9\frac{3}{5}$*d.* **52.** 22 yrs. ago ; 18 yrs hence.
53. *Rs.*21106. 8*a.* ; *Rs.*18232. 8*a* ; *Rs* 15000.
54. ·0102045. oz. ; 25·17 francs. **55.** 25 fr $53\frac{1}{2}$ c. ; 25 fr. $14\frac{1}{2}$ c.
56. £1687. 14*s.* 8*d.* **57.** *Rs.*5110. **58.** *Rs.*2600 ; *Rs* 3580.
59. £4092. **60.** £10 **61.** (1) 23515302409.
(2) 10192. **62.** He loses £240. **63.** $6\frac{5}{8}$ p. c.
64. £2275. **65.** (i) *Rs.*137. 8*a.* (ii) *Rs.*39. 4*a.* $6\frac{6}{7}$*p.*
66. *Rs.*12960 ; *Rs.*11220. **67.** *A* £150. ; *B* £144 ;
C £90 ; *D* £60. **68.** *Rs.*6500. **69.** 2*s.* 8*d.*
70. *Rs.*210 ; *Rs.*105 ; *Rs.*72 ; *Rs.*42. **71.** $18\frac{1}{8}$ p. c.
72. $17\frac{6}{7}$ p. c. **73.** £14700. **74.** $9\frac{1}{11}$ per cent.
75. *A* receives 6*s.* 8·0325*d.* more than *B.* **76.** Nothing.
77. Gain = $623\frac{1}{3}$ francs ; $37\frac{13}{128}$ p. c. **78.** Gain $74\frac{46}{61}$ p. c.
79. £600. **80.** 12 p. c. **81.** 9 5 ft. *nearly.*

Miscellaneous Exercises (pp. 520-557.)

1. 857628. **2.** 62968425. **3.** 37. **4.** £1. 3s 2d.
5. 12lbs. **6.** £2. 14s. $1\frac{1}{4}$d. **7.** 477 times. **8.** *A Rs* 48 ; *B Rs.*72 ; *C Rs* 112. **9.** *Rs.*500. **10.** 795.
11. 33 ; £3048751. 15s. **12.** Sum = 17624 ; product = 76945744.
13. 612. **14.** 54459281679. **15.** £73. 16s. $2\frac{1}{4}$d.
16. 2415 times. **17.** 76 times. **18.** 29. **19.** *Rs.*7 13a.
20 256 weeks. **21.** Five millions two hundred and ninety-nine thousand eight hundred and thirty.
22. 1670730025511134812. **23.** 2378. **24.** 477648483.
25. £33438069. 12s. $3\frac{1}{4}$d. **26.** Yes ; 6p. **27.** £392. 19s. $3\frac{1}{4}$d.
28. 25 hens. **29.** 262080 min. ; 797. **30.** £57. 12s 6d.
31. One hundred millions, seven hundred and ninety-six thousand, nine hundred and fifty-seven. **32.** 17.
33. 1861. **34** 95. **35.** 511 ac. 2 ro. 8 po. 5. sq. ft. 129 sq. in.
36. 4 gals. **37.** $\frac{228}{365}$ **38.** 7 horses ; 7 pigs ; 21 cows ; 105 sheep.
39. *Rs.*116666. 10a. 8p. **40** *Rs.*15. **41.** $668674698\frac{66}{83}$.
42. 1 ac. 10 po. 4 yds. 2 ft. **43.** £11. 4s. $6\frac{3}{4}$d. **44.** 57 years.
45. (i) *A Rs* $22222\frac{2}{9}$; *B Rs.*$33333\frac{1}{3}$; *C Rs.*$44444\frac{4}{9}$.
(ii) *D Rs.*$46153\frac{11}{13}$; *E Rs.*$30769\frac{3}{13}$; *F Rs.*$23076\frac{12}{13}$.
46. 119 ; 17. **47.** £52. 15s. 6d. **48.** $\frac{13}{16}$. **49.** £10. 8s.
50 *Rs.*170 4a. **51.** 2, 3, 7, 11, 13, 37 ; 71716.
52. Quot. = 17430 ; Rem. = 12. **53.** 90945547 miles 2 fur. 8 po. 1 yd. 2 ft. 3 in. **54.** $2\frac{1}{6}$. **55.** $1533\cdot\dot{3}$; $\frac{210}{743}$.
56. £11. 3s $10\frac{47}{48}$d. **57.** £233. 17s. 10d. ; $5\frac{1}{5}$d.
58. *A Rs.*22840 ; *B Rs.*11420 ; *C Rs.*3806. 10a. 8p. ; *D Rs.*7613. 5a. 4p.
59. $13\frac{1}{3}$ days. **60.** *Rs.*103. 1a. 3p. **61.** *Rs.*774. 0a. $6\frac{13}{61}$p.
62. £4. 0s. 9d. **63** 12 mds. **64.** 9 weeks.
65. $88\frac{8}{9}$; *Rs* 13500. **66.** 54. **67.** $7\frac{14}{25}$d.
68. *Rs.*333. 5a 4p. **69.** *A* £317. 9s. $2\frac{10}{21}$d. ; *B* £267. 17s. $1\frac{5}{7}$d. ; *C* £253. 19s. $4\frac{8}{21}$d. ; *D* £160. 14s $3\frac{3}{7}$d.
70. £2440 ; £73. 4s. **72.** 2967. **73.** 1. **74.** 756 cub. ft.
75. 55 sq. ft. 80 sq. in ; 18s. $6\frac{2}{9}$d. **76.** *A's Rs*$586\frac{13}{17}$; *B's Rs* $391\frac{3}{17}$; *C's Rs.*$352\frac{1}{17}$. **77.** £20. **78.** $4\frac{1}{6}$ ft. **79.** £8000 ; £7500.
80. £1239. 13s. $4\frac{80}{121}$d. **81.** 15099500. **82.** 1568
83. $1\cdot12345\dot{5}6932239110\dot{4}$ **84.** ·03212. **85.** *Rs.*767. 13a. $5\frac{387}{1795}$p.
86. 90 days. **87.** 17 ft. 4 in **88.** £104. **89** $44\frac{4}{9}$ p. c.
90. £4687. 10s. **91.** 763. **92.** 794997. **93.** ·03493.
94. £26. 13s. 4d. **95.** $\frac{27}{84}$. **96.** 1050. **97.** £1682.
98. $\frac{4}{7}$. **99** $\cdot8\dot{3}$; 5·91. **100.** $12\frac{1}{2}$ p. c.
101. 14 days 7 hrs. 11 min. 17 sec. ; 2674 days 9 min. 59 sec.
102. 3998936616. **103.** 4s. $7\frac{11}{21}$d. **104.** 2·721551. **105.** 1 : 4.
106. $33\frac{1}{3}$lbs. **107.** $4\frac{24}{119}$. **108.** 30 seers. **109.** 439·824 yds.
110. Gain *Rs.*770 ; $\frac{10}{11}$ p. c. **111.** 24754. **112.** 425.
113. 20 hrs. 16 min. **114.** 3 tons 4 cwt. 3qrs. 4lbs. 13 oz.
115. $307\frac{1097}{5925}$ days. **116.** 25 miles. **117.** *Rs.*9180. **118.** £1600.
119. *Rs.*1666. 10a. 8p. monthly. **120.** 72 men ; 288 women.

121. 45. **122.** 126. **123.** ·4806. **124.** $108\frac{9}{25}$ grs. ; $\cdot 00221\dot{1}4285\dot{7}$.
125. *Rs.*14586 **126** 11 ft $6\frac{1}{5}$ in.
127. *A* $3\frac{6}{25}$*s.* ; *B* $4\frac{8}{25}$*s.* ; *C* $5\frac{13}{25}$*s.* ; *D* $7\frac{17}{25}$*s.* **128.** *Rs.*9920. 15*a.* $6\frac{248}{329}$*p.*
129 £818. 8*s.* **130.** £250 ; 4 p. c. **131.** 73. **132.** $\frac{2125}{2504}$.
133. *Rs.*288. 12*a.* **134.** 2·068501. **135.** 150
136. *Rs.*52000. **137.** 5 p. c. **138.** *Rs* 25000.
139. *Rs.*72. **140** 32 **141.** 3612924 ; 357250824.
142. $\frac{8}{9}$; 11*s.* 5*d.* **143.** 600600 ; 6·006 ; $\frac{91}{90}$; 8·8.
144. 24. **145.** *Rs.*1500. **146.** $3\frac{111}{203}$ hours.
147. 5 per cent. **148.** $3\frac{18}{19}$ p. c. **149** $\frac{3}{8}$ gal **150.** £1350.
151. 14*a.* **152** 6 times. **153.** £7. 9*s* $7\frac{1}{2}$*d.*
154. £1. 13*s.* $6\frac{7201477}{8800000}$*d.* **155.** 11 oz. **156.** $14\frac{2}{7}$ days.
157. 12 lacs ; *Rs.*5. **158.** *Rs.*3 2*a.* **159.** *Rs.*1026.
160 29040. **161.** *Rs.*120. **162** Eldest *Rs.*5184 ; second *Rs.*2592 ; youngest *Rs.*1728. **163.** (i) $\frac{1}{2}$. (ii) ·0513 ..
164. 124 rings. **165.** 7 days 10 hrs. 52 min. 30 sec.
166. 20°. **167.** 6*a.* **168.** *Rs.*2319. **169.** 624.
170. Loss *Rs.*1000. **171.** 9 days. **172.** 320.
173. 10 half-crowns ; 25 shillings ; 50 six-pences ; 75 four pences.
174. *A* £1650 ; *B* £1540. **175.** The clock ought to have been set at 5 h. $30\frac{2170}{5761}$ min. P. M. **176.** £690.
177. 10 yds. 2 ft. $5\frac{82}{487}$ in. **178.** £31. 5*s.* **179.** Loss 80 p.c.
180. 11 : 2. **181.** 0. **182.** $4\frac{1}{6}$ meas.
183. *A* *Rs.*450 ; *B* *Rs* 900 ; *C* *Rs.*2250 **184.** 20. **185.** 16*s.* $5\frac{19}{25}$*d.*
186. $36\frac{13}{22}$ days. **187.** 12*s.* **188.** $72398369\frac{770}{1109}$ gals.
189. $391\frac{1}{9}$ rev. ; diamrs. 2 ft. $4\frac{7}{11}$ in. ; 4 ft. $3\frac{6}{11}$ in. **190.** 2 years.
191. $2\frac{10}{21}$. **192.** (i) $\frac{5}{8}$. (ii) 0.
193. 32 seers. **194.** 1. ; ·0447. **195.** 1*s.* $11\frac{1}{4}$*d.*
196. $8\frac{3}{4}$ days. **197.** 176 ac. 540 sq. yds. **198.** *Rs.* 2.
199. £1. **200.** 1*s.* $10\frac{29}{40}$*d.* **201.** 13025·22 ; an abstract number.
202. $\frac{6}{19}$. **203.** $50\frac{10}{19}$ days. **204.** $1513\frac{3}{4}$. **205.** $\frac{3}{11}$ in.
206. $20546\cdot\dot{3}$. **207.** £4600. **208.** 4 years.
209. Author *Rs.*5322 ; Publisher *Rs.*1008 ; Bookseller *Rs.*4920.
210. 9600 yds. **211.** 528093440. **212.** 1·25 lbs.
213. *A* £212. 2*s.*, *B* £353. 10*s.*, *C* £388. 17*s.*
214. $12\frac{4}{5}$ days. **215.** *A*, *B* and *C* each *Rs.*615 ; *D* *Rs.*410.
216. 23 boys. **217.** 4 cwt. **218.** £1. 0*s.* 10*d.*
219. £200. **220** £106. 8*s.* $5\frac{1}{19}$*d.* gain.
221. *B* is $\frac{1}{24}$ of a mile in advance of *A*. **222.** $\frac{1}{16}$.
223. $33\frac{8016197}{9410700}$ years. **224.** 336 yds. **225.** £80. 7*s.* 1*d.* ; $183\frac{3}{8}$ days.
226. $163\frac{1}{5}$ days. **227.** 29th March 1860 ; 7 hrs. 12 min.
228. 69·3109 miles. **229.** $26\frac{6}{19}$ p. c. **230.** Gain £24.
231. £10. 10*s.* ; $\frac{16}{45}$. **232.** 20 yds. **233** 1*s.* $10\frac{34}{105}$*d.*
234 $33\frac{1}{3}$ per cent. **235.** 3·029 in **236.** 15 ; $\frac{78}{4000}$ cub. in.
237. 15·404 ft. **238.** *Rs.*75. 5*a.* 8·08*p.* **239.** *Rs.*$621\frac{9}{11}$ each.
240. $3\frac{9}{10}$ miles. **241.** 25 mi. 6 fur. 6 po. 2 ft. 4 in. ; 42524 hhd. 57 gals. 3 qts. 1. pt. 2 gills. ; 146929459 sq. in. **242.** $12\frac{1}{2}$ ft.
243. 16 days. **244.** *Rs.*7992. **245.** $8^6 : 7^6$; £179. 4*s.*

246. 11 days. **247.** *Rs.*1032. **248.** £303. 15*s*.
249. 112·64 yds. **250.** 36. **251.** 360 ; 216 ; 20736 ; 4.
252. Tea *Re.*1. 5*a*. 4*p*. ; Coffee 13*a*. 4*p*. ; Sugar 2*a*. 4*p*. per lb.
253. 24 times. **254.** *A* 162 ; *B* 108 ; *C* 72 runs. **255.** $\frac{5}{8}$ mile.
256. Less by $7\frac{48}{81}$ per cent. **257.** £1000 ; £7000. **258.** 13*s* 4*d*.
259. £3. 17*s*. **260.** 10*d*. ; 4*s*. $3\frac{1}{4}$*d*. **261.** 4280730.
262. 189 tons 19 cwt. 1 qr. 19 lbs. ; 6 lbs. $6\frac{317}{324}$ oz.
263. 3 : 2. **264.** Loss $\frac{71}{88}$ p. c **265.** $\frac{2}{3}$.
266. *A* 15 days ; *B* 10 days ; *C* 12 days. **267.** *Rs.*1061. 7*a*.
268. £272. 5*s*. **269.** 6 years. **270.** The quantities are in order of magnitude. **271.** $\cdot\dot{8}\dot{7}$. **272.** £75 7*s*. $9\frac{3}{4}$*d*.
273. £6. 7*s*. $10\frac{3}{4}$*d*. **274.** *A* 9 days ; *B* 18 days ; *C* 9 days.
275 *C* just passes. **276.** 10 ft. **277.** £3661. 13*s*. 4*d*.
278. $1\frac{1}{2}$ miles ; 1 hour. **279** *Rs.*50 ; *Rs.*75 ; *Rs.* 125.
280. Gain £17. 3*s*. 6*d*. **281.** $(3009)^2+(4012)^2=(5015)^2$.
282. 5· $\frac{1}{8}$. **283.** 23 : 68. **284.** *Rs.*7. 8*a*.
285. $18\frac{3}{4}$ p. c. **286** (i) ·779 ; 3 ; ·866.
287. 87 and $43\frac{1}{2}$ yds. **288.** 76 ac. **289.** £7. 15*s*. $7\frac{661}{1152}$*d*.
290. 13 ft. 7 in. **291.** Divisor 561 ; Quot. 943.
292. 1071 and 1547. **293.** $\frac{7}{9}$. **294** 276 tons. **295.** ·1115718.
296. $22\frac{1}{2}$ days. **297.** £1500. **298.** 4 p c.
299. £70. **300.** 2 miles per hour. **301.** 42.
302 256 ; £3. 7*s*. 6*d*. **303** 150000 yds. **304.** $687\frac{1}{2}$ rev.
305. $3\frac{37}{61}$ hours. **306.** The steamer ; 16 hours.
307. 32 days **308.** £871. **309.** $7\frac{7}{11}$ min or $57\frac{9}{11}$ min. past 6.
310. £2387. **311.** Divisor 779 ; Rem. 270. **312.** 11*s*. $7\frac{2}{13}$*d*.
313. $46\frac{821}{864}$ cub. ft. ; $10\frac{815}{864}$ cub. ft. **314** 40·75. **315.** $4\frac{16}{21}$ p c.
316. *A* and *B* each takes $74\frac{2}{17}$ days , *C* $157\frac{1}{2}$ days. **317.** £200.
318. *Rs.*$24\frac{202}{465}$. **319.** 3*s*. $9\frac{5}{11}$*d*. ; £1782. **320.** 5 min ; 10 hrs. 50 min. 16 sec. **321.** 7 ac. 2 ro. 28 po. 22 sq yds. 5 sq. ft. $113\frac{1}{7}$ in. **322** 2*s*. $7\frac{1}{2}$*d*. **323.** Zinc 1·512 tons ; lead 1·161 tons ; tin ·327 ton.
324 Land $2\frac{694}{12153}$ ac. ; iron 110 tons $6\frac{506}{1049}$ cwt. ; Int. = *Rs.*3090 13*a* $1\frac{1}{2}$*d*.
325. $\frac{1}{30}$ per min. **326** £214 5*s*. $8\frac{1}{2}$*d* $\frac{2}{7}$*q*. **327.** £16.
328. *Rs.*180. **329** 46980 **330.** 3 yrs 8 mo. 24 days.
331. 93324. **332** 200. **333.** 2880. **334.** 75
335. 8 days ; $12\frac{1}{4}$ days. **336.** (i) 1·3660. (ii) 2·3434. (iii) ·12702.
337. The $3\frac{1}{2}$ per cents. ; $28\frac{414}{137}$ **338.** £1. 1*s*.
339 *Rs.*344. 7*a*. 5·088*p*. **340** (i) £1260 (ii) £31010.
341. Horse *Rs* 350 ; Cow *Rs* 140. **342.** 240 sq. ft. **343.** *Re.*1. 12*a* 4*p*.
344 1 ft. 5 in. **345.** Man *Rs.*8. 5*a*. 4*p* ; woman *Rs.*2. 11*a*. 4*p*.
346. 104 boys. ; 46 girls. **347.** 48 days. **348.** 2·34375 per cent.
349. £ 30000. **350.** 12 per cent. **351.** 780, 468, 520 acres.
352. £718. 15*s*. **353.** *A* wins by $\frac{1}{4}$ mile. **354.** £15400.
355. 63 years **356.** 60 p. c. **357.** £92.
358. £8. 5*s*. $4\frac{1}{2}$*d* ; ·7875. **359.** $52\frac{1}{12}$ per cent. **360.** Yes ; side of cube = 15·9 ft. ; side of square = 194·4 in. **361.** 15 + 16 + 17 + 18.

362. 16·1... **363.** 8 : 9. **364.** $\cdot 8\dot{6}$; $2\frac{4}{11}$.
365. ·0061. **366.** (i) $\frac{3}{4}d.$; 20 p. c. **367.** 9 gals.
368. (i) 2 hrs. (ii) $1\frac{1}{4}$ hrs. **369.** $5\frac{173}{180}$ grs. **370.** £14440.
371. 82 ; 820 ; 1053. **372.** $2\frac{4}{5}$. **373.** 90 miles
374. *Rs.*8250 each. **375.** *Rs.*10210 4*a.* $0\frac{12}{25}p$. **376.** 54 times.
377. *B* walks a mile in $13\frac{1}{4}$ min. ; he loses by $11\frac{1}{4}$ min. ; and by $\frac{45}{58}$ of a mile. **378.** 45 days from the commencement.
379. One of the latter = two of the former. **380.** $\frac{1}{3}$.
381. 999663 ; 100203. **382.** 12 **383.** 1015.
384. 9 hours. **385.** 90 quarters. **386.** 6.
387. 3 ft. $11\frac{1}{18}$ in. **388** *Rs.*112. 6 *a.* **389.** $7\frac{1}{3}$ miles.
390. $13\frac{11}{13}$ min or $16\frac{4}{11}$ min. past 3. **391.** 13221, 12231, 15201, 10251, 14211, 11241, 15291, 16281, 17271, 18261, 19251. **392.** 1057.
393. (i) $6\frac{5}{6}$. (ii) 9. **394.** $1\frac{1}{2}$ ft. **395.** 15 masters ; 345 boys.
396. 11 : 2. **397.** £6000. **398.** 25.
400. Length 27 ft. ; breadth 18 ft. ; height 12 ft.
401. $2^4 . 3^3 . 5^2 . 7 . 11$; 118580. **402** *Rs.*4800.
403. 8 miles per hour. **404.** *Rs.*162. 8*a.* **405.** £15. 15*s.*
406. 660 ft. ; 495 ft. ; area = $7\frac{1}{2}$ ac. **407.** 35 days.
408. One orange 6*p.* ; one peach 3*p.* ; one mango 12*p.* or 1*a.*
409. 13·6801 cub. yds. **410.** £$94\frac{7}{8}$. **411.** Multiplier 4253.
412. 45 times. **413.** 3. **414.** *Rs.*27. 15*a.* $0\frac{38}{43}p$.
415. 25 p. c. loss. **416.** 30 min. $41\frac{37}{43}$ sec. **417.** $8\frac{3}{4}$ days.
418. 12 ft. $7\frac{1}{2}$ in. **419.** $3\frac{1}{2}$ p c. ; £275. 10*s.* **420.** 6 mo.
421. 48 days. **422.** Brus. : Kid. :: 9 : 8. **423.** 11*s.* 3*d.* ; 5.
424. *A* ought to pay *Rs.*34. 2*a.* 8*p.* ; *B* *Rs.*44. 2*a.* 8*p.* ; *C* *Rs.*50. Rent per acre per annum = *Rs.*55. 9*a.* $9\frac{1}{3}p$.
425. *A*'s *Rs.*1295 ; *C*'s *Rs.*3885. **426.** £6. 15*s.*
427. *A* $10\frac{29}{30}$ miles ; *B* $10\frac{8}{15}$ miles. **428.** 18 ft. **429.** $8\frac{1}{3}$ in.
430. Thrice. **431.** 45 sq. miles. **432.** 230 stones. **433.** $3\frac{4}{5}$ days.
434. 20 yds. ; 60 coolies. **435.** Dist. 150 miles ; rate 25 miles per hr.
436. $16\frac{1}{2}$ hours. **437.** 50. **438.** *Rs.*3. $9\frac{1}{15}a$. **439.** *Rs.*3 ; *Rs.*2. 8*a.* ; *Rs.*2. **440.** 17*s.* 6*d.* **441.** ·28284 mile.
442. 8 yds. **443.** *Rs.*$38\frac{332}{741}$. **444.** £4800.
445. 1 : 8. **446.** $9\frac{23}{25}d$. **447.** 33·24 in. **448.** $49\frac{1}{2}$ p. c.
449. (i) $2\frac{1}{4}$ min. (ii) $2\frac{4}{13}$ min. **450.** *Rs.*489. 10*a.* 8*p.*
451. 13212. **452.** 640 and 350. **453.** $\cdot\dot{6}$. **454.** ·25.
455. 52 days. **456.** *Rs.*3312 + *Rs.*219. **457.** *Rs.*7678. 2*a.* ; 10*a.* 2·85*p.*
458. 2 seconds. **459.** 500 cub. ft. **460.** $23\frac{529}{627}$ miles per hour.

Problems (pp. 558-582.)

1. £275. **2.** $\frac{6}{13}$ of a mile. **3.** *Rs.*400. **4.** $1166\frac{2}{3}$; 1169 ; 1000 ; 1002.
5. 397 yds. per min. **6.** 4*a.* 8*p.* ; 8*p.* **7.** £2394. **8.** £1000.
9. *Rs.*9. 7*a.* 3*p.* **10.** $36\frac{2}{7}$ miles per hour ; 37 min. past 8 A. M.
11. *Rs.*$4942\frac{8}{41}$. **12.** $23\frac{1}{2}$ months, *nearly*. **13.** £2 per cent.
14. *Rs.*2933. 5*a.* 4*p.* **15.** 18 miles per hour. **16.** The first..
17. 360 gals. ; gain 1 gal. per hour. **18.** £125000. **19.** 10 per cent.

20. £463. 1*s*. **21.** $6\frac{2}{3}$ min. **22.** $3\frac{3}{4}$ min. ; 10 miles an hour.
23. 3·231 in. **24.** $1437\frac{1}{2}$ bricks. **25.** £1000.
26. £322 ; £627.4*s*. ; £2060. 16*s* **27.** 3016. **28.** $34\frac{1}{8}$ miles.
29. 8*s* 4*d*. **30.** $998\frac{2}{15}$ oz. **31.** $8\frac{1}{3}$ min. **32.** $16\frac{2}{3}$ miles.
33. 1 o'clock. **34.** 6 hr 30 min P. M. **35.** £2000.
36. 50000000 quarters. **38.** £5. 17*s*. **39.** *Re*.1. 8*a* ; *Re*.1.; 12*a*.
40. *Rs*.80. **41** 51 men. **42.** Gain £2. 16*s*.
43. 7496192000 cub. yds. **44.** 65 per cent. **45.** 12^{*} weeks.
46. $10\frac{1}{8}$ hrs. **47.** 291600 **48.** *Rs*.6000 **49.** 55·5 yds. *nearly*.
50 *Rs*.850 ; 26 p. c. **51.** 7 men. **52.** *Rs*.95. **53** *Rs*.144.
55. 8960 in. **56** *A*'s cap. *Rs*.450000 ; *B*'s cap. *Rs*.300000 ;
A's profit *Rs*.19500 ; *B's Rs*.13000. **57.** 112 days
58. *A* $16\frac{1}{2}$ rounds ; *B* 17 rounds ; 405 hrs. **59.** 5 gals.
60. Express 2 hrs. 13 min ; Ordinary 3 hrs. 10 min.
61 1300 ; no. in stalls 300 ; boxes 500 ; pit 400 ; gallery 100.
62. *A* 4*s*. $2\frac{1}{2}$*d*. ; *B* 6*s* $7\frac{1}{2}$*d* ; *C* 1*s*. 9*d*. **63.** 26 men.
64. £38. **65.** *Rs*.$549\frac{61}{101}$. **66.** 18942 pipes.
67. $4\frac{5}{13}$ min. spaces. **68** $16\frac{3}{7}$ min. **69.** 123·2 grs.
70 10-30 P. M. yesterday ; 10-$30\frac{5}{6}$ P. M. ; 45 days later.
71. 133. **72.** 12 hrs. **73.** *B* ; $\frac{1}{512}$. **74.** £19. 14*s*. $3\frac{3}{7}$*d*.
75. 33·75 dollars. **76.** 3*s*. 4*d*. and 5*s*. 4*d*. **77.** £121.
78. $4\frac{1}{3}$ days. **79.** Equal. **80.** 9 miles 1035 yds.; 9 miles.
660 yds. **81.** 15 min. 15 sec. past 11. **82.** £26. 7*s*. $7\frac{5}{8}$*d*.
83. $2\frac{8}{11}$ sec. **84.** (1) 20·7 yds. (ii) 120·7 yds. **85.** Rad of inner circles=31·0813...ft.; rad. of 2nd circle=46·622...ft.
86. 253 : 377. **87.** $\frac{81}{256}$. **88.** 4 mds. 8 sr. **89.** 1000 ft. **90** 8*s*.
91. 135 days. **92.** 121 yds. ; $6\frac{7}{8}$ sec.
93. 15 men ; 18 women ; 27 children. **94** 14*s*. 6*d*. and 15*s*. 6*d*. per gal.
95. 8 horses. **96.** $2\frac{3}{4}$ miles. **97.** 352 ft. slower ; 264 ft. faster.
98. 2 men. **99.** £1334 11*s*. $9\frac{4}{5}$*d*. **100** £1782. **101.** 12 days.
102. 10 ft. **103** $7\frac{13}{81}$ seers. **104** 15 lbs. 10 oz.
105. No. of fruits, 288 ; no. of two anna pieces, 287. **106.** 230 min.
the boys bringing pint and gallon coming up 1 min. too late.
107. $11\frac{71}{305}$ sec. **108.** 14·076 min. ; $\frac{1593}{2930}$ of the cistern.
109. *Rs*.12600. **110.** $\frac{1}{2}$*d*. **111.** *Rs* 8 ; *Rs*. 16 ; *Rs*.24.
112. £72. 6*s*. 8*d*. **113.** *Rs* 8 ; *Rs*.16. **114** $10\frac{1}{2}$ pies.
115. £556. 10*s*. **116.** 21 ft. **117.** First class, *Rs*.5 ; 2nd class
Rs.3. 5*a*. 4*p*. ; rate per mile=1*a*. 4*p*. **118.** 10 miles.
119. *Rs*.400. **120.** *Rs*.387. 8*a*. **121.** 12 min. before 9 o'clock ;
12 hours 24 min. **122.** *Rs*.106666. 10*a*. 8*p*. **123.** *Rs*.18984. 6*a*.
124. *A Rs*. 162 ; *B Rs*. 324 ; C *Rs*.405 ; *D Rs*. 486 ; *E Rs*. 648.
125. 2 ft. **126.** $1\frac{1}{2}$ yds. **127.** 22 miles. **128.** £5565. **129.** $10\frac{1}{3}$ in.
130. *Rs*.5376 ; *Rs*.3584. **131.** $7\frac{2}{3}$ months. **132.** $9\frac{1}{21}$ miles per hour.
133. £1 11*s*. 3*d*. **134.** 14 cwt. 3 qrs. 13 lbs. ; 9 cwt. 3 qrs. 18 lbs.
8 cwt. 3 qrs. 19 lbs. **135.** 6 min. 6·08 sec.
136. 6 miles per hour. **137.** £$9309\frac{1}{11}$; £$271\frac{7}{15}$; £$10\frac{8}{15}$.
138. £264. 6*s*. 8*d*. **139.** Gold $46\frac{7}{8}$ lbs. ; silver 64 lbs. ; coppex $6\frac{1}{8}$ lbs.

140. Width = 13 ft. 2 in. ; length = 271 ft. 5 in. ; height = $14\frac{17778}{87808}$ ft.
141. (i) 2hrs. $37\frac{1}{2}$min. (ii) 2hrs. $11\frac{1}{4}$ min. ; 3hrs. $3\frac{3}{4}$ min. **142.** 37800.
143. *Rs.*1000. **144.** 14 years. **145.** For ploughing the field with oxen, £4. 7*s.* 9*d.* ; for ploughing with horses, £3. 18*s.* $9\frac{3}{4}$*d.*
146. *Rs.*127. 8*a.* **147.** £60 ; $7\frac{9}{13}$ p. c. **148.** *A Rs.*10500 ; *B Rs.*7000 ; *C Rs.*7350. **149.** 6 miles. **150.** 15*s.* 6*d.*
151. 15 oxen. **153.** £150. 15*s.* **154.** £97826$\frac{2}{23}$; £459$\frac{18}{23}$.
155. $7\frac{1}{3}$ hours. **156.** 62 hrs. 33 min. 45 sec. ; 48 min. 45 sec.
157. 25 min. $18\frac{3}{4}$ sec. **158.** 3 hrs. 25 min.
159. They first cross at $\frac{20}{31}$ mi. from starting-post, *A* going towards it ; they next cross at $\frac{10}{31}$ mi. from starting-post, *A* going from it ; they next cross at $\frac{16}{31}$ mi. from starting-post, *A* going towards it ; and so on. **160.** $1\frac{1}{2}$ miles ; 2 hrs. **161.** 2*a.*
162. £147. **163.** First $11\frac{3}{7}$ p. c. ; 2nd $8\frac{4}{7}$ p. c. ; 3rd $8\frac{4}{7}$ p. c.
164. 10 times ; $1\frac{4}{5}$ hrs. or $\frac{3}{5}$ hr. **165.** *Rs.*525. **166.** 10 in.
167. $5\frac{18}{23}$ miles from Dover ; 6 o'clock. **168.** 4 miles $73\frac{1}{3}$ yds.
169. (i) *A*'s work : *B*'s work :: 3 : 2 ; *A*'s rate : *B*'s rate :: 2 : 1. (ii) *A*'s work = 0 ; *A* does nothing.
170. £1. 10*s.* $1\frac{1}{2}$*d.* **171.** 13 : 20. **172.** 19 miles.
173. £1640. 12*s.* 6*d.* **174.** $266\frac{2}{3}$ per cent. **175.** 1114·4826 oz.
176. *Rs.*2170. **177.** 13 times. **178.** *A* 90 min. : *B* 72 min.; *C* 60 min.
179. At the summit. **180.** £2000. **181.** £7. 11*s.* 3*d.*
182. 20 miles an hour. **183.** $94\frac{1}{2}$ miles from Arconum.
184. 200 years. **185.** First day $1\frac{5}{8}$*d.* ; second day $\frac{39}{40}$*d.*
187. 75 days. **188.** 1 mile ; 6 : 5. **189.** (i) £5. 5*s.* (ii) $30\frac{1}{12}$ days.
190. *Rs.*4000. **191.** Length = 1 ft. 11 in. ; breadth = 1 ft. 5 in. ; depth = 1 ft. 1 in. ; 2236 cases. **192.** $90\frac{90}{139}$ days.
193. *A Rs.*60 ; *B Rs.*70 ; *C Rs.*84 ; *D Rs.*105 ; *E Rs.*101.
194. *A* and *C* each travels 8 miles per hour.
195. 24 men, 20 women and 15 boys. **196.** 1 hr. 21 min. $18\frac{1}{22}$ sec.
197. $4\frac{1}{2}$ years. **198.** (i) 48 hrs. (ii) 54 hrs. **199.** $\frac{5}{168}$.
200. *Rs.*180. **201.** $27\frac{29}{33}$ days. **202.** Indian ; *Rs.*4500. **203.** 12 cub. ft.
204. 20 and 30 miles per hour. **205.** $30\frac{18}{41}$ and $17\frac{18}{41}$ miles per hour.

EXAMINATION PAPERS.

CALCUTTA ENTRANCE PAPERS.

85. **1.** $\frac{13}{5}$; $\frac{87}{650}$. **2.** ·12 ; ·2 ; ·30472... **3.** $3·\dot{4}6153\dot{8}$; £1. 10*s.*
4. £513. 6*s.* $6\frac{1}{4}$*d.* ; 3·1225, ·2828. **5.** 18 ; $8\frac{9}{13}$ per cent.
6. The first investment is better ; £1342. 10*s.* : $3\frac{68}{175}$ per cent.
86. **1.** $\frac{875}{972}$. **2.** $\frac{209}{6000}$. **3.** $\frac{158}{761}$; $\frac{279}{800}$. **4.** ·5 ; $·0011\dot{3}\dot{6}$.
5. £36. 17*s.* 6*d.* **6.** R28659. 6*a.* **7.** R12-12-$9\frac{3}{5}$; gain R133-5-4.
87. **1.** (*a*) $\frac{1}{8}$; (*b*) 350. **2.** ·0203125. **3.** (*a*) £17-12-$2\frac{1}{4}$; (*b*) R2000.
4. 10. **5.** R510. **6.** 13·31 ; ·471...
88. **1.** $\frac{3}{17}$. **2.** 11200 ; 37·96. **3.** $138·449\dot{7}\dot{1}$; £20. 16*s.* $9\frac{7}{15}$*d.*
4. £1034. 14*s.* $4\frac{18}{25}$*d.* **5.** $15\frac{5}{8}$ days. **6.** $6\frac{1}{4}$; £100.

89. 1. 51·59139412. 2. 8·62126... 3. £5247. 2*s*. $6\frac{29}{84}d$.
4. 1·000127. 5. £6705. 14*s*. 7*d*.

90. 1 3 ; R23931. 7*a*. 7*p*. 2. 7305·$\dot{4}$0$\dot{5}$; $\frac{189}{370}$. 9. R1771.
4. 60 days. 5 R104. 4*a*.

91. 1. (*a* $\frac{18}{35}$; (*b*) $\frac{3}{4}$. 2. 2·202642. 3. R408. 3*a*. $4\frac{11}{12}p$.
4. $9\frac{9}{13}$ hrs. 5. R19992. 6. $8\frac{1}{3}$ yds.

92. 1. $\frac{79}{135}$. 2. 26219. 3. ·312 ; ·098 ; ·998.
4. R1232. 14*a*. $0\frac{24}{73}p$. 5. £2500.

93. 1. (1) $5\frac{67}{140}$. (2) 3. 2. ·0789 ; $\frac{226}{495}$; $\frac{38}{55}$; 1·$\dot{1}$. 3. £315. 7*s*. $3\frac{3}{4}d$.
4. R238. 3*a*. $2\frac{6}{7}p$. 5 R90000 ; R73000.

94. 1. £37. 0*s*. $8\frac{1}{4}d$. 2. £491. 8*s*. 3. 16*s*. 0·37$\dot{5}$01$\dot{3}$*d*.
4. ·9998... 5. R6 each. 6. R124-10 $1\frac{247}{1825}$.

95. 1. 1·00001. 2. R12345. 3. 3 fr 84 centimes.
4. $\frac{7}{12}$. 5. R47, increase ; ·68$\dot{3}$287671$\dot{2}$.

96. 1. 23704543, 8143. 2. (i) $\frac{5}{12}$. (ii) ·075088. 6 2·2677...
4 R531. 3*a*. 10·064*p*. 5. Loss, 1·6 p c 3. R21735.

97. 1. ·0725. (*a*) $\frac{2}{15}$. 2. Yes ; $\frac{1}{320}$ part ; *Rs* 32. 9*a*. $1\frac{1}{3}p$.
3. 20. 4. $33\frac{1}{3}$ years 5. *Rs*.6. 6. 1 7724 ..

98. 1. 20150. 2. (*a*) $\frac{11}{18}$. (*b*) ·08$\dot{3}$. 3. 234 ; 8·06007...
4. *Rs*.250. 5. $1\frac{1}{2}$ per cent. gain. 6. *Rs*.23400.

99. 1. 25. 2. (*a*) $1\frac{11}{18}$. 3. *Rs*.606 11*a* $9\frac{3}{4}p$.
4. 226 ; ·226. 5. 60 mds. ; $3\frac{13}{28}$ p. c. 6. *Rs*.13.

1900. 1. 42 min. 2. (*a*) 6 (*b*) $\frac{1}{24}$. 3. 8.
4 £55. 10*s*. $3\frac{17}{72}d$. 5. 125. 6. £100 gain.

1901. 1. (*a*) 1·41$\dot{6}$. (*b*) ·565. 2. (*a* Yes ; $\frac{1}{840}$. (*b*) £68. 15*s* 9*d*.
3. $4\frac{1}{2}$ lbs. 4. (*b*) 4 p. c. 5. 86·42. 6. *Rs*.122169.

1902. 1. (*a* Terminating decimal. (*b*) $\frac{1}{28}$; ·0$\dot{3}\dot{6}$.
2. *Rs*.15326. 10*a*. 8*p*. ; *Rs* 7340. 3. 35 b ys.
4. $2\frac{1}{4}$; 8·729. 5 (*b*) £100. 6 4 p. c ; *Rs*.60.

1903. 1. (*a*) 1. (*b*) ·00056$\dot{8}\dot{1}$. 2. (*a*) Yes ; $\frac{1}{840}$. (*b*) £170. 19*s*. $4\frac{1}{2}d$.
3. 1 min. 40 sec. 4. (*b*) $3\frac{1}{8}$; 1 7728. 5. $\frac{13}{48}$ gal. 6. (*b*) *Rs*.50.

1904. 1. (*a*) 997920. 2. $\frac{0}{10}$ or 0, $\frac{1}{100}$, $\frac{0}{1000}$ or 0, $\frac{2}{10000}$, $\frac{0}{100000}$ or 0, $\frac{3}{1000000}$. (*a*) $\frac{1}{2}$ 3. $22\frac{1}{2}$ days. 4. 39·6 poles. 5. £700.
6. *Rs*.151710.

1905. 1. 165 or 154, 176, 187,... 2 (2) $\frac{12}{49}$. 3. 4·4666 ; ·7905.
4. *Rs*.4. 1*a*. 5. *Rs*.110. 7*a*. $2\frac{1}{4}p$. ; 3 per cent. 6. £3000.

1906. 1. (2) 99679. 2. (2) (1) 1. (2) 2. 3. *Rs*.17. 2*a*.
4. ·1$\dot{6}$; ·7745 or ·7746 *nearly*. 5. 17*s*. 6*d*. 6. $11\frac{1}{9}$ p. c.

1907. 1. 37128. 2. (1) $\frac{1}{9}$: (2) $\frac{1}{40}$. 3. *Rs*.46. 11*a*. $10\frac{43}{50}p$.
4. 110400. 5. *Rs*.760. 8*a*. 6. 13*s*. $3\frac{80581}{88881}d$.

1908. 1. (2) *Re*.1. 8*a*. 2. £934 18*s*. 2*d*. 3. (1) $11\frac{109}{121}$ min ;
(2 $7\frac{1819}{9488}$ yds. 4. (1) $2\frac{8}{23}$ (2) ·5345.
5. *Rs*.1000. 6. *Rs*.425.

1909. 1. 2865149859 ; 385671. 2. (1) 7. (2) 65.
3. *Rs*.939. 13*a*. 6*p*. III. ~~371·137~~. 4. *Rs*.1228. 2*a*.
IV. £588. 10*s*. 10*d*. 5. 1947, 3, 11, 59, 33, 177, 649.
V. 20 p. c.

Supplementary Examination.

1. 786522510 ; 12837. 2. (*a*) $\frac{4}{21}$. (*b*) $10\cdot58\dot{3}$. 3. *Rs.*8833. 5*a.* 4*p.* III. *Rs.*1080. 4. 12. IV. $8\frac{1}{2}$ yrs. 5. 13579. V. *Rs* 300.

MATRICULATION EXAMINATION.

1910

Compulsory Paper.

1. 326137981294 ; 756445. I. 504 ; 17280. 2. (1) $15\frac{8}{10}$ (2) $\cdot0\dot{0}\dot{9}$. II. 40 men. 3. (1) *Rs.*3816. 10*a.* 8*p.* (2) $16\frac{2}{3}$ yrs. III. 2·115 cubic inches.

Additional Paper.

1. 2501317. I. 15 ft. 2. (1) 1·718. (2) 24855·2967.

1911.

Compulsory Paper.

1. 620526133723107 : Quot. 621225011, rem. 860482. (Alt.) *Rs.*610
2. (1) 1. (2) $\cdot005208\dot{3}$ of a rupee. (Alt.) $\cdot0002\dot{7}$ of an hr.
3. (1) *Rs.*31. 15*a.* 5*p.* (2) £227. 12*s.* (Alt.) 180 men.

Additional Paper.

1. 469246. (Alt.) $54\cdot03\dot{2}1$. (Alt.) 579.
2. (1) 3·14159. (Alt.) ·78539..... (2) 28.

1912.

Compulsory.

1. 573299983476 ; 3276. (Alt.) $\frac{88}{119}$.
2. (1) $\frac{7}{12}$; (2) £12. 13*s.* 2*d.* (Alt.) (1) $3\frac{1}{4}$ p. c. ; (2) *Rs* 442. 7*a.* $7\frac{1}{2}$*p.*
3. 25 men. (Alt.) 2624 sq. ft. ; *Rs.*95. 10*a.* 8*p.*

Additional.

1. 371·173. (Alt.) 117 ft. 2. 24855. (Alt.) ·54925.

1913.

Compulsory.

1. (1) 38590032276 ; (Alt.) 6907 ; *Rem.* 587. (2) 504 ; (Alt.) 1890.
2. (1) $\frac{1}{8}$. (2) ·2907. (Alt.) (1) ·0015625 ; (2) *Rs.*2362. 8*a.*
3. (1) 3 p. c. (2) 28 yds.

Additional.

1. 5·403 ; (Alt.) *Rs* 366. 10*a.* 8*p.*
2. 239·197... ; (Alt.) 71, 73, 79, 83, 89.

1914.

Compulsory.

1. $278523\frac{81}{879}$.   2. 247 ; (Alt.) 2160.

3. $1\frac{63223}{67520}$; (Alt.) *Rs.*14650. 3*a.* $6\frac{3}{4}p$.

4. 45·408*d.* ; (Alt.) *Rs.*564. 5*a.* 4*p.*

Additional.

1. 2·645.   2. $20\frac{12560620}{15499969}$.

1915.

Compulsory Paper.

1. (1) 75154060188. (Alt.) 7908.   (2) 504. (Alt.) 28000.

2. (1) $\frac{2}{3}$. (2) 70·2702 and 85·8. (Alt.) (1) $11\cdot 9\dot{3}8461\dot{5}$.
   (2) *Rs.*5615. 9*a.* 3*p.*

3. (1) 10 p. c.   (2) *Rs* 3072.

Additional Paper.

1. 13·057.   2. 15499969 slabs. (Alt.) ·41937.

1916.

Compulsory Paper.

1. (1) 55[illegible]35194285. (Alt.) 83205.   (2) 119. (Alt.) 2520.

2. (1) 1. (2) $\cdot 0041\dot{6}$. (Alt.) (1) ·1035546875.   (2) £173. 8*s.*

3. (1) $12\frac{1}{2}$ p. c.   (2) 60 men.

Additional Paper.

1. ·06435.   2. $\cdot 0972838130\dot{0}571428\dot{}$. (Alt.) 1·732.

1917.

Compulsory Paper.

1. (*a*) 272428968896. (Alt.) 101793.
   (*b*) 756. (Alt.) 29 yds. 2 ft. 3 in.

2. (*a*) $\frac{3}{4}$. (Alt.) 5*s.* $0\frac{1}{4}d$.   (*b*) 30. (Alt.) $\cdot 58\dot{3}$.

3. (*a*) *Rs.*106. 7*a.* (Alt.) *Rs.*37. 2*a.* 8*p.*
   (*b*) $12\frac{1}{2}$ p. c. (Alt.) 23 p. c.

Additional Paper.

1. 7589. (Alt.) 1414 m. m.   2. 1·6487. (Alt.) 1·2499.

1918.

Compulsory Paper.

1. (1) 129601503654. (Alt.) 157.
   (2) 5910 cisterns filled and 33 gallons left.

2. (1) $2\frac{1}{2}$. (2) 2·4.
3. (1) £2. 2s. (Alt.) £12 15s. $11\frac{1}{4}$d.
(2) Rs.1350. (Alt.) 20 days.

Additional Paper.

1. (1) 31623. (2) 1·875 metres by ·625 metres.
2. ·5750. (Alt) ·3770.

1919.

Compulsory.

1. (1) 60843854981484. (2) 391. Alt. (1) 50709.
(2) 104329. 2. (1) Rs.2. 13a. $2\frac{2}{3}$p. (2) ·00027.
3. (1) £43 15s. $10\frac{1}{2}$d. (2) 4 years.

Additional Paper.

1. 554·2001. Alt. 1·024. 2. ·2836. Alt. Rs.73. 12a.

1920.

Compulsory.

1. (1) 2728005955734400. (2) 97. Alt. (1) 6079. (2) 2520.
2. (1) 1. (2) $\frac{1}{24}$; ·0416. 3. (1) £84. 9s. $4\frac{1}{2}$d. (2) Rs.5000.

Additional Paper.

1. ·1824804. Alt. ·0859. 2. 9 hours. Alt Rs.80000.

1921.

Compulsory Paper.

1. (1) 4596969201. (2) 133. Alt. (1) 67089. (2) 6 mins.
2. (1) (i) $\frac{1}{24}$, (ii) ·0416. (2) 60 ft.
3 (1) Rs.317. 4a. $6\frac{3}{4}$p. (2) $5\frac{1}{2}$ per cent. per annum.

Additional Paper.

1 5·378...cm. Alt. 1s. 11d. $1\frac{5597}{82928}$q. (more in England). 2. ·0259.
Alt. A—£1139.
B—£ 438.
C—£ 275.

MADRAS MATRICULATION PAPERS.

85. 1. $\frac{1}{3}$. 2. $\frac{95}{192}$, Rs.64. 3. £1·6-$0\frac{1}{2}$. 4. £19-3-$10\frac{7}{8}$.
5. 11s. $10\frac{1}{2}$d. 6. R1920. 7. 3s. 9d. 8. 4 yrs.
9. £5000. 10. ·9196 ; £16. 10s. 11. 3500000.

86. 1. 1. 2. ·9705. 3. £2-11-$7\frac{3}{4}$. 4. Rs.3955-3-11.
5. 1-13 P. M., 2nd July. 6. 80 men. 7. £1000.
8. £180. 9. R17. 8a. 10. 520344000 cub. ft. ; $1\frac{47}{88}$ in.

88. **2.** $\frac{3}{4}$. **3.** 1*a*. 4*p*. ; ·11458$\dot{3}$ **4.** £721-15-6$\frac{9}{16}$
5. ₨335000. **6.** £416. 13*s*. 4*d*. **7.** ₨7. 2*a*.
8. Increase, ₨502. 8*a*. **9.** ₨500000. **10** 500400.
89. **2.** $\frac{241}{261}$. **3.** ·0827$\dot{3}$02$\dot{9}$; 6*s*. 9$\frac{3}{4}$*d*. **4.** ₨1730-13-6$\frac{1}{4}$. **5.** ₨48. 2*a*.
6. £1694-13-9. **7.** ₨280000. **8** 10 days.
9. 7500274. **10.** £39-3-9. **11.** 2·08008.
90. **1.** 342 ac. 2 ro. 39 po. 2 sq. ft. 36 sq. in. : 160 yds. **2.** 1·5.
3. ₨975358 9-2$\frac{1}{5}$ **4.** 30 wks. **5** ₨6744273. **6.** 4 mo
7. Increase, £397. **8.** 12 cwt. 1 qr. 19 lbs. 4 oz. ; £33. 2*s*. 6*d*
9. 343 : 169. **10** 19487 171.
91. **2.** ₨1-11-8. **3.** ₨1-10-2$\frac{41}{181}$. **4.** 9 ; 46·94$\dot{7}$17$\dot{7}$.
5. 12 min **6** ₨6-6-4 ; ₨158. **7.** £291 9 5$\frac{1}{4}$ *nearly*
8. 20 min. (afternoon). **9.** 10*d*. **10.** ₨9180.
92. **2.** $\frac{3}{4}$. **3.** 5*s*. 3*d*. ; ·0037115625. **4.** ₨67567-9-7$\frac{1}{8}$.
5. £416-13-4. **6.** 3·700965. **7.** 3221625 tons.
8. ₨355-13-4. **9.** ₨55-8-4. **10.** 3$\frac{3}{35}$ p.c. **11.** 25640000.
94. **2.** $\frac{1}{4}$. **3** 5*s* 2$\frac{18477}{18000}$*d*. **4.** ₨1593. 4*a*. 5 6*p*.
5. ₨93333 5*a*. 4*p*. **6.** £976. 11*s*. 3*d*. **7.** ₨23 12*a* 4*p*.
8. 4$\frac{1}{2}$ mi. per hour. **9.** ₨1062. **10.** 14·1625 p c.
11. 9·0073210.

BOMBAY MATRICULATION PAPERS.

85-86. **1.** ·$\dot{4}$85714$\dot{2}$. **2** 113 boys. **3.** 1$\frac{1}{3}$ cwt. ; £1. 1*s*. 6*d*.
4. £72. 6*s*. 8*d*. **5.** In the latter ; £457. 5*s*. 1$\frac{4}{5}$*d*.
86-87. **1,** 5,7,11,13 ; 22·999$\dot{9}$8912084530386257958$\dot{8}$ **2.** 192$\frac{1}{2}$ ft.
3. $\frac{1}{5}$. **4.** (i) 7 ft. 2 in. (ii) 3·5752. **5.** £5103. **6** 3$\frac{179}{187}$.
87-88. **1.** 6. **2.** 19*s*. 3*d*. **3.** 20 months. **4.** 20 : 7 ; 5*s*. 1$\frac{1}{2}$*d*. **5.** 5*a*. 4*p*
88-89. **1.** 10*d*. ; 4*s*. 3$\frac{1}{4}$*d*. **2.** 8*s*. 4*d*. **3.** 108*s*.
4. 94298·11. **5.** 4 p.c.
89-90. **1.** 2. **2.** 5$\frac{2}{5}$ ft. × 5$\frac{3}{5}$ ft. × 5$\frac{5}{7}$ ft. **3.** 5-15′.
4. ₨32000. **5.** 3 : 13.
91-92. **1.** (i) $\frac{1}{8}$. (ii) ·8$\dot{3}$. **2.** 100 lbs.; 2 cwt., 3 cwt.
3. 10-30 P. M. yesterday ; 10 h. 30′ 50″. **4.** ₨640.
92-93. **1.** ·0050208$\dot{3}$; 15*a*. 7·44*p*. ; $\frac{1}{7}$. **2.** 10. **3.** 3$\frac{1}{2}$ hrs.
4. £259. 3*s*. 5$\frac{1}{4}$*d*. **5.** 401 : 544.
93-94. **1.** 20577 ; 39690 ; ·5184429327286470143613OOO7......
(Mof.) **2.** £11. 11*s*. 6$\frac{3}{4}$*d*. **3.** 2 cwt. 2 qrs. 20 lbs. **4.** £83. 6*s*. 8*d*
5. 12$\frac{1}{2}$ hrs. **6.** 4 lbs. of the infr. to 5 lbs. of the supr. quality
93-94. **1.** (i) 24 ; (ii) $\frac{58}{115}$. **2.** £32 14-3$\frac{6}{7}$. **3.** 3$\frac{9}{13}$ mo.
(Bom.) **4.** £3-2-0$\frac{24}{29}$. **5.** $\frac{19}{24}$. **6.** 2 P. M., 23 Aug.; 1-46′,2-16′
94-95. **1.** 146097 days (actual 11 days less). **2.** 156. **3.** 30.
4. 1$\frac{1}{15}$ days. **5.** ₨2160. **6.** 7·72 ; 15*a*. 3*p*.

PUNJAB ENTRANCE PAPERS.

85. **1.** 1 ; 123 times. **2.** ·08125 ; ·00$\dot{0}\dot{3}$; ·$\dot{0}$3896$\dot{1}$. **3.** R884. 1$\dot{5}$*a.* 3*p.* **4.** £9. 15*s.* **5.** 2963520 ; 2420 sq. yds. **6.** The latter ; R49000.

86. **1.** ·375 ; ·612. **2.** ·7895. **3.** $\frac{1}{40}$. **4.** She loses. **5.** *Rs.*195. 13*a.* 0·96*p.* ; *Rs.*172. 8*a.* $5\frac{17}{227}$*p.*

87. **1.** (*c*) 3·025. **2.** £133. 6*s.* 8*d.* **3.** R9. 6*a.* $2\frac{1}{4}$*p.* a md. **4.** *Rs.*480, *Rs.*405, *Rs.*336, *Rs.*297. **5.** *Rs.*11250. **6.** ·04.

88. **1.** $1\frac{4}{25}$. **2.** ·0625. **3.** 16 days. **4.** $16\frac{52}{103}$ p. c. **5.** 36·284.

89. **1.** ·32208$\dot{3}$; 7·99. **2.** $2\frac{3}{4}$ miles. **3.** 3·76$\dot{5}\dot{1}$; 2 min. 6 sec. **4.** R14. 8*a.* $6\frac{6}{11}$*p.* ; the latter. **5.** $\sqrt[3]{3}$ is the greater.

90. **1.** (*a*) 1 ; (*b*) ·0$\dot{3}$. **2.** $\frac{8}{75}$; ·390625. **3.** £1274. **4.** 8320 men. **5.** R$86\frac{31}{54}$. **6.** 2*p.*

91. **1.** (i) $1\frac{1}{24}$. (ii) 11. **2.** ·00622... **3.** 10 years. **4.** £200 ; 5 yrs. **5.** 45 gals.

92. **1.** $\frac{1}{761}$; $\frac{355}{113}$. **2.** 5·90625. **3.** 218972·16 gals. **4.** £$10166\frac{2}{3}$; £6000. **5.** 17*s.* 3*d.*

93. **2.** 13·713729902. **3.** ·41421. **4.** R17-5-$9\frac{1}{3}$. **5.** 32 mi.

94. **1.** ·$\dot{5}$71428$\dot{8}$; ·$\dot{4}$2857$\dot{1}$. **2.** 44 ft. ; 33 ft. **3.** £511. **4.** R987-10-$5\frac{17}{27}$; R987. 8 *i*. **5.** 10·04987.

95. **1.** $1\frac{1}{14}$, 1·0$\dot{7}$1428$\dot{5}$. **2.** 1·25 lbs. **3.** £391, £529, £1311. **4.** 2·2360680. **5.** 76·2 yds. ; 38·1 yds.

96. **1.** *Rs.*63. 10*a.* $2\frac{2}{5}$*p.* **2.** $\frac{65}{333}$. **3.** 2·2136 lbs. **4.** 11400000 ac. ; 17100000 ac. ; 3800000 ac. **5.** *Rs.*3930.

97. **2.** Men £63, women £60, children £54. **3.** 2·4142136. **4.** $14\frac{2}{7}$ weeks. **5.** 540.

98. **1.** 2 thousand million, 9 hundred thousand, 9 ten-thousand, 1 hundred, 2 tens, 5 units ; 107·0129615898. **2.** Monday. **3.** *Rs.*66. 10*a.* 8*p.* **4.** 1. **5.** *Rs.*66. 10*a.* 8*p.* gain.

99. **1.** ·764. (*a*) 102·1916$\dot{3}$49780924038$\dot{8}$. **2.** 6 yds. **4.** 4 sov. ; 6 h. cr. ; 22 sh. **5.** The $3\frac{1}{2}$ per cents.

1900. **1.** 2000·301. **2.** *Rs.*7352. $15\frac{1}{17}$*a.* **3.** 10 yds. by 22 yds.

1901. **1.** 7 hrs. **2.** £5. 0*s.* 4*d.* **3.** 32490 ; 20577 ; 1·579... **4.** 12 per cent. **5.** 79 : 49.

1902. **1.** 3.5.7.11.13.37. **2.** 5760. **3.** *Rs.*4800. **4.** *Rs.*2. 7*a.* 9*p.* **5.** 15 per cent.

ALLAHABAD ENTRANCE PAPERS.

89. **1.** 1·$\dot{3}$8461$\dot{5}$. **2.** (*a*) $3\frac{8}{11}$. (*b*) ·00$\dot{0}\dot{3}$; ·0029644268··· **3.** ·0316 ; ·01. **4.** R1300. **5.** 12 min. $40\frac{10}{11}$ sec. **6.** 8 days. **7.** $39\frac{3}{8}$ miles from the starting place.

90. **1.** ·697684787; 36·62387$\dot{3}$. **2.** $\frac{1}{24}$. **3.** ₨8600. 13*a*. $10\frac{1}{12}$*p*.
4. ·99999. **5.** 2·115 cub. in. **6.** $11\frac{9}{41}$ in.

91. **2.** $\frac{1}{20}$. **3.** $62\frac{7}{16}$ gals. **4.** 5. **5.** 9999; $7\frac{10}{13}$.

92. **2.** (*a*) 12. (*b*) $\frac{436}{333}$, $\frac{7}{24}$; ·846714... **3.** $56\frac{2}{3}$ days.
4. £3. **5.** 1·0001.

93. **2.** 2 fur. 12 po. 1 yd. **3.** ₨3. 1*a*. 6*p*. **4.** £14. 1*s*. $3\frac{5}{8}$*d*.
5. £350. 11*s*. 8*d*. **6.** 1869. **7.** 79·032; $8\frac{7}{15}$.

94. **1.** (*a*) 999 × 807 = 806193. (*b*) − 1. **2.** (*a*) ·00$\dot{0}\dot{9}$...; (*b*) 5·0596.
3. 444 mi. **4.** ₨555.

95. **1.** (*b*) 4 feet square. **2.** (*a*) $1\frac{511}{8160}$; 1·7724... **3.** *Rs*.5700.
4. *Rs*.7. 6*a*. $11\frac{1}{7}$*p*. and *Rs*.4. 15*a*. $3\frac{3}{7}$*p*. **5.** 150 yds.

96. **1.** (*a*) $1\frac{3}{19}$. (*b*) 25. **2.** (*a*) ·0203125. (*b*) 200·001.
3. 178 hrs. 52 min. 30 sec. **4.** $7\frac{1}{7}$ miles. **5.** *Rs*.2800.

97. **1.** 47; 127041. **2.** ·28$\dot{3}$. **3.** £291. 17*s*. 6*d*. *nearly*.
4. £315. **5.** 2203·90625 francs.

98. **1.** (i) 2*a*. (ii) $\frac{1}{12}$. (*a*) 29. **2.** (*a*) $3\frac{2161}{3162}$. (*b*) $-4\frac{1}{2}$.
3. $3\frac{1}{7}$; ·001. **4.** £1. 9*s*. $4\frac{176}{357}$*d*. **5.** £125.

99. **1.** $4\frac{1}{4}$; ·63. **2.** 42·68. **3.** £425. 1*s*. $5\frac{253}{1120}$*d*. **4.** 60 days.
5. 2·2360; ·7071. **6.** $3\frac{149}{505}$*d*. **7.** (*a*) 25 per cent. loss. (*b*) 80.

1900. **1.** ·0067143... **2.** (*a*) ·2$\dot{9}$9567$\dot{0}$. (*b*) $\frac{1}{40}$. **3.** (*a*) *Rs*.197. 12*a*. $0\frac{3}{4}$*p*.
(*b*) 3·165. **4.** *Rs*.3200; *Rs*.3889·62 **5.** *Rs*.$289\frac{1529}{4839}$

1901. **1.** (*a*) $1\frac{3}{140}$ ft. (*b*) £8. 1*s*. **2.** (*a*) $\frac{3}{35}$. (*b*) ·03391...
3. 160 yds. **4.** *Rs*.625. **5.** *Rs*.16800.

1902. **1.** G. C. M. = ·000279; L. C. M. = 25234·713.
2. 44. **3.** *Rs*.328·5. **4.** *A* 48, *B* 84.
5. $7\frac{7}{80}$ days. **6.** $8\frac{1}{3}$ years.

APPENDIX.

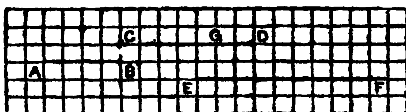
GRAPHS.

1. In this section a few arithmetical sums will be solved graphically.

Examples worked out.

Ex. 1. Find the G. C. M. of 4, 6 and 10 graphically.

Let **AB** represent four units of length, **CD** six units and **EF** ten units of length.

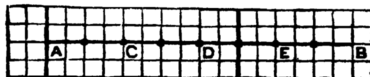


First, place **AB** along **CD** and let **CG** be equal to **AB**. Then it is evident that **GD** represents two units of length. Place **GD** along **AB** successively. Since **AB** denotes four units of length, therefore **AB** contains **GD** exactly twice. Similarly place **GD** along **EF** and it is evident that **EF** contains **GD** an exact number of times. Hence **GD** which represents two units of length is the G. C. M. of **AB**, **CD** and **EF**, that is, the G. C. M. of 4, 6 and 10 is 2.

(*N. B.*—From the solution it is clear how to divide an arithmetical sum graphically.)

Ex. 2. Shew graphically that $\frac{3}{4}=\frac{6}{8}$.

Let **AB** represent the unit of length. Divide **AB** into four equal parts. Then from the fig. it is evident that **AE** $=\frac{3}{4}$ of **AB**. Again



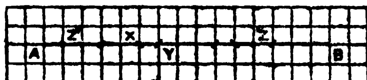
let each of **AC**, **CD**, **DE** and **EB** be divided into two equal parts, then **AB** contains eight of these parts. But **AE** contains 6 of these parts.

$\therefore$ **AE** $=\frac{6}{8}$ of **AB**. But **AE** $=\frac{3}{4}$ of **AB**.

$\therefore \frac{3}{4}=\frac{6}{8}$.

Ex. 3. Shew graphically that $\frac{2}{5}+\frac{1}{3}=\frac{6+5}{15}=\frac{11}{15}$ and $\frac{2}{5}-\frac{1}{3}=\frac{1}{15}$.

Let AB represent the unit of length. Divide it into five equal parts and also into three equal parts.



Therefore $AY=\frac{2}{5}$ of AB and $AX=\frac{1}{3}$ of AB. Now divide AB into fifteen equal parts and take YZ and YZ′ equal to AX on the opposite sides of Y.

$\therefore AZ=AY+YZ=AY+AX.$

$\therefore$ 11-fifteenths = 6-fifteenths + 5-fifteenths ; *i.e.*, $\frac{11}{15}=\frac{6}{15}+\frac{5}{15}$;

$\therefore \frac{2}{5}+\frac{1}{3}=\frac{11}{15}$.

Similarly,

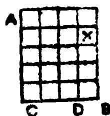
$AZ'=AY-YZ'$,

$\therefore$ 1-fifteenth = 6-fifteenths − 5-fifteenths ;

$\therefore \frac{2}{5}-\frac{1}{3}=\frac{1}{15}$.

Ex. 4. Shew graphically that $\frac{3}{4}\div 5=\frac{3}{20}$.

Suppose the unit is represented by the rectangle AB. Its length contains five equal parts and breadth four equal parts.



Then the rect. is divided into 20 equal parts as shewn in the fig.

Now $\frac{1}{4}$ is represented by the column AC ; $\therefore \frac{3}{4}$ of the unit is represented by AD ; $\therefore \frac{3}{4}\div 5$ is represented by AX and it contains *three* of such equal parts ;

$\therefore \frac{3}{4}\div 5=\frac{3}{20}$.

(*N. B.*—Similarly multiplication of fractions can be graphically solved.)

Examples I.

1. Add together 3, 5, 7 and 9 ; 4, 8, 12 and 20 ; 125, 140 and 175.

2. Represent graphically the result of :—

$15-7$; $25-10$; $149-100$; $1225-525$.

3. Multiply graphically :— 3×5 ; 7×9 ; 10×15 ; $2\times 5\times 3$.

4. Divide graphically :—

15 by 5 ; 33 by 9 ; 289 by 17 ; 510 by 115.

5. Find graphically the G. C. M. of :— 3, 12 and 21 ; 12, 20 and 24 ; 35, 49 and 77 ; 125, 275 and 325.

6. Establish the truth of the following graphically :—

$\frac{1}{5}=\frac{3}{15}$; $\frac{1}{4}=\frac{4}{16}$, $\frac{1}{9}=\frac{9}{81}$; $\frac{2}{11}=\frac{8}{44}$; $\frac{3}{13}=\frac{12}{52}$.

7. Simplify the following graphically :—

(1) $\frac{3}{4}+\frac{1}{8}$; $\frac{1}{5}+\frac{2}{7}$; $\frac{1}{11}+\frac{1}{13}$; $\frac{8}{17}+\frac{2}{9}$; $\frac{5}{7}+\frac{2}{3}$.

(2) $\frac{5}{8}-\frac{1}{4}$; $\frac{3}{4}-\frac{1}{6}$; $\frac{5}{9}-\frac{2}{5}$.

(3) $\frac{2}{3}\times\frac{1}{4}$; $\frac{1}{5}\times\frac{1}{6}$; $\frac{2}{7}\times\frac{3}{4}$; $\frac{3}{13}\times\frac{1}{2}$.

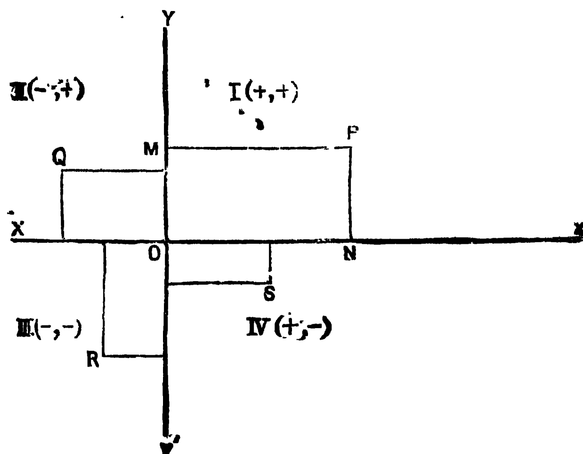
(4) $\frac{2}{7}\div 5$; $\frac{2}{9}\div 3$; $\frac{3}{11}\div 4$; $\frac{1}{5}\div 3$.

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2. APPLICATION OF GRAPHS.

Axes of Co-ordinates.

Let the horizontal line **XOX′** be cut at right angles by the vertical line **YOY′** at the point **O**, thus dividing the plane in which they are into four equal spaces called *Quadrants*, viz. **XOY, YOX′, X′OY′** and **Y′OX** respectively.



Take any point **P** in their plane and draw **PM** and **PN** parallel to **OX** and **OY** respectively.

Let **PM** or **ON** $=x$

and **PN** or **OM** $=y$

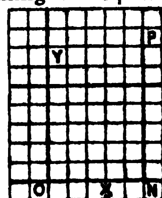
then for **P** x and y are fixed.

The numbers x and y are called the *co-ordinates* of the point P, the lines XOX′ and YOY′ are called the **axes of co-ordinates** and generally the **axes**, they are the lines of reference and are respectively called the **axes of X and Y.**

The point O is called the **origin,** x is called the *abscissa* and y the *ordinate* of the point P. It is common in graphs to denote a point by its co-ordinates. Hence in this case the point P is described as the point (x, y).

The values of x are measured from O along the axis of x, the values are *positive* when drawn to the *right* of O along OX and *negative* when drawn to the *left* of O along OX′. The values of y are *positive* when drawn *above* XX′ and *negative* when drawn *below* XX′.

Thus a point (5, 7,) is represented by first taking ON equal to 5 units along the axis of x to the **right** of O and then by drawing a perp. at N **above** OX and measuring in it PN equal to 7 units of length. The pt. required is the pt. P.

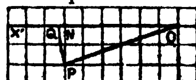


(For a fuller description see **Graphs of Algebraical Functions.**)

Examples worked out.

Ex. 1. A pedestrian walks westward 6 miles and then turns south and walks 2 miles; how far is he from the starting pt.?

Suppose O be the starting point. Measure ON equal to 6 units of length along OX′. Draw NP perp. to and below the line OX′, and cut off from it NP equal to 2 units of length. Then evidently P is the halting place. Join OP and with centre O and radius OP describe a circle cutting OX′ at Q. Then OP = OQ. But OQ = 6·3 approximately. Therefore he is 6·3 miles away from the starting place.



Ex. 2. Given that 1 kilogramme = 2·2 lbs. Draw a graph which will enable you to read off any number of lbs. in kilogramme (up to 50 lbs.), and read off the value of 25 kilogrammes in lbs. and of 32·5 lbs. in kilogrammes.

Measure lbs along OX and kilogrammes along OY.

If x lbs. $= y$ kilogrammes

then $\frac{x}{2 \cdot 2} = y$

i. e., $y = \frac{5}{11}x$;

Therefore the graph is a st. line passing through the origin.

Therefore OP represents the graph.



Since corresponding to the ordinate 25 we have the abscissa 55 therefore 25 kilogrammes are equivalent to 55 lbs.

Similarly, corresponding to the abscissa 32·5 we have the ordinate 14·7 approximately, therefore 32·5 lbs. are equivalent to 14·7 kilogrammes.

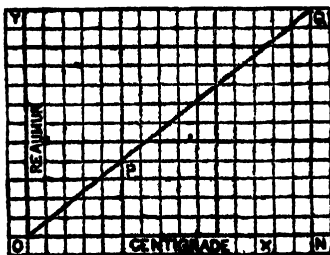
Ex. 3. In a Reaumur thermometer the freezing pt. stands at **0°** and the boiling pt. at **80°**; in a Centigrade the freezing point at **0°** and the boiling at **100°**. Draw a chart to convert Centigrade degrees into Reaumur degrees and *vice-versâ* and read off 15° **C** in **R**.

If x degrees in Centigrade be equivalent to y degrees in Reaumur, then

$$\frac{x}{100}=\frac{y}{80}$$

$$\therefore y=\frac{80}{100}x$$

$$\therefore y=\frac{4}{5}x.$$



Therefore the graph is a st. line passing through the origin.

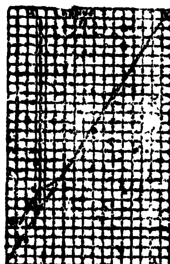
Measure the Centigrade degrees along the axis of x and the Reaumur degrees along the axis of y. Since corresponding

to the abscissa 15, we have the ordinate 12, therefore 15° C = 12° R.

Ex. 4. In a certain business it was arranged that the rate of wages of the labourers would rise or fall with the rise or fall of the price of the merchandise. The following table shows corresponding prices and wages.

Price	*Rs.*5	*Rs.*12. 8*a.*	*Rs.*15	*Rs.*17. 8*a.*	*Rs.*20.
Wages	*Rs.*8	20	24	26	30

Determine graphically whether the wages are proportional to the prices; if not, take the first pair of prices and wages as the standard, and find graphically what wages would be proportional to the remaining prices.

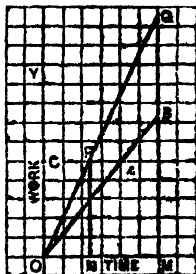


Measure prices along the axis of x and the wages along the axis of y and let us begin from the pt. which represents (*Rs.*5 and *Rs.*8).

From the fig. it is evident that the last two wages are not proportional. The wages *Rs.*28 and *Rs.*32 are proportional to the prices.

Ex 5. A can do a piece of work in 4 days, B can do it in 6 days; how many days will they take to finish it jointly?

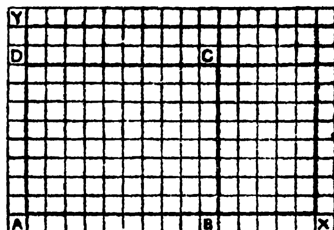
Measure time along the axis of x and let OC represent the whole amount of work. Then OA is the graph of the work done by A. From B, the point where it meets the vertical line take BQ equal to OC. Join OQ, then OQ is the graph of the work done by them jointly.



From the fig. since PN = OC = the amount of work, therefore ON represents the time in which the work is done and ON is evidently 2·4 hrs.

Ex. 6. A rectangle ABCD contains an area of 80 sq. ft. If the length is increased in the ratio of 3 : 2 and the breadth increased in the ratio of 5 : 4, what is the new area?

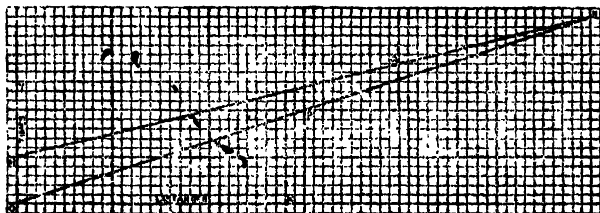
First, if the length be increased in the ratio of 3 : 2, *i.e.* from AB to AX, then if the breadth remains the same, DX is the new area. Secondly, if the breadth be increased in the ratio of 5 : 4 and the length AX remains the same, YX is the new area ; for



$$\text{rect. } XY = \tfrac{5}{4} \text{ of rect. } XD.$$
$$= \tfrac{5}{4} \cdot \tfrac{3}{2} \text{ of rect. } BD.$$
$$= \tfrac{5}{4} \cdot \tfrac{3}{2} \cdot 80 \text{ sq. ft.}$$
$$= 150 \text{ sq. ft.}$$

Ex. 7. A starts walking at the rate of 3 miles per hour ; 30 min. after B starts from the same place at the rate of 4 miles per hour. Find graphically when and where B overtakes A.

Measure distances along OX to the scale of 10 spaces to



1 mile and times along OY to the scale of 10 spaces to 1 hour.

Since in 1 hour A goes 3 miles,

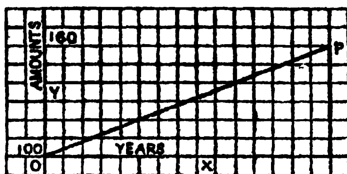
∴ OP represents the graph of A's motion. B starts 30 min. after, therefore M denotes B's starting pt. As before let MQ represent the graph of B's motion. Then the intersection of MQ and OP *viz.* N will give the place and the time when B overtakes A. From the fig. it is clear that B will overtake A. 2 hrs. after A's starting and at 6 miles from the place where A started.

Ex. 8. Shew graphically the sums to which *Rs.* 100 will amount at the end of 3, 6, 9, 12 and 15 years at 4 p. c. Simple Interest the following dates being given.

Years	3	6	9	12	15
Amount at S. I.	112	124	136	148	160 .

The amounts of Simple Interest for 3, 6, 9, 12 and 15 years are *Rs.*12, *Rs.*24, *Rs.*36, *Rs.*48, *Rs.*60.

Therefore representing the years by the axis of x and the amounts by the axis of y, we get the following graph.



Examples II.

1. Draw by straight lines a sketch of a section of a mountain from the following data :—

Horizontal distance.	1000	2000	2800	3000	3700
Corresponding height.	820	2090	2540	3440	2500

2. The readings of a centigrade thermometer in degrees and the corresponding readings on a Fahrenheit thermometer in degrees are given in the following table :—

C	5	10	15	20	30	50	80
F	41	50	59	68	86	122	176

Illustrate graphically the connection between the two scales. Express 140° F. in Centigrade.

Ans. 60° C.

3. If a man first walks 8 miles east and then 5 miles north, how far is he from the starting point ?

Ans. 9·4 miles nearly.

4. Represent graphically the increase of Velocity of a train starting from rest.

Distance travelled	50	200	800	1800	3200	5000	feet
Velocity	5	10	20	30	40	50	miles per hr.

5. Shew graphically the amounts at Simple Interest and Compound Interest of £1 at 5 p. c. for different periods.

No. of years.	0	5	10	15	20	25
Amt. at S. I.	1	1·25	1·50	1·75	2·00	2·25
Amt. at C. I.	1	1·28	1·63	2·68	2·65	3·39

6. A man bicycles from **A** to **B** at 10 miles an hour; and returns from **B** to **A** at 15 miles an hour. If he takes 5 hours to go there and back, find the distance from **A** to **B**. Find also his average speed per hour. **Ans.** 30 miles ; 12 miles.

7. **A** starts from Calcutta to walk to Burdwan a distance of 68 miles at 3 miles an hour ; two hours later **B** starts from Burdwan for Calcutta at 5 miles per hour. When will **A** and **B** meet? When will they be 20 miles apart ?

Ans. $9\frac{3}{4}$ hrs. from **A**'s start, $7\frac{1}{2}$ hrs. and $12\frac{1}{4}$ hrs. from **A**'s start.

8. A policeman sets off after a thief at 7 A. M., the thief having had ($2\frac{1}{2}$ hrs. start). If the thief goes 4 miles an hour and the policeman $5\frac{1}{4}$ miles an hour, when will the policeman catch the thief? **Ans.** At 3 P. M.

9. A monkey climbing up a greased pole, ascends 5 ft. and slips down 2 ft. in alternate seconds, until he reaches the top of the pole. If the pole be 17 ft. high, how long will it take him to reach the top ? **Ans.** 9 sec.

10. Two taps *A* and *B* will fill a cistern in 10 and 20 hours respectively. In what time will they fill it together ?

Ans. 6·7 hrs.

11. At what times between 6 and 7 are the hands of a clock (i) together, (ii) at right angles ?

Ans. (i) $32\frac{8}{11}$ min. past 6.
(ii) $16\frac{4}{11}$ min. and $49\frac{1}{11}$ min. past 6.

12. The expenses of a family when rice is at 20 seers for a rupee are *Rs.*50 a month ; when rice is at 25 seers for a rupee the expenses are *Rs.*48 a month (other expenses remaining the same) what will they be when rice is at 30 seers for a rupee ? Also find how much rice can be had for a rupee when the expenses are *Rs.*60.

Ans. *Rs.*46. 10*a.* 8*p.* and 10 seers per rupee.

(**N. B.**—For solution see **Graphs of Algebraical Functions**, p. 20.)

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